



NATURAL CONVECTION HEAT TRANSFER IN AN INCLINED CIRCULAR CYLINDER

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ABSTRACT

Experiments were carried out to investigate natural convection heat transfer in an inclined uniformly heated circular cylinder . The effects of surface heat flux and angle of inclination on the temperature and local Nusselt number variations along the cylinder surface are discussed . The investigation covers heat flux range from 92 W/m² to 487 W/m², and angles of inclination 0° (horizontal) , 30° , 60° and 90° (vertical) . Results show an increase in the natural convection as heat flux increases and as angle of inclination moves from vertical to horizontal position. An empirical equation of average Nusselt number as a function of Rayleigh number was deduced for each angle of inclination .

الخلاصة

أجريت تجارب لدراسة عملية انتقال الحرارة بالحمل الحر في أسطوانة دائرية مائلة مسخنة تسخين منتظم. تم مناقشة تأثيرات الفيض الحراري للسطح وزاوية الميل على تغيرات درجة الحرارة و رقم نسلت الموقعي خلال سطح الأسطوانة. غطت الدراسة مدى الفيض (عمودي) . بينت النتائج بأن الحمل الحر يزداد 90°، و 60°، 30° (افقي)، 0°، الحراري من (92 الى 487) واط/متر مربع و زوايا ميل كلما زاد الفيض الحراري و كلما تغيرت زاوية الميل من الوضع العمودي إلى الوضع الأفقي. تم استنباط معادلة تجريبية لمعدل رقم نسلت كدالة لرقم رايلي لكل زاوية ميل.

KEY WORDS: Heat Transfer, Natural Convection, Cylinder.

INTRODUCTION

The problem of natural convection heat transfer across a channel of various cross-section (rectangular, circular, concentric annulus and parallel plates) has received considerable attention in view of its fundamental importance germane to numerous engineering applications such as electronic systems, chemical process equipments, combustion chambers, environmental control systems and so on. Bae and Hun (2003) carried out a study on air cooling in an unsteady laminar natural convection in a vertical rectangular channel with three flush mounted heat sources on one vertical wall. The results show the effects of the thermal conditions of the lowest source on the downstream sources. The study emphasizes that the transient temperatures may exceed average values in time. This is important for designing electronic equipment projects. Madhavan and Sastri (2003) developed a parametric study of natural convection in a set of boards inside an enclosure. Each board has heat sources. This layout has direct application on electronic equipment cooling. It is noted that the Rayleigh and the Prandtl numbers as well as the boundary conditions strongly affect the fluid flow and heat transfer features. Vande Sande and Hamer (1979) have obtained empirical correlations for natural convection heat transfer in concentric and eccentric annuli of constant heat flux inner cylinder while the outer cylinder was subjected to the ambient temperature. An empirical equation of average Nusselt number as a function of Rayleigh number was deduced. There are no available literatures concerning the heat transfer by natural convection in an inclined insulated circular cylinder. The present study covers this lack and gives a clear view to actual physical behavior in the heat transfer process by natural convection.

EXPERIMENTAL APPARATUS

The apparatus consists essentially of settling chamber (D), cylinder as a test section mounted on an iron frame, which can be rotated around a horizontal spindle (to adjust the inclination angle of cylinder as required) as shown in Fig.(1). A well designed teflon bell mouth (H) was fitted at the beginning of aluminum cylinder (I) and bolted in the other side inside the settling chamber (D). Another

Teflon piece (M) represents the cylinder exit and has the same dimensions as the inlet piece. The teflon was chosen due to its low thermal conductivity to reduce the heat loss from the aluminum cylinder ends. The inlet air temperature was measured by one thermocouple located in the settling chamber (J) while the outlet bulk air temperature was measured by two thermocouples located in the test section exit (Z). The local bulk air temperature was calculated by using a straight line interpolation between the measured inlet and outlet bulk air temperatures. The test section consists of 3.5 mm wall thickness, 59.3 mm outside diameter and 1.2 m long aluminum cylinder. The cylinder was heated electrically using an electrical heater which consisted of a 1 mm in diameter and 60 m in length nickel-chrome wire electrically isolated by ceramic beads, wound uniformly as a coil with 10 mm pitch. The outside of the test section was then thermally insulated, covered with 60 mm and 5.7 mm as thickness for asbestos rope layer and fiber glass, respectively. To enable the calculation of heat loss through the lagging (P) to be carried out, six thermocouples are inserted in the lagging as two thermocouples at three points along the heated section 390 mm apart. Using the average measured temperature drop and thermal conductivity of lagging the heat losses through lagging can be calculated.

The cylinder surface temperature were measured by eighteen asbestos sheath thermocouples (type K), arranged along the cylinder. The thermocouples were fixed by drilling 18 holes of 2mm diameter and approximately 3 mm deep in and along the cylinder wall while the ends of the holes chamfered by drill then the measuring junction were secured permanently in the holes by a high temperature application epoxy steel adhesive. The excess adhesive was removed and the cylinder surface was cleaned carefully by fine grinding paper. All the thermocouple wires and heater terminals were taken out the test section. All thermocouples were used with leads and calibrated using the melting point of ice made from distilled water as



reference point and the boiling points of several pure chemical substances. To determine the heat loss from the test section ends, two thermocouples were fixed in each teflon piece. Knowing the distance between these thermocouples and the thermal conductivity of the teflon, the heat ends loss thus can be calculated.

Experimental Procedure

To carry out the experiments the following procedure was followed:

1. The inclination angle of the cylinder was adjusted as required.
2. The electrical heater was switched on and the heater input power then adjusted to give the required heat flux.
3. The apparatus was left at least two hours to establish steady state condition. The thermocouples readings were measured every half an hour by means of the digital electronic thermometer until the reading became constant, a final reading was recorded. The input power to the heater could be increased to cover another run in a shorter period of time and to obtain steady state conditions for next heat flux. Subsequent runs for other ranges of cylinder inclination angles were performed in the same previous procedure.
4. During each test run , the following readings were recorded:
 - a. The angle of inclination of the cylinder in degree.
 - b. The readings of the thermocouples in °C.
 - d. The heater current in amperes.

- e. The heater voltage in volts.

Data Analysis

Simplified steps were used to analyze the heat transfer process for the air flow in a cylinder when its surface was subjected to a uniform heat flux.

