AN INVESTIGATION INTO THERMAL PERFORMANCE OF A TAPERED GLASS WINDOW

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ABSTRACT

The conjugate heat transfer problem through a glass window with non-uniform thickness is presented. The problem was studied using the fully implicit finite difference numerical technique. The tapered glass window is subjected to a convective boundary condition on the outer uniform side of the window and the heat transfer coefficient depends on the direction of the flow. While the inside boundary condition is at constant comfortable temperature. The upper and lower edge of the tapered section is also at constant temperature but equal to the mean temperature between the indoor and the outdoor temperature. In the first part a rectangular cross section an area of 4mm width and 1m height was taken under study. While, in the second part of the study, a tapered cross section with different tapered angles of 0.05, 0.1, 0.15, and 0.2 degree was held. Another case of the equal cross section area of the tapered and the rectangular cross section was also studied. Good results were obtained and reported graphically. It was found that the ratio of the heat loss by convection from the linear tapered to the rectangular section increases with the increasing of time and the tapered angle.

الخلاصة

يتناول البحث تحليل مسألةَ انتقال الحرارة خلال نافذة زجاجية ذو سُمك غير منتظم باستخدام الطريقة العددية للفروقات المحددة. إنّ النافذةَ الزجاجيةَ المخروطية تُخضَعُ إلى فقدان للحرارة بالحمل الحر من الجانب الخارجي المنتظم للنافذة كشرط حدودي و معامل انتقالِ الحرارةَ يَعتمد على اتجاه التدفقِ الهوائي بينما الشرط الحدودي الداخلي عبارة عن درجةِ الحرارة المريحةِ الثابتة. كما إن الحافة العليا والسفلى للشكل المخروطي أيضا تملك درجة حرارة ثابتةِ لكن مساوية إلى درجةِ الحرارة المتوسطةِ لدرجة الحرارة الداخليــة ودرجــة حرارة الهواء الخارجي. لقد تم الاكتفاء بدراسة جزء تمثيلي من النافذة الزجاجية. فــي الجــزء الأولِ مــن الدراسة تم اخذ منطقة مقطع عرضي مستطيلة منْ 4مليمترِ عرضِ و 1 متر ارتفاع. بينما، في الجزءِ الثــاني للدراسة، أخذ مقطع عرضي مستدق بقيم الزوايا المُستَدَقّة المختلفة 0.05، 0.1، 0.15، و0.2 درجة. كما أجريت دراسة أخرى لحالة المساواة في المساحة لمنطقةِ المقطع العرضي المُستَدَقّ والمقطع المستطيل. لقد ذُكِــرت النتائج بشكل تخطيطي و تم الحصول على النتائجِ الجيدة. و قد وُجدَ من هذه النتائج إن النسبةَ بين الحــرارة المفقودة بالحمل الحر منْ الشكل المُستَدَقّ الخطيّ و الشكل المستطيلي يزداد بزيادَةِ الوقتِ والزاويةِ المُستَدَقّة.

KEY WORDS
Tapered Glass Window, Heat Transfer, Numerical Solution.

INTRODUCTION

Studies on tapered heat transfer problems were first accomplished by [Lim et al. 1992]. They studied the conjugate heat transfer through a wall with non uniform thickness; which is lined on one side by a boundary layer. In the first part, they shows by using the variational calculus that the total heat transfer rate is minimized when the wall thickness decreases in an optimal manner in the direction of flow. While in the second part of the study, the complete problem of a laminar forced convection boundary layer coupled with conduction through a variable thickness wall was solved numerically. Also, means for calculating the total heat transfer rate were reported graphically.

In order to study the thermal performance of a tapered glass window with specified boundary and initial conditions, a glass window cross section model is taken into account in this paper. First, a mathematical formulation for this model is presented in the next section and two problems are presented. One of the problems is for a glass of unit depth and rectangular cross section and the other problem is for a glass of unit depth and cross section area that is linearly tapered at the outdoor edge of the glass window. Then, a numerical solution of the differential equation and boundary and initial conditions is formulated briefly. Also, the results are reported graphically and discussed in the later section.

Although this kind of thermal performance investigation of a cross section that is linearly tapered is not significant for normal glass window because the cost of the material and manufacturing is not expensive, but it is very important for the manufacturing of the glass resistant to bullets because of the decrease in the cost of this type of glass cost and its manufacturing process cost.

MATHEMATICAL FORMULATION

In the present work two problems are taken in consideration. The first problem is a glass of unit depth and rectangular cross area with a width B and height H . This uniform thickness cross-section and the boundary conditions are illustrated in **Fig. (1)**. The right hand outer boundary condition is a convection boundary condition with outdoor ambient temperature T_{ao} and the left side is of constant indoor comfortable temperature T_{ai}. While the upper and the lower part is at constant temperature equal to the mean value of the indoor and the outdoor temperatures.

This problem is represented by the two-dimensional, rectangular coordinates, transient heat conduction equation as shown in the following equation:

$$\frac{\partial^2 T}{\partial^2 x}+\frac{\partial^2 T}{\partial^2 y}=\frac{1}{\alpha_g}\frac{\partial T}{\partial t} \qquad (1)$$

Where $\alpha_g=\dfrac{K_g}{\rho_g Cp_g}$

The applicable boundary conditions are:

$$T=T_u=\frac{T_{ai}+T_{ao}}{2} \qquad \text{at } 0\le x\le B \text{ and } y=0\text{ , } t>0 \qquad (2)$$

$$T = T_L = \frac{T_{ai} + T_{ao}}{2} \quad \text{at } 0 \le x \le B \text{ and } y = H,\ t > 0 \tag{3}$$

$$\frac{\partial T}{\partial x} = \frac{-h}{K}(T - T_{ao}) \quad \text{at } 0 \le y \le H \text{ and } x = 0,\ t > 0 \tag{4}$$

$$T = T_{ai} \quad \text{at } 0 \le y \le H \text{ and } x = B,\ t > 0 \tag{5}$$

and the initial condition is:

$$T = T_o \quad \text{for } t = 0 \tag{6}$$

In the other hand, the second problem of this study is a glass of unit depth and cross section area that is linearly tapered at the outdoor edge of the glass window. The schematic diagram of this non uniform thickness cross section and its boundary conditions are shown in **Fig. (2)**. The same kind of boundary conditions for the first problem is employed in this problem in order to make a comparison between the two types of cross section areas. This comparison can be very useful in the performance and cost balancing of the rectangular cross section and the non uniform tapered cross section.

