FINITE ELEMENT ANALYSIS OF STRIP FOOTING RESTING ON GIBSON-TYPE SOIL BY USING MATLAB

Ahmed Qasim Obaid [1]; Firas Hussain Ali [1]; and Adnan Qahtan Mohammed [1]

[1] **Assistant Lecturer, Department of Engineering Affairs, Baghdad University, Iraq.**

ABSTRACT

This research presents a method of using MATLAB in analyzing a nonhomogeneous soil (Gibson-type) by estimating the displacements and stresses under the strip footing during applied incremental loading sequences. This paper presents a two-dimensional finite element method. In this method, the soil is divided into a number of triangle elements. A model soil (Gibson-type) with linearly increasing modulus of elasticity with depth is presented. The influences of modulus of elasticity, incremental loading, width of footing, and depth of footing are considered in this paper. The results are compared with authors' conclusions of previous studies.

الخلاصة

هذا البحث يقدم طريقة استخدام الـ MATLAB في تحليل تربة غير متجانسة (Gibson-type) بواسطة حساب الإزاحات والإجهادات تحت أساس شريطي أثناء تطبيق تحميل تزايدي متعاقب. هذا البحث يقدم طريقة العناصر المحددة الثنائية الأبعاد. في هذه الطريقة تم تقسيم التربة إلى عدد من العناصر المثلثة. تربة النموذج المقدمة (Gibson-type) ذات معامل مرونة متزايد بشكل خطي مع العمق. تم دراسة تأثير معامل المرونة، والتحميل المتزايد، وعرض وعمق الأساس في هذا البحث. إن النَتائج تمت مقارنتها مع استنتاجات مؤلفين من الدراسات السابقة.

KEYWORDS: nonhomogeneous soil, Gibson-type, MATLAB, finite element method, strip footing.

INTRODUCTION

In general, the magnitude and distribution of the displacements and stresses in soil are predicted by using solutions that model soil as a linearly elastic, homogeneous and isotropic continuum. From the standpoint of practical considerations in engineering, anisotropic soils are often modeled as orthotropic or isotropic medium. Besides, the effects of deposition, overburden, desiccation, etc., can lead geotechnical media, which exhibit both nonhomogeneity and anisotropic deformability characteristics.

The type of elastic nonhomogeneity is a useful approximation for modeling certain problems of geotechnical interest (Selvadurai, 1998).

In this work, an elastic static loading problem for a continuously nonhomogeneous isotropic medium with Young's modulus varying linearly with depth is relevant.

The solutions of displacements and stresses for various types of applied loads to homogeneous and nonhomogeneous isotropic/anisotropic full/half-spaces have played an important role in the design of foundations. However, it is well known that a strip load solution is the basis of complex loading problems for all constituted materials. A large body of the literature was devoted to the calculation of displacements and stresses in isotropic media with the Young's or shear modulus varying with depth according to the linear law, the power law, and the exponential law, etc. A more recent survey of the existing solutions for a nonhomogeneous isotropic is summarized in Table 1 (Wang et al., 2003).

A closed-form expression for a footing resting on a soil with stiffness linearly increasing with depth (Gibson, 1967) is given only for undrained loading (ν=0.5). To resolve drained loading cases with ν<0.5, finite element solutions were developed for estimating the displacements for nonhomogeneous cases (e.g. Carrier and Christian, 1973; and Boswell and Scott, 1975).

A numerical technique was used by Stark and Booker (1997) for the analysis of surface displacements of a non-homogeneous elastic half-space subjected to vertical and/or horizontal surface loads uniformly distributed over an arbitrarily shaped area.

In geotechnical engineering practice, it is usual to consider only the change in stresses (Δσ) when computing displacements.

In practice, most foundations are flexible. Even very thick ones deflect when loaded by the superstructure loads (Bowles, 1996).

- NONHOMOGENEOUS "GIBSON-TYPE" SOIL PROFILE

In natural soil deposits, the variation of soil modulus with depth may assume any of a number of possible scenarios which shown in Table 1. Since many soils exhibit stiffness increasing with depth because of the increase in overburden stress, the displacement and stress will be evaluated for a Gibson type soil. A footing resting on a nonhomogeneous elastic medium with modulus increasing with depth is a more generalized problem (Boswell and Scott, 1975; and Stark and Booker, 1997).