The total input power supplied to the cylinder can be calculated:

$$Q_i = V \times I \quad \dots(1)$$

The convection and radiation heat transferred from the cylinder is :

$$Q_{cr} = Q_r - Q_{cond} \quad \dots(2)$$

where Q_{cond} is the conduction heat loss which was found from the following equation:

$$Q_{cond} = \frac{\Delta T_{oi}}{\ln \frac{r_o}{r_i}} \quad \dots(3)$$

$$\frac{1}{2 \pi k_a L}$$

where:

$$\Delta T_{oi} = T_o - T_i$$

The convection and radiation heat flux can be represented by:

$$q_{cr} = Q_{cr} / A_o \quad \dots(4)$$

The local radiation heat flux can be calculated from the expression [Holman 1984]:

$$q_r = F_{1-2} \varepsilon \sigma \left[(T_{s_z} + 273)^4 - (T_{s_z} + 273)^4 \right] \quad \dots(5)$$

but, $F_{1-2} \approx 1$

hence the convection heat flux at any position is:

$$q = q_{cr} - q_r \quad \dots(6)$$

The radiation heat flux is very small and can be neglected.

hence:

$$q_{cr} \approx q = \text{convection heat flux}$$

The local heat transfer coefficient can be obtained as:

$$h_z = \frac{q}{(T_s)_z - (T_b)_z} \quad \dots(7)$$

All the air properties are evaluated at the mean film air temperature

$$(T_f)_z = \frac{(T_s)_z + (T_b)_z}{2} \quad \dots(8)$$

The local Nusselt number (Nu_z) then can be determined as:

$$Nu_z = \frac{h_z D_h}{k} \quad \dots(9)$$

The average values of Nusselt number Nu_m can be calculated as follows:

$$Nu_m = \frac{1}{L} \int_0^L Nu_z dz \quad \dots(10)$$

The average values of the other parameters can be calculated based on calculation of average cylinder surface temperature and average bulk air temperature as follows:

$$\bar{T}_s = \frac{1}{L} \int_{z=0}^{z=L} (T_s)_z dz \quad \dots(11)$$

$$\bar{T}_b = \frac{1}{L} \int_{z=0}^{z=L} (T_b)_z dz \quad \dots(12)$$

$$\bar{T}_f = \frac{\bar{T}_s + \bar{T}_b}{2} \quad \dots(13)$$

$$Gr_m = \frac{g \beta D_h^3 (\bar{T}_s - \bar{T}_b)}{\nu^2} \quad \dots(14)$$

$$Pr_m = \frac{\mu C_p}{k} \quad \dots(15)$$

$$Ra_m = Gr_m Pr_m \quad \dots(16)$$

where;

$$\beta = 1 / (273 + \bar{T}_f)$$

All the air physical properties ρ , μ , ν , and k were evaluated at the average mean film temperature (\bar{T}_f) [Grimson 1971].

Error Analysis

The accuracy of obtaining experimental results depends upon two factors: the accuracy of measurements and the nature of rig design. There is no doubt that, the maximum portion of errors in calculations referred essentially to the errors in the measured quantities. Hence, to calculate the error in the obtained results, Kline and McClintock method [Holman 1984] is used in this field.

Let the result R be a function of n independent variables: v_1, v_2, \dots, v_n

$$R = R(v_1, v_2, \dots, v_n)$$

...(17)

For small variations in the variables, this relation can be expressed in linear form as [Holman 1984]:

$$\delta R = \frac{\partial R}{\partial v_1} \delta v_1 + \frac{\partial R}{\partial v_2} \delta v_2 + \dots + \frac{\partial R}{\partial v_n} \delta v_n$$

...(18)

Hence, the uncertainty interval (w) in the result can be given as:

$$w_R = \left[\left(\frac{\partial R}{\partial v_1} w_1 \right)^2 + \left(\frac{\partial R}{\partial v_2} w_2 \right)^2 + \dots + \left(\frac{\partial R}{\partial v_n} w_n \right)^2 \right]^{1/2}$$

...(19)

Eq.(19) is greatly simplified upon dividing by R to nondimensionalize:

$$\left(\frac{w_R}{R} \right)^2 = \left(\frac{\partial R}{\partial v_1} \frac{w_1}{R} \right)^2 + \left(\frac{\partial R}{\partial v_2} \frac{w_2}{R} \right)^2 + \dots + \left(\frac{\partial R}{\partial v_n} \frac{w_n}{R} \right)^2 \quad (20)$$

Hence, the experimental errors that may happen in the used variables are given in Table(1) which is taken from measuring devices as follows:

Independent variables (v)	uncertainty interval (w)
Surface to bulk air temperature	± 0.16 °C
Voltage of the heater	± 0.04 volt
Current of the heater	± 0.0003 Amp
Hydraulic diameter	± 0.0002 m
Average outer and inner lagging surface temperature	± 0.11 °C

The local Nusselt number equation can be written as follows:

$$Nu_z = \frac{(Q_t - Q_{cond}) D_h}{\Delta T_s A_s k}$$

$$Nu_z = \frac{\left(VI - \frac{\Delta T_{oi}}{\ln \frac{r_o}{r_i}} \right) D_h}{\Delta T_s A_s k} = \frac{\left(VI - \frac{\Delta T_{oi}}{C} \right) D_h}{\Delta T_s A_s k} \quad \dots(21)$$

$$\text{Where } C \text{ is constant} = \frac{\ln \frac{r_o}{r_i}}{2 \pi k_a L}$$

The experimental errors in the local Nusselt number calculation can be expressed in the following manner:



$$\begin{aligned} \frac{\partial Nu_z}{\partial V} &= \frac{I \cdot D_h}{\Delta T_s A_s k} \\ \frac{\partial Nu_z}{\partial I} &= \frac{V \cdot D_h}{\Delta T_s A_s k} \\ \frac{\partial Nu_z}{\partial D_h} &= \frac{\left(V \cdot I - \frac{\Delta T_{oi}}{C} \right)}{\Delta T_s A_s k} \\ \frac{\partial Nu_z}{\partial \Delta T_s} &= - \frac{\left(VI - \frac{\Delta T_{oi}}{C} \right) D_h}{(\Delta T_s)^2 A_s k} \\ \frac{\partial Nu_z}{\partial \Delta T_{oi}} &= - \frac{D_h}{C \Delta T_s A_s k} \\ \frac{\partial Nu_z}{\partial A_s} &= - \frac{\left(V \cdot I - \frac{\Delta T_{oi}}{C} \right) D_h}{(A_s)^2 \Delta T_s k} \\ w_{Nu_z} &= \left[\left(\frac{\partial Nu_z}{\partial V} w_V \right)^2 + \left(\frac{\partial Nu_z}{\partial I} w_I \right)^2 + \left(\frac{\partial Nu_z}{\partial D_h} w_{D_h} \right)^2 + \right. \\ &\quad \left. \left(\frac{\partial Nu_z}{\partial \Delta T_s} w_{\Delta T_s} \right)^2 + \left(\frac{\partial Nu_z}{\partial A_s} w_{A_s} \right)^2 + \left(\frac{\partial Nu_z}{\partial \Delta T_{oi}} w_{\Delta T_{oi}} \right)^2 \right]^{1/2} \end{aligned} \tag{22}$$