The problem for tapered glass window is also expressed by the two-dimensional, rectangular coordinates, transient heat conduction equation:

$$\frac{\partial^2 T}{\partial^2 x} + \frac{\partial^2 T}{\partial^2 y} = \frac{1}{\alpha_g}\frac{\partial T}{\partial t} \tag{1}$$

The applicable boundary conditions are:

$$T = T_u = \frac{T_{ai} + T_{ao}}{2} \quad \text{at } 0 \le x \le B \text{ and } y = 0,\ t > 0 \tag{7}$$

$$T = T_L = \frac{T_{ai} + T_{ao}}{2} \quad \text{at } 0 \le x \le B - H\tan\theta \text{ and } y = H,\ t > 0 \tag{8}$$

$$\frac{\partial T}{\partial x} = \frac{-h}{K_g}(T - T_{ao}) \quad \text{at } 0 \le y \le H \text{ and } x = 0,\ t > 0 \tag{9}$$

$$T = T_{ai} \quad \text{at } 0 \le y \le H \text{ and } x = B - y\tan\theta,\ t > 0 \tag{10}$$

and the initial condition is :

$$T = T_o \quad \text{for } t = 0 \tag{11}$$

The convective heat transfer coefficient can be calculated by utilizing the following natural heat convection empirical formula, (Kothandaraman and Subramangan 1994):

$$h = \frac{K_a}{y}\left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1+(0.492/\mathrm{Pr}_a)^{9/16}\right]^{8/27}}\right\}^2 \tag{12}$$

where Rayleigh number, Ra_L, is given as :

$$Ra_L = \frac{g\beta_T \rho_a (T_{sur} - T_{ao}) L^3}{\alpha_a \mu_a} \tag{13}$$

In the present work, a fully implicit finite difference numerical technique is used in order to solve the above heat conduction equations. The finite difference form of the heat conduction equation and boundary and initial conditions are formulated in the next section.

NUMERICAL SOLUTION

To solve the above partial differential equations, a finite difference numerical technique is employed. A grid of points is first established throughout the calculation domain. However, there is a unique difficulty when establishing the grid in the tapered part because the inner surface will not coincide with the rectangular coordinate lines. This difficulty may be overcome through the use of boundary conditions eq. (8) and eq. (10).

The fully implicit method of solution is utilized as recommended by [(Patanker 1980) and Myers 1971)]. The partial differential eq. (1) can be written in the discretization finite difference form as:

$$\frac{T_{i-1,j}^{n+1} - 2T_{i,j}^{n+1} + T_{i+1,j}^{n+1}}{(\Delta x)^2} + \frac{T_{i,j-1}^{n+1} - 2T_{i,j}^{n+1} + T_{i,j+1}^{n+1}}{(\Delta y)^2} = \frac{1}{\alpha_g}\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t} \tag{14}$$

The constant temperature boundary conditions are written in the following finite difference approximation for the first problem case as below:-

$$T_{i,0}^{n+1} = T_u \quad \text{at } 0 \le x \le B \text{ and } y = 0 \tag{15}$$

$$T_{i,m}^{n+1} = T_L \quad \text{at } 0 \le x \le B \text{ and } y = H \tag{16}$$

$$T_{p,j}^{n+1} = T_{ai} \quad \text{at } 0 \le y \le H \text{ and } x = B \tag{17}$$

Whereas for the case of tapered glass window, the constant temperature boundary conditions are expressed by the following finite difference approximation as:-

$$T_{i,0}^{n+1} = T_u \quad \text{at } 0 \le x \le B \text{ and } y = 0 \tag{18}$$

$$T_{i,m}^{n+1} = T_L \quad \text{at } 0 \le x \le B - H\tan\theta \text{ and } y = H \tag{19}$$

$$T_{p,j}^{n+1} = T_{ai} \quad \text{at } 0 \le y \le H \quad \text{and } x = B - y\tan\theta \tag{20}$$

While the convective boundary condition, which is appropriate for any time increment, is represented by a central finite difference approximation. This boundary condition is applicable for both uniform thickness, eq. (2c), and tapered (non uniform thickness), eq. (4c), of glass window and it can be formulated for each boundary grid point as:

$$\frac{1}{(\Delta x)^2}\left[T_{1,j}^{n+1} + \frac{2\Delta x h}{K_g}\left(T_{ao}^{n+1} - T_{0,j}^{n+1}\right) - 2T_{0,j}^{n+1} + T_{1,j}^{n+1}\right] + \frac{1}{(\Delta y)^2}\left[T_{0,j-1}^{n+1} - 2T_{0,j}^{n+1} + T_{0,j+1}^{n+1}\right] = \frac{1}{\alpha_g \Delta t}\left(T_{0,j}^{n+1} - T_{0,j}^{n}\right) \tag{21}$$

In addition, the initial condition for each case can be formulated in the following form:

$$T_{i,j} = T_o \tag{22}$$

It is important to know that the boundary conditions eq. (15), eq. (16), eq. (17), eq. (18), eq. (19), eq. (20), and eq. (21) are appropriate for any time increment.
The values of thermal conductivity, density, and specific heat are substituted for the glass window for the specified point in the mesh from values given in Appendix (A). In equations (12) and (13) the air thermo physical properties are needed. These are given in appendix A. The convective heat transfer coefficients were taken from (Kothandaraman and Subramangan 1994) and the thermo physical properties of air were taken from (Perry 1984) at the mean value of the ambient outdoor and respective surface temperatures.
Upon substituting the known values of ρ_g, Cp_g, K_g, Δt, Δx, Δy and the heat transfer coefficient in eq. (12) and employing the boundary condition and initial condition equations, a number of linear algebraic equations will be obtained for the whole domain. These equations were solved by the Gauss-Seidel iterative technique.
A computer program written in FORTRAN was used. The error criteria for the Gauss-Seidel iterative technique, found by trial and error, was equal to 10^{-3}.
After solving the above heat conduction problem and obtaining the temperature distribution, the heat loss by convection from the outdoor left side of the glass window for both the rectangular cross section and tapered cases is calculated. This heat loss is obtained for each grid point on the left side of the model and then finding the summation of the heat loss of these grids for each time step in order to make a comparison between the above two cases. The convective heat loss can be found by using the following equation:

$$Q = h_y \times A \times \left(T_{i,j} - T_{ao}\right) \tag{23}$$

RESULTS AND DISCUSSION

In order to investigate the performance of the tapered glass window, a uniform thickness rectangular cross section with 4mm width and 1m height was first considered. Then, a testing for the optimum distance between grid points in y-direction was done. As a result, the relation between the total heat loss by convection and the number of grid points in y-direction was sketched in **Fig. (3)**. It can be shown from this figure that a 60 grid point is suitable to choose for the present case study. Also, another test for the time step was done and illustrated in **Fig. (4)**. It can be deduced from this figure that any time step can be used and a time step of 1 second was chosen in this paper.
The variation of three selected mesh point's temperature versus time is presented in **Fig. (5)**. It is clear from this figure that the temperature of these selected points increases until it reaches the

steady state average temperature of 21.7 ^{o}C after 290 second. In addition, from **Fig. (6)** which shows the convective heat loss variation with time it can be observed that the heat loss by convection is increasing until maximum heat loss is equal to 97.3 watt when the steady state condition is reached. The increasing of the heat loss is due to the conduction heat transfer from the internal part of the glass window toward the external part of it and then a temperature difference between the glass surface and the ambient outdoor temperature. As a result, a heat loss is occurring and this loss is increasing with time because of the increasing in the temperature difference.