The variation of elastic modulus for a generalized Gibson soil (Gibson, 1967) is expressed by:

$$E_s = E_o + kz \qquad (1)$$

where E_s = the elastic soil modulus increasing linearly with depth; E_o = Young's modulus of elasticity of soil directly underneath the foundation base; k = linear rate of increase of elastic modulus with depth (units of E per unit depth); and z = depth, (Figure 1) (Mayne and Poulos, 1999; and Das, 2002).

- FINITE ELEMENT ANALYSIS

The finite element method provides an extremely powerful method for the analysis of elastic materials. Many natural soil deposits can be considered to have been deposited in a sequence of horizontal layers and, thus, there is no variation of elastic properties in any horizontal plane.

One of the essential ingredients for a successful finite element analysis of a geotechnical problem is an appropriate soil constitutive model (Potts and Zdravkovic, 2001).

The advantage of an arbitrary triangular shape is to approximate to any boundary shape (Zienkiewicz and Taylor, 2000). So the triangular element shape is considered in this research.

A finite element computer program for a rigid circular plate resting on a non-homogeneous elastic half-space was presented by Carrier and Christian (1973).

Boswell and Scott (1975) presented a finite element solution for a flexible circular footing resting on a semi-infinite half space.

In the present study, 3-noded triangles elements with two degree of freedom at each node have been used to model soil. In each increment of the analyses, the stress-strain behavior of the soil is treated as being linear, and the relationship between stress and strain is assumed to be governed by the generalized Hooke's law of elastic deformations, which may be expressed as follows for conditions of plane strain case:

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\tau_{xy} \end{Bmatrix} = \frac{E_s(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\gamma_{xy} \end{Bmatrix} \tag{2}$$

where $\Delta\sigma_x$, $\Delta\sigma_y$ and $\Delta\tau_{xy}$ = the increments of stress during a step of analysis; $\Delta\varepsilon_x$, $\Delta\varepsilon_y$ and $\Delta\gamma_{xy}$ = the corresponding increments of strain; E_s = the value of Young's modulus; and ν = the value of Poisson's ratio.

About MATLAB

The MATLAB programming language is useful in illustrating how to program the finite element method due to the fact it allows one to very quickly code numerical methods and has a vast predefined mathematical library. A simple two dimensional finite element program in MATLAB need only be a few hundred lines of code whereas in Fortran or C++ one might need a few thousand (Chessa, 2002).

MATLAB can be very useful as a solution tool for the finite element method which Matrix and vector manipulations are essential parts in the method (Kwon and Bang, 1997).

- The Finite Element Computer Program by MATLAB

A computer program designed by the authors was used in the finite element analysis carried out during this research. The program allows for triangle type of elements to be used in the finite element mesh in solving soil problems under plane conditions (strain or stress). The behavior of the soil can be approximated by Gibson model (Gibson, 1967). The model that is considered in this work is nonhomogeneous, isotropic on primary loading with a different modulus.

The sign convention for the stresses and the convention for numbering the nodes of elements are shown in Figure (2). The program presents the results of analysis as the displacements of the nodal points, and the value of stresses developed at the centre of each element at the end of each solution increment.

Figure (3) is a flowchart that illustrates the main features of the solution procedure adopted in the finite element computer program used.

Verification of the Computer Program

The authors have used this program in different problems (Figure 4) presented by another researchers (e.g. Brown, 1984; and Smith and Griffiths,1988).

The results obtained by the program modified in this research were compared with results presented by Brown (1984); and Smith and Griffiths (1988). In all these comparisons, excellent agreement was found between the present work results and those published, as shown in Table 2.

- PROBLEM GEOMETRY

The case study is treated as plane strain two-dimensional problem for simplicity when analyzed by the finite element method. The shape of elements used is the triangular element because of its suitability to simulate the very important behavior of soils under strip footing.

The basic problem chosen for the parametric study shown in Figure (5.a), involves a soil stratum, 21.0 m thick and 28.0 m width, of a silty clay soil underlaid by bedrock and loads by strip sequence loadings (80, 160, 240 kN/m^2) with base width equal to 1.0 m.

