MAGNETO HYDRODYNAMIC NATURAL CONVECTION FLOW ON A VERTICAL CYLINDER WITH A PRESENCE OF HEAT GENERATION AND RADIATION

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ABSTRACT

The present work investigates the effect of magneto – hydrodynamic (MHD) laminar natural convection flow on a vertical cylinder in presence of heat generation and radiation. The governing equations which used are Continuity, Momentum and Energy equations. These equations are transformed to dimensionless equations using Vorticity-Stream Function method and the resulting nonlinear system of partial differential equations are then solved numerically using finite difference approximation. A thermal boundary condition of a constant wall temperature is considered. A computer program (Fortran 90) was built to calculate the rate of heat transfer in terms of local Nusselt number, total mean Nusselt number, velocity distribution as well as temperature distribution for a selection of parameters sets consisting of dimensionless heat generation parameter $(0.0 \le Q \le 2.0)$, conduction – radiation parameter $(0.0 \le N \le 10.0)$, and the dimensionless magneto hydrodynamic parameter $(0.0 \le M \le 1.0)$. Numerical solution have been considered for a fluid Prandtl number fixed at (*Pr*=0.7), Rayleigh number $\left(10^2 \le Ra_l \le 10^5\right)$. The results are shown reasonable representation to the relation between Nusselt number and Rayleigh number with other parameters (M, N and Q). Generally, *Nu* increase with increasing *Ra*, M, N and Q separately. When the MHD, N, and Q effect added to the heat transfer mechanism, the heat transfer rate increased and this effect increased with increasing in Ra, MHD, N, and Q. The effect of magneto hydrodynamic, heat generation and heat radiation on the rate of heat transfer is concluded by correlation equations. The results are found to be in good agreement compared with the results of other researchers.

الخلاصة

تمت دراسة تأثير الهيدروديناميكية المغناطيسية على الحمل الحر للجريان الطبـاقي علـى اسـطوانة عمودية بوجود تولد للطاقة الحرارية والإشعاع. المعادلات التي تتحكم بالبحث هـي الاسـتمرارية والـزخم ومعادلة الطاقة وتم تحويل المعادلات الى معادلات لا بعدية باستخدام دالـة الانـسياب الدواميـة وتـم حـل المعادلات التفاضلية الجزئية عدديا باستخدام طريقة الفروق المحددة والظروف الحدية الحرارية التي أخـذت بنظر الاعتبار هي ثبوت درجة حرارة الجدار. تم بناء البرنامج الحاسوبي (Fortran 90) لحـساب معـدل انتقال الطاقة الحرارية بدلالة رقم نسلت الموضعي ورقم نسلت الكلي وتوزيـع الـسرعة ودرجـة الحـرارة

للمعايير التي تتضمن مجموعة من القيم الابعدية.الطاقة الحراريـة المتولـدة اللابعديـة $(0.0 \le Q \le 2.0)$ والإشـعاع الحـراري اللابعـدي $(0.0 \le N \le 10.0)$ والهيدروديناميكيـة المغناطيـسية اللابعديـة $(0.0 \le M \le 1.0)$ اخذ بنظر الاعتبار الحل العددي لمـائع ذو رقـم برانـدتل $(Pr=0.7)$ ورقـم رايلـي $(10^2 \le Ra_l \le 10^5)$. النتائج أعطت تمثيل مقبول لعلاقة رقم نسلت مع رقم رايلي بتأثير المعايير الأخـرى (M,N,Q) وبصورة عامة يزداد رقم نسلت بزيادة رقم رايلي وعند إضافة تأثير(M,N,Q)على ميكانيكيـة انتقال الطاقة الحرارية يزداد معدل انتقال الطاقة الحراريـة. النتـائج أعطـت تمثيـل لـرقم نـسلت ضـد الهيدروديناميكية المغناطيسية، التولد الحراري والإشعاع الحراري. . المقارنة مع البحوث الأخـرى أعطـت نتائج بتوافق جيد مع نتائج باحثين آخرين.