Then, in the second problem the linear variation of thickness (i.e. tapered cross section) is taken under study. Results for different tapered angles are presented in several figures. In **Fig.** (7) the relation between the temperature of five selected points and time for tapered angle of 0.05 degree is sketched. From this figure, it is clear that 3080 second is spent in order to reach a steady average temperature of $21.8^{o}C$. In addition, by referring to **Fig. (8)** an amount of 85.5 watt heat is loosed by convection until the same steady temperature is attained. As expected, because the heat problem is not changed physically, the curves resulted for the uniform and non uniform thickness has the same trend.

Results for an angle of 0.1 degree are illustrated in **Fig. (9)** and it shows that the steady state condition is reached after 3180 second with an average temperature of $21.9^{o}C$. Whereas, there is 90.17 watt convection heat is loosed in order to reach the steady state condition as shown in **Fig. (10)**.

It is clear from the figures above that the convective heat loss is increasing with the tapered angle increase. This behavior is due to the decreasing in the cross section area and as a result an increasing in the temperature difference. This increase cause a more heat is loose by convection. While the time required to achieve the steady state condition remains constant in spite of the increasing of tapered angle. This is also ensured from the description of the figures below.

The variation of the five selected point's temperature with time for 0.15 degree tapered angle is shown in **Fig. (11)**. This figure shows that 3180 second is required to reach a steady temperature of $21.9^{o}C$. While it can be deduced from **Fig. (12)** that 94.62 watt of heat loss by convection is required for steady state condition. This means that the heat loss is still increasing with the tapered angle as before and for the same reason.

It is important to know that the maximum tapered angle that can be taking as a case study is 0.2 degree. The results of this angle case are shown in **Fig. (13)** and **Fig. (14)**. From these two figures, it can be observed that the steady state temperature is equal to $21.97^{o}C$. This temperature is reached after 3180 second also with convective heat loss of 97.1 watt.

For each tapered angle case an equal rectangular cross sectional area case was taken in consideration. This was done in order to make a comparison between the performance of the tapered and uniform rectangular cross section. This equal uniform cross section can be found by calculating the average value of the upper and lower width of the tapered section and make it equal to the new width and taking the same height of the tapered section.

For different time periods until reaching the steady state condition for all cases at approximately maximum time of 3180 second, the effect of the variation of the convective heat loss versus the tapered angle is illustrated in **Fig. (15)**. As expected, the heat loss by convection is increasing with the increase of the tapered angle for each period of time. As it is explained before this behavior is due to the increase in the temperature difference between the surface glass window and the ambient outdoor temperature. Also, it may be observed from this figure that the heat loss is increasing with time as it has been discussed from the above figures at each value of the tapered angle.

Finally, an attempt was made to study the effect of the increasing tapered angle on the ratio between the convective heat loss from tapered section and the uniform section with same cross section area at different period of time until reaching the steady state condition. Results for this attempt are

presented in Fig. 16. From this figure, it can be observed that the tapered heat loss to uniform heat loss ratio is increasing with the tapered angle increasing.
If the results of the rectangular cross section and the non uniform linear tapered cross section of a glass window are analyzed, it may be noticed that the tapered glass window takes longer time to reach the steady state condition. However, the convection heat loss is less in the tapered angle case. But this heat loss increases with the increasing of the tapered angle until it achieves the same heat loss of the rectangular cross section model.

CONCLUSIONS

In this paper the conjugate heat transfer problem through a glass window with non-uniform thickness was studied using the fully implicit finite difference numerical technique. It may be concluded that the temperature of the tapered glass window and the heat loss by convection increase with increasing period of time until reaching the steady state condition. Also, the angle variation, for different tapered angles, has no significant effect on the time required to reach the steady state temperature while the convective heat loss increases with the increasing of the tapered angle. In addition, increasing the tapered angle at different periods of time until the steady state condition is reached causes an increase in the ratio between the heat loss from tapered window and the equal cross section uniform thickness one. Finally, the linear tapered glass window takes longer time than the rectangular cross section to reach the steady state condition. Also, it looses less convective heat than the rectangular model.

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NOMENCLATURE

Symbols

A : Grid point cross section area $(= \Delta y \times depth)$ $[m^2]$.
B : Width $[m]$.
Cp : Specific heat $[J/kg.K]$.
h : Heat transfer coefficient $[W/m^2.K]$.
h_y : Heat transfer coefficient for a specified grid point $[W/m^2.K]$.
H : Height $[m]$.

g : Acceleration due to gravity (9.81) [m/s^2].
K : Thermal conductivity [$W/m.K$].
m: The last mesh point in y-direction.
p: The last mesh point in x-direction.
Pr : Prandtl number.
Q : The heat loss by convection for each grid point [W].
Ra_L Rayleigh number.
t : Time [s].
T : Temperature [^oC].
T_o : Initial temperature [^oC].

Greek Symbols

α Thermal diffusivity = $\frac{K}{\rho Cp}$ [m^2/s].
β_T : Coefficient of volumetric expansion [K^{-1}].
Δ: Small increment.
μ Dynamic viscosity of air [$Pa.s$].
ρ : Density [kg/m^3].
θ : Tapered angle [deg].

Subscript

a: Air.
ai: Indoor ambient temperature.
ao: Outdoor ambient temperature.
g: Glass.
i: Denotes mesh position in x-direction.
j: Denotes mesh position in y-direction.
L: lower.
sur: Surface temperature.
u: upper.

Superscript

n: Time-step index.

APPENDIX A

A.1 Glass Window Properties

The glass window thermo physical properties were taken from reference (Yunus 1998) as given below:

$K_g = 0.7 W/m.K$

$Cp_g = 2800 J/kg.K$

$\rho_g = 750 kg/m^3$

A.2 Air Properties

The properties of air were derived from the data of the Chemical Engineering Handbook (Perry

1984). The various properties were fitted with the following equations using the Grapher software as expressed below:

$$\mu_a = \begin{bmatrix} -.00639618 + 0.000833948T - 8.24177E-07T^2 + 6.26527E-10T^3 \\ -2.05474E-13T^4 \end{bmatrix} \Big/ 10000 \qquad (A.1)$$

$$K_a = -0.011639 + 0.000216555T - 4.92475E-07T^2 + 8.34825E-10T^3 \\ -7.19228E-13T^4 + 2.38678E-16T5 \qquad (A.2)$$

$$Cp_a = \begin{bmatrix} 0.842058 + 0.00240217T - 1.41644E-05T^2 + 4.29282E-08T^3 \\ -7.23575E-11T^4 + 7.04261E-14T^5 - 3.71999E-17T^6 \\ +8.26405E-21T^7 \end{bmatrix} * 1000 \qquad (A.3)$$

$$\rho_a = 4.78427 - 0.0271474T + 8.23404E-05T^2 - 1.4458E-07T^3 + \\ 1.472748E-10T^4 - 8.07719E-14T^5 + 1.84433E-17T^6 \qquad (A.4)$$

$$\Pr_a = 0.805584 - 0.000606581T + 1.3898E-06T^2 - 2.16057E-09T^3 \\ + 2.19458E-12T^4 - 8.97427E-16T^5 \qquad (A.5)$$

where T is in Kelvin [K].