The finite element mesh (Figure 5.b) used consists of 989 nodal points and 1848 triangular two-dimensional elements. The nodal points along the bottom boundary of the mesh are assumed to be fixed both horizontally and vertically. The nodes on the right and left ends of the mesh are fixed in the horizontal direction while they are free to move in the vertical direction. All interior nodes are free to move horizontally and vertically.

- MATERIAL CHARACTERIZATION

The stratum is silty clay soil and the properties of the soil are reported in Table 3, (Das, 2002). The behavior of soil material is a nonhomogeneous elastic medium with modulus increasing linearly with depth.

- RESULTS AND DISCUSSIONS

In this study, a model of silty clay soil was analyzed under uniformly flexible strip loading with soil modulus increasing linearly with depth. In order to develop more knowledge about the behavior of soils under strip loading problems, a parametric study is performed by varying the basic problem parameters and comparing these results with the original basic problem results. The results of increasing the load, constant soil modulus with depth, and changing the footing depth (D_f) and width (B) are presented as follow:

For uniformly flexible strip loaded area the vertical displacement along the surface of the model is shown in Figure (6) and the contact settlement under the strip footing is shown in Figure (7). The settlement at the center is much larger than the settlement at the edge of the loaded area. These results agree with the results founded by Wu (1974) and Das (2002). Also the vertical displacement increases in direct proportion to the pressure of the loaded area, as shown in Figures (6) and (7), which agrees with that reported by Craig (1987).

The vertical stress contours throughout the soil to depth (5B) under the strip loadings (80, 160, 240 kN/m^2) with base width (B) equal to 1.0 m are shown in Figure (8). It can be seen that the vertical stress values along the depth of the layer decrease throughout the layer for each increment and increase throughout the loading sequence stages.

From the settlements at depth (4B) from the top of model (Figure 9), it can be seen that the settlement for soil with modulus increasing linearly with depth is less than the settlement for soil with constant modulus (E_s = E_o = 9000 kN/m^2). The results agree with that mentioned by Terzaghi in Wu's book (Wu, 1974).

The immediate settlement at the center of the loaded area is reduced when the strip footing is placed at some depth ($D_f \leq B$) in the ground, depending on footing width (B), as shown in Figure (10). These results agree with that mentioned by Fox in Bowels' book (Bowles, 1996).

The vertical displacement (immediate settlement) increases in direct proportion to the width of the loaded area (size of the footing), as shown in Figure (11), which agrees with that reported by Wu (1974) and Craig (1987).

CONCLUSIONS

The results obtained from this study can lead that the computer program can simulate the analysis of the nonhomogeneous silty clay soil (Gibson-type), which had a soil modulus increasing linearly with depth and loaded with incremental strip loading.

This paper shows how the computer solutions may be used to improve the prediction of settlements and stresses beneath a strip footing resting on Gibson-type soil.

Displacements and stresses can be calculated with knowledge of soil stiffness beneath the footing, rate of increase of soil stiffness with depth, soil Poisson’s ratio, depth to an incompressible layer, and footing width.

The immediate settlement at the center is much larger than the settlement at the edge of the strip loaded area. The immediate settlement increases in direct proportion to the pressure and the width of the strip loaded area. The vertical stress values (stress bulb) under the strip loaded area decrease throughout the layer for each increment and increase throughout the loading sequence stages. The vertical displacement for soil with modulus increasing linearly with depth (Gibson-type) is less than the vertical displacement for the same soil with constant modulus which leads to that the soil with (Gibson-type) modulus is more approximate simulation for soil modulus. The immediate settlement of the strip loaded area decreases when the depth of strip footing increases. The results compare favorably with available published analytical and numerical solutions.

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Table 1. Existing analytical/numerical solutions for nonhomogeneous isotropic media, (Wang et al., 2003)