KEY WORDS: Natural Convection, Radiation, Magneto hydrodynamic, Vertical cylinder

INTRODUCTION

The problem of free convection due to a heated or cooled vertical cylinder provides one of the most basic scenarios for heat transfer theory and thus is of considerable theoretical and practical interest. The free convection boundary-layer over a vertical cylinder is probably the first buoyancy convective problem which has been studied and it has been a very popular research topic for many years.

Cooling of electronic devices to that of solar energy collectors, missile reentry, rocket combustion chambers, power plants for interplanetary flight and gas cooled nuclear reactors, have focused attention on thermal radiation as a mode of energy transfer and emphasized the need for an improved understanding of radiative transfer in these processes on the other hand magneto hydrodynamics is also motivated by its widespread application to the description of space (within the solar system) and astrophysical plasmas (beyond the solar system).

[Ganesan and Loganathan, 2002] analyzed the interaction of free convection with thermal radiation of a viscous incompressible unsteady flow past a moving vertical cylinder with mass and heat transfer. It was found that at small values of the Prandtl number and radiation parameter (N), the velocity and temperature of the fluid increased sharply near the cylinder as the time t increase, which is totally absent in the absence of radiation effects.

[Molla, 2005] describes the effect of magneto hydrodynamic natural convection flow on a sphere in present of heat generation. Results indicated that the local Nusselt number (Nu_x) decreases owing to increase the value of heat generation Q and the local rate of heat transfer Nu_x decreased slightly as the value of magnetic parameter M increased at different positions. The velocity distribution decreased slightly as the magnetic parameter M increased, but near the surface of the sphere velocity increased and became maximum and then decreased and finally approached to zero.

[Filar et.al., 2005] computed numerically three-dimensional convection of air inside a vertical cylinder isothermally heated and cooled from a side wall for both magnetic and gravity fields. A single electric coil was placed around the cylinder to generate a magnetic field. Convection was calculated for various coil level and magnetic strengths. The results indicated that effect of Magneto hydrodynamic increased and lead to increase the rate of heat transfer and this is clear at higher values of Rayleigh number and both coil elevation and Rayleigh number affect the heat transfer rate extensively. The maximum Nusselt number could be obtained for the coil located at about half of cylinder.

In the present study, the magneto hydrodynamic, effect was investigated for steady state laminar natural convection external flow with presence of heat generation and radiation on a vertical cylinder, for thermal boundary condition of constant wall temperature and for ($10^2 \le Ra_l \le 10^5$), ($0 \le M \le 1.0$), ($0 \le Q \le 2.0$) and ($0 \le N \le 10$).

To the best of our knowledge there is no previous work was found that studies the case of external flow on a cylinder taking the effect of MHD and the effect of all the parameters gathered a comparison was done between the result of the present study for the variation of Nu with Ra_l and the correlation of **[Mc Adams, 1954]** with deviation of 5% for the case of no

MHD, radiation and heat generation and another comparison between the present study and the work of **[Ganesan, 2002]** with and without radiation and for no MHD and no heat generation which gives a deviation of 1.7% with radiation and 1.5% without radiation..

MATHEMATICAL MODEL

The mathematical modeling will be set for laminar natural convection heat transfer on a vertical cylinder. The buoyancy effect caused by the density variation produces natural circulation resulting in the fluid motion relative to the bounding solid surface. The buoyancy forces behave as body forces and are included as such in the momentum equation. Under these conditions the continuity, momentum and energy equations are coupled.