or,

$$\left(\frac{w_{Nu_z}}{Nu_z} \right)^2 = \left[\left(\frac{w_V}{V} \right)^2 + \left(\frac{w_I}{I} \right)^2 + \left(\frac{w_{D_h}}{D_h} \right)^2 + \left(\frac{w_{\Delta T_s}}{\Delta T_s} \right)^2 + \left(\frac{w_{A_s}}{A_s} \right)^2 + \left(\frac{w_{\Delta T_{oi}}}{\Delta T_{oi}} \right)^2 \right]$$

RESULTS AND DISCUSSION
TEMPERATURE VARIATION

The variation of cylinder surface temperature for different heat flux and for angle of inclination $\alpha = 0^\circ$ (horizontal) , 30° , 60° , and 90° (vertical) are shown in Figs.(2- 5) ; respectively . It is obvious from these figures that the surface temperature increases as heat flux increases because of faster increasing of the thermal boundary layer as heat flux increases (i.e. , increasing of buoyancy effect) . Fig.6 & 7 show the effect of angle of inclination on the temperature distribution along the cylinder surface for low heat flux $q=103 \text{ w/m}^2$ and high heat flux $q=420 \text{ w/m}^2$; respectively .It is clear that the

surface temperature increases as angle of inclination moves from horizontal to vertical position . This behavior can be attributed to the fact that says as the air is heated and dilates , the difference between air density near the wall and the cylinder center causes a circulation which displaces the wall air in a direction parallel to the gravity vector . When the heat transfers through the wall of a horizontal cylinder , the warmer air moves upward along the side walls , and by continuity the heavier air near the center of the cylinder flows downward . As a result , a two symmetrical spiral, like motion is formed along the cylinder . The circulation is driven by radial temperature variation , and at the same time it reduces this temperature variation . These two spiral vortex weak as the angle of inclination moves from horizontal to vertical position to be single vortex only . Therefore ; it is expected that the heat transfer process in horizontal position is better than that in other positions .

LOCAL NUSSLETT NUMBER

The local Nusselt number variation along the cylinder surface for different heat flux and for angle of inclination $\alpha = 0^\circ$ (horizontal) , 30° , 60° , and 90° (vertical);are shown in Figs.(8-11) ; respectively. Generally , It is obvious from these figures that the local Nusselt number values increase as the heat flux increases because of increasing natural convection currents which improves the heat transfer process . The effects of angle inclination on the local Nusselt number variation are shown in Fig. (12 & 13) for low heat flux $q = 103 \text{ w/m}^2$, and high heat flux $q = 420 \text{ w/m}^2$; respectively . As be expected ,it is clear from these two figures that , the local Nusselt number increases relatively as angle of inclination moves from vertical to horizontal position for the same heat flux . This fact appears more pronounced in low heat flux than high heat flux.

AVERAGE NUSSLETT NUMBER

Figs. (14–17) show the logarithmic of mean Nusselt number versus logarithmic Rayleigh number for $\alpha = 0^\circ$ (horizontal) , 30° ,

60° , and 90° (vertical) ; respectively . An empirical equations have been deduced from these figures as follows :-

$$\begin{aligned} \text{Nu}_m &= 5.7099 \text{ Ra}^{-1.1553} & \alpha &= 0^\circ \text{ (horizontal)} \\ \text{Nu}_m &= 6.1366 \text{ Ra}^{-1.2168} & \alpha &= 30^\circ \\ \text{Nu}_m &= 3.1263 \text{ Ra}^{-1.6637} & \alpha &= 60^\circ \\ \text{Nu}_m &= 2.5031 \text{ Ra}^{-1.3296} & \alpha &= 90^\circ \text{ (vertical)} \end{aligned}$$

CONCLUSIONS

1. The extent of the local mixing increases as the heat flux increases .
2. The heat transfer process improves as heat flux increases and as angle of inclination moves from vertical to horizontal .
3. The effect of buoyancy is small at the cylinder entrance and increases downstream.

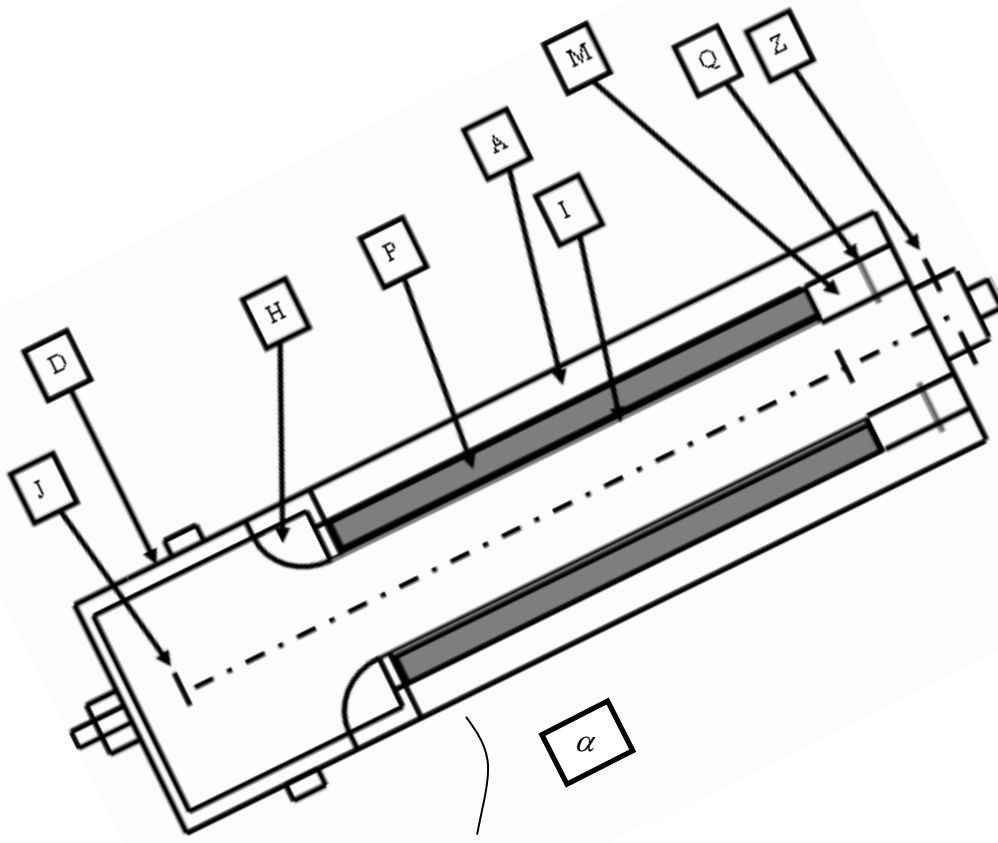
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NOMENCLATURE