Types of nonhomogeneity	Author
$E = m_E z^\alpha$ or $G = m_G z^\alpha$ $(0 \leqslant \alpha \leqslant 1)$	Rostovtsev ; Lekhnitskii ; Popov ; Zaretsky and Tsytovich ; Kassir ; Rostovtsev and Khranevskaia ; Carrier and Christian ; Puro ; Popov ; Booker et al. ; Oner ; Booker ; Giannakopoulos and Suresh ; Stark and Booker ; Yue et al. ; Holzlohner .
$E = E_0(a+bz)^c$ or $G = G_0(a+bz)^c$	Plevako ; Chuaprasert and Kassir ; Kassir and Chuaprasert ; Dhaliwal and Singh ; Harnpattanapanich and Vardoulakis ; Rajapakse and Selvadurai ; Jeng and Lin .
$E = E_0 + \lambda z$ or $G = G_0 + \lambda z$	Gibson ; Gibson et al. ; Brown and Gibson ; Awojobi and Gibson ; Carrier and Christian ; Alexander ; Calladine and Greenwood ; Rajapakse ; Chow ; Rajapakse and Selvadurai ; Dempsey and Li ; Yue et al. .
$E = E_0 + E_1 e^{\xi z}$ or $G = G_0 + G_1 e^{\xi z}$	Ter-Mkrtich'ian ; Rowe and Booker ; Row and Booker ; Selvadurai et al. ; Vrettos ; Selvadurai ; Giannakopoulos and Suresh ; Jeng and Lin .
$G = G_0 e^{-\xi r}$	George .
$G = G_0 r^\alpha z^\beta$	Singh ; Dhaliwal and Singh .
$G = G_0 * h/(h - z)$	Awojobi .
$G = \text{constant}$	Gibson and Sills .

