# APPLICATION OF VIBRATION MEASUREMENT TO DETECT DAMAGE IN CASTING

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## ABSTRACT

Modal testing has become commonplace in many industries today as a research and development tool. In this capacity, it is used primarily during product prototype development and for vibration problem in general. Many types of structural or parts faults will cause changes in the measured dynamic response of a structure. These changes will, in turn, cause change in the structure's modal parameter.

The purpose of the present work is to propose an improved damage detection and location based on the measurement of modal parameter (natural frequency and mode shape) before and after faults, which they have varying extents, for three different sizes of Aluminum casing plates. This local damage can be translated into or characterization as a reduction of the local stiffness which, simulated in the presented numerical models using software package. After measured natural frequency, if a change is detected a statistical method is used to make the best match between the measured changes in frequencies and the family of the theoretical predictions. This predicts the most likely defect location. Analytical results are also used to check numerical results, which showed a good agreement with it. Standard Aluminum plates were also investigated in this work. It results were compared with casting results for two boundary conditions. Also, the defect location charts that plotted with the support of deriving stiffness sensitivity equation showed a good agreement between the predicted defect site and the actual defect location for most of the study cases.

## الخلاصة

إن الاختبارات الشكلية أصبحت هذه الأيام شيئا مألوفا في عدة صناعات كبحوث أو أداة تطّور. وضمن هذه ألا مكانيه ، فقد استخدمت أولا أثناء إنشاء النموذج الأولي للأجزاء وكذلك لمشاكل الاهتزاز بشكل عام. أن أنواع عديدة من العيوب البنيوية سوف تحدث تغيرات في الاستجابة الديناميكية المقاسه لتركيب هيكلي ما. و هذه التغيرات سوف تحدث، تباعا، تغيّر في الباراميترات الشكلية للتركيب الهيكلي. إن الغرض من العمل الحالي هو اقتراح تقنية مع إدخال بعض التحسينات لكشف وتعيين موقع العيب مستنده على قياس البارامیتر الشكلي (التردد الطبيعي وشكل الموجه) قبل وبعد حدوث العيوب، والتي لها أحجام أو مديات مختلفة، لثلاثة أحجام مختلفة من ألواح الألمنيوم المسبوكة. إن هذه العيوب الموضعية يمكن نقلها أو تمثيلها على شكل تقليل في الجساءة الموقعية، والتي مثلت في النماذج العددية المقدّمة باستخدام برنامج جاهز. بعد قياس الترددات الطبيعية، إذا تم كشف تغيّر في الترددات الطبيعية فسوف يتم استخدامها ومن خلال طريقه إحصائية لحساب أفضل توافق بين التغيرات المقاسة والمجموعة المتنبأ بها نظريا. هذه الطريقة تنبأ عن الموقع الأكثر احتمالا

للعيب. كذلك تم استخدام النتائج النظرية لفحص دقة النتائج العددية، والتي أظهرت تطابق جيد معها. ذلك تم استخدام ألواح قياسية من الألمنيوم حيث تم التحقق منها في هذا العمل. هذه النتائج تم مقارنتها مع نتائج الألواح المسبوكة ولنوعين من الحدود الشرطية. كذلك فأن مخططات موقع العيب، والتي رسمت وبمساعدة المعادلة المشتقة لحساسية الجساءة، أظهرت توافق جيد بين مكان العيب المتنبأ به والموقع الحقيقي للعيب لاكثر الحالات المدروسة.

**KEY WORDS**
Non- destructive test, Modal Analysis, Damage, Detection, Location, Casting.

## INTRODUCTION

The physical mass, stiffness, and damping properties of a structure determine how this structure can vibrate. Vibration is caused by an exchange of energy between the mass (inertia) property and stiffness (restoring) property of the structure. The damping property dissipates vibrational energy.

A structure's modal properties are directly related to its physical properties. That is, changes in structure's mass, stiffness, or damping properties will cause change in its modal properties (modal frequencies, modal damping, and mode shapes). Also, changes in the structure's boundary conditions (mountings) can be viewed as changes in the mass, stiffness, or damping plus its surroundings, and will change its modal parameter.

The modal parameter solutions to the differential equation of motion which are themselves functions of the mass, stiffness, and damping of the structure [Mannan 1991].

Changes in structure's modal parameters are to be used as a reliable means of detecting, and possibly even locating and quantifying structural faults. "Faults" means the following occurrences:

- Flaws voids, cracks, thin spots, etc. caused during manufacturing processes such as casting or forming operations. Faults of casting are the type of fault that we need it in this paper.
- Failure of the structural material, e.g. cracking, breaking, or delamination.
- Loosening of assembled parts, e.g. loose bolts, rivets, or glued joints.
- Improper assembly parts during manufacturing.

Now we can asked that what is the smallest physical change in a structure that can be detected, located, and quantified from changes in its modal parameters? The best answer to this question is "the smaller the better". This answer presumes that it is always better to detect the onset of structural faults as early as possibly when it is still small.

Non destructive damage detection can be broken up into four categories. *Level I* test for the presence of damage. *Level II* test for the presence and location of the damage on the structure. *Level III* test for the presence, location, and severity of the damage. Finally *level IV* tests for the presence, location, and level of the imparted damage as well as predicting the change in physical properties of the structure due to the damage. In this work, we used level II techniques to detect and locate faults in casting.

