RELIABILITY ANALYSIS OF THE SEISMIC STABILITY OF EMBANKMENTS REINFORCED WITH STONE COLUMNS

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ABSTRACT

Geotechnical engineers have always been concerned with the stabilization of slopes. For this purpose, various methods such as retaining walls, piles, and geosynthetics may be used to increase the safety factor of slopes prone to failure. The application of stone columns may also be another potential alternative for slope stabilization. Such columns have normally been used for cohesive soil improvement. Most slope analysis and design is based on deterministic approach i.e a set of single valued design parameter are adopted and a set of single valued factor of safety (FOS) is determined. Usually the FOS is selected in view of the understanding and knowledge of the material parameters, the problem geometry, the method of analysis and the consequences of failure. This results in different FOS obtained by different designers. This inherent variability characteristic dictates that slope stability problem is a probabilistic problem rather than deterministic problem. Furthermore, the FOS approach cannot quantify the probability of failure or level of risk associated with a particular design situation. The objective of this study is to integrate probabilistic approach as a rational means to incorporate uncertainty in the slope stability analysis. The study was made through a hypothetical problem which includes a sensitivity analysis. The methodology is based on Monte Carlo simulation integrated in commercially available computer program SLOPE/W. The output of the analysis is presented as the probability of failure as a measure of the likelihood of the slope failure. Results of this study have verified that the probability of failure is a better measure of slope stability as compared to the factor of safety because it provides a range of value rather than a single value.

الاعتمادية في تحليل الاستقرارية تحت تاثير الهزات الارضية للسداد الترابية المسلحة بالاعمدة الحجرية

الخلاصة

ان المهندسين الجيوتكتيك غالبا ما يتعاملون مع مسائل تثبيت المنحدرات و تستخدم لهذا الغرض عدة طرق منها الجدران الساندة اوالركائز لزيادة معامل الامان لمنحدرات المائلة الى الفشل. حيث ان استخدام الاعمدة الحجرية قد تعتبر احد الطرق البديلة المحتملة لتثبيت المنحدرات و التي تستخدم لتحسين الترب التي تمتلك خاصية التماسك.

ان معظم طرق التحليل و التصميم مبنية على اساس طرق حسابية تقريبية بمعنى اخر تبني قيمة مفردة للتصميم مبنية على اساس وضع قيمة حسابية واحدة لمعامل الامان.ان قيمة معامل الامان غالبا ما تختار بعد فهم و معرفة متغيرات خواص المادة و الشكل الهندسي للمنحدر و طريقة التحليل وتتابع الفشل و الذي يؤدي الى حصول نتائج مختلفة لمعامل الامان باختلاف المصممين ولذلك فان هذا التباين الموروث في الخواص يملي علينا اعتبار مسالة استقرارية المنحدرات هي مسالة احتمالية اكثر من كونها مسالة حسابية فقط.

ان الهدف من هذا البحث هو ايجاد تقريب احتمالي كمعنى عقلاني يتضمن الشكوك في تحليل السداد الترابية المسلحة بالاعمدة الحجرية حيث ان الدراسة اجريت على مسالة افتراضية تتضمن الحساسية في تباين خواص المواد في التحليل. ان الدراسة مبنية على نموذج Monte Carlo الموجود ضمنيا في البرنامج SLOPE/W. وقد وجد في هذه الدراسة ان احتمالية الفشل احسن مقياس لاستقرارية المنحدر اذا ماقورنت مع معامل الامان بسب انها توفر مجموعة من قيم معامل الامان بدلا من حصول على قيمة واحدة.

Keywords: stone column, slope stability, probability, reliability index, seismic analysis.

INTRODUCTION

Soils are naturally formed materials; consequently their physical properties vary from point to point. This variation occurs even in an apparently homogeneous layer. The variability in the value of soil properties is a major contributor to the uncertainty in the stability of a slope. Laboratory results on natural soils indicate that most soil properties can be considered as random variables conforming to the normal distribution function (Lumb, 1966, Tan et al. 1993).

Deterministic slope stability analyses compute the factor of safety based on a fixed set of conditions and material parameters. If the factor of safety is greater than unity, the slope is considered to be stable. On other hand, if the factor of safety is less than unity, the slope is considered to be unstable or susceptible to failure. Deterministic analyses suffer from limitations such as the variability of the input parameters.

In general, a factor of safety is really an index indicating the relative stability of a slope. It does not imply the actual risk level of the slope due to the variability of input parameters. With probabilistic analysis, two useful indices are available to quantify the stability or the risk level of a slope. These two indices are known as the probability of failure and the reliability index.

METHODS FOR SEISMIC SLOPE STABILITY ANALYSES

Surveys of earth dam performance during earthquakes suggest that embankments constructed of materials that are not vulnerable to severe strength loss as a result of earthquake shaking (most well compacted clayey materials, unsaturated cohesionless materials, and some dense saturated sands, gravels, and silts) generally perform well during earthquakes (Seed et. al., 1978). The embankment, however, may undergo some level of permanent deformation as a result of the earthquake shaking with well-built earth embankments experiencing moderate earthquakes, the magnitude of permanent seismic deformations should be small, but marginally stable earth embankments experiencing major earthquakes may undergo large deformations that may jeopardize the structure’s integrity. Simplified procedures have been developed to evaluate the potential for seismic instability and seismically induced permanent deformations (Seed, 1979; Makdisi and Seed, 1978), for the evolution of the seismic stability of natural slopes in clayey materials in most often carried out using various modifications of the following two methods (Duncan and Wright, 2005):

1. Pseudo-static method.
2. Sliding block method.

Pseudo Static Analyses

One of the earliest procedures of analysis for seismic stability is the pseudo static procedure, in which the earthquake loading is represented by a static force, equal to the soil weight multiplied by a seismic coefficient, k. The pseudo static force is used in a conventional limit equilibrium slope stability analysis. The seismic coefficient may be thought of loosely as an acceleration (expressed as a fraction of the acceleration, g, due to gravity) that is produced by the earthquake. However, the pseudo static force is treated as a static force and acts in only one direction, whereas the earthquake accelerations act for only a short time and change direction, tending at certain instances in time to stabilize rather than destabilize the soil.

The term pseudo static is a misnomer, because the approach is actually a static approach that is more correctly termed pseudo dynamic; however. The vertical components of the earthquake accelerations are usually neglected in the pseudo static method, and the seismic coefficient usually represents a horizontal force. Application of a seismic coefficient and pseudo static force in limit quilibrium slope stability analyses is relatively straightforward from the perspective of the mechanics: The pseudo static force is assumed to be a known force and is included in the various equilibrium equations as shown in Figure (1) for an infinite slope with the shear strength expressed in terms of total stresses.

Sliding Block Analyses

Newmark (1965) first suggested a relatively simple deformation analysis based on a rigid sliding block. In this approach the displacement of a mass of soil above a slip surface is modeled as a rigid block of soil sliding on a plane surface as shown in figure (2). When the acceleration of the block exceeds yield acceleration, a_y, the block begins to slip along the plane. Any acceleration that exceeds the yield acceleration causes the block to slip and imparts a velocity to the block relative to the velocity of the underlying mass. The block continues to move after the acceleration falls below the yield acceleration. Movement continues until the velocity of the block relative to the underlying mass goes to zero, as shown in figure (3). The block will slip again if the acceleration again exceeds the yield acceleration. This stick-slip pattern of motion continues until the accelerations fall below the yield acceleration and the relative velocity drops to zero for the last time. To compute displacements, the accelerations in excess of the yield acceleration are integrated once to compute the velocities and a second time to compute the displacements as shown in figure (3).