The density is considered as linear function of temperature so that the usual Boussinesq's approximation is taken as **[Singh and Ajay, 2003]**:

$$\rho = \rho_o \left(1 - \beta \left[T - T_\infty \right] \right) \qquad (1)$$

$$g_r = 0 \qquad (2)$$

$$g_z = g\beta (T - T_\infty) \qquad (3)$$

Continuity Equation

The equation of conservation of mass in the cylindrical coordinates is given as:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,u) + \frac{\partial w}{\partial z} = 0 \qquad (4)$$

Momentum Equation

By using Navier-Stokes' equation in the cylindrical coordinates (r, z), the equation of conservation of momentum in the cylindrical coordinates (the radial (r) direction) is in the following form:

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P^*}{\partial r} + \nu\left(\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru)\right] + \frac{\partial^2 u}{\partial z^2}\right) + f_r \qquad (5)$$

Where (f_r) is the electromagnetic force in (r) direction **[Branover, 1978]**:

$$f_r = \frac{\sigma_o B_o^2 u}{\rho} \qquad (6)$$

The equation of conservation of momentum in the cylindrical coordinates (in the axial (z) direction) is in the following form:

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P^*}{\partial z} + \nu\left(\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial w}{\partial r}\right] + \frac{\partial^2 w}{\partial z^2}\right) + g\beta(T - T_\infty) + f_z \qquad (7)$$

Where (f_z) is the electromagnetic force in (z) direction **[Branover, 1978]**:

$$f_z = \frac{\sigma_o B_o^2 w}{\rho}$$

Energy Equation

The energy equation in the cylindrical coordinates takes the following form:

$$u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right]+\frac{\partial^2 T}{\partial z^2}\right)+\frac{q'''}{\rho\ cp}-\frac{1}{\rho\ cp}\frac{1}{r}\frac{\partial}{\partial r}\left(r\ q_r\right)$$

(8)

The volumetric rate of heat generation is given as:

$$q''' = Q_o(T - T_\infty)$$

(9)

Where Q_o is the heat generation/absorption and is a constant. **[Molla, 2005].**
The radiative heat flux q_r is given as: **[Brewster, 1992]**.

$$q_r = -\frac{4\sigma\partial T^4}{3K^*\partial r}$$

(10)

Where (K^*) is the mean absorption coefficient and (σ) is the Stefan-Boltzmann constant.

DIMENSIONLESS PARAMETERS AND EQUATIONS

$$\left(R=\frac{r}{l}\right),\quad \left(Z=\frac{z}{l}\right),\quad \left(U=\frac{u\,l}{\alpha}\right),\quad \left(W=\frac{w\,l}{\alpha}\right),$$

$$\left(\theta=\frac{T-T_\infty}{T_w-T_\infty}\right),\quad \left(P=\frac{pl^2}{\rho\,\alpha^2}\right)\ \left(Q=\frac{Q_o l^2}{\rho cp\,\alpha}\right),$$

$$\left(N=\frac{K^* K}{4\sigma T_\infty^3}\right),\quad \left(M=\frac{\sigma_\circ B_\circ^2 l^2}{\rho\alpha}\right)$$

Dimensionless Continuity Equation

$$\frac{1}{R}\frac{\partial(RU)}{\partial R}+\frac{\partial W}{\partial Z}=0$$

(11)

Dimensionless Momentum Equation In (r) Direction

$$U\frac{\partial U}{\partial R}+W\frac{\partial U}{\partial Z}=-\frac{\partial P}{\partial R}+\Pr\left(\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial(R\,U)}{\partial R}\right)+\frac{\partial^2 W}{\partial Z^2}\right)+MU$$

(12)

Dimensionless Momentum Equation In (z) Direction

$$U\frac{\partial W}{\partial R}+W\frac{\partial W}{\partial Z}=-\frac{\partial P}{\partial Z}+\Pr\left(\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial W}{\partial R}\right)+\frac{\partial^2 W}{\partial Z^2}\right)+\Pr Ra_l\,\theta+MW$$

(13)

Dimensionless Energy Equation

$$U\frac{\partial\theta}{\partial R}+W\frac{\partial\theta}{\partial Z}=\left(1+\frac{4}{3N}\right)\left[\frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial\theta}{\partial R})\right]+\frac{\partial^2\theta}{\partial Z^2}+Q\theta \qquad (14)$$