The main objective of the current work is to build a vibration technique capable of detecting and locating faults in Aluminum casting plates as a mean of quality control, where there are large numbers of the same component produced.

A computer model is developed to study the effect of damage in the casting plates on the natural frequencies and mode shapes using a package deal with finite element analysis. This technique is highly accurate, and can be used for analysis in any structure with any type of structural vibration. Also calculate the change in the model parameters of the casting plates due to the introduction of suggested defect models. The defect model must be able to represent the defect that have a sensible effect on the structure stiffness and also must be simple for calculating.

Test rigs will be designed and built to study the vibration behavior of the model structures before and after damage to predict the measured change in natural frequencies. Finally, develop a statistical method for the best match between the measured changes in frequencies and the family of numerical predictions. This model in true will then locate the most likely damage sites. A comparison was made between the experimental and the theoretical results and between casting plates and Aluminum plates.

**Advantages of Modal Parameter Measurements as Continuous Monitoring**

Measurement and the estimation of modal parameter changes have some inherent advantages for continuous monitoring applications that are not available with other methods.

1- Modal parameters can be measured on any structure that vibrates.
2- Modes of vibration are sensitive indicators of physical changes.
3- Changes in modes can localize a fault.
4- Faults can be detected in a measured region of the structure (measurements at single point in the structure is enough for detection).
5- Only a small number of measurements are required.
6- A wide variety of excitation and signal processing methods can be used.
7- Modal testing is non-destructive test.

## ANALYTICAL ANALYSIS

A plate is a two-dimensional sheet of elastic material, which lies in a plane. The general assumptions and equations used in the analysis of plates in this section are:

For plate laying in the x-y plane the normal strains ($\varepsilon_x$ and $\varepsilon_y$) and shear strain ($\varepsilon_{xy}$) in the plane of the plate are [Srinivasan 1982]:

$$\varepsilon_x = z\frac{\partial^2 w}{\partial x^2}\ ,\ \varepsilon_y = z\frac{\partial^2 w}{\partial y^2};\ \varepsilon_{xy} = 2z\frac{\partial^2 w}{\partial x \partial y} \qquad (1)$$

The out of plane strains are zero:

$$\varepsilon_{xz} = \varepsilon_{yz} = \varepsilon_{zz} = 0$$

These strains are associated with the following stresses for a homogenous isotropic material:

$$\sigma_{xx} = \frac{E}{1-\upsilon^2}(\varepsilon_x + \upsilon\varepsilon_y)\ ;\sigma_{yy} = \frac{E}{1-\upsilon^2}(\varepsilon_y + \upsilon\varepsilon_x) \qquad (2)$$

$$\sigma_{xy} = G\varepsilon_{xy};\ \sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$$

If $w$ is the transverse deflection of the plate, the elementary kinetic energy $dT$ and the elementary potential energy $dV$ of the plate is given by:

$$dT = \frac{1}{2}\rho\left(\frac{\partial w}{\partial t}\right)^2 dx\,dy\,dz \qquad (3)$$

$$dV = \frac{1}{2}(\sigma_x\, dydz)[\varepsilon_x\, dx] + \frac{1}{2}(\sigma_y\, dxdz)[\varepsilon_y\, dy] + \frac{1}{2}(\sigma_{xy}\, dydz)[\varepsilon_{xy}\, dx] \qquad (4)$$

The total kinetic energy $T$ and potential energy $V$ of the plate is given by integrating:

$$T = \frac{1}{2}\int_0^a \int_0^b \rho h \left(\frac{\partial w}{\partial t}\right)^2 dx\, dy \quad (5)$$

$$V = \frac{D}{2}\int_0^a \int_0^b \left[\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\upsilon \frac{\partial^2 w}{\partial x^2} \times \frac{\partial^2 w}{\partial y^2} + 2(1-\upsilon)\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right] dxdy \quad (6)$$

Where: $D = \frac{Eh^3}{12(1-\upsilon^2)}$ is called the plate bending stiffness element

Forming the Lagrangian $L=T-V$ and applying the Hamilton's principle: $\delta \int_{t_1}^{t_2} L\, dt = 0$, we obtain:

$$\frac{1}{2}\delta \int_{t_1}^{t_2} \int_0^a \int_0^b \rho h w_t^2 - D\left[w_{xx}^2 + w_{yy}^2 + 2\upsilon w_{xx} w_{yy} + 2(1-\upsilon) w_{xy}^2\right] dxdy = 0 \quad (7)$$

Performing the operations as per the rules of the calculus of variation term by term, we use the variational operation (δ) to extract the analytical term for vibrating plate:

$$D\nabla^4 w + \rho h \ddot{w} = 0 \quad (8)$$

Suppose the plate is supported on all the four edges, and the plate is vibrating with a frequency $p$ given by:

$$w = w_0 \cos(pt) \quad (9)$$

The equation of motion of the plate (8) becomes:

$$\nabla^4 w = \gamma^2 w : \qquad \text{Where } \gamma^2 = \frac{\rho h}{D} p^2$$

We shall assume the deflection $w_0$ as [2]:

$$w_0(x,y) = \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \quad ; m, n = 1,2,3,.... \quad (10)$$

So that frequency equation for simply supported plate becomes:

$$P_{m,n} = \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)\sqrt{\frac{D}{\rho h}} \qquad \text{i, j=m, n=1,2,3,...} \quad (11)$$

Equation (11) represent the analytical term for finding natural frequency for the above boundary condition. In the same manner we can find the frequency equation of a plate subjected to other boundary conditions.