EMBANKMENTS STABILIZED WITH STONE COLUMNS

A number of factors and parameters such as soil properties, pore water pressure resume, slope geometry, earthquake, and vibration can influence the slope stability. Engineering slope stabilization is generally referred to stop or decrease the possible of instability process of slopes. Preventing the movement of a slope or increasing the safety factor (SF) is possible by using structural or geotechnical methods. Stone columns are method for slope stabilization. Such columns have been used since 1950 normally for cohesive soil improvement. It is a hole with circular section which is filled by gravel, rubble and etc and is an effective method to increase the shear strength on the slip surface of clayey slopes. The most important cases for utilizing stone columns (Barksdale and Bachus, 1983) are:

1. Improving slopes stability of both embankment and natural slopes.
2. Increasing the bearing capacity of shallow foundations constructed on soft soils.
3. Reducing total and differential settlements.
4. Decreasing the liquefaction potential of sandy soils.

RELIABILITY AND PROBABILITY OF FAILURE

The probability of failure can be interpreted in two ways (Mostyn and Li, 1993):

- If a slope is to be constructed many times, what percentage of such slopes would fail.
- The level of confidence that can be placed in a design.

The first interpretation may be relevant in projects where the same slope is constructed many times, while the second interpretation is more relevant in projects where a given design is only constructed once and it either fails or it does not. Nevertheless, the probability of failure is a good index showing the actual level of stability of a slope.

There is no direct relationship between factor of safety and probability of failure. In other words, a slope with a higher factor of safety may not be more stable than a slope with a lower factor of safety (Harr, 1987). For example, a slope with factor of safety of 1.5 and a standard deviation of 0.5 will have a much higher probability of failure than a slope with factor of safety of 1.2 and a standard deviation of 0.1.

The reliability of a slope (R) is an alternative measure of stability that considers explicitly the uncertainties involved in stability analyses. The reliability of a slope is the computed probability that a slope will not fail and is 1.0 minus the probability of failure (Duncan and Wright, 2005):

$$\mathbf{R=1-P_{\mathit{f}}} \qquad (1)$$

Where:

P_f is the probability of failure and **R** is the reliability or probability of no failure.

Reliability calculations provide a means of evaluating the combined effects of uncertainties and a means of distinguishing between conditions where uncertainties are particularly high or low.

The reliability index provides a more meaningful measure of stability than the factor of safety. The reliability index (β) is defined in terms of the mean (μ) and the standard deviation (σ) of the trial factors of safety as (Christian et al., 1994):

$$\beta = \frac{\mu - 1}{\sigma} \qquad (2)$$

The reliability index describes the stability of a slope by the number of standard deviations separating the mean factor of safety from its defined failure value of 1.0. It can also be considered as a way of normalizing the factor of safety with respect to its uncertainty.

STATISTICAL ANALYSIS OF SOIL DATA

Probability Density Function

A normal distribution function, often referred to as the Gaussian distribution function, is the most commonly used function to describe the variability of input parameters in probabilistic analyses. The normal distribution is so prevalent because many physical measurements provide frequency distributions that closely approximate a normal curve. A normal distribution function can be represented mathematically as:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(\chi-\mu)^2}{2\sigma^2}} \quad -\infty\langle\chi\rangle\infty \qquad (3)$$

Where:

$f(x)$ = relative frequency

σ = standard deviation

μ = mean value

A normal curve is bell shaped, symmetric and with the mean value exactly at middle of the curve. A normal curve is fully defined when the mean value, m and the standard deviation, s are known. A probability density function (PDF) shown in Figure (4) which describes the relative likelihood that the variable will have a certain value within the range of potential values. In this case the random variable is continuously distributed. A PDF can be fitted over the frequency diagram, which is a modified histogram whose ordinate has been scaled, so that the area under the histogram is unity.

Random Number Generation

The random numbers generated from the function are uniformly distributed with values between 0 and 1.0. In order to use the uniformly generated random number in the calculations of the normally distributed input parameters, it is necessary to transform the uniform random number to a normally distributed random number. This "normalization" process is done using the following transformation equation as suggested by SLOPE/W manual (2005):

$$N = \sqrt{(-2\ln R_1)} * (2\pi R_2) \qquad (4)$$

Where:

N = normalized random number

R_1 = uniform random number 1

R_2 = uniform random number 2

The transformation equation requires the generation of two uniform random numbers. The normalized random number can be viewed as the standard normal deviate in a normal curve with a mean value of 0 and standard deviation of 1.

Correlation Coefficient

A correlation coefficient expresses the relative strength of the association between two parameters. Laboratory tests on a wide variety of soils (Lumb, 1970) show that the shear strength parameters c and f are often negatively correlated with correlation coefficient ranges from -0.72 to 0.35. Correlation between strength parameters may affect the probability distribution of a slope. SLOPE/W allows the specification of c and f correlation coefficients for all soil models using c and f parameters. Furthermore, in the case of a bilinear soil model, SLOPE/W allows the specification of correlation coefficient for f and f2.

Correlation coefficients will always fall between -1 and 1. When the correlation coefficient is positive, c and f are positively correlated implying that larger values of c are more likely to occur with larger values of f. Similarly, when the correlation coefficient is negative, c and f are negatively correlated and reflects the tendency of a larger value of c to occur with a smaller value of f. A zero correlation coefficient implies that c and f are independent parameters.

In SLOPE/W, when estimating a new trial value for f and f2, the normalized random number is adjusted to consider the effect of correlation. The following equation is used in the adjustment:

$$N_A = N_1 k + (1 - |\ k\ |) N_2 \quad (5)$$

Where:

k = correlation coefficient between the first and second parameters

N1 = normalized random number for the first parameter

N2 = normalized random number for the second parameter

Na = adjusted normalized random number for the second parameter.

Method of Probabilistic Analysis

Monte Carlo method

The Monte Carlo method is a simple but versatile computational procedure. In general, the implementation of the method involves the following (Yang et al., 1993):

- The selection of a deterministic solution procedure, such as the Spencer's method or the finite element stress method.
- Decisions regarding which input parameters are to be modelled probabilistically and the representation of their variability in terms of a normal distribution model using the mean value and standard deviation.
- The estimation of new input parameters and the determination of new factors of safety many times.
- The determination of some statistics of the computed factor of safety, the probability density and the probability distribution of the problem.

In SLOPE/W, the critical slip surface is first determined based on the mean value of the input parameters using any of the limit equilibrium and finite element stress methods. Probabilistic analysis is then performed on the critical slip surface, taking into consideration the variability of the input parameters. The variability of the input parameters is assumed to be normally distributed with user-specified mean values and standard deviations.

During each Monte Carlo trial, the input parameters are updated based on a normalized random number. The factors of safety are then computed based on these updated input parameters. By assuming that the factors of safety are also normally distributed, SLOPE/W determines the mean and the standard deviations of the factors of safety. The probability distribution function is then obtained from the normal curve.

The number of Monte Carlo trials in an analysis is dependent on the number of variable input parameters and the expected probability of failure. In general, the number of required trials increases as the number of variable input increases or the expected probability of failure becomes smaller. It is not unusual to do thousands of trials in order to achieve an acceptable level of confidence in a Monte Carlo probabilistic slope stability analysis (Mostyn and Li, 1993).

Number of Monte Carlo Trials

Probabilistic slope stability analysis using the Monte Carlo method involves many trial runs. Theoretically, the more trial runs used in an analysis the more accurate the solution will be. How many trials are required in a probabilistic slope stability analysis? Harr, (1987) suggested that the number of required Monte Carlo trials is dependent on the desired level of confidence in the solution as well as the number of variables being considered. Statistically, the following equation can be developed (Harr, 1987):

$$N_{mc} = \left[\frac{(d^2)}{(4(1-\varepsilon)^2)} \right]^m \quad (6)$$

where :

N_{mc} = number of Monte Carlo trials,

ε = the desired level of confidence (0 to 100%) expressed in decimal form,

d = the normal standard deviate corresponding to the level of confidence, and

m = number of variables.