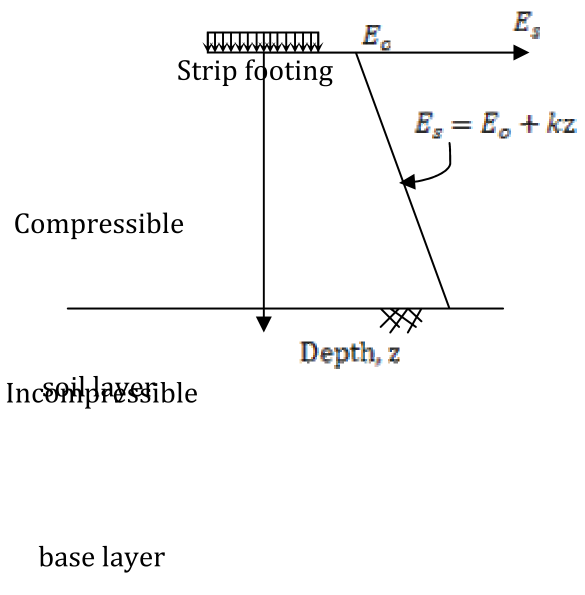


Fig. 1. Variation of soil modulus with depth

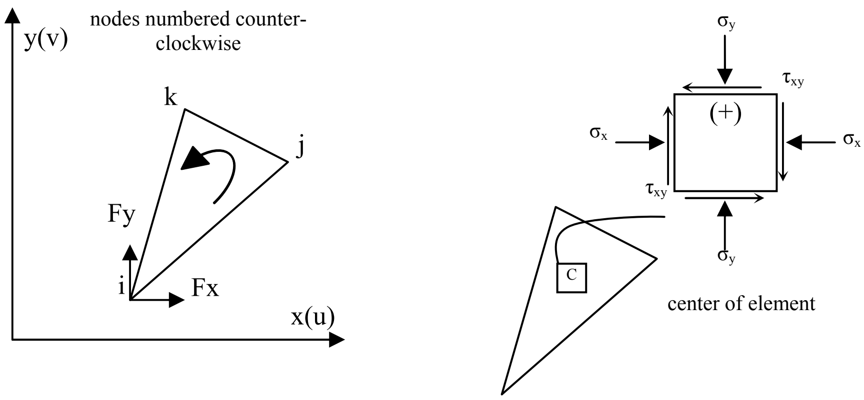


Fig. 2. Sign convention and element numbering

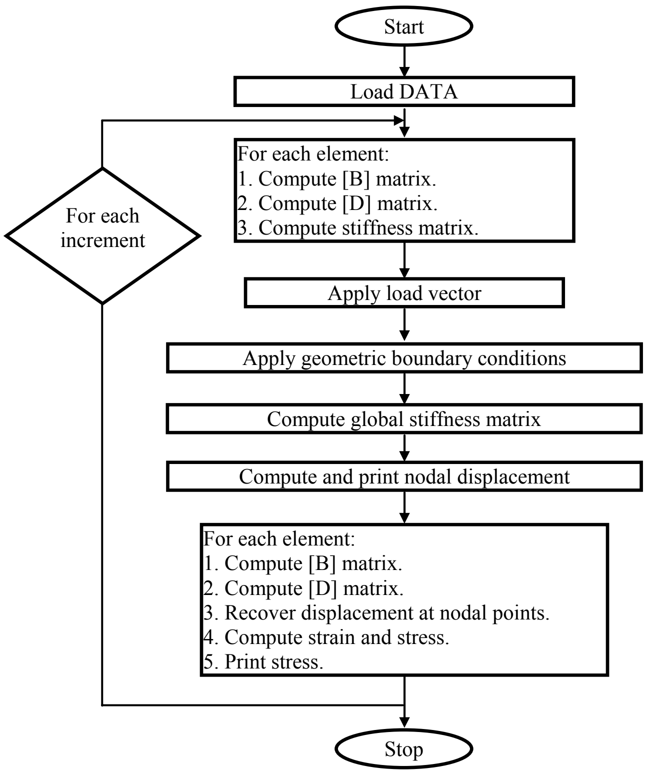
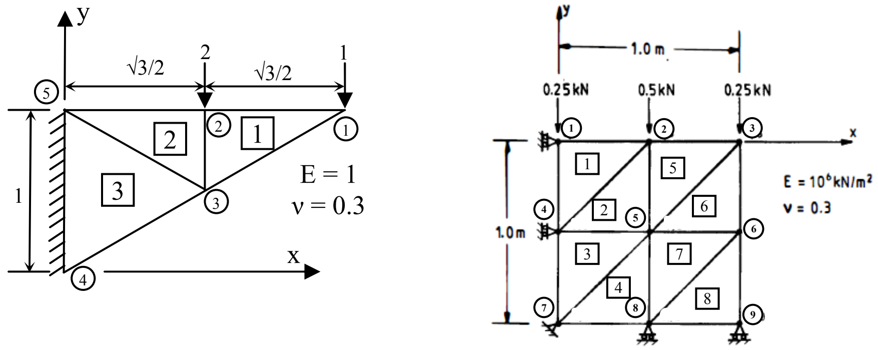


Fig. 3. Simplified flow chart of the finite element program



(a) after Brown (1984). (b) after Smith and Griffiths (1988).

Fig. 4. Mesh and data for different problems

Table 2. Comparison with the theoretical results

Item considered	Brown results	Authors results
Hor. Disp. of node 1	7.712	7.7122134E+000
Ver. Disp. of node 3	- 13.582	- 1.3583206E+001
Ver. Stress at elem. 2	-2.461	- 2.4605374E+000
Hor. Stress at elem. 2	6.816	6.8155730E+000

Item considered	Smith and Griffiths results	Authors results
Ver. Disp. of node 2	-0.1000E-05	-1.000000E-006
Hor. Disp. of node 6	0.3000E-06	3.000000E-007
Ver. Stress at elem. 1	-0.1000E+01	- 1.000000E+000
Shear Stress at elem. 8	0.0000E+00	0.000000E+000