While the subject of vibration analysis of the completely free rectangular plate has a history, which goes back nearly two centuries, it remains a fact that most theoretical solutions to this case are considered to be at best approximate in nature. This is because of the difficulties, which have been encountered in trying to obtain solutions that satisfy the free edge conditions as well as the governing differential equation [Gorman 1978]. Since no analytical solution exist for this boundary [Victor 2001], so that the eigenvalues for plate vibrate under this boundary condition is governed by the approximate natural frequency expression (in Hertz) of the form presented below [Blevins 1979]:

$$Pij = \frac{\pi}{2}\left[\frac{G_1^{\,4}}{a^4} + \frac{G_2^{\,4}}{b^4} + \frac{2J_1J_2 + 2\upsilon(H_1H_2 - J_1J_2)}{a^2b^2}\right]^{1/2}\left[\frac{Eh^3}{12\gamma(1-\upsilon^2)}\right]^{1/2} \qquad (12)$$

Where, $i = 1,2,3,....$ and $j = 1,2,3...$

The dimensionless parameter G, H and J are functions of the indices i and j and the boundary conditions on the plate.The approximate natural frequencies predicted by equation (12) are directly analogous to the analytical solutions. The approximate natural frequencies can be expected to be within 5% of the exact solutions [Blevins 1979].

## NUMERICAL ANALYSIS

The finite element analysis applied to vibration problems is now becoming well known and the study of vibrational behavior of a structure is of great importance because it can be used to determine the eigenvalue and the eigenvector solutions.

In contrast to the early days, it can use computer software to generate complex geometry, at either component or reassemble level. It can (with some restrictions) automatically generate elements and nodes, by merely indicating the desired nodal density. Software is available that work in conjunction with finite element to generate structure of optimum topology, shape, or size.

In order to select the suitable mesh for the plates, in the numerical calculation, the plates must be investigated for different number of (D.O.F.). **Figs. (1), (2), and (3)** show the variation of natural frequency with the number of (D.O.F.) for the three different sizes of free-free plates used in this work respectively. For the work purpose, natural frequencies have been calculated numerically using ANSYS FE software package and the mode shapes have been observed for all the Aluminum casting plates and Aluminum plates tested in this work. The plates were drawn to the scale 1:1 on this software and a study was performed in two boundary conditions, free-free and simply supported boundaries. Natural frequencies for some cases have been listed in **tables (1), and (2)** which are compared with the theoretical natural frequencies. Also the first three dynamic modes are shown in **Figs. (4), and (5)** for some cases.

## EXPERIMENTAL WORK

The objective of this section is to detail the testing procedure, which gives as possible as accurate values of the natural frequencies of the structure. The procedure requires that the structure under test is excited by harmonic force and the response at various points of the structure must be measured. An arbitrary choice of the point of excitation could be lead to difficulties in producing the resonance at certain natural frequencies of the structure. For example, exciting the structure at a nodal point (point of zero amplitude of vibration) of a certain natural frequency would result in missing out the resonance at the natural frequency. It useful to change the position of the excitation of a structure to exist the different modes of vibration or by drawing the theoretical mode shapes.

A natural frequency was distinguished by observing the sharp increase in amplitude of the pickup output and by the intensity of the tone emitted, which was amplified and displayed on the oscilloscope. This procedure was represented to measure the pre and post damage natural frequencies. The test procedure must establish homogeneous conditions throughout all phases of the experimental work. Thus, the method used for supporting the structure during each investigation should be simple to set-up and must be reproducible.
The test plates was suspended horizontally in the test rig using a very soft elastic cords attached to the mid-points of its four sides in order to approximate the free-free boundary condition, the procedure was used by several researchers [Lauwagie 2002]. Also in order to approximate the simply-supported conditions the tested plates was putted on a four sides sharp edge frame made for the purpose. Each plate was set-up in precisely the same fashion in attempt to minimize the effects of human factors on the frequency changes. Damage was represented in all tests carried out by putting the saw cut and then change the dimension of the saw cut by extends it. Also, a drill was used to make all sizes of holes damage in the plates. The natural frequencies have been measured for undamaged free-free and simply supported rectangular sand casting Aluminum plates, also we use a standard Aluminum plates to compare there results with casting Aluminum plates results. Then natural frequencies for the same plates have been damaged by making six damage cases which used in the all tests. The cast Aluminum used is C443.0: S5C ASTM, which it mechanical and physical properties are ( Tensile strength is 230 $Mn/m^2$, Yield strength is 110 $Mn/m^2$, Modulus of elasticity is 71.0 $Gn/m^2$, Percentage elongation is 9%, Density is 2690 $Kg/m^3$, and Poisson's ratio is 0.33).