- A_o =outer surface area of cylinder = $2\pi r_o L$
 A_s = cylinder surface area = $\pi D_i L$
 c , specific heat of fluid at constant pressure ;
 g , gravitational body force per unit mass ;
 Gr , modified Grashof number , $q''g\beta L^4/kv^2$;
 L =length of cylinder
 k_a =thermal conductivity of asbestos= $0.161 \text{ W/m}^3 \cdot ^\circ\text{C}$
 k =thermal conductivity of air = $0.6099 \text{ W/m}^2\text{C}$
 Nu_z ; local Nusselt number
 Pr , Prandtl number , $\mu c/k$;
 q , heat flux;
 r_o =the distance from center of cylinder to the outer lagging surface
 r_i =the distance from center of cylinder to the beginning lagging (radius of outer cylinder surface)
 T_o =average outer lagging surface temperature
 T_i =average inner lagging surface temperature
 $(T_s)_z$ = local temperature of cylinder.
 $\overline{(T_s)}_z$ = average temperature of cylinder.
 $(T_b)_z$ = Local bulk air temperature.
 T_f = Local mean film air temperature.
 $\Delta T_s = T_s - T_b$
- Greek symbols**
 β , volumetric coefficient of thermal expansion ;
 ρ , fluid density ,
 μ , dynamic viscosity of fluid ;
 ν , kinematic viscosity of fluid ;
 σ = Stefan beltzman constant = $5.66 \times 10^{-8} \text{ W/m}^2 \text{ } ^\circ\text{K}^4$
 ϵ = emissivity of the polished aluminum surface=0.09



- A WOODEN BOARD
- D SETTLING CHAMPER
- H BELL MOUTH
- I CYLINDER
- J,Z THERMOCOUPLES
- M TEFLON PIECE
- P LAGGING
- Q SCREW
- ANGLE of INCLINATION α

Fig.(1): Diagram of Experimental Apparatus.

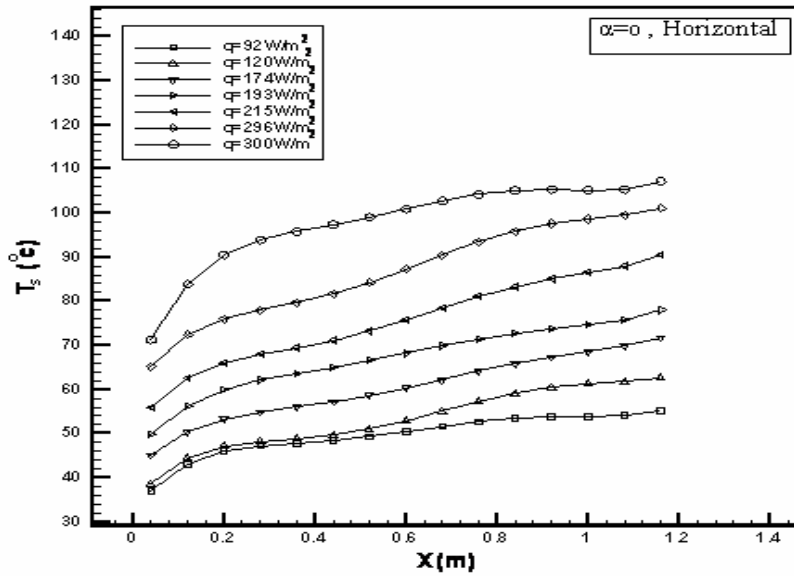


Fig. (2): Experimental Variation of the Surface Temperature with the Axial Distance, $\alpha = 0^\circ$ (Horizontal).

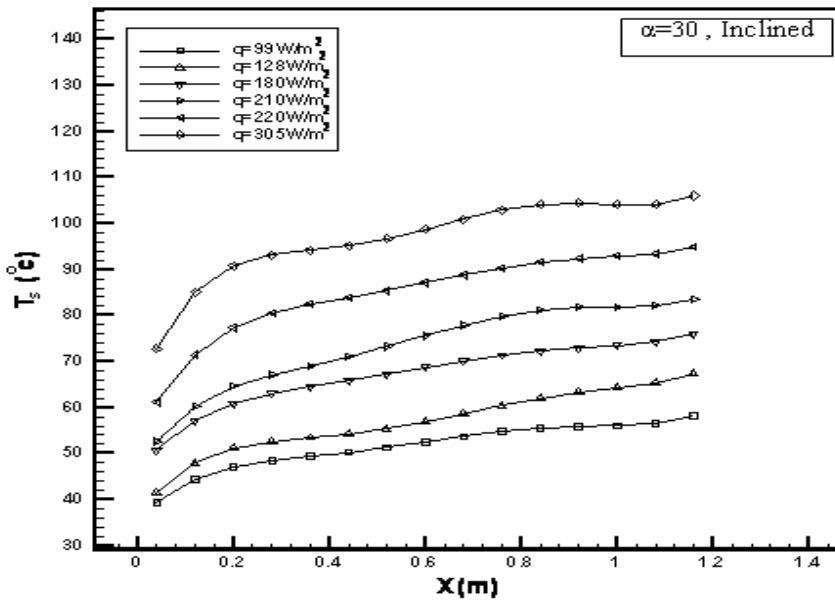


Fig. (3): Experimental Variation of the Surface Temperature with the Axial Distance, $\alpha = 30^\circ$ (Inclined).