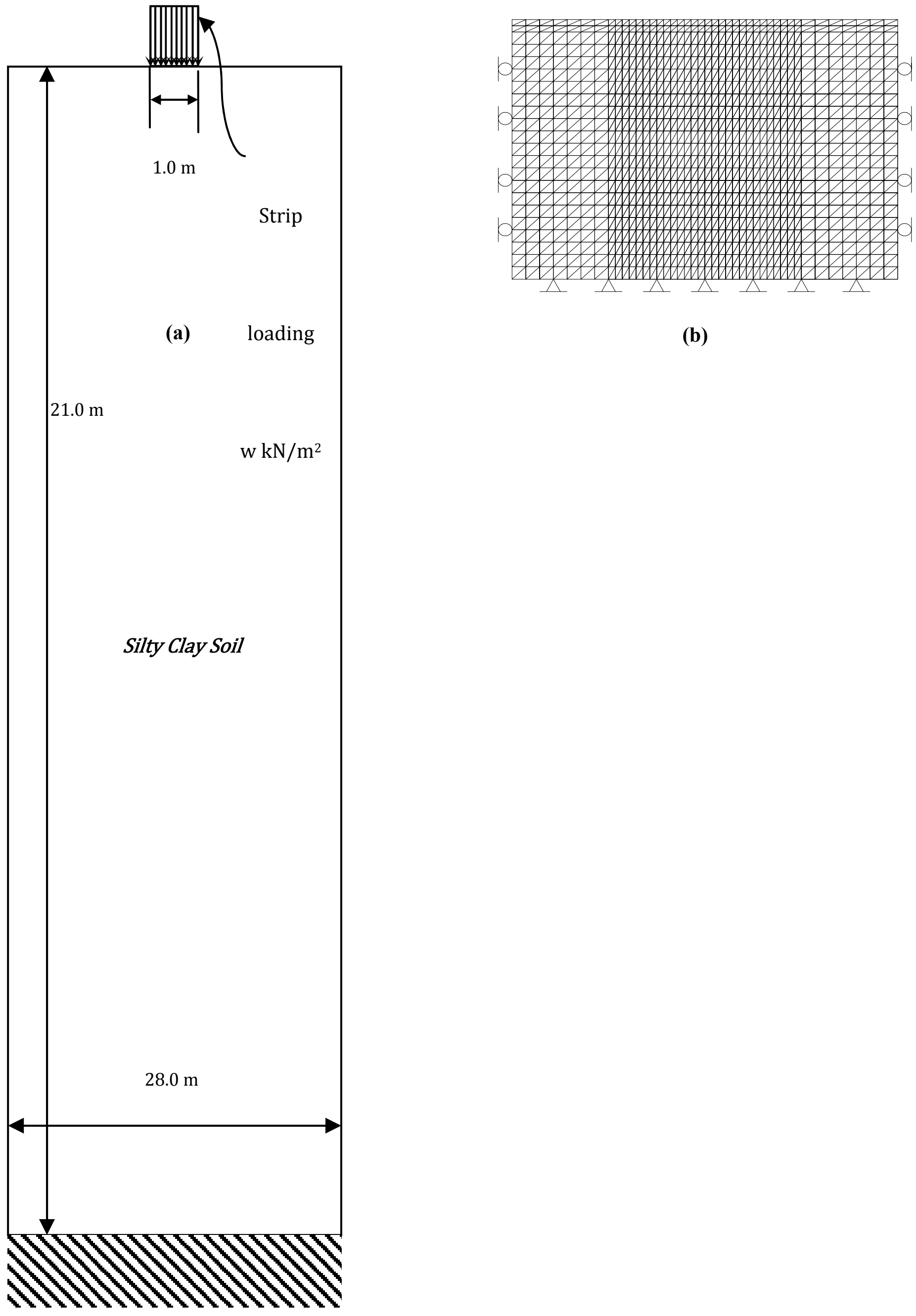


Fig. 5. The basic problem for the parametric study

Table 3. The soil properties

E_o	9000 kN/m^2
k	500 kN/m^2/m
ν	0.3

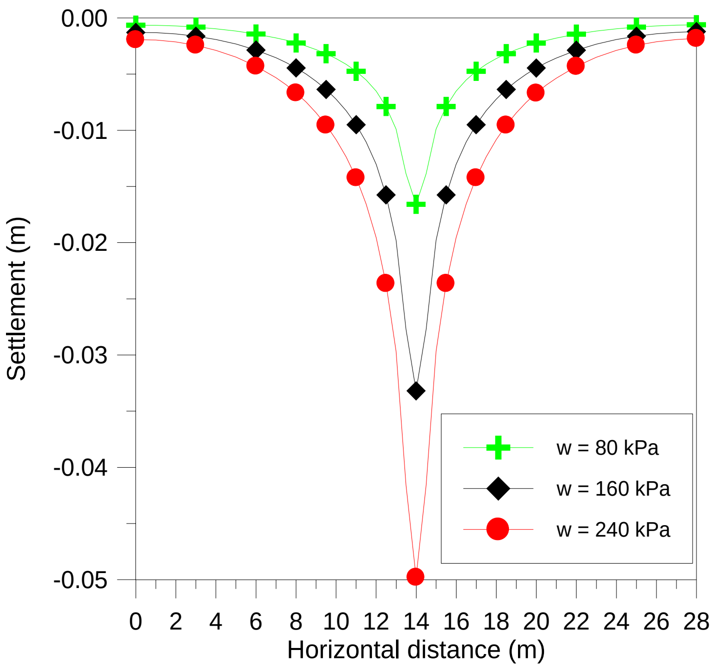


Fig. 6. Settlements along the surface of the model under the strip loadings (80, 160, 240 kN/m^2) with base width equal to 1.0 m