## DEFECT LOCATION TECHNIQUE

The stress distribution through a vibrating structure is non-uniform and is different for each natural frequency (mode shape). This means that any localized defect would affect each mode differently, depending on the particular location of the defect. The defect may be modeled as a local decrease in stiffness of the structure. So, if it is situated at a point of zero stress such as the nodal lines in a given mode, it will have no effect on the natural frequency of that mode. On the other hand if it is at a point of maximum stress, it will have the greatest effect. Therefore the location of the defect site requires the computation of the relative effect on several modes of vibration at different sites within the structure .The experimentally measured changes in natural frequencies may be then be compared with the theoretically calculated changes for defect at different sites and the position of the defect deduced. If natural frequency measurements are to be curried out, the effect of defect may be determined by modeling the defect as a local decrease in the stiffness, rigidity, or thickness of the structure and currying out the dynamic analysis of the system.
Following Cawley et.al. [Cawley 1979], it is assumed that, in theory, the change in the natural frequency of mode ($i$) of a structure due to damage in the structure is a function of the position vector of the damage ( $r$), and the reduction in stiffness caused by the damage ( $\delta K$ ), thus:

$$\delta\omega_i = f(\delta K, r) \tag{13}$$

A formula expansion about the undamaged state ( $\delta k = 0$ ), and ignoring second and higher-order terms, yields:

$$\delta\omega_i = f(0,r) + \delta K \frac{\partial f}{\partial(\delta K)}(r,0) \tag{14}$$

Assuming that there is no frequency change with out damage, it follows that $f(r, 0)=0$ for all ($r$) and so, writing the partial derivative as $g_i(r)$, equation (14) then simplified to:

$$\delta\omega_i = \delta K\, g_i(r) \qquad (15)$$

If it is assumed further that ( $\delta K$ ) is independent of frequency, it follows that the ratio of frequency changes is dependent only upon the damage location as specified by ($r$):

$$\frac{\delta\omega_i}{\delta\omega_j} = \frac{g_i(r)}{g_j(r)} = h(r) \qquad (16)$$

Measurements of the frequency changes in one pair of modes will yield in a locus of possible damage sites, that the point where the ration of the experimentally determined changes equals the theoretical ratio. With symmetrical structures, two or more sites will be predicted, the number depending on the degree of geometric symmetry. So that an "error "which is denoted by ($e_{rij}$), in assuming the defect to be at position ($r$), given frequency changes ( $\delta_{wi}$ ) and ( $\delta_{wj}$ ) in modes ($i$) an ($j$) respectively, as:

$$e_{rij} = \frac{S_{ri}/S_{rj}}{\delta_{\omega i}/\delta_{\omega j}} - 1, \qquad \frac{S_{ri}}{S_{rj}} > \frac{\delta_{\omega i}}{\delta_{\omega j}} \qquad (17)$$

The value of the error function ( $e_{rij}$ ) is computed for each mode pair according to equation (17). These values are then assumed to give a measure ($e_{rt}$) of the total error in assuming the damage to be at position ($r$) given the experimentally measured frequency changes. Thus:

$$e_{rt} = \sum_{all\ pairs\ (i,j)} e_{rij} \qquad (18)$$

The most probable defect site is taken to be the one at which the value ($e_{rt}$) is minimum .Let this minimum value is ($e_{min}$). This was then used to normalize each total error, which was expressed as the "normalized error" for failure at theoretical position ($r$), defined as:

$$ne_r = 100 \times e_{\min}/e_{rt} \qquad (19)$$

A very attractive alternative to repeat the full dynamic analysis in order to compute the changes in the natural frequencies due to localized damage is to use a sensitivity (perturbation) analysis. The basic principles of the method are described by, for example, Courant and Hilbert [Courant 1953]. By this method, the sensitivities of the natural frequencies of a system to small changes in the stiffness matrix, mass matrix, and damping matrix are calculated from mode shapes of the unmodified structure (structure with no faults) produced by the initial full dynamic analysis.

The orthogonality property of the modes "almost" simultaneously diagonalizes the mass, stiffness, and damping matrices, and therefore "almost" uncouples the equations of motion. The term "almost' is used because strict diagonalization occurs if there is no damping ([$C$] = [0]). Probably the most sought after cause of a structural fault is a reduction in local stiffness, which might be caused by the formation of a crack, delamination, voids, or a loose fastener.

$$\{U_k\}^t[dK]\{U_k\} = \omega_{1k}^2 - \omega_{0k}^2 \qquad (20)$$

This formula only required the mode shapes for the unmodified structure plus changes in the stiffness matrix [$dK$]. A fault that causes local stiffness change can then be detected and located by simply tracking the stiffness change of the structure, and using equation (20).

## RESULTS AND DISCUSSION

The validity of any theoretical approach may be examined by one of two methods, the first method is by making a comparison between the suggested approaches and well known analytical methods, while in the second method, the comparison is made with the experimental results.

In this section, the performance of the proposed NDT has been checked. Several tests were carried out for different casting plate structures with readily qualifiable forms of damage in order to check the operation of the technique and the supporting analysis.

