TWO-PARAMETER GAMMA DISTRIBUTION AND LOG NORMAL DISTRIBUTION FOR DERIVATION OF SYNTHETIC UNIT HYDROGRAPH

Mohammed Rashid Dhahir
Computer center ,Baghdad University

ABSTRACT

Most available methods for unit hydrographs (SUH) derivation involve manual, subjective fitting of a hydrograph through a few data points. The use of probability distributions for the derivation of synthetic hydrographs had received much attention because of its similarity with unit hydrograph properties. In this paper, the use of two flexible probability distributions is presented. For each distribution the unknown parameters were derived in terms of the time to peak(**tp**), and the peak discharge(**Qp**). A simple **Matlab program** is prepared for calculating these parameters and their validity was checked using comparison with field data. Application to field data shows that the **gamma** and **lognormal** distributions had fit well.

INTRODUCTION

The term ‘synthetic’ in synthetic unit hydrograph (SUH) denotes the unit hydrograph (UH) derived from watershed characteristics rather than from rainfall-runoff data. **Chow V.T.(1964)** and **Viessman *et al.* (2007)** provide a good review of the various methods available for (SUH) derivation. Among the available approximate methods for (SUH) derivation as mentioned in **Singh.(1988)**, the method of fitting a smooth curve manually through a few salient points of (UH) is generally practiced. For example, the methods of **Snyder(1938) [**Quoted from **Bhunya *et al.*(2007)]** and **Espey and Winslow (1974)** utilize empirical equations for the estimation of peak flow(Q_p) $[L^3T^{-1}]$,lag time(t_L)[T], time to peak(t_p)[T], UH widths at 0.5 Qp and 0.75 Qp.Thus, beside the involvement of a great degree of subjectivity in such manual fitting, the fitted curves require simultaneous adjustment for the area under SUH to represent unit runoff volume. Due to similarity in shapes, several attempts have been made in the past to use some probability density functions, (pdf) for UH and its derivation, e.g. **Gray (1961)**, **Sokolov *et al.*(1976) and Ciepieelowki(1987)**.**[** Quoted from **Bhunya *et al.*(2007)].** The pdf of the gamma and beta distributions to represent the UH shape were used by **Gottschalk *et al.*(1998) and Haktanir and Sezen(1990) [**Quoted from **Saralees Nadarajah.(2007)].**

Synthetic unit hydrograph methods

- **Snyder’s method**

Snyder(1938) [Quoted from **Singh.(2000)],** used five variables dependent on catchment characteristics to define a (SUH),(1) catchment lag t_L; (2) peak discharge rate Q_p; (3) base time t_b;(4) width of UH at Q=0.5Q_p, W_{50};

and(5) width of UH at $Q=0.75Q_p$, W_{75}. The expressions for t_L and Q_p are as follows:

$$t_L=C_t(LL_C)^{0.3} \quad (1)$$

$$Q_p= C_P \frac{645A}{t_L} \quad (2)$$

in which Q_p = peak discharge rate (ft^3/s); t_L = catchment lag in hours measured from the center of the effective rainfall to the peak of the SUH; L =length of the main stream in miles from the outlet to the upstream divide; L_C = distance in miles from the outlet to a point on the stream nearest to the centroid of the catchment; A = area of the catchment (square miles); and C_t and C_P are coefficients. The coefficient C_t varies from 1.8 to 2.2, generally assumed to be equal to 2. Equation (1) and (2) were obtained from the study of catchments varying in size from 26 to 26,000 km^2 (10 to 10,000 mi^2) in the United States. The expression for time to peak of the SUH, t_p, is:

$$t_p = t_L + t_r /2$$

STATISTICALS DISTRIBUTIONS

TWO PARAMETER LOG NORMAL DISTRIBUTION

The probability density function **(pdf)** of this distribution is given by [Quoted from **Saralees Nadarajah.(2007)].**

$$f(x) = \frac{1}{x\sigma_y\sqrt{2\pi}} \exp\left(-\frac{(lnx-\mu_y)^2}{2\sigma_y^2}\right) \quad (6)$$