Measure of Random Variables

SLOPE/W assumes that the trial factors of safety are normally distributed. As a result, statistical analysis can be conducted to determine the mean, standard deviation, the probability density function and the probability distribution function of the slope stability problem. The equations used in the statistical analysis are summarized as follows (Lapin, 1983):

Mean factor of safety, μ:

$$\mu = \left(\frac{\sum_{i=0}^{n} F_i}{n} \right) \tag{7}$$

Standard deviation, σ:

$$\sigma = \sqrt{\left(\frac{\sum_{i=0}^{n} (F_i - \mu)^2}{n} \right)} \tag{8}$$

PARAMETRIC STUDY

The parametric study contains the analysis of embankment constructed on soft clays. The material of the embankment body is the same as that of its foundation but strengthened with stone columns. In this section, a one row or two rows (at distance 1.7m from first row) of stone columns are used to reinforce the slope and parametric study has been performed to determine the effect of uncertainties in the geotechnical properties of the slope soil materials and stone column material on the slope stability. The embankment to be analyzed is shown in figure (5). The height of embankment is 10m with 30^0 side slopes and 10m crest width.

The geotechnical properties of the clayey soil and stone column are shown in Tables (1) and (2).

Typically, the strength parameters (C and Φ) and the unit weight could be treated as variables. Table (3) shows a summary of typical reported values of coefficient parameters.

In this section, a study is to be carried out on embankment constructed using different conditions (with and without stone columns). Reliability is studied and different states of standard division are discussed.

Case (1)

Four soil parameters are considered as variables, the strength of the embankment and its foundation, angle of internal friction of the stone column and saturated unit weight of the soil and stone column as shown in Table (4) by making use of the data of Table (3)

The results obtained from analysis of case (1) where the standard deviation with lower limit are shown in Tables (5) and (6) for static and seismic conditions, respectively. In general, the mean factor of safety increases as compared to the factor of safety obtained from state without using stone columns analysis. The probability of failure decreases or the reliability index increases when the stone column of one or two rows is used.

The density function and cumulative distribution function of the factor of safety for this case as obtained by the program Slope/W are shown in Figures (6) to (17) for static and seismic analysis respectively.

Case (2)

In this case the soil is analyzed with a maximum limit of standard division for the strength, angle of internal friction and unit weight of soil as shown in Table (7)

Tables (8) and (9) show the result of analysis where the standard deviation is calculated with upper limit for static and seismic analysis. The effect of increasing the standard deviation on the probability density function and cumulative distribution function of factor of safety are demonstrated in Figures (18) to (29).The reliability index obtained for this case is much less than the reliability index obtained from case (1).

The density function and cumulative distribution function of the factor of safety for this case as obtained by the program Slope/W are shown in Figures (18) to (29) for static and seismic analysis, respectively.

Form static slope stability analysis, it can be noticed from the results based on lower limit and upper limit of standard deviation that the use of one row of stone columns increases the reliability index by about (93) % and (58) %, respectively. An increase in the reliability index to about (94) % and (61) % is obtained when using two rows of stone columns, while when adopting seismic load in slope stability analysis, the increase in reliability index is about (90) % and (83) for one raw of stone column and increase in the reliability index is about (94) % and (91) % for two rows of stone columns. This means that the best improvement in stability is obtained when using one row, then limited benefit is obtained when increasing the number of rows.

CONCLUSIONS

1. A reduction in the probability of failure in the order of about (41-100) % can be obtained when using two rows of stone columns in the embankment with two limits of standard deviation for static slope stability analysis.
2. The effect of seismic load on the probability failure reduction is in the order of about (26-56) % when using two rows of stone columns in the embankment with upper and lower limits of standard deviation.
3. The safety factor values and reliability index of stone column reinforced slopes are influenced by various parameters including geotechnical properties of the stone column material and number of rows.
4. The results obtained from seismic analysis of cases 1 and 2 show that the mean factor of safety increases as compared to the minimum factor of safety obtained from deterministic analysis.
5. The mean safety factor does not change much when standard deviations are varied in the static slope stability analysis However, the probability of failure increase gradually when the standard deviation of the soil parameters increases.
6. There is no direct relationship between the factor of safety and probability of failure, In other words the slope of higher factor safety; it does not mean that the slope is safe because of high probability of failure or low reliability index.

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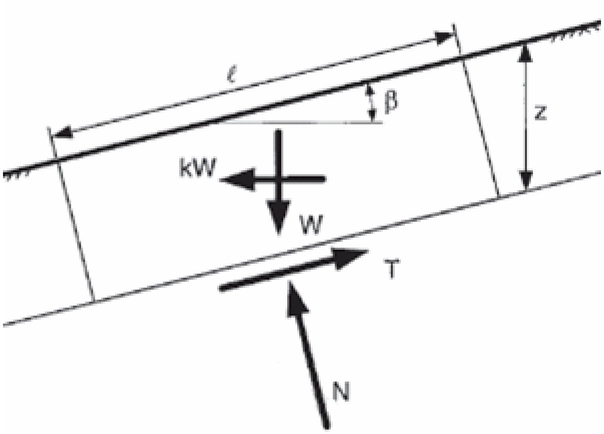

Resolving forces perpendicular to slip plane:

$$N = W\cos\beta - kW\sin\beta$$

Resolving force parallel to slip plane:

$$T = W\sin\beta + kW\cos\beta$$

Weight of sliding block:

$$W = \gamma \ell z \cos\beta$$

Substituting (3) into (1) and (2):

$$N = \gamma \ell z \cos^2\beta - k\gamma \ell z \cos\beta \sin\beta$$

$$T = \gamma \ell z \cos\beta \sin\beta + k\gamma \ell z \cos^2\beta$$

For the stresses on the slip plane:

$$\sigma = \frac{N}{\ell} = \gamma z \cos^2\beta - k\gamma z \cos\beta \sin\beta$$

$$\tau = \gamma z \cos\beta \sin\beta + k\gamma z \cos^2\beta$$

Finally, for the factor of safety (total stresses):

$$F = \frac{s}{\tau} = \frac{c + \sigma\tan\phi}{\tau} = \frac{c + \left(\gamma z \cos^2\beta - k\gamma z \cos\beta \sin\beta\right)\tan\phi}{\gamma z \cos\beta \sin\beta + k\gamma z \cos^2\beta}$$

Figure (1) Derivation of the equation for the factor of safety of an infinite slope with a seismic force (kW)—total stress analyses, after (Duncan and Wright, 2005)

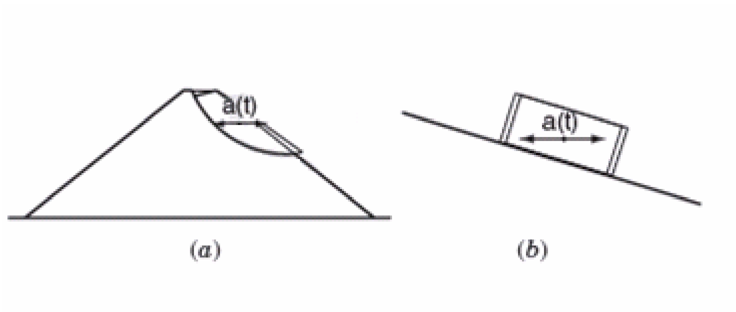

Figure (2) (a) Actual slope; (b) sliding block representation used to compute permanent soil displacements in a slope subjected to earthquake shaking, after (Duncan and Wright, 2005).