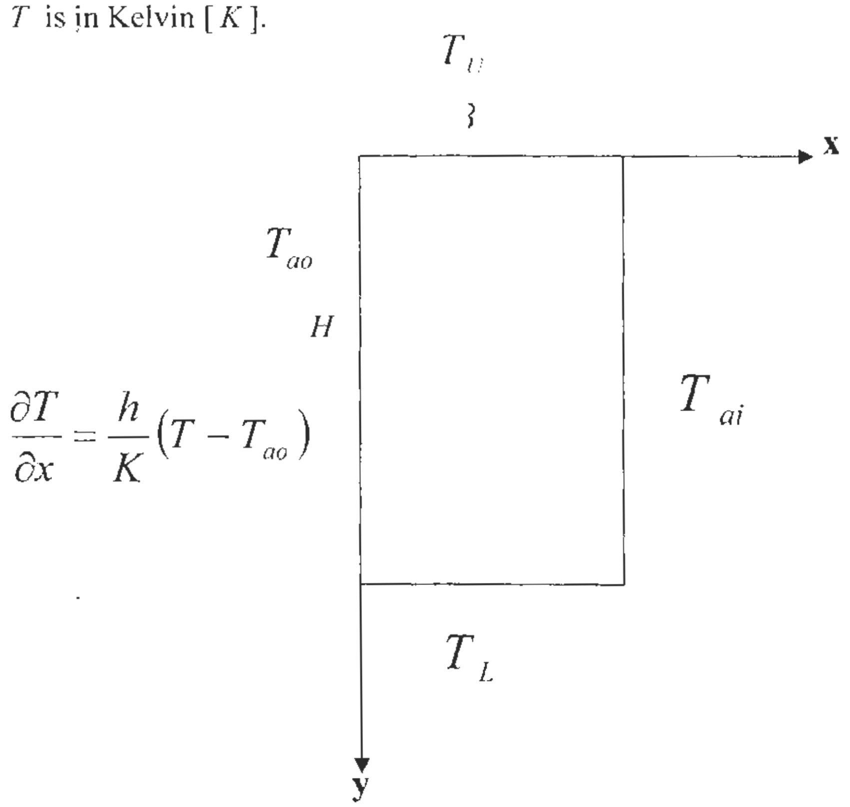

Fig. (1) The Rectangular Cross Section and Its Boundary Conditions.

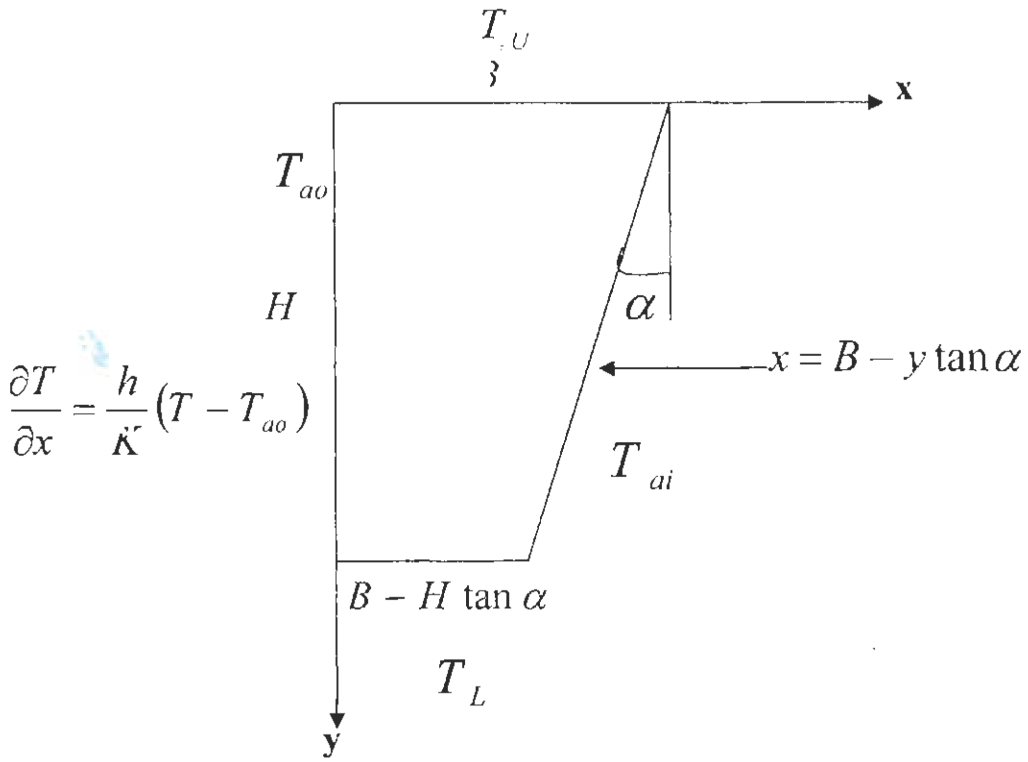

Fig. (2)The Linear Tapered Non Uniform Thickness Cross Section and Its Boundary Conditions.

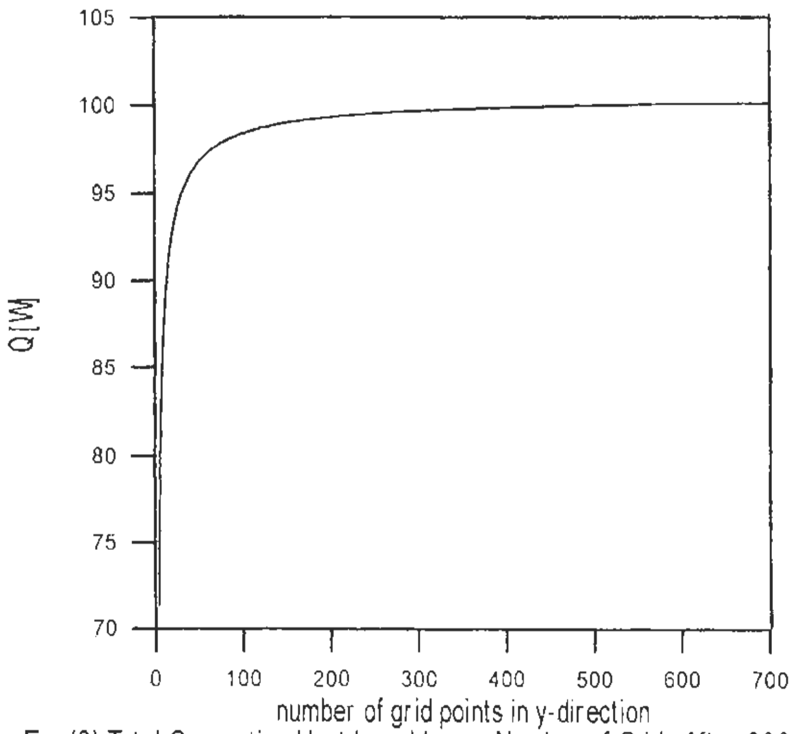

Fig. (3):Total Convective Heat Loss Versus Number of Grids After 600 Second.