Vorticity Transport, Stream Function and Energy Equations

The governing equations in dimensionless form above were written in terms of dependant variables (U, W, P and θ). It may be recommended to eliminate pressure term (because it will be a non linear term in momentum equation) **[Patanker, 1980]**. By converting momentum equations to vorticity transport equation by differentiate momentum equation in (r) direction with respect to (z) and momentum equation in (z) direction with respect to (r) and subtract them from each other and make use of continuity equation and vorticity definition:

$$\omega=\frac{\partial W}{\partial R}-\frac{\partial U}{\partial Z} \qquad (15)$$

$$\frac{\partial(U\omega)}{\partial R}+\frac{\partial(W\omega)}{\partial Z}=\Pr Ra_l\frac{\partial\theta}{\partial R}+M\omega+\Pr\left(\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial(R\omega)}{\partial R}\right)+\frac{\partial^2\omega}{\partial Z^2}\right) \qquad (16)$$

Also, by use of vorticity definition (15) and the definition of stream function, (ψ) which satisfy continuity equation, the vertical and radial velocities can be written as follows respectively:

$$W=-\frac{1}{R}\frac{\partial\psi}{\partial R} \qquad (17)$$

$$U=\frac{1}{R}\frac{\partial\psi}{\partial Z} \qquad (18)$$

So by substituting the velocity components (17) and (18) in equation (15), stream function equation resulted as:

$$-\omega=\frac{1}{R}\left(\frac{\partial^2\psi}{\partial R^2}-\frac{1}{R}\frac{\partial\psi}{\partial R}+\frac{\partial^2\psi}{\partial Z^2}\right)=\nabla^2\psi \qquad (19)$$

The dimensionless energy equation (14) can be transformed to another form by substituting the continuity equation (11) in it as follows:

$$\frac{1}{R}\frac{\partial(RU\theta)}{\partial R}+\frac{\partial(W\theta)}{\partial Z}=\left(1+\frac{4}{3N}\right)\left[\frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial\theta}{\partial R})\right]+\frac{\partial^2\theta}{\partial Z^2}+Q\theta \qquad (20)$$

Boundary Conditions;

The imposed boundary conditions (illustrate in **Fig. (1)** and **Table (1)**), rewritten in terms of stream function and vorticity

(No slip condition)
$\omega = \psi = U = W = 0$

$\theta = 1$ (Constant wall temperatures)

NUMERICAL SOLUTION

The method of the numerical solution taken is the Finite Difference technique for solving the set of equations**[Patanker, 1980]**.

$$a_1\theta_{i-1,j} + a_2\theta_{i+1,j} + a_3\theta_{i,j} + a_4\theta_{i,j-1} + a_5\theta_{i,j+1} = 0 \quad (21)$$

Where:

$$a_1 = \frac{\left(U_b + |U_b|\right)\left(\Delta R(1-2i) - R_i\right)}{4\Delta R(R_i + i\Delta R)} + \left(1 + \frac{4N}{3}\right)\left(-\frac{1}{(\Delta R)^2} + \frac{1}{2\Delta R(R_i + \Delta R)}\right) \quad (22)$$

$$a_2 = \frac{\left(U_f - |U_f|\right)\left(R_i + \Delta R(1+2i)\right)}{4\Delta R(R_i + i\Delta R)} - \left(1 + \frac{4}{3N}\right)\left(\frac{1}{\Delta R^2} + \frac{1}{2\Delta R(R_i + \Delta R)}\right) \quad (23)$$

$$a_3 = -Q + \frac{\left[\left(U_f + |U_f|\right)\left(R_i + \Delta R(1+2i)\right) + \left(U_b - |U_b|\right)\left(\Delta R(1-2i) - R_i\right)\right]}{4\Delta R(R_i + i\Delta R)} + \frac{\left(W_f + |W_f| - W_b + |W_b|\right)}{2\Delta Z} + \frac{2}{(\Delta R)^2} + \frac{2}{(\Delta Z)^2} \quad (24)$$

$$a_4 = \frac{\left(-W_b + |W_b|\right)}{2\Delta Z} - \frac{1}{(\Delta Z)^2} \quad (25)$$

$$a_5 = \frac{\left(W_f - |W_f|\right)}{2\Delta Z} - \frac{1}{(\Delta Z)^2} \quad (26)$$

$$b_1\omega_{i-1,j} + b_2\omega_{i+1,j} + b_3\omega_{i,j} + b_4\omega_{i,j-1} + b_5\omega_{i,j+1} + c = 0 \quad (27)$$