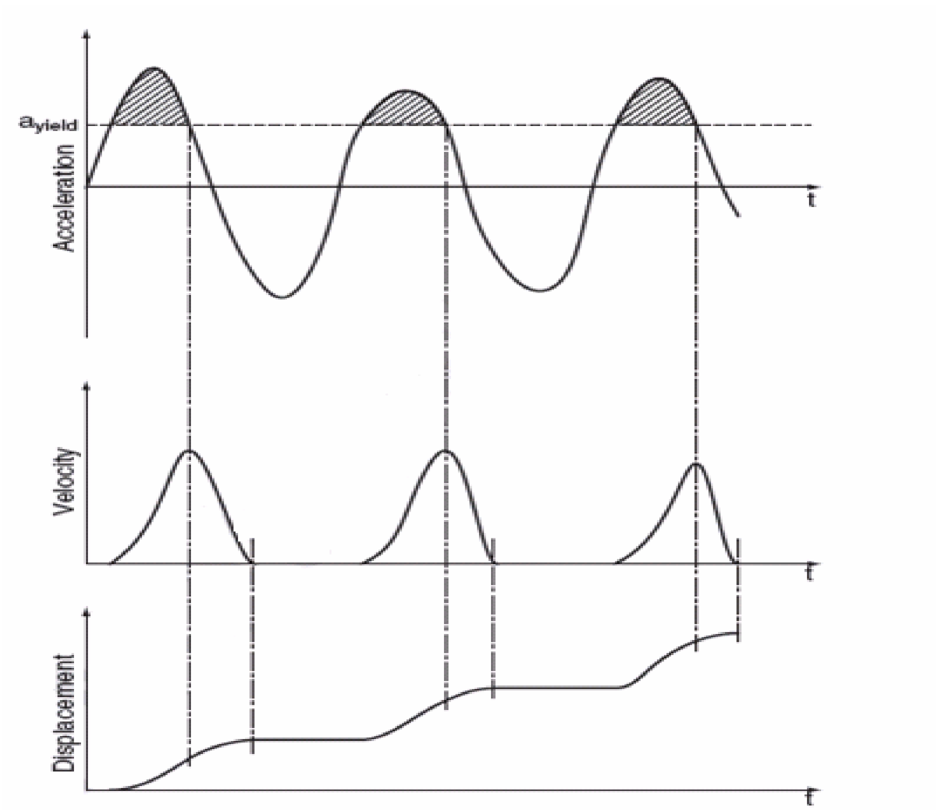

Figure (3) Double integration of acceleration–time history to compute permanent displacements, after (Duncan and Wright, 2005).

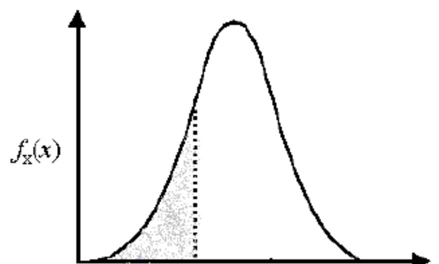

Figure (4) Probability density function

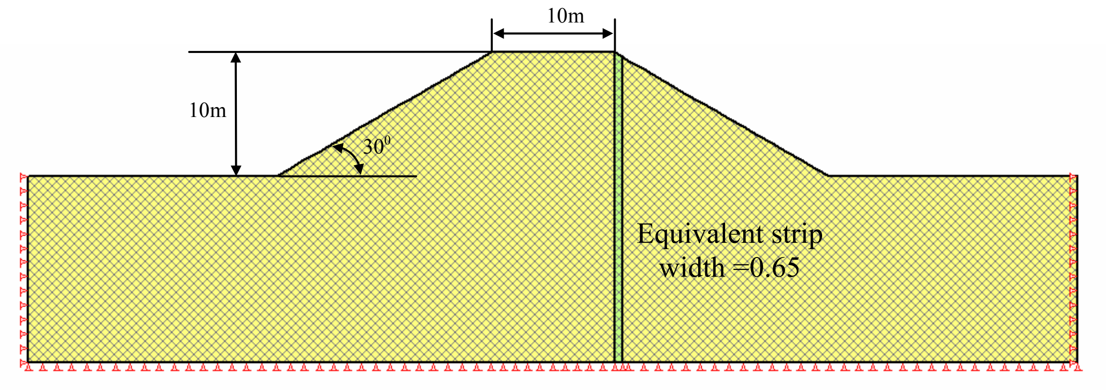

Figure (5) Geometrical specification of slope with stone column (after Ghazavi and Shahmandi, 2008).

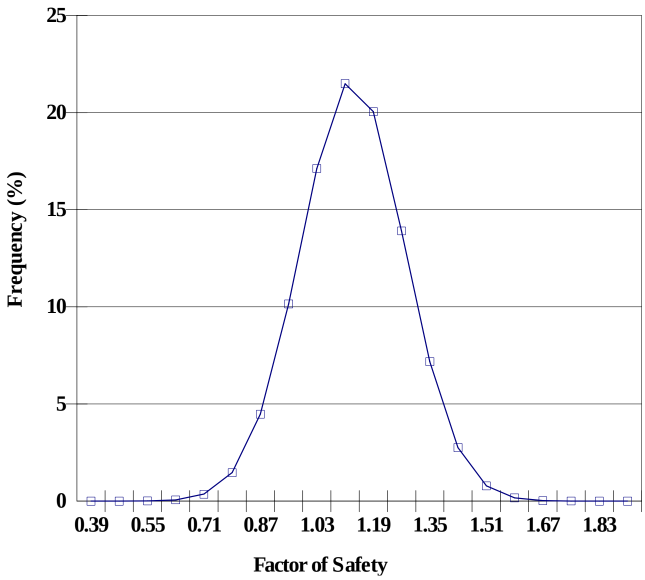

Figure (6) Probability density function without stone columns for static analysis

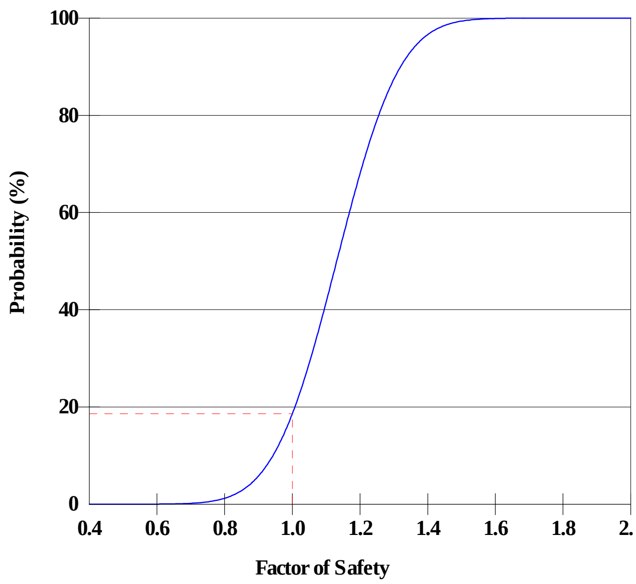

Figure (7) Probability distribution function without stone columns for static analysis

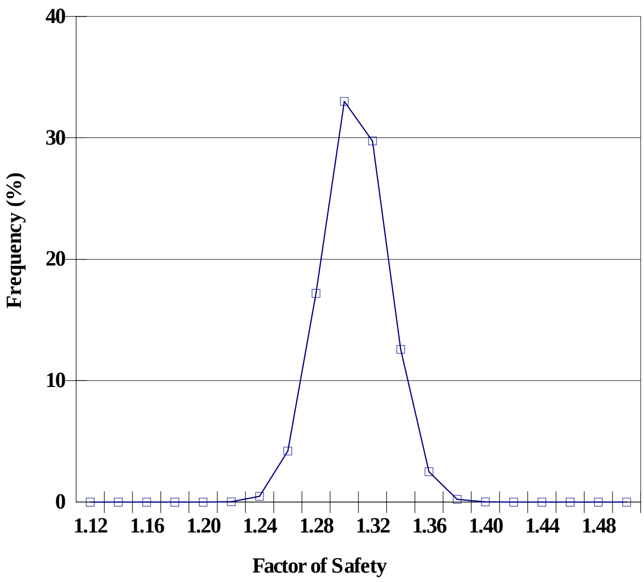

Figure (8) Probability density function with one stone column for static analysis

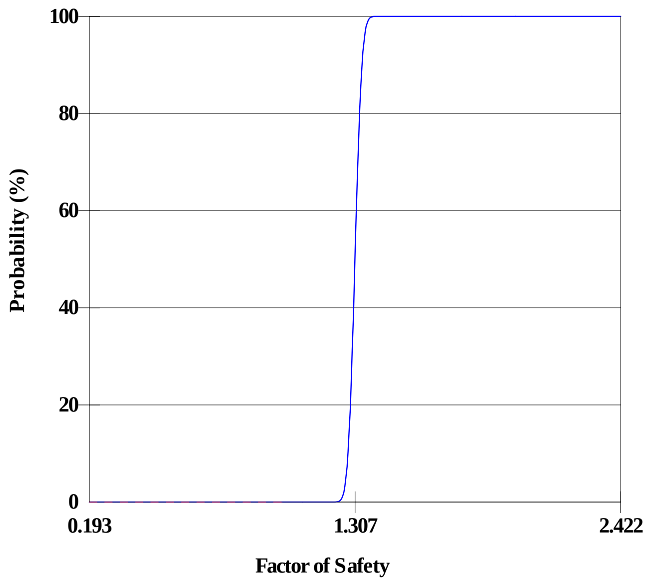

Figure (9) Probability distribution function with one stone column for static analysis