Where μ_y and σ_y are the mean and standard deviation of the natural logarithms of x.
for $x > 0$, $-\infty < \mu_y < \infty$ and $\sigma_y > 0$. For this distribution, it is known (Johnson and Kot (1970a) that the mode is given by

$$\text{Mode} = \exp(\mu - \sigma^2) \quad (7)$$

Substituting (7) into (6), Yields:

$$t_p q_p = \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(-\frac{\sigma_y^2}{2}\right) \ldots\ldots\ldots$$

$$\mu_y = \sigma_y^2 + \ln t_p$$

GAMMA DISTRIBUTION

Use of a two-parameter Gamma distribution for representing the UH has a long hydrologic history that started with **Edson. (1951)** [Quoted from **Singh. (2000)]**, who presented a theoretical expression for the unit hydrograph assuming Q to be proportional to($t^x e^{-yt}$) as:

$$Q=\frac{cAy(yt)^x e^{-yt}}{\Gamma(x+1)} \quad (10)$$

where Q=discharge (cfs) at time t; A=drainage area (square miles); x and y=parameters that can be represented in terms of peak discharge; and **Γ(x+1)** is the Gamma function of (x+1). **Nash (1959) and Dooge (1959)**, based on the concept of n linear reservoirs with equal storage coefficient K, expressed the instantaneous UH (IUH) in the form of a Gamma distribution as:

$$q = \frac{1}{K\Gamma(n)}\left(\frac{t}{K}\right)^{n-1} e^{-t/k} \quad (11)$$

in which n and K are parameters defining the shape of the IUH; and q is depth of runoff per unit time per unit effective rainfall. These parameters have been referred to as Nash model parameters in the subsequent literature. With the suitable change of variables and applying dimensional homogeneity, Eq. (11) can be derived from Eq. (10). The area under the curve defined by Eq. (11) is unity; thus the rainfall and runoff depths are equal to unity. To obtain the

SUH, the parameters of Eq. (11) were related to catchment characteristics[**Nash (1960)]**. other attempts to fit a Gamma distribution to hydrographs were by **Croley (1980)**, **Aron and White (1982)**, [Quoted from **Saralees Nadarajah,(2007)]**and **Singh (1998)** . The procedure given by **Croley (1980)**, to calculate n for known values of (**q_p**) and (**t_p**) requires programming to iteratively solve for n. **Croley (1980)** also proposed procedures to obtain a UH from other observable characteristics.

The method by **Aron and White (1982)** involves reading the values from a graph, in which errors are introduced.Based on their method, **Bhunya (2003)** listed a step-by-step procedure to obtain UH, which may be briefly described by the following equations:

$$n = 1.045 + 0.5f + 5.6 f^2 + 0.3f^3 \qquad (12)$$

in which

$$f = Q_p t_P / A \qquad (13)$$

where Q_p is in cubic feet per second; t_p is in hours; and A is in acres. Equations (12) and (13) require careful attention for the units, and these cannot be used as such when $Q_p t_p$ is required to be computed for a value of (n)known from other sources. **Hann et al. (1994)[**Quoted from **Bhunya (2003)**]gave the following expression to calculate(n):

$$n = 1 + 6.5\left(\frac{Q_p t_p}{V}\right)^{1.92} \qquad (14)$$

where V=total volume of effective rainfall. An equation provided by Singh (1998) to obtain the value of (n) may be written as:

$$n = 1.09 + 0.164\,\beta + 6.19\,\beta^2 \qquad (15)$$

where $\beta = q_p t_p$ (dimensionless), in which (q_p) is the peak runoff depth per unit time per unit effective rainfall. Singh observed that the error in n obtained from Eq. (12) is (0.53%)when (b = 0.25) and (0.05%) when(b = 1.0).The error in n calculated from Eq.(15) decreases with increasing values of β.

APPLICATION

The applicability of the proposed method was examined for two cases,(A) and (B). In case (A), the UH was derived from the actual hydrograph; (Q_p) and (t_p) are used from the observation. In case (B), the partial data only, few observations from the actual data, were used only to find q_p and t_p.