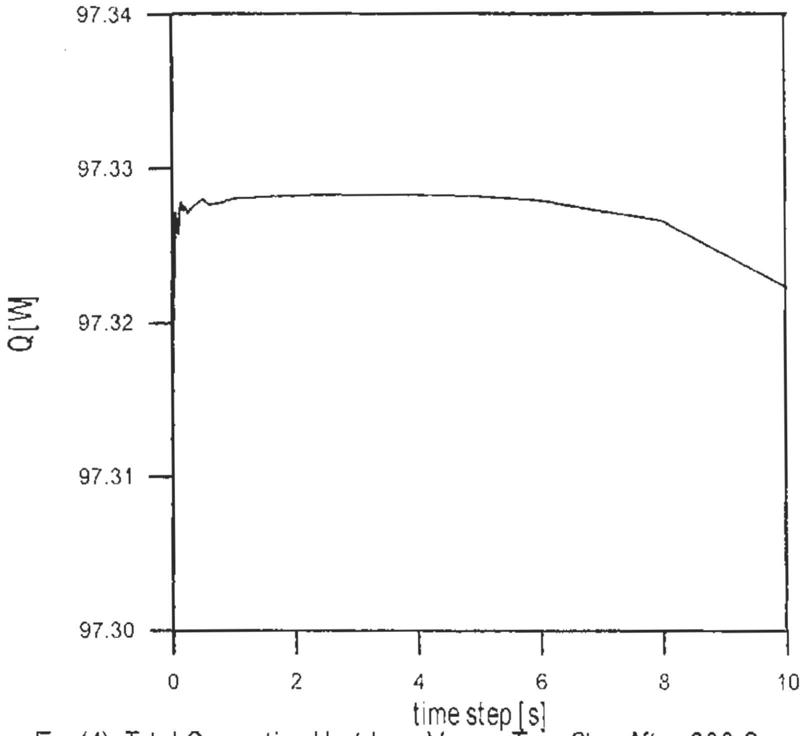

Fig. (4): Total Convective Heat Loss Versus Time Step After 600 Second.

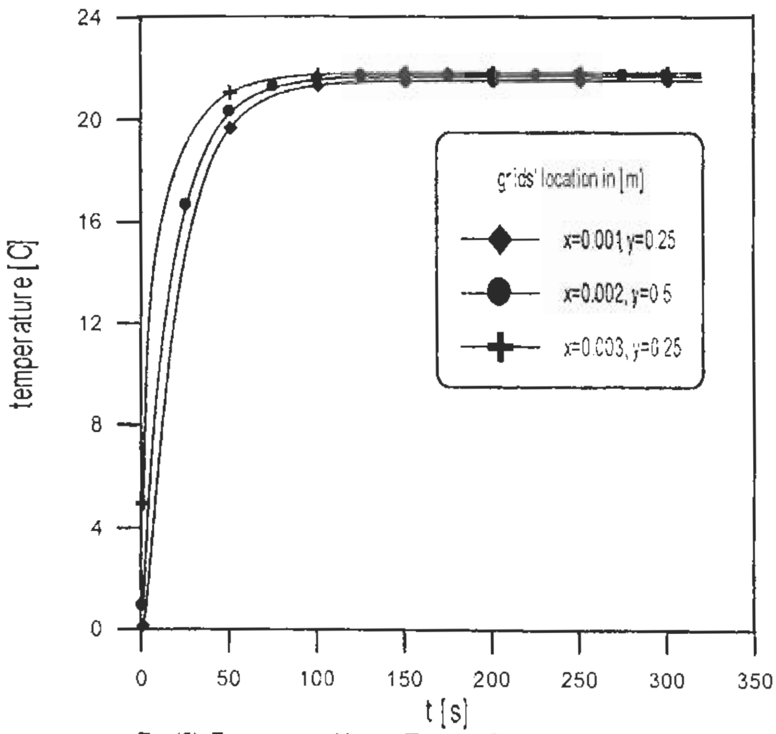

Fig. (5): Temperature Versus Time for Rectangle B=4mm and H=1m.

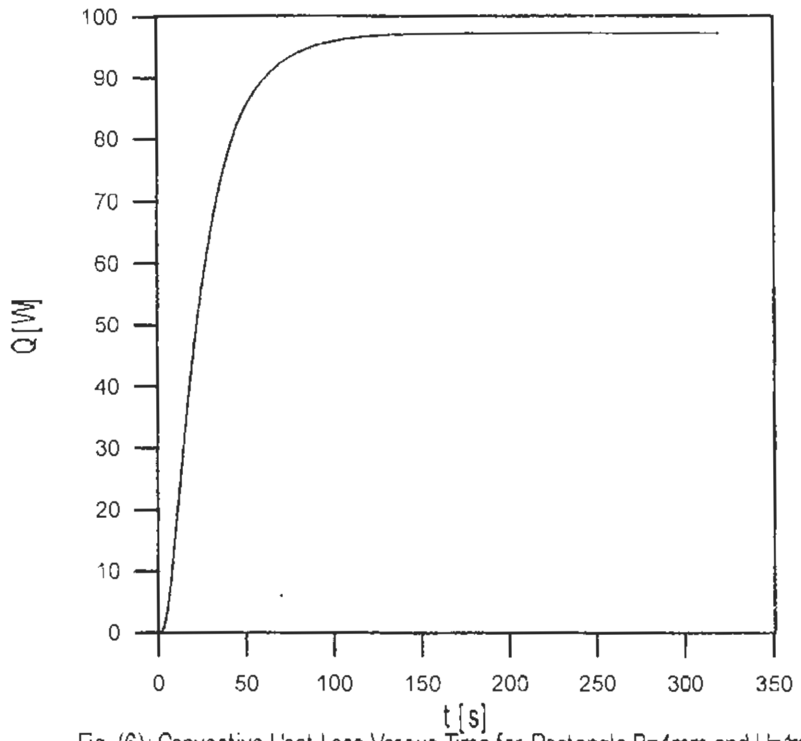

Fig. (6): Convective Heat Loss Versus Time for Rectangle B=4mm and H=1m.