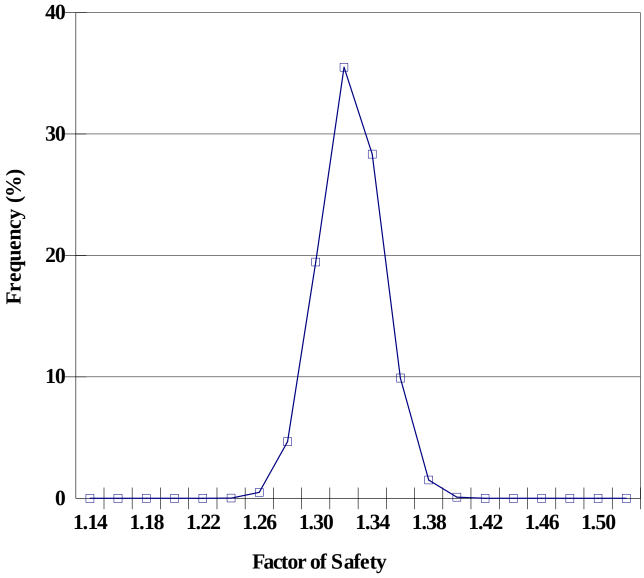

Figure (10) Probability density function with two stone columns for static analysis

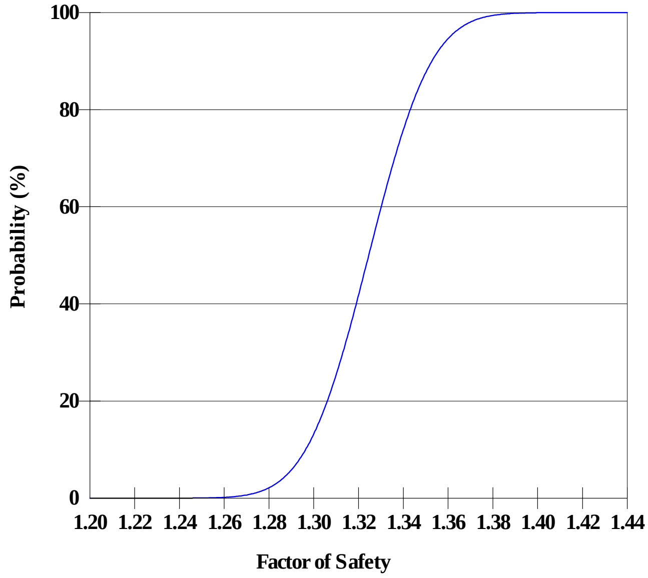

Figure (11) Probability distribution function with two stone columns for static analysis

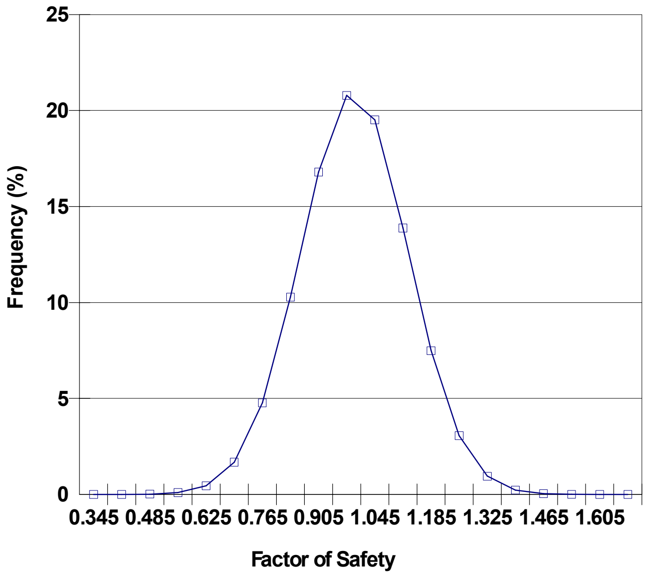

Figure (12) Probability density function without stone columns for seismic analysis

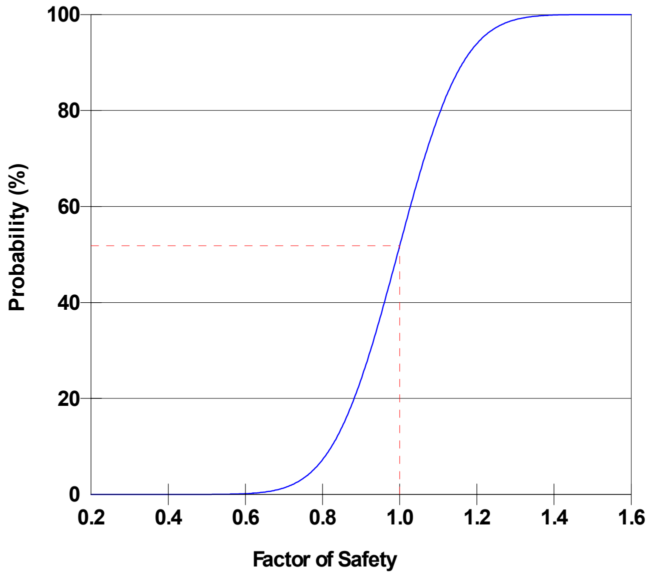

Figure (13) Probability distribution function without stone columns for seismic analysis

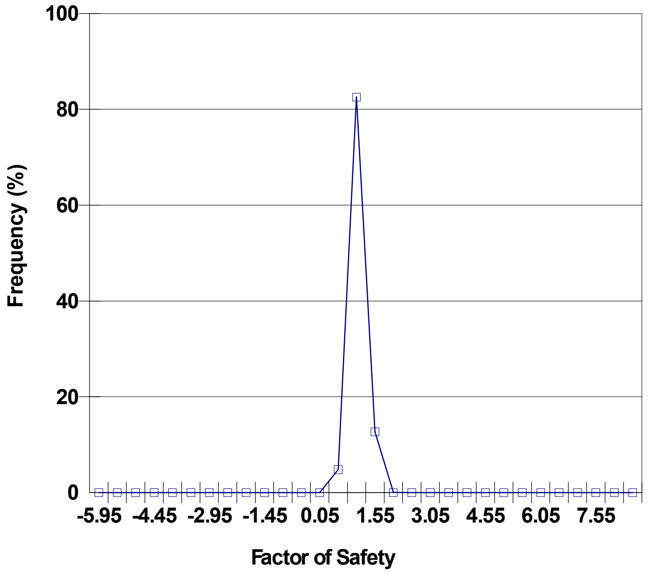

Figure (14) Probability density function with one stone column for seismic analysis

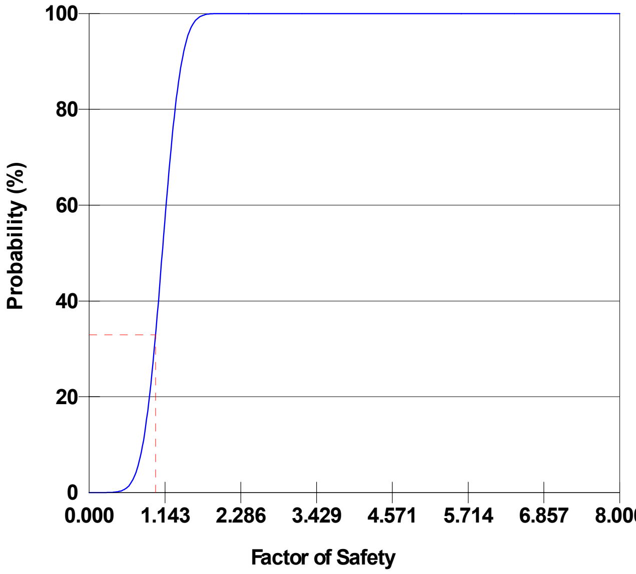

Figure (15) Probability distribution function with one stone column for static analysis