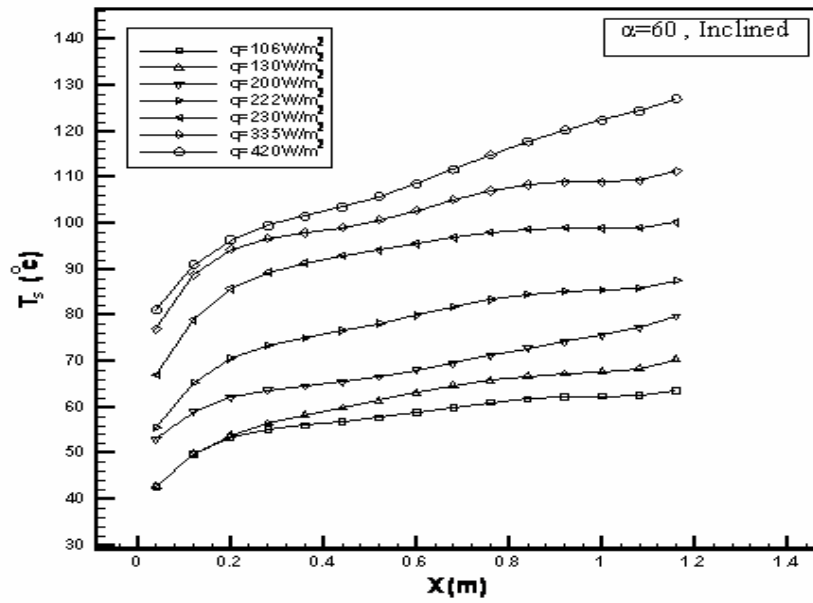


Fig. (4): Experimental Variation of the Surface Temperature with the Axial Distance, $\alpha = 60^\circ$ (Inclined).

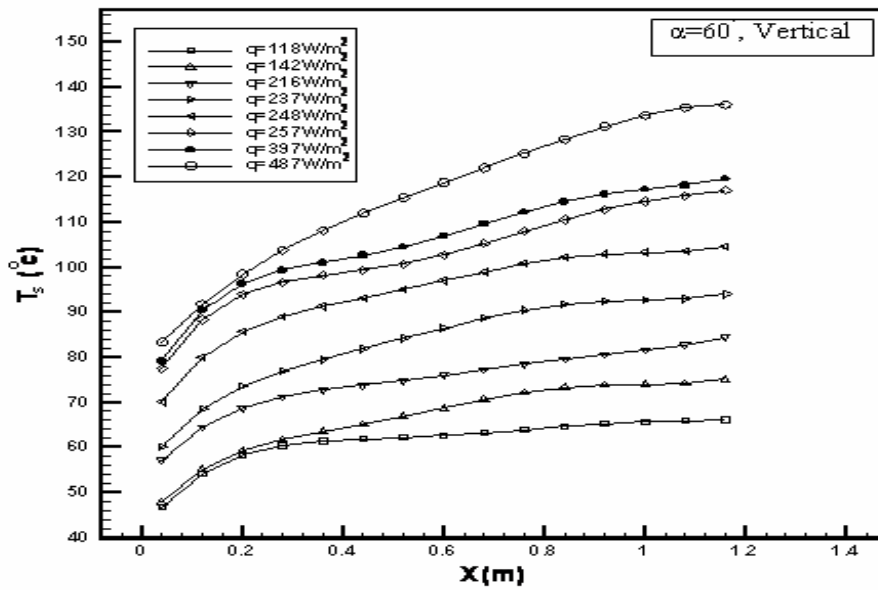


Fig. (5): Experimental Variation of the Surface Temperature with the Axial Distance, $\alpha = 90^\circ$ (Vertical).

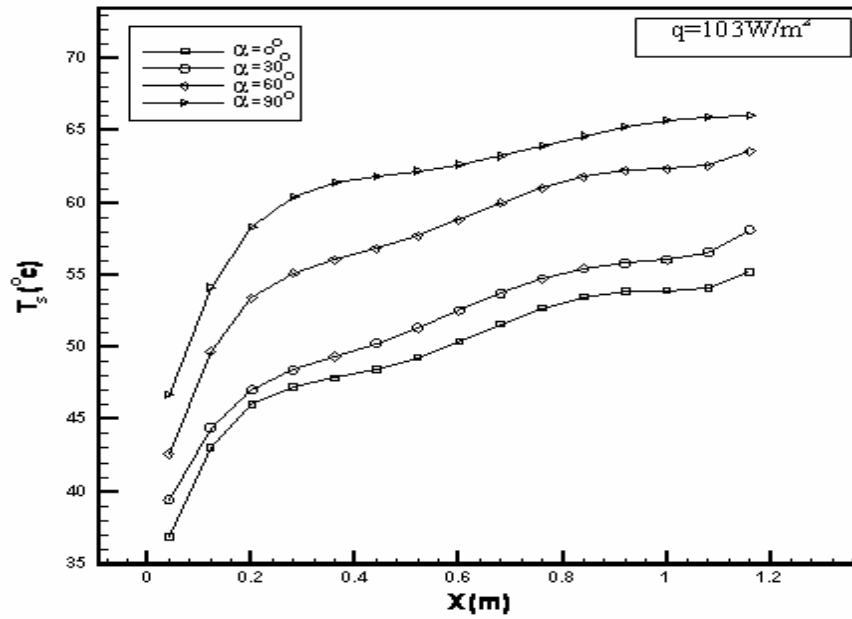


Fig.(6): Experimental Variation of the Surface Temperature with the Axial Distance for Various Angles, $q=103\text{W/m}^2$.