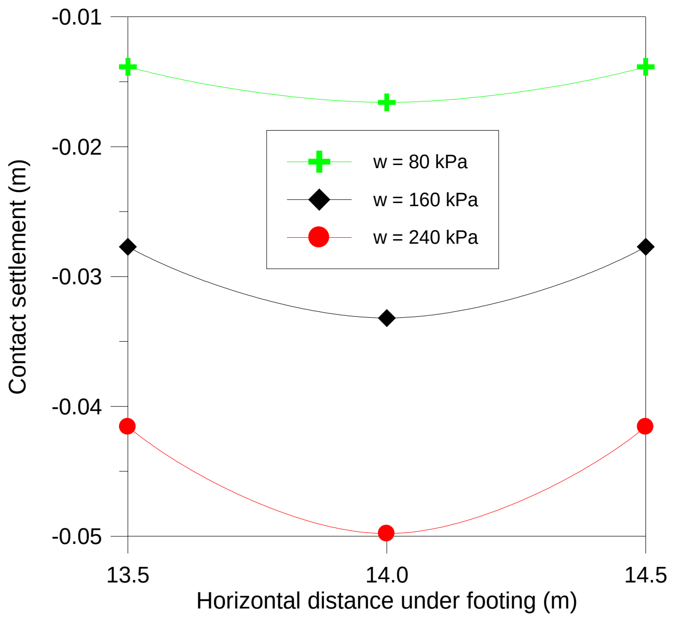


Fig. 7. Contact settlements under the strip loadings (80, 160, 240 kN/m^2) with base width equal to 1.0 m

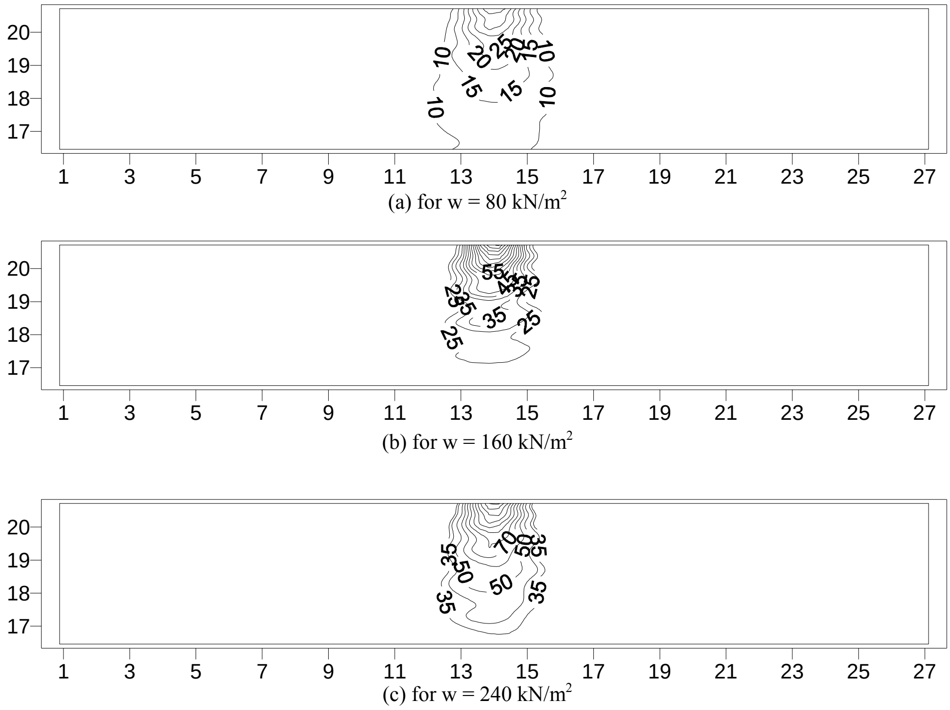


Fig. 8. Vertical stress contours for the soil model to depth (5B) throughout the loading sequences