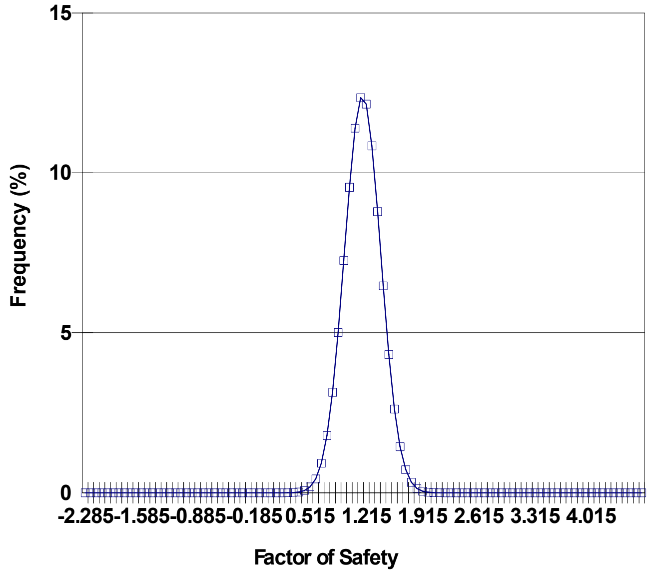

Figure (16) Probability density function with two stone columns for seismic analysis

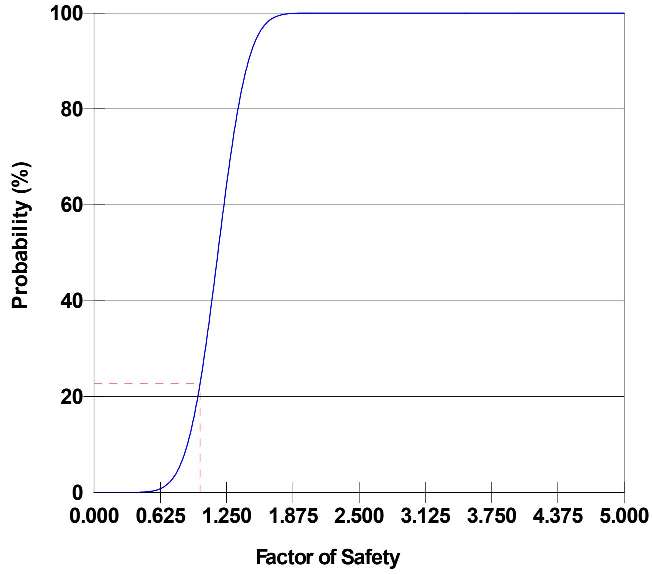

Figure (17) Probability distribution function with two stone columns for seismic analysis

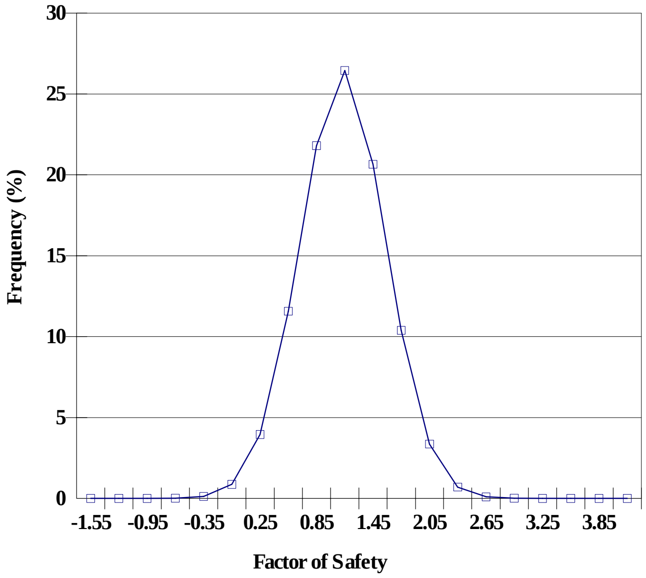

figure (18) Probability density function without stone columns for static analysis

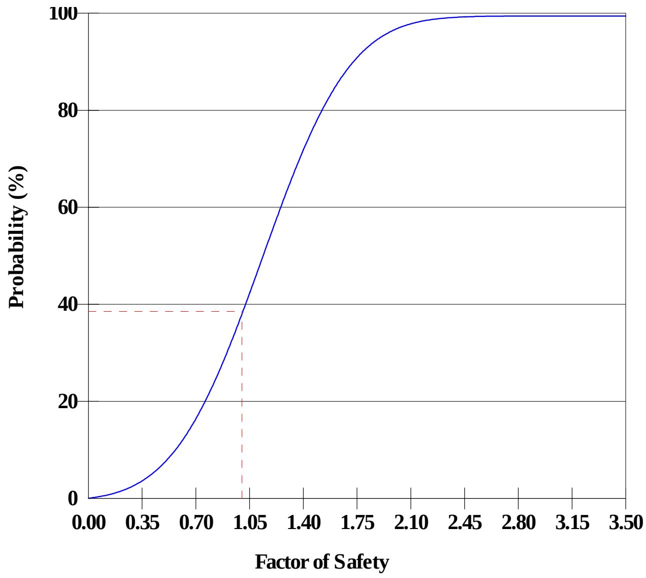

Figure (19) Probability distribution function without stone columns for static analysis

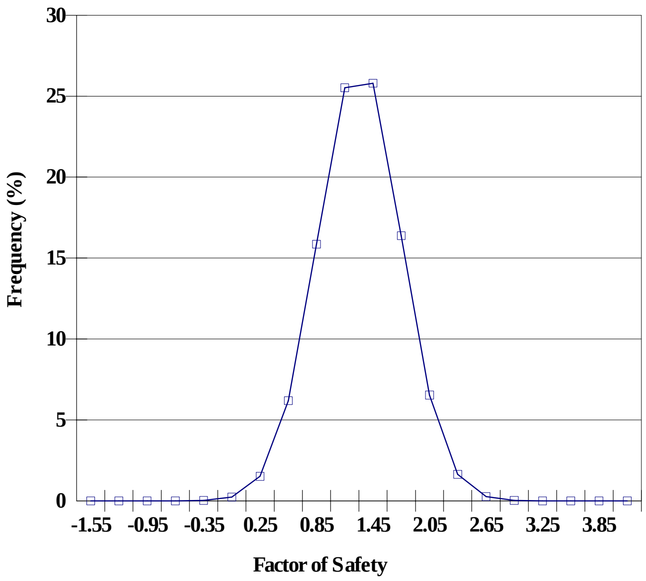

Figure (20) Probability density function with one stone column for static analysis

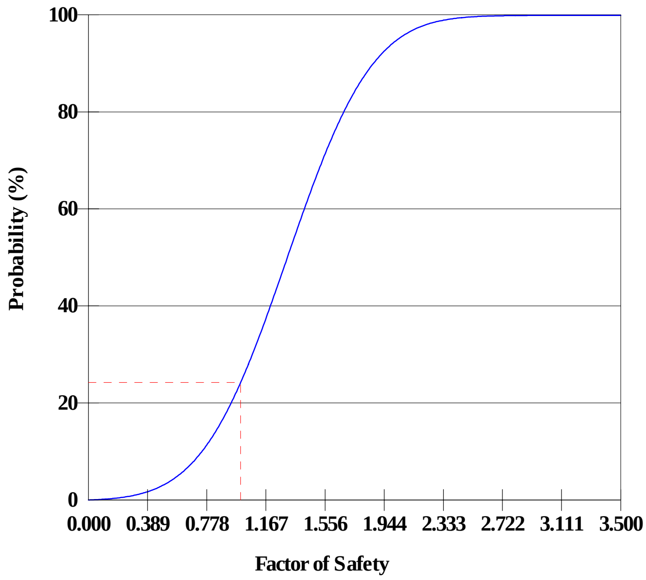

Figure (21) Probability distribution function with one stone column for static analysis