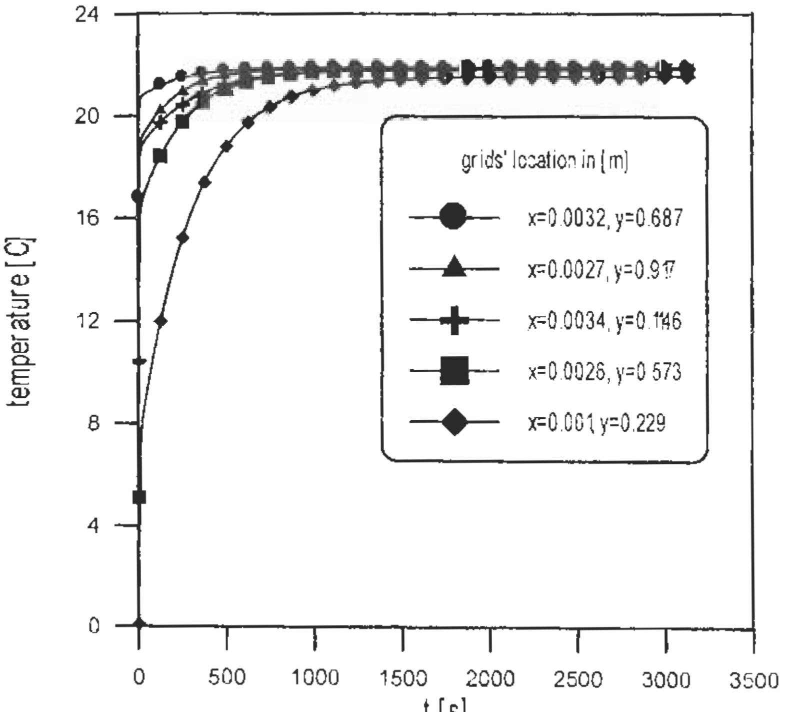

Fig. (7):Temperature Variation with Time for Tapered Angle=.05 Deg.

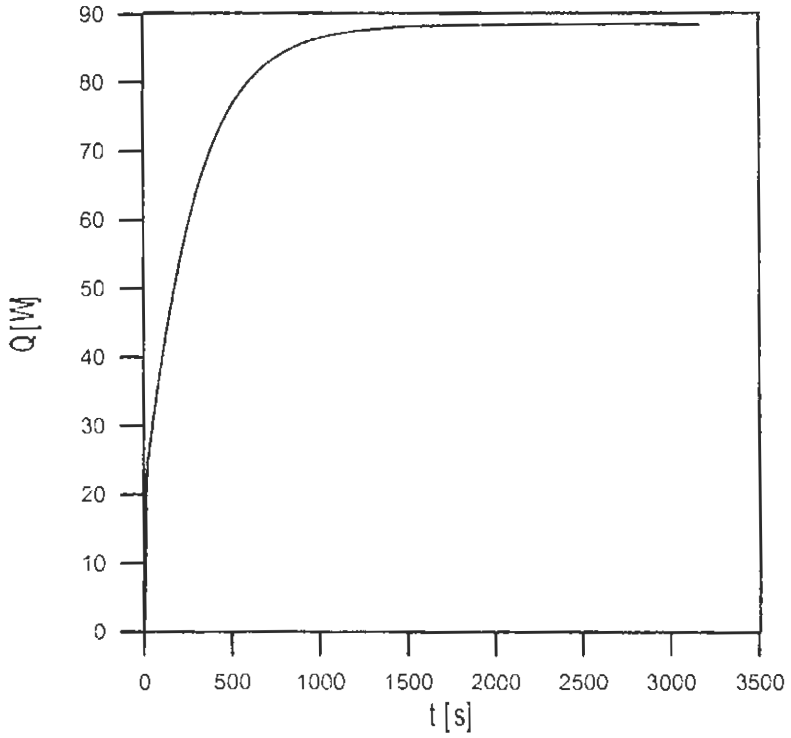

Fig. (8): Heat Loss Variation with Time for Tapered Angle=.05 Deg.

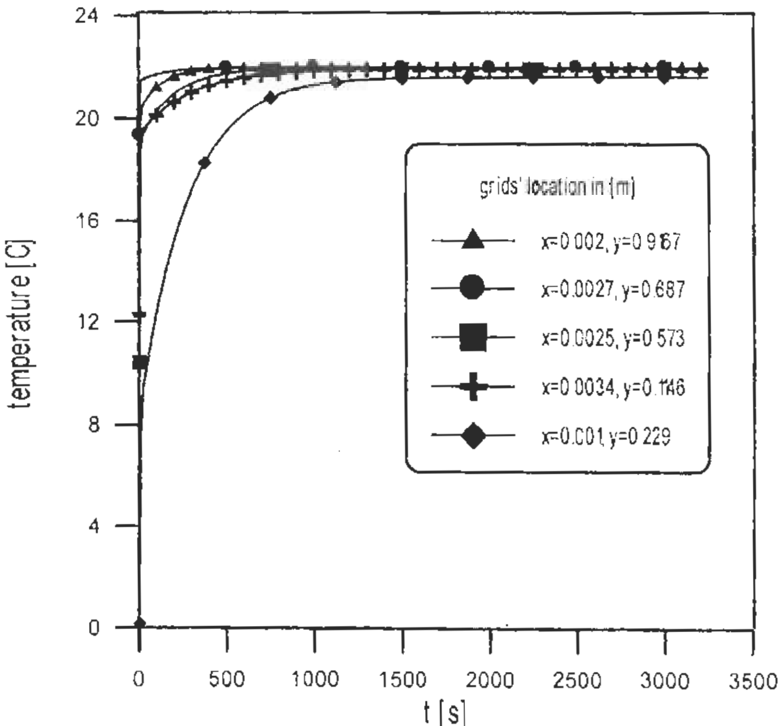

Fig. (9): Temperature Relation with Time for Tapered Angle=0.1Deg.

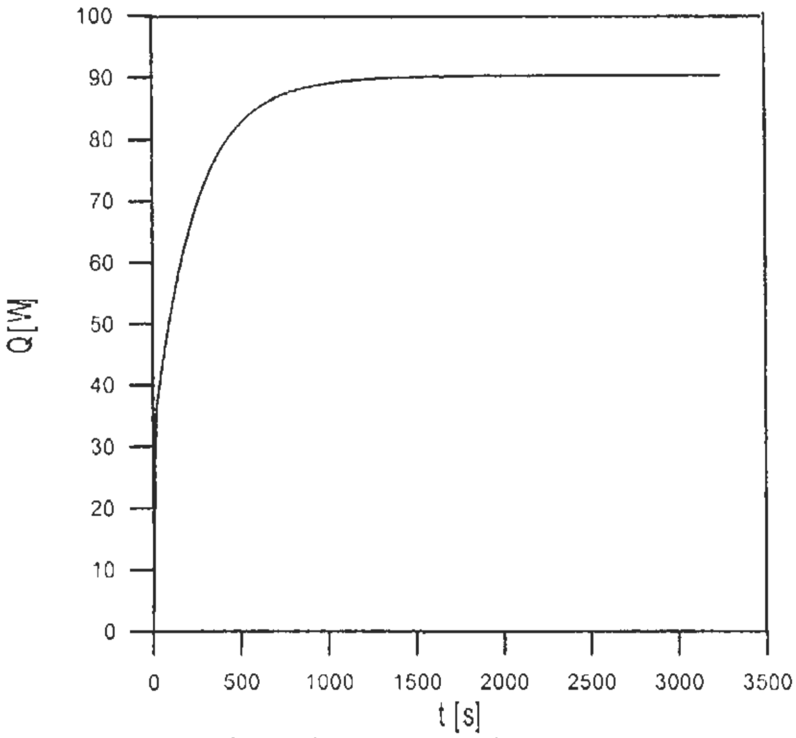

Fig. (10): Heat Loss Versus Time for Tapered Angle=0.1Deg.