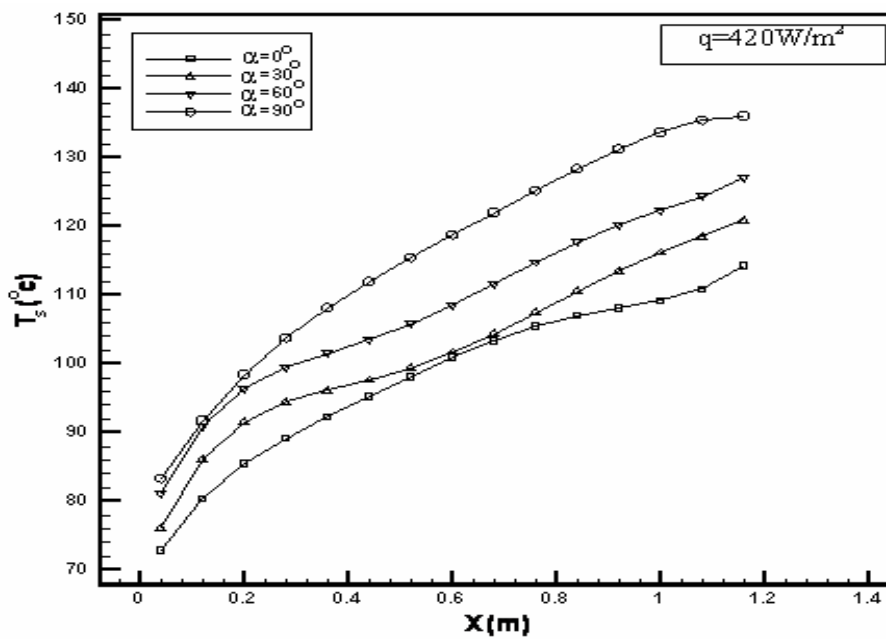


Fig.(7): Experimental Variation of the Surface Temperature with the Axial Distance for Various Angles, $q=420\text{W/m}^2$.

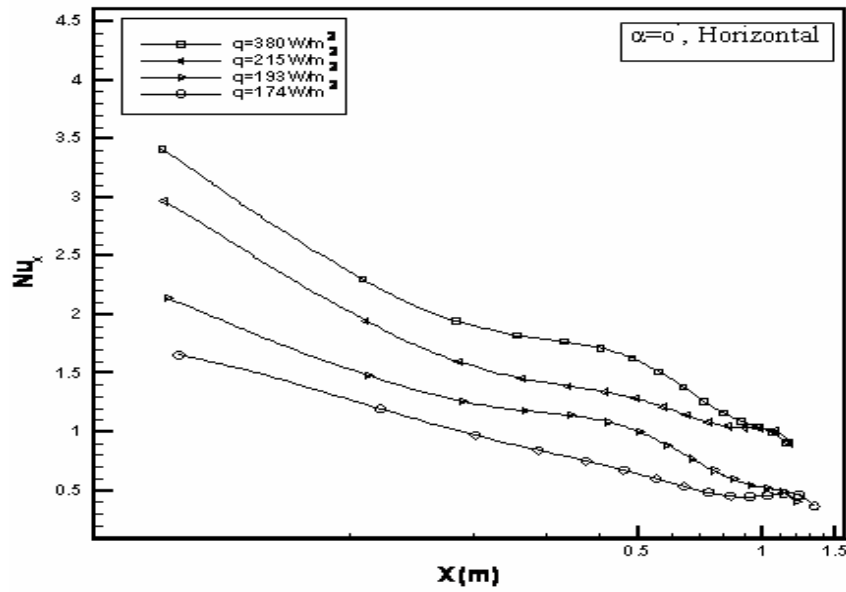


Fig.(8): Experimental, Variation of the local Nusselt number with the Axial Distance, $\alpha = 0^\circ$ (Horizontal).

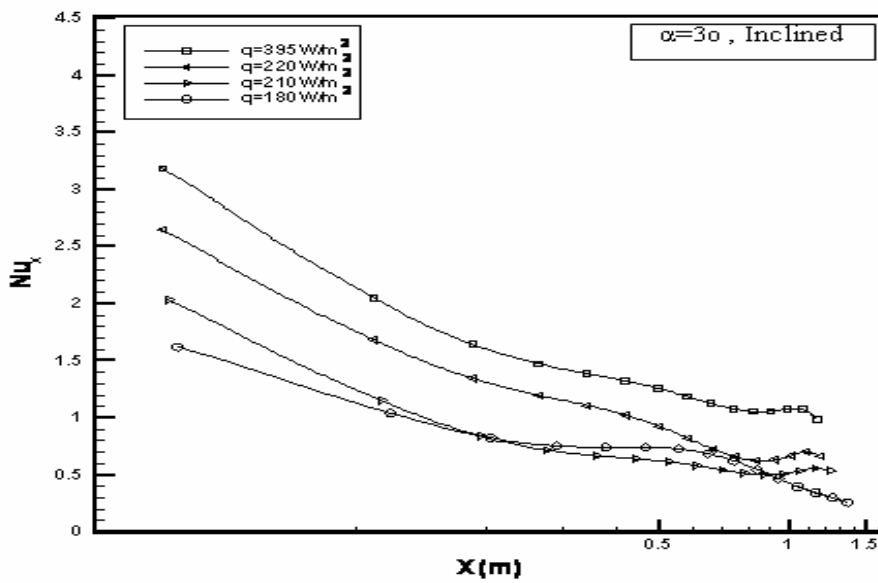


Fig.(9): Experimental, Variation of the local Nusselt number with the Axial Distance, $\alpha = 30^\circ$ (Inclined).

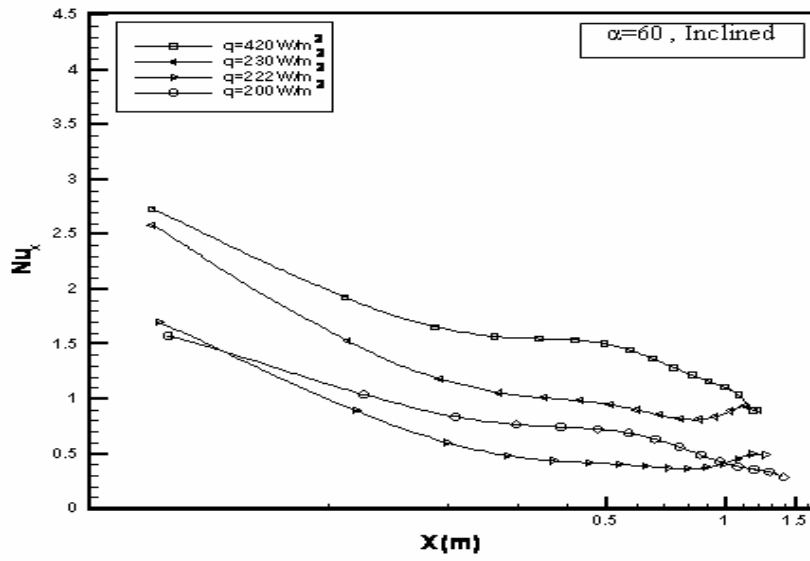


Fig.(10): Experimental, Variation of the local Nusselt number with the Axial Distance, $\alpha = 60^\circ$ (Inclined).