**Fig. (6), (7)** shows a comparison between the exact and approximate natural frequencies. It may be observed from the figure that the results compare very favorably with the exact values. **Table (3)** give the predicted and measured natural frequencies (Hz) for the undamaged state of the 600*500*6 mm free free casting plate. The maximum error in predicting natural frequencies was (1.969) in the second mode. These errors are from variations in dimensions due to casting process and material properties. The data in table shows that the FEA model of the casting plate without fault tended to give slightly higher natural frequency than the test data as the modes increased in frequency. Instead of soaking time in frequency measurement experiment by placing the shaker and the accelerometer at different positions in order to avoid the possibility of having the accelerometer and / or the shaker at a nodal line. The best position can be predicted from the mode shapes of the tested plate. The node lines are drawn by connecting node points, which are computed as points where the mode shapes is zero in a normal direction to the surface of the plate.

The results obtained from ANSYS FE software package showed that the maximum displacement (especially in the first mode) is in the corners of the plate (for free free boundaries). Therefore, the shaker and the accelerometer were mounted in one of the Four Corners of the tested plate. Also the results showed that the maximum displacement is around the center of the plates (for simply supported boundaries). Therefore, the shaker and the accelerometer were mounted in the center of the tested plate. If we want to plot the mode shapes after the fault made in the plate. We expected that there are large differences with the shapes for unmodified plate, but also we expected that they don't pinpoint the location of the fault. One explanation for this is that all of these modes are "global" in nature (which is true for most simple structures and hence will change globally even due to a "local" change such as the hole damage.

Different cases of damage have been investigated for this casting plate. **Table (3)** also gives the experimentally measured frequency reduction (Hz) for all cases of damage. It is apparent that the reduction in the natural frequencies increased as the damage size increased. From the analysis it is observed that at least three modes are needed to detect damage existing any were in the casting plate. Individual modes have relatively different sensitivities to potential damage location in casting plate. For example modes 1 and 3 are sensitive to the location near the center while modes 2 and 4 are not sensitive. The table values reveal that the frequency reduction clearly indicates the presence of the 10 mm and 5 mm holes, by the frequency shift of the modes. Due to the relatively small magnitude of damage they don't detect the 3 mm hole. Also **Table (4)** has been presented for another case.

The defect location chart will consist of a plot of the plate, with elements labeled using the normalized error. The predicted damage site being represented by the value 100 (see eq. 19). Series of tests for defect location analysis were carried out on the casting plates using different damage models. Sensitivity analysis was used to simulate damage by the reduced thickness of the whole element. From these results it will be possible to establish the generality and validity of the method used in order to deal with such structures. **Fig. (8)** shows the location chart produced from a test on a rectangular Aluminum casting plate of dimension 600*500*6 mm. The analysis used a 16*16 finite element mesh with a total of 1024 grid points. The plate was damage by drilled a 10 mm hole at site A. It will be observed that, due to symmetry, the location chart shows four possible damage sites. It can be seen from the figure that the damage was correctly located.

## CONCLUSIONS

This paper presented a method of non-destructively detect faults in casting plates for which only a few natural frequencies are available. Also, we presented an improved damage location algorithm which was used a set of modal parameters for an unmodified(undamaged) casting plates with experimental data with the support of sensitivity equations, which consider the orthogonality conditions of the undamaged plate mode shape.

The final scheme has the advantage that only one dynamic FEA need to perform on the casting plate structure. The dynamic analysis may be stored on disc and used as input to the damage location program, along with the experimentally determined natural frequencies.

The major conclusions that can be obtained from the present work can be summarized as follows:

1- For this work, in frequency measurement the best position of attachment between the shaker/ accelerometer and the tested plate was predicted from the theoretical mode shape of the tested plate. It was found that this position was in one of its four corners for free free boundaries and around the center for simply supported boundaries, where maximum displacement was found numerically at these positions.
2- Any set of measurements that are repeatedly made over time will exhibit variations. These variations are caused either by the "natural" statistical variation in the measurement process, due to numerous sources of measurement error, or they are caused by a physical change in the structure, i.e. an "assignable cause".
3- It is possible to use the method of health monitor without need to have measured the frequency of the virgin structure by using a damaged state as the baseline for future measurement. This is an important property in the job field, where there are a large number of products. Also a key advantage of this technique is that it can be used on the type of data, namely natural frequency, which is commonly measured in a structure testing laboratory using the experiments system.
4- The experimental results indicated that the smallest defect size that can be located, using the proposed defect location method, depends upon the accurate measurements of natural frequency. The error in the measurement of natural frequency is come from the error in simulating the actual boundary conditions, the change in material properties, and the error in dimensions of casting plates due to casting process.
5- Individual modes have relatively different sensitivities to potential damage location. Thus, we observed that they were appreciable changes in some natural frequencies and comparatively small in other. The effect depends on the location of damage and increases as the damage size increase.
6- Good agreement was obtained between the experimental and theoretical results both for undamaged and damaged plates.