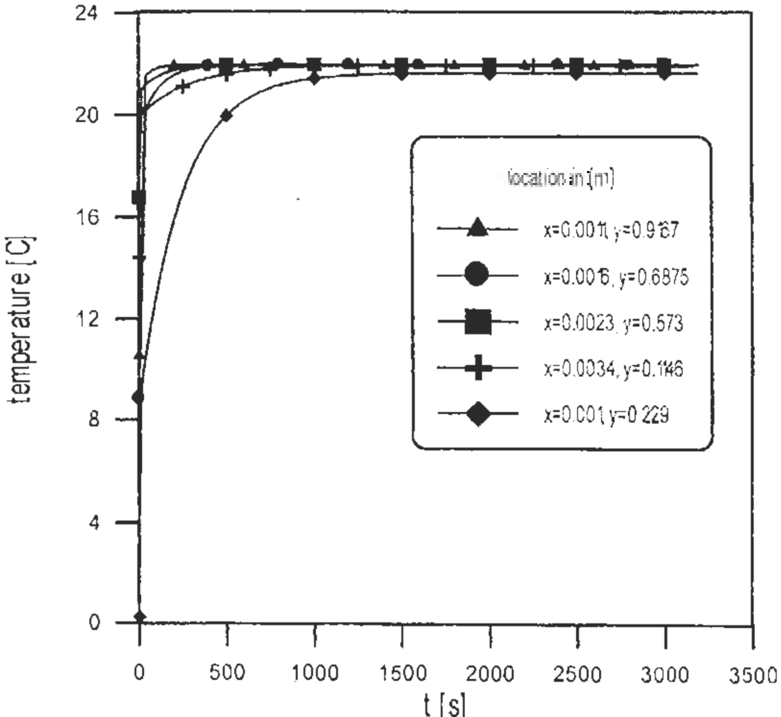

Fig. (11): Temperature Versus Time for Tapered Angle=0.15 Deg.

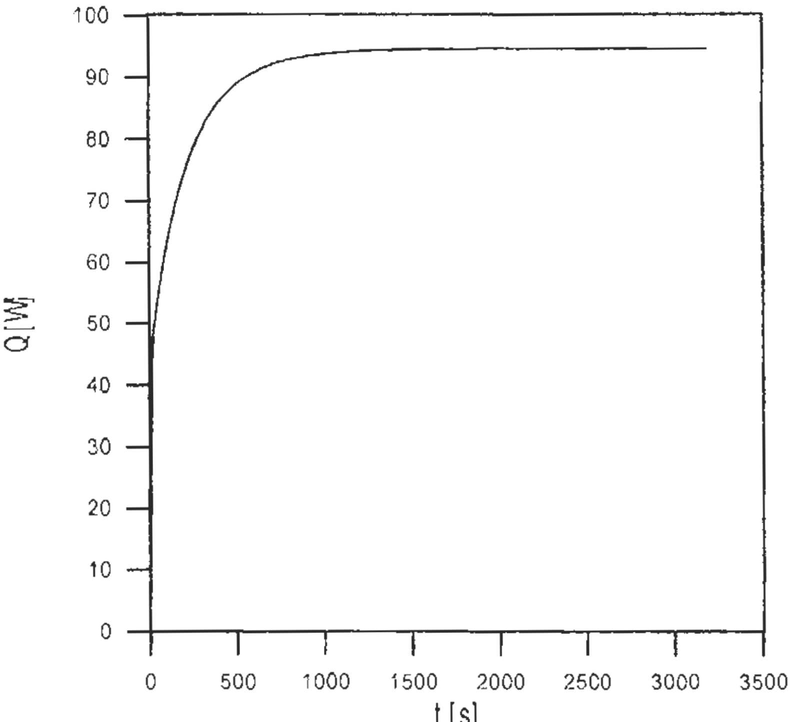

Fig. (12): Heat Loss Variation with Time for Tapered Angle=0.15 Deg.

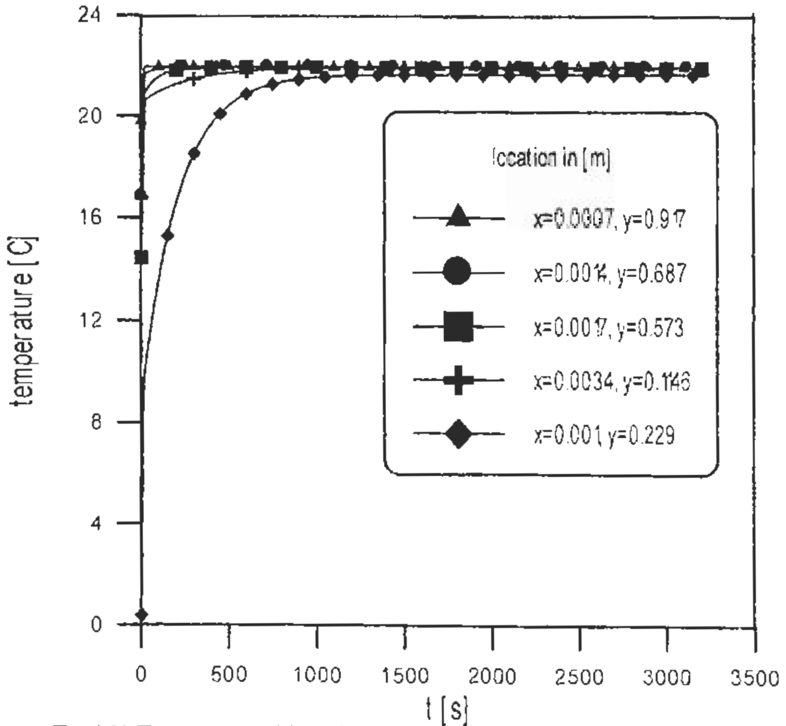

Fig. (13) Temperature Variation with Time for Tapered Angle=0.2 Deg.