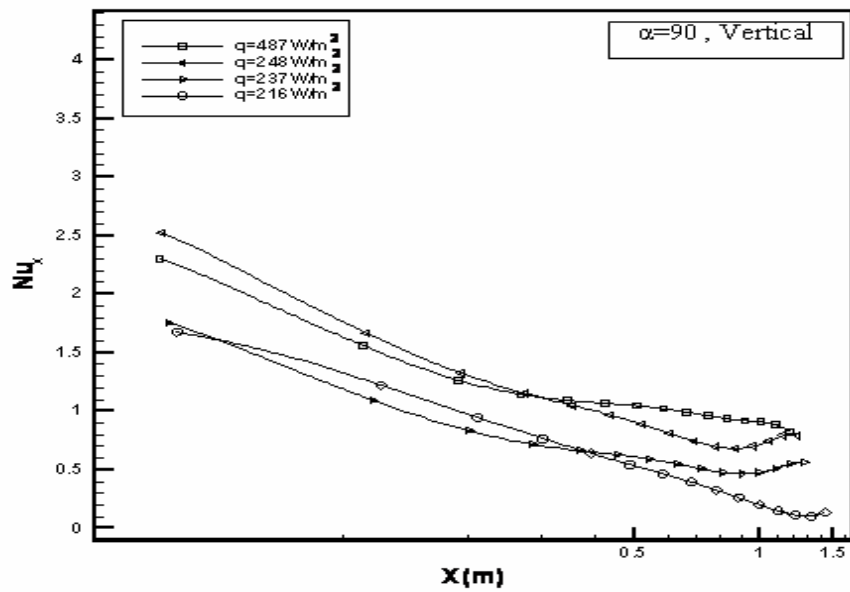




Fig.(11): Experimental, Variation of the local Nusselt number with the Axial Distance, $\alpha = 90^\circ$ (Vertical).

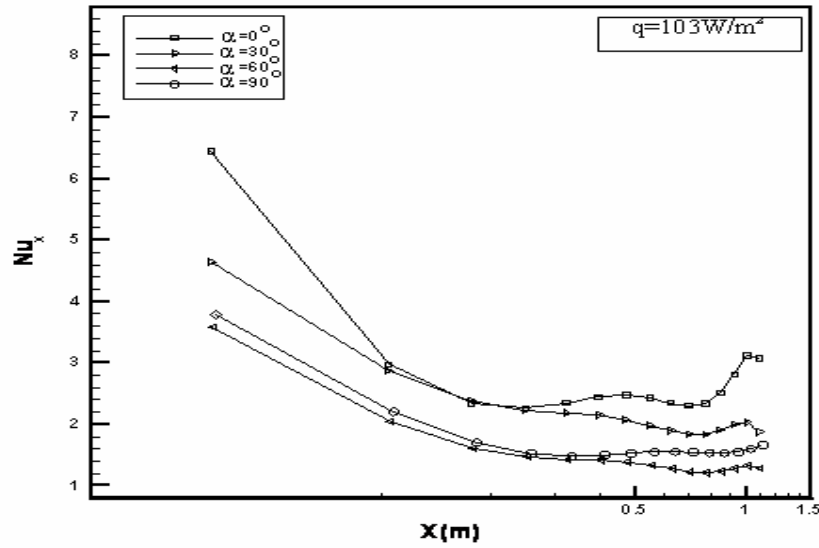


Fig.(12): Experimental Variation of the Local Nusselt number with the Axial Distance for Various Angles, $q=103\text{W/m}^2$.

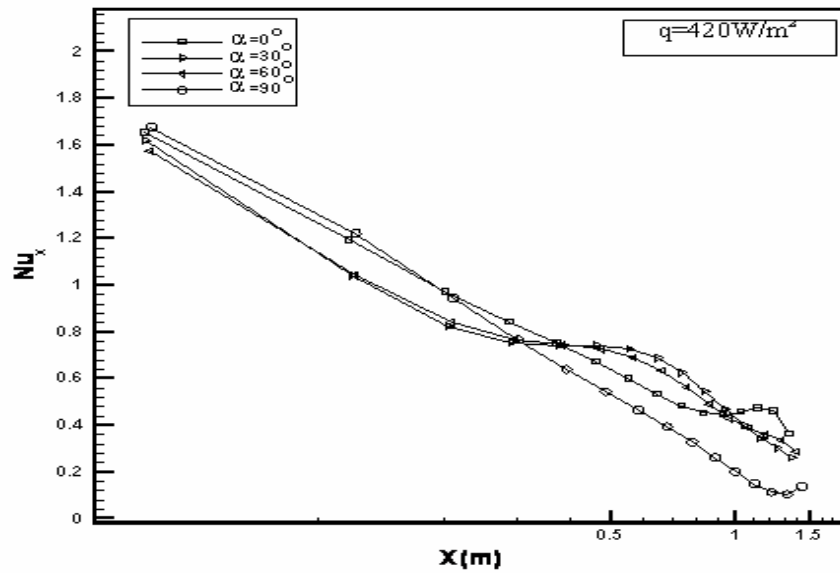


Fig.(13): Experimental Variation of the Local Nusselt number with the Axial Distance for Various Angles, $q=420\text{W/m}^2$.