Case A

For this case the watershed area is (A) =201.6 KM^2.The calculations for the base flow, direct runoff, and unit hydrograph are shown in the table (1) [Quoted from. salas (2006)], also the plot of the observed unit hydrograph is shown in fig(1)

Case B

A watershed area of 54km^2, according to Snyder model the t_p=5 (hour,) q_p=0.13 (1/hour)

Fig. (3): SUH for case B.

RESULTS:

The results arrived can be summarized as follows.

- The gamma distribution results are closer to the actual data as shown in the fig(2) which shows Comparisons of observed unit hydrograph with SUH obtained from Gamma and Log Normal distributions, Also table(2) shows that the estimated depth of excess rainfall for the Gamma distribution is 0.9913 which is too close to 1 than Log normal distribution with value of 0.9912
  Fig(3) shows the comparison between the Gamma , Log Normal distributions and the Snyder model whereas table (3) shows that the estimated depth of excess rainfall for the Gamma distribution is 0.9910 which is very close to 1 as compared to Log normal distribution which value is 0.9851

CONCLUSIONS

The following conclusions are derived from the study:

- The pdf computed by probability distribution gave results better than the existing synthetic methods i.e methods suggested by **Synder**(1938), and gave accurate results of the actual pdf parameters, as verified by using observed data.

- The comparison between the gamma distribution and the log normal distribution shows that the gamma distribution is more flexible than lognormal distribution since the estimated depth (0.9913), is nearest to the actual data.

REFERENCES

- **Aron, G., White, E.L., (1982)**. Fitting a gamma-distribution over a synthetic unit-hydrograph. Water Resources Bulletin 18, 95–98.
- **Bhunya, P. K., Mishra, S. K. and Berndtsson, R., (2003).** Simplified of synthetic unit hydrograph. J. Hydrol. Eng. ASCE 8 (4), 226–230.
- **Bhunya, P. K., Mishra, S. K., Ojha, C. S. P. and Berndtsson, R., (2004)**. Parameter estimation of Beta-distribution for unit hydrograph derivation. J. Hydrol. Eng. ASCE 9 (4), 325–332.
- **Bhunya, P.K., Berndtsson, R., and Ojha, C.S.P., 2007**. Suitability of Gamma, Chi-square, Weibull, and beta distributions as synthetic unit hydrographs. Journal of Hydrology 334, 28–38.
- **Chow, V. T., 1964.** Handbook of Applied Hydrology. Mc Graw-Hill Book Co. Inc., New York.
- **Croley II, T.E., (1980)**. Gamma synthetic hydrographs. J. Hydrol. 47,Distributions, vol. 1 Houghton Mifflin Company, Boston.
- **Dooge, J.C.I., (1959)**. A general theo Geophys. Res. 64 (2), 241–256.ry of the unit hydrograph. J.
- **Espey, W. H. Jr. and Winslow, D. E. (1974).** ‘‘Urban flood frequency characteristics.’’ J. Hydraul. Div., 100(2), 279–293.
- **Gray, D. M. (1961).** ‘‘Synthetic hydrographs for small drainage areas.’’ J. Hydraul. Div., Am. Soc. Civ. Eng., 87(4), 33–54.
- **Johnson, N.L., Kotz, S., (1970a)**, first ed. Continuous Univariate Mgmt., Alexandria Univ., Egypt, 104–110.
- **Nash, J. E. (1960).** ‘‘A unit hydrograph study with particular reference to British catchnets.’’ Proc., Inst. Civ. Eng., London, 17, 249–282.
- **Nash, J.E., (1959**). Synthetic determination of unit hydrograph parameters. J. Geophys. Res. 64 (1), 111–115.
- **Salas, (2006) .**Notes on Unit Hydrographs, Colorado State University, Department of Civil and Environmental Engineering.
- **Saralees Nadarajah, (2007).** Probabilty models for unit hydrograph derivation. Journal of Hydrology 334, 185–189.
- **Singh, S. K. (1998).** ‘‘Reconstructing a synthetic unit hydrograph into a Gamma distribution.’’ Proc., Int. Conf. on Integrated Water Resour. Mgmt., Alexandria Univ., Egypt, 104–110.
- **Singh, S. K. (2000).** ‘‘Transmuting synthetic unit hydrographs into gamma distribution.’’ J. Hydrologic Eng., 5(4), 380–385.
- **Singh, V. P. (1988).** Hydrologic systems: Rainfall-runoff modeling, Vol. 1, Prentice-Hall, Englewood Cliffs, N.J.