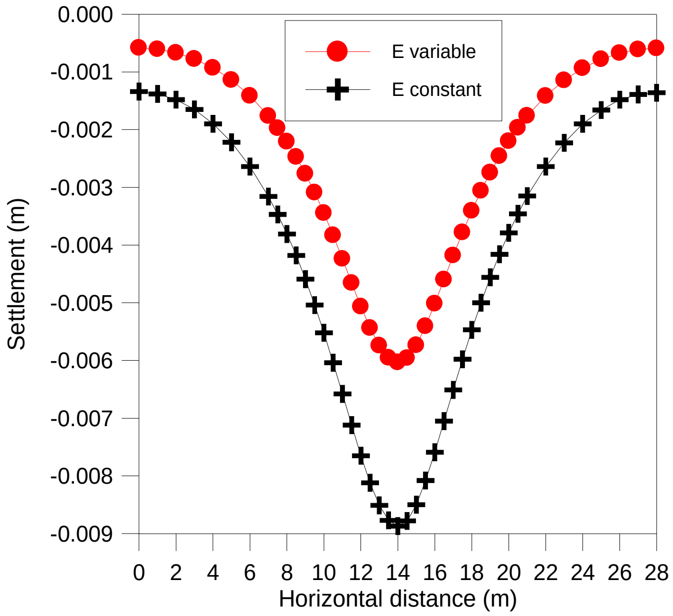


Fig. 9. Settlements along the horizontal distance at depth (4B) under strip loading (80 kN/m^2)

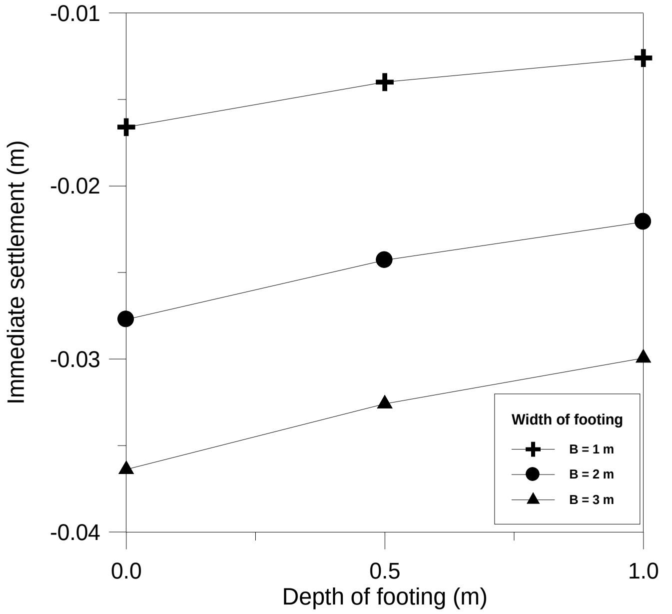


Fig. 10. Immediate settlements at the center of the strip loading (80 kN/m^2) with different width of footing according to depth of footing

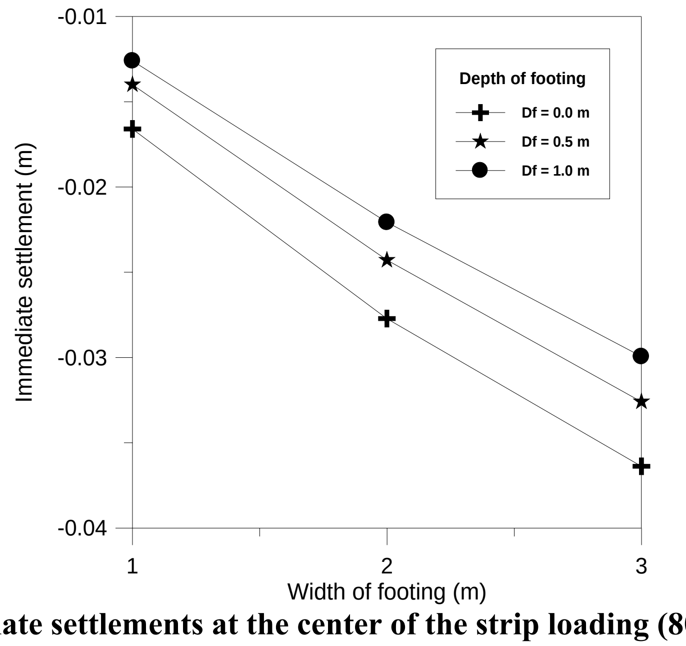


Fig. 11. Immediate settlements at the center of the strip loading (80 kN/m^2) with different depth of footing according to width of footing