Where:

$$b_1 = -\frac{\left(U_b + |U_b|\right)}{2\Delta R} + \frac{1}{2\Delta R(R_i + i\Delta R)} - \frac{1}{(\Delta R)^2} \quad (28)$$

$$b_2 = \frac{\left(|U_f| - U_f\right)}{2\Delta R} - \frac{1}{2\Delta R(R_i + i\Delta R)} - \frac{1}{(\Delta R)^2} \quad (29)$$

$$b_3 = -M + \frac{\left[\left(U_f + |U_f|\right) - \left(U_b + |U_b|\right)\right]}{2\Delta R} + \frac{\left(W_f + |W_f| - W_b + |W_b|\right)}{2\Delta Z} + \frac{2\Pr}{(\Delta R)^2} + \frac{2\Pr}{(\Delta Z)^2} + \frac{\Pr}{(R_I + i\Delta R)^2} \quad (30)$$

$$b_4 = -\frac{\left(W_b + |W_b|\right)}{2\Delta Z} - \frac{\Pr}{(\Delta Z)^2} \quad (31)$$

$$b_5 = \frac{\left(W_f - |W_f|\right)}{2\Delta Z} + \frac{\Pr}{(\Delta Z)^2} \quad (32)$$

$$c = -\frac{\Pr Ra\left(\theta_{i+1,j} - \theta_{i-1,j}\right)}{2\Delta R} \tag{33}$$

$$\psi_{i,j}^{it+1} = (1-\Omega)\psi_{i,j}^{it} + \frac{\Omega}{4}\begin{bmatrix} (R_i + i\Delta R)(\Delta R)^2 \omega'_{i,j} + \\ \left(\frac{R_i + \Delta R(i-0.5)}{(R_i + i\Delta R)}\right)\psi_{i+1,j}^{it} \\ \left(\frac{R_i + \Delta R(i+0.5)}{(R_i + i\Delta R)}\right)\psi_{I-1,J}^{it+1} + \\ \left(\psi_{i,j+1}^{it+1} + \psi_{i,j-1}^{it+1}\right) \end{bmatrix} \tag{34}$$

Where the parameter (Ω) is the over relaxation coefficient and its value is $(1 \le \Omega \le 1.5)$.
The local Nusselt number at the heated wall:

$$Nu_l = -\left(\frac{\partial \theta}{\partial R}\right) \tag{35}$$

The average Nusselt number along a single channel wall is defined by**[Schwab,1970]**:

$$N\bar{u} = -\frac{1}{l}\int_0^l Nu \; dZ \tag{36}$$

The overall heat transfer can be calculated by:

$$Q = 2\pi \; N\bar{u} \tag{37}$$

RESULTS AND DISCUSION

Effect of Different Parameters on Heat Transfer.

Streamlines and Isotherms.

Fig.(2a and b) show the streamlines and isotherms for different values of Ra_l without radiation, heat generation and magneto hydrodynamic (MHD). The mechanism of the flow occurs when the fluid near the hot wall is heated causing the density to be decreased and the fluid will be start to move upward nearby the hot wall towards the cold wall. It can be seen that the values of streamlines and isotherms at the cylinder surface increased when Ra_l increased. The isotherms will be closer to the cylinder and its value decreased from the surface to the ambient as Ra_l increased.

Fig. (3) and **Fig.(4)** show the effect of radiation for different values of Ra_l and with no heat generation and MHD. It is clear that the increase of radiation cause a distinct increase in the values of streamlines and a wide region of temperature distribution in the lateral direction for Ra_l (10^2) but for higher Ra_l the region of temperature distribution will be decreased and the streamlines will be closer to the cylinder.