- **Viessman, W. Jr., Lewis, G. L., and Knapp, J. W. (2007**). Introduction toHydrology, 5rd Ed.,
- **Yue, S., Taha, B.M.J., Bobee, B., Legendre, P., and Bruneau, P., (2002)**. Approach for describing statistical properties of flood hydrograph. J. Hydrol. Eng. ASCE 7 (2), 147–153.
- **Soil Conservation Service _SCS_. (1972)**. SCS national engineering handbook, Section 4: Hydrology, U.S. Dept. of Agriculture,Washington, D.C.

Table (1): The unit hydrograph calculations for case study (A)[salas (2006)]

Time (hr)	Total Flow (m^3/s)	Base Flow (m^3/s)	Direct runoff Qt (m^3/s)	Unit hydrograph (m^3/s)
0	125	--	--	--
1	100	100	0	0
2	150	100	50	12.5
3	250	100	150	37.50
4	415	100	315	78.75
5	600	100	500	125.00
6	515	100	415	103.75
7	400	100	300	75.00
8	300	100	200	50.00
9	225	100	125	31.25
10	175	100	75	18.75
11	150	100	50	12.5
12	135	100	35	8.75
13	125	100	25	6.25
14	100	100	0	0
15	100	100	0	0
			$\sum Q_t=2240$	$\sum u_t=560$

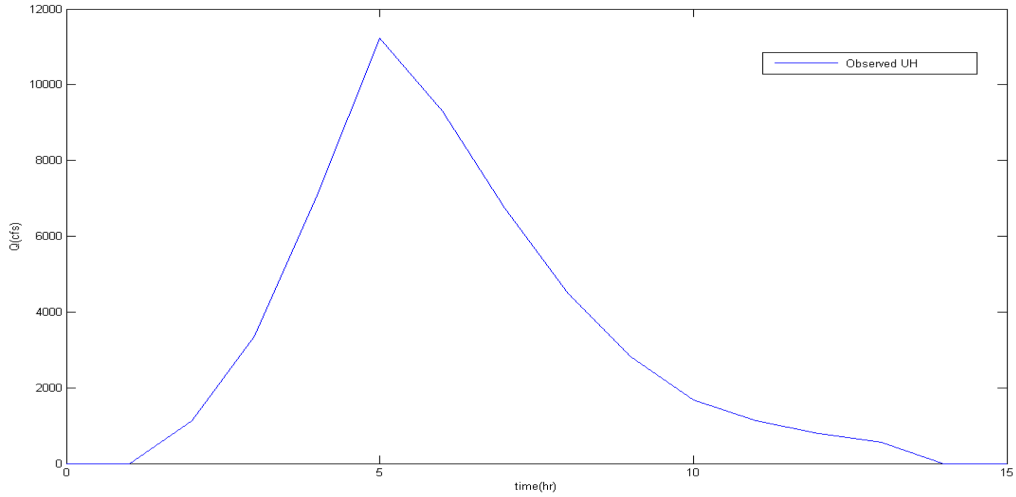

Fig. (1): Unit Hydrograph from observed data table1 [salas(2006)].