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**NOMENCLATURES**

| | |
|---|---|
| a | Length of plate (mm) |
| b | Width of plate (mm) |
| f | Functions |
| h | Thickness of plate (mm) |
| i , j | Modes i , j |
| $L$ | Lagrangian multiplier |
| $G_1, G_2$ $H_1, H_2$ $J_1, J_2$ | Dimensionless parameter for natural frequencies formulas |
| $e_{rij}$ | Error function |
| $e_{rt}$ | Total error function |
| $ne$ | Normalized error |
| $P_{i,j}$ | Natural frequency (Rad/Sec) |
| n | Number of DOFs of the Structure |
| m | Number of Modes |
| **r** | Position vector of damage |
| $Sr_i$ | Sesitivity of mode i to damage at **r** |
| $[dK]$ | changes in the stiffness matrix |
| $T$ | Total kinetic energy |
| $V$ | Total potential energy |
| [U] | Mode shape matrix |
| D.O.F. | Degree of freedom |
| N.D.T. | Non destructive testing |
| $w$ | Transverse deflection of The plate midsurface |
| $\sigma$ | Stress Vector |
| $\varepsilon$ | Strain Vector |
| $\rho$ | Density (Kg/m$^3$) |
| $\nu$ | Poisson's Ratio |
| $\delta K$ | Reduction in stiffness caused by damage |
| $\delta \omega_i$ | Change in natural frequency (Rad/Sec) |
| $\omega_i$ | Natural frequency (Rad/Sec) |
| $\omega_{0k}$ | Natural frequency of the unmodified structure (Rad/Sec) |
| $\omega_{ik}$ | Natural frequency of the modified structure (Rad/Sec) |

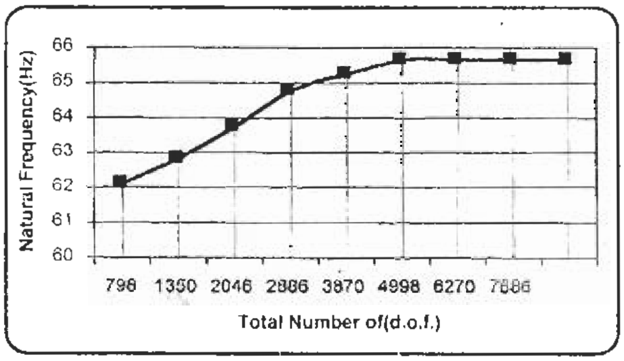


Fig. (1) Variation of natural frequency with total number of (d.o.f.) for 600*500*6 mm free-free casting plate.

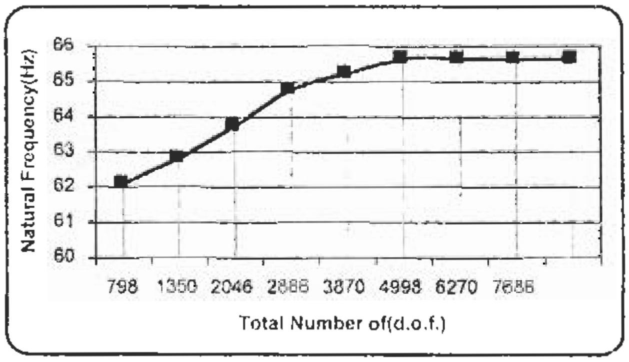


Fig. (2) Variation of natural frequency with total number of (d.o.f.) for 600*500*3 mm free-free casting plate.

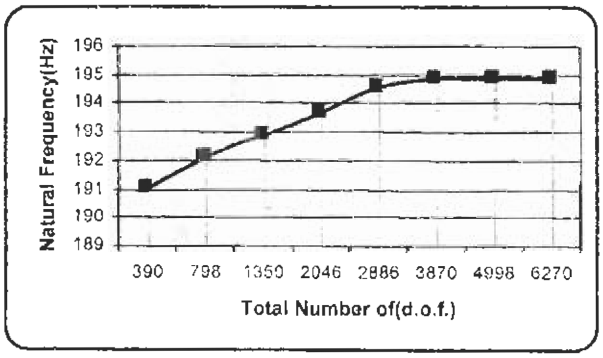


Fig. (3) Variation of natural frequency with total number of (d.o.f.) for 400*250*6 mm free-free casting plate.