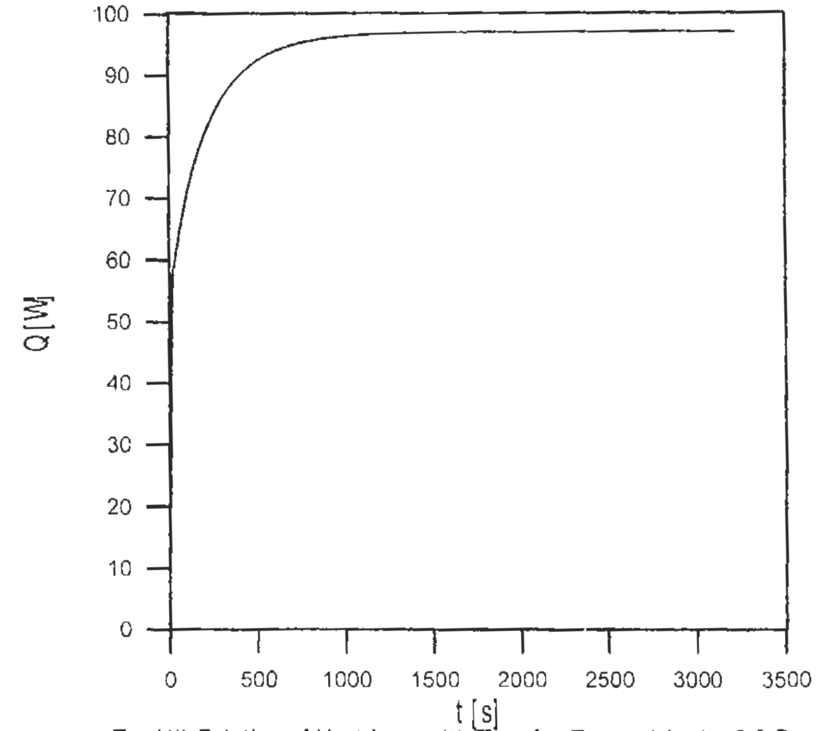

Fig. (14) Relation of Heat Loss with Time for Tapered Angle=0.2 Deg.

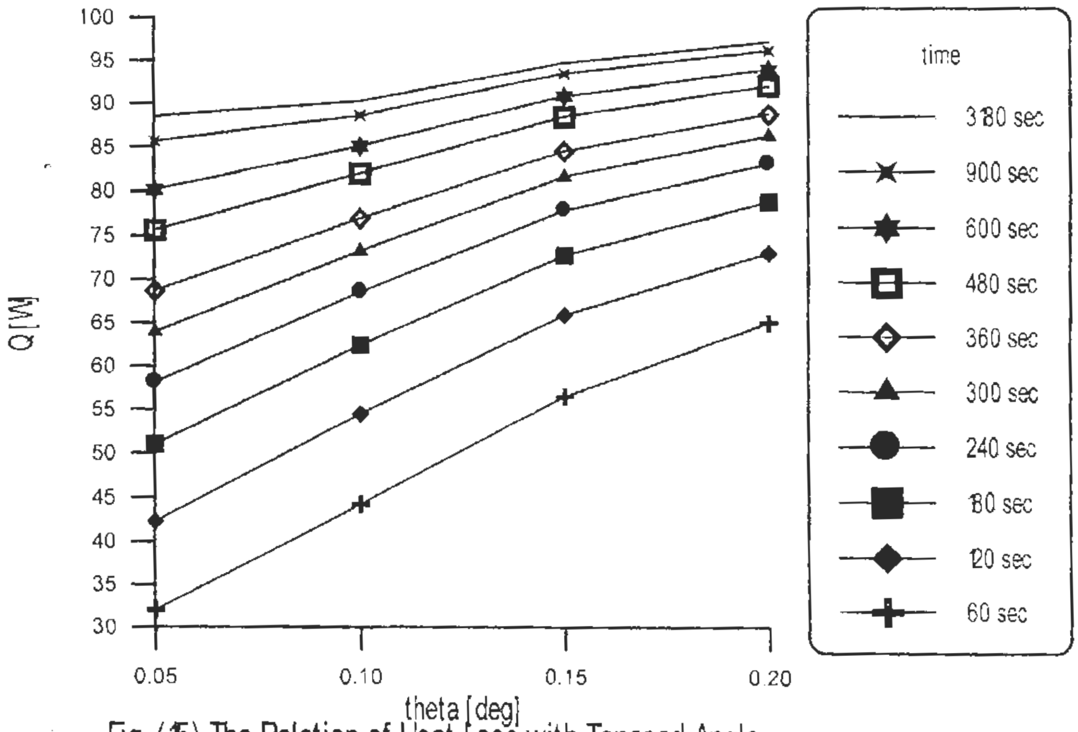

Fig. (15) The Relation of Heat Loss with Tapered Angle.

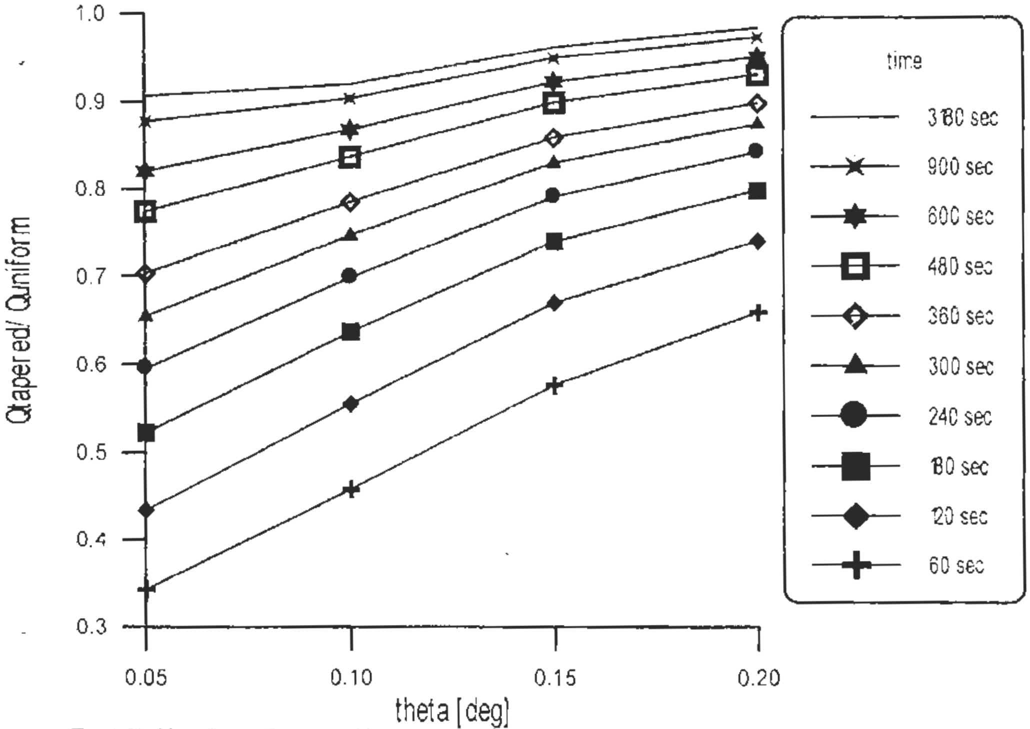

Fig. (16): Heat Loss Tapered/ Heat Loss Uniform Versus Tapered Angle.