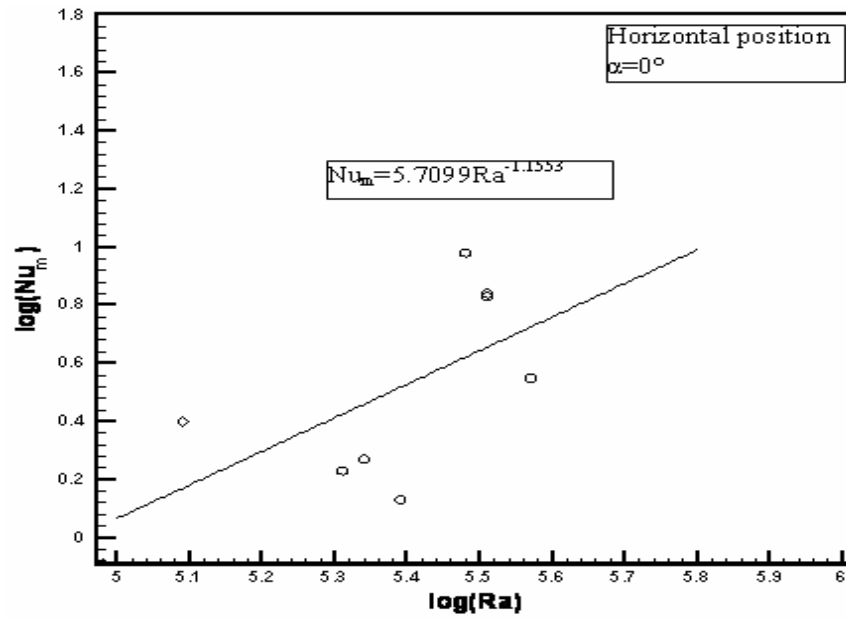


Fig.(14): Experimental Average Nusselt number Versus Ra, $\alpha=0^\circ$.

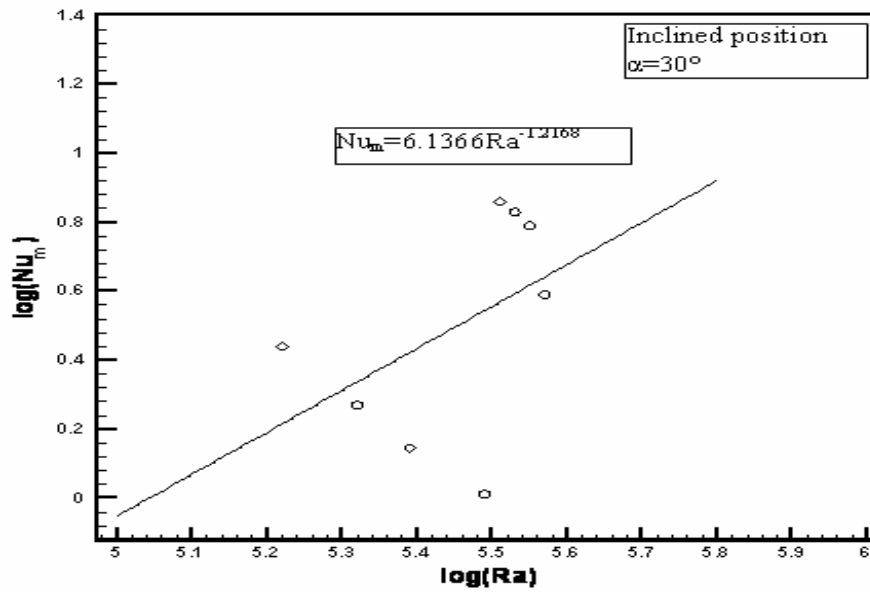


Fig.(15): Experimental Average Nusselt number Versus Ra, $\alpha=30^\circ$.

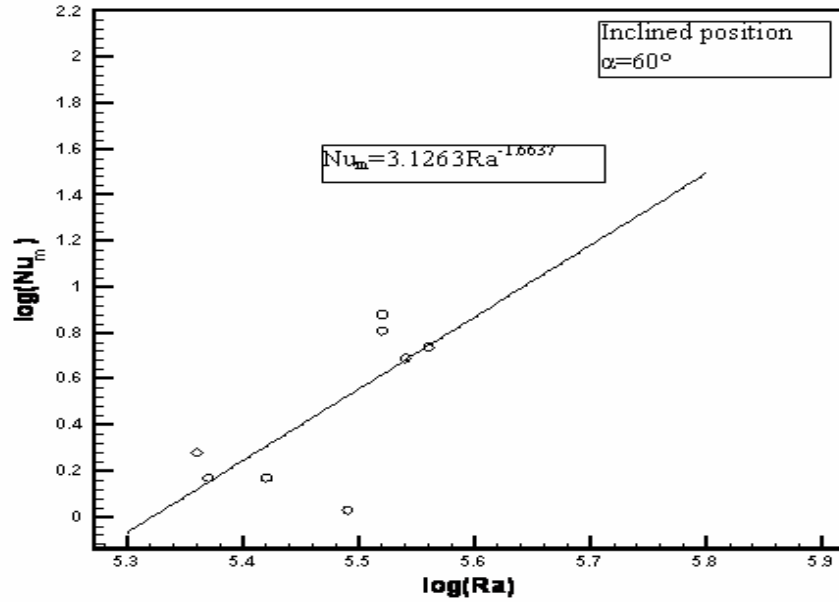


Fig.(16): Experimental Average Nusselt number Versus Ra, $\alpha = 60^\circ$.

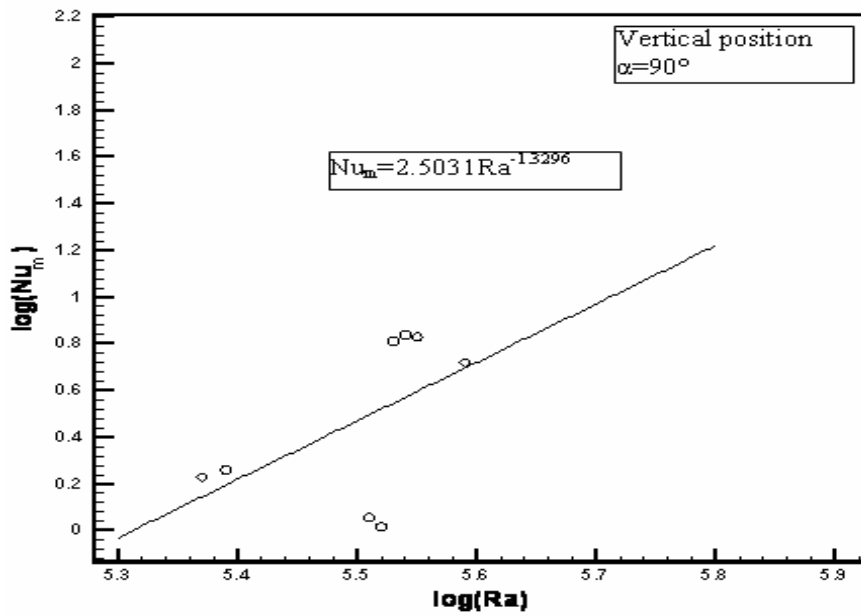


Fig.(17): Experimental Average Nusselt number Versus Ra, $\alpha = 90^\circ$.

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