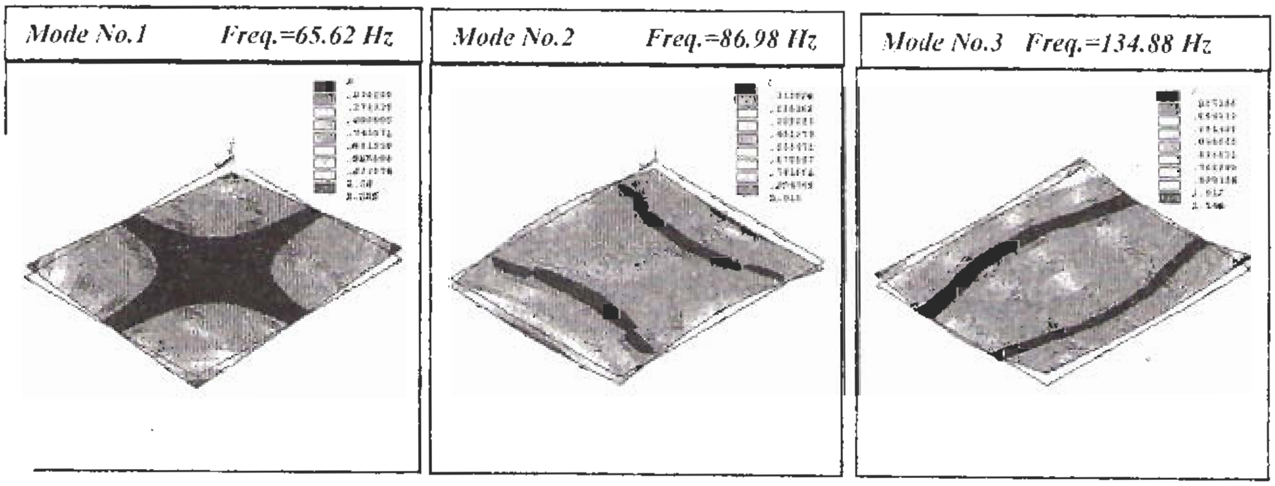


Fig. (4) Mode shape for the 0.6*0.5*0.006m free-fee casting plate.

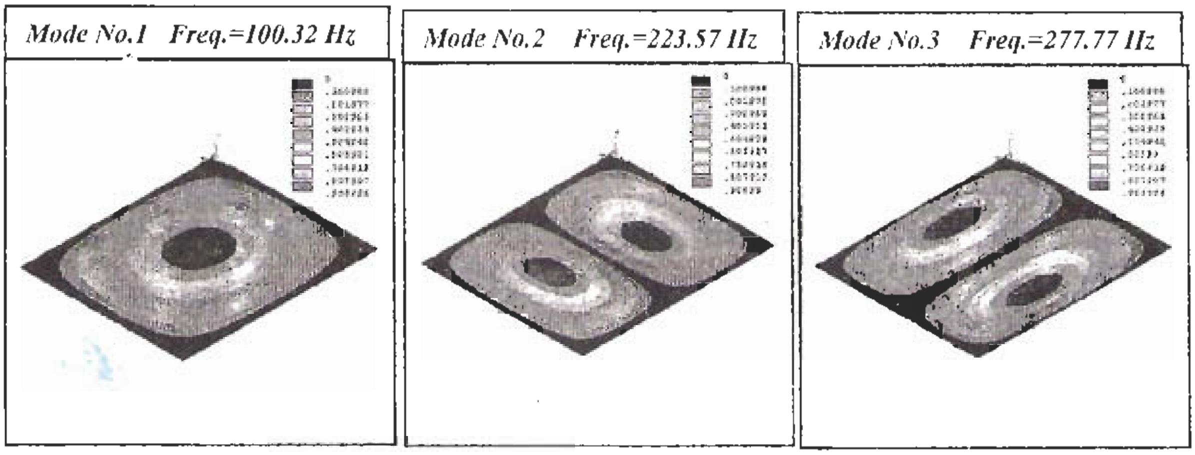


Fig. (5) Mode shapes for the0.6*0.5*0.006m simply-supported casting Plate.

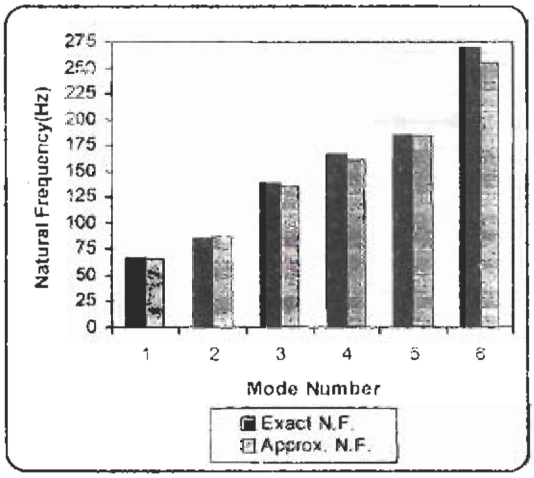


Fig (6) Comparison between the exact and approximate natural frequency for 600*500*6 mm free free casting plate.

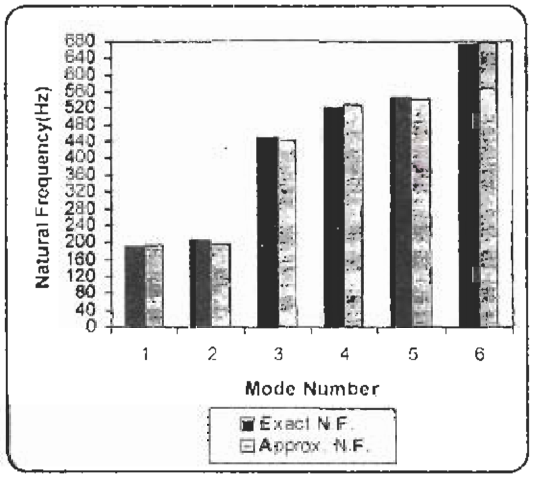


Fig. (7) Comparison between the exact and approximate natural frequency for 400*250*6 mm free free casting plate.