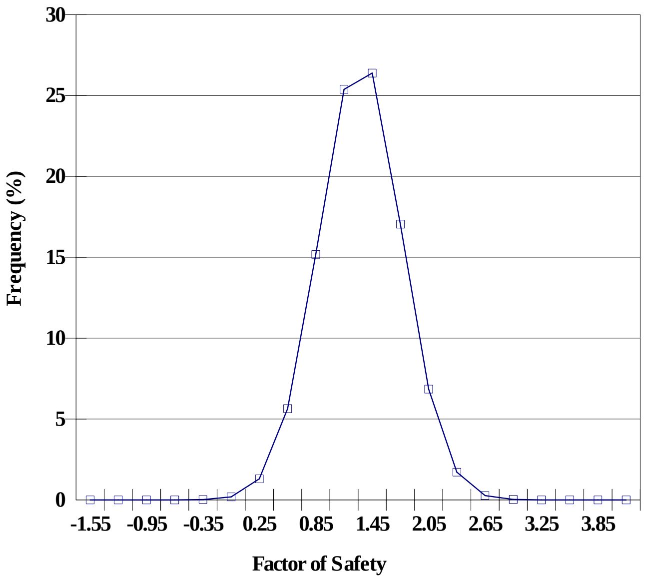

Figure (22) Probability density function with two stone columns for static analysis

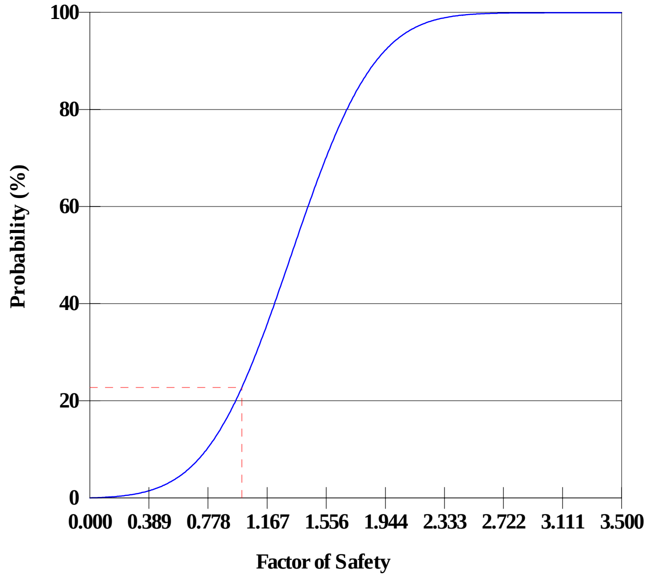

Figure (23) Probability distribution function with two stone columns for static analysis

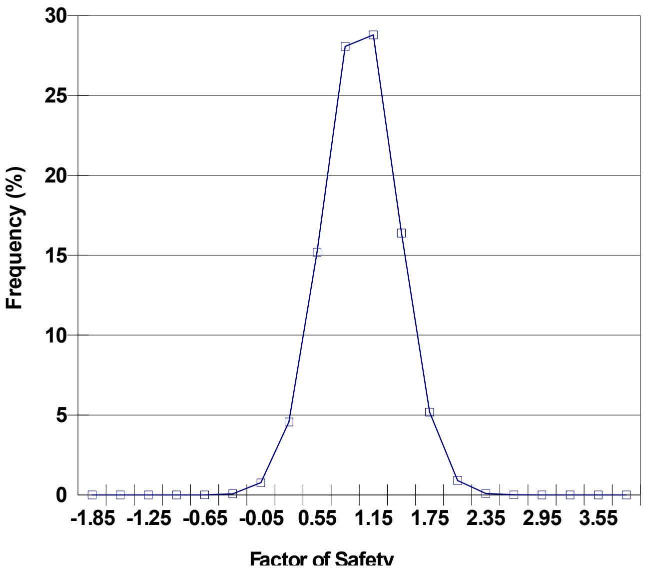

figure (24) Probability density function without stone columns for seismic analysis

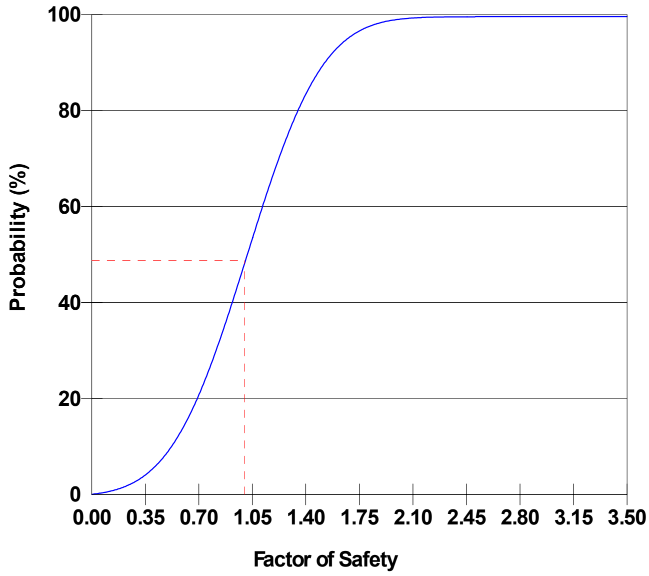

Figure (25) Probability distribution function without stone columns for seismic analysis

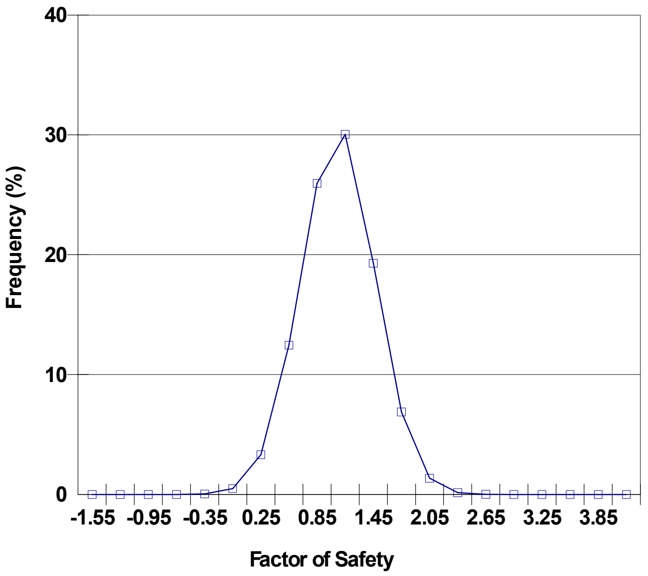

Figure (26) Probability density function with one stone column for seismic analysis

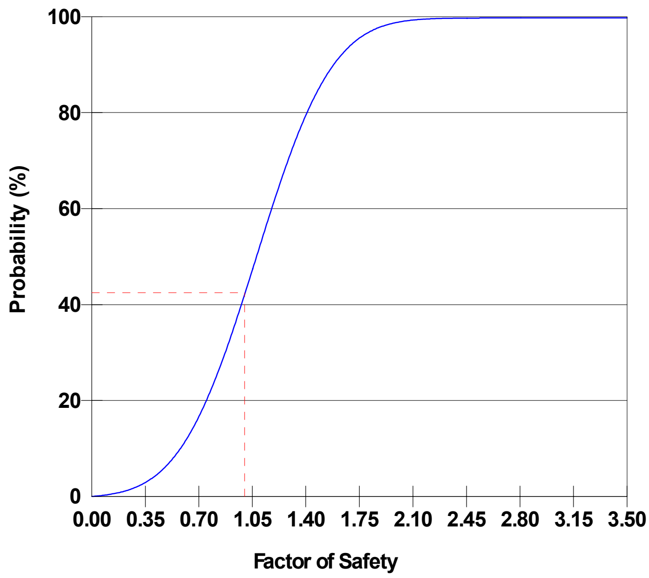

Figure (27) Probability distribution function with one stone column for seismic analysis