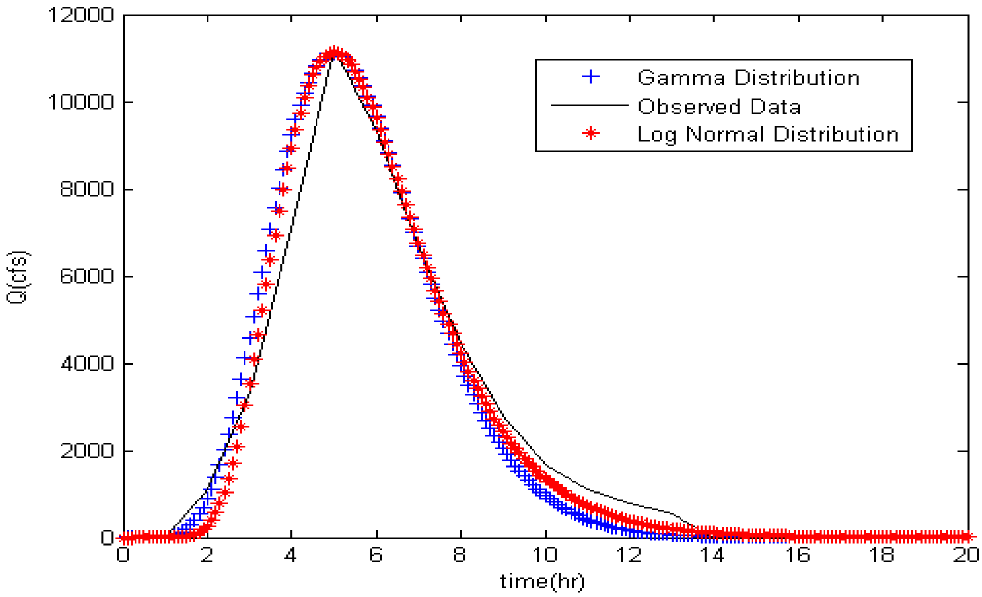

Fig. (2): Comparisons of observed unit hydrograph with SUH obtained from Gamma and Log Normal distributions

Table (2): Estimated depth of excess rainfall over the watershed area using different methods for case (A).

Type of method	Average depth (in)
Observed data	1
Gamma distribution	0.9913
Log normal distribution	0.9912

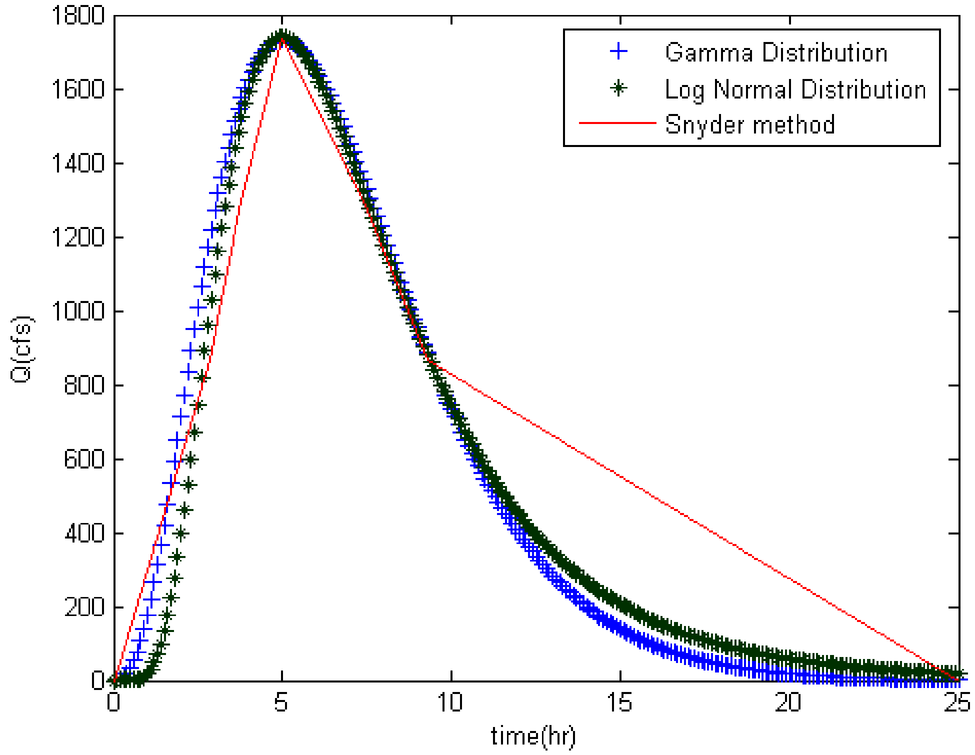

Fig. (3): Comparisons of Snyder unit hydrograph with SUH obtained from Gamma and Log Normal distributions

Table (3): Estimated depth of excess rainfall over the watershed area for different method.

Type of method	Average depth (in)
Snyder method	1.231
Gamma distribution	0.9910
Log normal distribution	0.9851