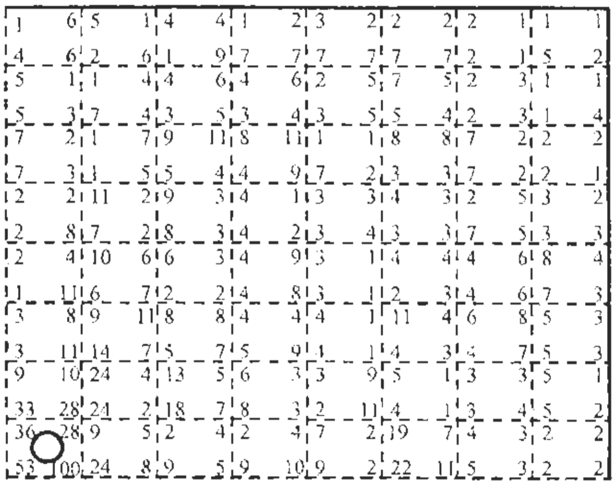

Fig. (8) Location chart for quarter of the 600*500*6 mm casting plate with 10mm hole damage at site A.

Table (1) Numerical and theoretical pre-damage frequency of the first six modes for the 600*500*6 mm fre-free casting plate.

| Mode No. | Natural Frequency (Hz) | | Percentage Error % |
|---|---|---|---|
| | ANSIS 5.4 | Analytical | |
| 1 | 65.62 | 67.28 | 2.4673 |
| 2 | 86.98 | 85.49 | 1.7429 |
| 3 | 134.88 | 138.85 | 2.8592 |
| 4 | 160.42 | 166.39 | 3.5879 |
| 5 | 183.23 | 184.58 | 7.2658 |
| 6 | 255.25 | 269.35 | 5.2348 |

Table (2) Numerical and theoretical pre-damage frequency of the first six modes for the 400*250*6 mm fre-free casting plate.

| Mode No. | Natural Frequency (Hz) | | Percentage Error % |
|---|---|---|---|
| | ANSIS 5.4 | Analytical | |
| 1 | 194.86 | 190.168 | 2.4674 |
| 2 | 198.80 | 203.80 | 2.4533 |
| 3 | 445.00 | 450.07 | 1.1265 |
| 4 | 529.20 | 522.42 | 1.2978 |
| 5 | 542.93 | 545.32 | 0.4382 |
| 6 | 676.23 | 671.23 | 0.7449 |

Table (3) Experimental and theoretical frequencies for 600*500*6mm free free casting plate with six damage case.

| | **Unmodified Frequency (Hz)** | | | | | |
|---|---|---|---|---|---|---|
| **Mode No.** | **Mode 1** | **Mode 2** | **Mode 3** | **Mode 4** | **Mode 5** | **Mode 6** |
| **Exp.** | 64.8 | 85.3 | 134.0 | 159.1 | 182.5 | 254.5 |
| **Theo.** | 65.62 | 86.98 | 134.88 | 160.42 | 183.23 | 255.25 |
| **Error %** | 1.265 | 1.969 | 0.656 | 0.829 | 0.400 | 0.0.294 |
| ***Damage Case*** | **Frequency Reduction (Hz) Exp.** | | | | | |
| **3 mm Hole** | 0.2 | 0.3 | 0.2 | 0.1 | 0.2 | 0.4 |
| **5 mm Hole** | 3.2 | 2.1 | 3.6 | 2.4 | 3.6 | 4.1 |
| **10 mm Hole** | 7.0 | 5.9 | 6.8 | 5.1 | 8.1 | 6.2 |
| **All Holes** | 11.4 | 8.6 | 11.2 | 8.1 | 11.7 | 10.6 |
| **20mm Saw Cut** | 2.6 | 4.2 | 4.5 | 8.9 | 5.8 | 3.9 |
| **40mm Saw Cut** | 6.2 | 8.2 | 10.8 | 11.9 | 10.9 | 8.3 |

Table (4) Experimental and theoretical frequencies for 400*250*6mm free free casting plate with six damage case.

| | **Unmodified Frequency (Hz)** | | | | | |
|---|---|---|---|---|---|---|
| **Mode No.** | **Mode 1** | **Mode 2** | **Mode 3** | **Mode 4** | **Mode 5** | **Mode 6** |
| **Exp.** | 192.3 | 196.0 | 442.8 | 526.8 | 540.3 | 673.7 |
| **Theo.** | 194.86 | 198.80 | 445.00 | 529.20 | 542.93 | 676.23 |
| **Error %** | 1.331 | 1.428 | 0.496 | 0.455 | 0.486 | 0.370 |
| ***Damage Case*** | **Frequency Reduction (Hz) Exp.** | | | | | |
| **3 mm Hole** | 0.3 | 0.2 | 0.3 | 0.5 | 0.3 | 0.6 |
| **5 mm Hole** | 4.0 | 3.1 | 4.9 | 3.5 | 2.8 | 2.2 |
| **10 mm Hole** | 6.5 | 5.8 | 7.2 | 6.3 | 7.9 | 5.6 |
| **All Holes** | 11.1 | 9.1 | 9.0 | 9.4 | 11.5 | 12.0 |
| **25mm Saw Cut** | 6.3 | 7.2 | 5.0 | 7.2 | 9.6 | 7.3 |
| **40mm Saw Cut** | 7.6 | 9.7 | 8.5 | 11.6 | 13.0 | 13.6 |