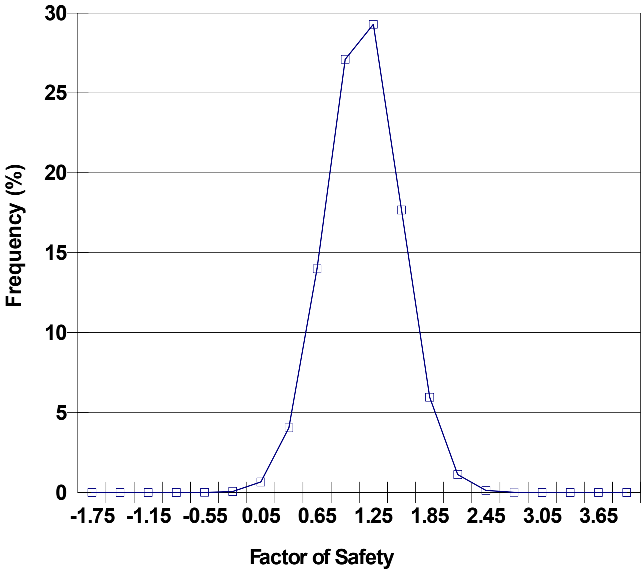

Figure (28) Probability density function with two stone columns for seismic analysis

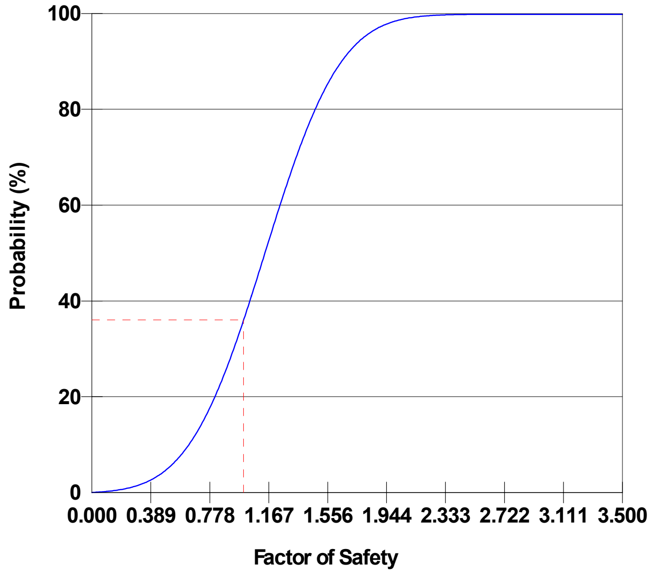

Figure (29) Probability distribution function with two stone columns for seismic analysis

Table 1. Geotechnical properties of clayey soil. (after Ghazavi and Shahmandi, 2008)

Modulus of elasticity [kN/m²]	Poisson's ratio	Undrained cohesion [kN/m²]	Friction angle [degree]	Saturated unit weight [kN/m³]
5000	0.48	25	0	17

Table 2. Geotechnical and geometrical properties of stone column materials (after Ghazavi and Shahmandi, 2008).

Modulus of elasticity [kN/m²]	Poisson's ratio	Undrained cohesion [kN/m²]	Friction angle [degree]	Saturated unit weight [kN/m³]	equivalent strip width [m]
50000	0.3	0	45	22	0.65

TABLE 3. Values of coefficient of Variation for geotechnical properties and in situ tests (after Duncan and Honorary, 2000).

Property or in situ test result	Coefficient of variation (%)	Source
Unit weight (γ)	3-7%	Harr (1984), Kulhawy (1992)
Buoyant unit weight (γ_b)	0-10%	Lacasse and Nadim (1997), Duncan (2000)
Effective stress friction angle (Φ')	2-13%	Harr (1984), Kulhawy (1992)
Undrained shear strength (*Su*)	13-40%	Harr (1984), Kulhawy (1992), Lacasse and Nadim (1997), Duncan (2000)
Un drained strength ratio ($Su/\sigma'v$)	5-15%	Lacasse and Nadim (1997), Duncan

		(2000)
Compression index (*Cc*)	10-37%	Harr (1984), Kulhawy, (1992), Duncan (2000)
Preconsolidation pressure (*Pp*)	10-35%	Harr (1984), Lacasse and Nadim (1997), Duncan (2000)
Coefficient of permeability saturated clay (k)	68-90%	Harr (1984), Duncan(2000)
Coefficient of permeability of partly saturated clay (k)	130-240%	Harr (1984), Benson et al. (1999)
Coefficient of consolidation (C_v)	33-68%	Duncan (2000)
Standard penetration test blow count (N)	15-45%	Harr (1984), Kulhawy (1992)
Electric cone penetration test (q_c)	5-15%	Kulhawy (1992)
Mechanical cone penetration test (q_c)	15-37%	Harr (1984), Kulhawy (1992)
Dilatometer test tip resistance (q_{DTM})	5-15%	Kulhawy (1992)
Vane shear test undrained strength (*Sv*)	10-20%	Kulhawy (1992)

Note: the coefficient of variation is the ratio of the standard deviation to the mean

Table (4) Soil properties used for cases with different standard deviation

Parameter	Mean	Coefficient of variation (lower limit)/ standard deviation
Cohesion, c (kN/m^3) (soil)	25	13/3.25
Angle of Friction, Φ (stone column)	45	2/0.9
Unit Weight, γ (kN/m^3) (soil)	17	3/0.51
Unit Weight, γ (kN/m^3) (stone column)	22	3/0.66
Horizontal and vertical seismic acceleration	0.05	-----

Table (5) Analysis results of probability for case (1) for static condition.

parameters	values		
	Without stone column	With one raw of stone column	With two row of stone column
FoS(FEM)	1.131	1.307	1.325
Mean F of S	1.131	1.307	1.325
Reliability Index	0.891	13.561	14.433
P (Failure) (%)	18.597490	0.000000	0.000000
Standard Dev.	0.147	0.023	0.022
Min F of S	0.43149	1.2138	1.2217
Max F of S	1.7955	1.4217	1.4333

Table (6) Analysis results of probability for case (1) for seismic condition.

parameters	values		
	Without stone column	With one raw of stone column	With two row of stone column
FoS(Bishop method)	0.993	1.062	1.133
Mean F of S	0.99396	1.1016	1.168
Reliability Index	0.046	0.441	0.748

P (Failure) (%)	51.819581	32.924610	22.684731
Standard Dev.	0.133	0.23	0.225
Min F of S	0.44148	0.57608	0.60839
Max F of S	1.5991	7.3574	4.3708

Table (7) Soil properties used for cases with different standard deviations.

Parameter	Mean	Coefficient of variation (upper limit)/ standard deviation
Cohesion, c (kN/m^3) (soil)	25	40/10
Angle of Friction, φ (stone column)	45	13/5.85
Unit Weight, γ (kN/m^3) (soil)	17	7/1.19
Unit Weight, γ (kN/m^3) (stone column)	22	7/1.54
Horizontal and vertical seismic acceleration	0.05	-----

Table (8) Analysis results of probability for case (2) for static condition

parameters	values		
	Without stone column	With one row of stone column	With two row of stone column
FoS(FEM)	1.131	1.307	1.325
Mean F of S	1.1316	1.307	1.3244
Reliability Index	0.291	0.697	0.746
P (Failure) (%)	38.535780	24.232920	22.752750
Standard Dev.	0.452	0.44	0.435
Min F of S	-1.0388	-0.66468	-0.73564
Max F of S	3.3028	3.3895	3.3771

Table (9) Analysis results of probability for case (2) for seismic condition

parameters	values		
	Without stone column	With one row of stone column	With two row of stone column
FoS(Bishop method)	0.993	1.062	1.133
Mean F of S	1.0128	1.0743	1.14
Reliability Index	0.033	0.19	0.357
P (Failure) (%)	48.689261	42.445001	36.041120
Standard Dev.	0.391	0.391	0.393
Min F of S	0.10973	0.10584	0.10947
Max F of S	3.0975	3.2048	3.2034