A slight increase in streamlines and isotherms are shown in **Fig. (5)** and **Fig. (6)** when MHD increases for different values of Ra_l for the case of no radiation and heat generation. The flow exhibits a simple circulating pattern rising a long the hot wall and descending along the cold wall of the cavity. It is interesting to note that as the strength of the magnetic field increases the central streamlines are elongated horizontally and the temperature stratification in the core diminishes. The isotherms are almost parallel and are nearly conduction like and this is due to the suppression of convection by the magnetic field. For higher Rayleigh number and low MHD, the thermal boundary layers are well established along the side walls and the temperature stratification exists. This is because convection is the dominant mode of heat transfer at high Rayleigh number. From these figures, it is also observed that for higher Rayleigh number the effect of MHD on the temperature distribution is not prominent compared to that in the case of small Ra_l.

The Variation of Mean Nu with Ra_l.

The variation of total mean Nusselt number Nu with Ra_l is shown in **Fig.(7a)** with and without radiation for no heat generation and MHD. The increase of Nu is very clear when radiation effect is included and it is about 18%.

The variation of total mean Nusselt number Nu with Ra_l is shown in **Fig.(7b)** with and without heat generation for no radiation and MHD. The increase in Nu when the effect of heat generation is included and it is about 2.68 % for $Ra = 10^2$ and this percent continued until $Ra > 10^4$ then decreased and the curves coincides.

Fig.(7c) shows the variation of total mean Nu with Ra_l with and without MHD for no radiation and heat generation. The increase in Nu by 0.2% with MHD included is insignificant and the curves seem to be coincides.

The Effect of MHD on Nu Including Other Parameters

Fig.(8 a and b) show the variation of Nu with MHD for different values of Ra_l and heat generation with no radiation. When heat generation included the effect of MHD increased and cause to increase the rate of heat transfer and this is clear at higher values of Ra_l but for the situation of applying radiation, Nu increase as the heat generation and Ra_l increase but MHD has no effect.

Fig.(9 a and b) show the variation of Nu with MHD for different values of Ra_l and heat radiation with no heat generation. For all values of Ra_l the effect of heat radiation to increase the rate of heat transfer when MHD increase.

The Variation of Dimensionless Temperature with Cylinder Radius.

Fig.(10a, b and c) shows variation of dimensionless temperature with R at center length of cylinder Z=0.5 and $Ra_l = 10^3$. **Fig (10a)** shows the effect of heat radiation on dimensionless temperature at no Q and MHD where increasing the value of heat radiation increase the dimensionless temperature and maximum increases at R=0.916 by 32.4% as N increases from 0.0 to 10.0. **In Fig (10b)** the effect of heat generation Q on dimensionless temperature is shown for no MHD and N. increasing the value of heat generation increase the dimensionless temperature and maximum increases at R=0.713 by 18.48% as Q increases from 0.0 to 2.0. **Fig (10c)** shows the effect of MHD on dimensionless temperature for no Q and N where the increase of MHD cause to increase the dimensionless temperature and maximum increases at R=0.651 by 11.83% as M increases from 0.0 to 1.0.

The Variation of Dimensionless Velocity profile with Cylinder Radius

Fig.(11a, b and c) shows the variation of velocity profiles with R at center length of cylinder Z=0.5 and $Ra_l = 10^3$. **Fig. (11a)** shows that the velocity profile is influenced considerably and increase when the value of the heat generation Q increases. But near the surface of the cylinder velocity increases significantly and then decreases slowly. The maximum values of the velocity are 0.652, 0.743, and 0.805 for Q=0.0, 1.0, and 2.0 respectively which occur at R=0.248, here it is observed that the velocity increases by 24.48% as Q increases from 0.0 to 2.0. In **Fig (11b)** the velocity distribution decrease slightly as the magnetic parameter M=(0.0,0.5,1.0), increase in the region $R \in \{0,2\}$ but near the surface of the cylinder velocity increases and become maximum and then decreases slowly to the end of the cylinder. The maximum value of the velocity are 0.652, 0.581, and 0.4808 for M=0.0, 0.5 and 1.0 respectively which occur at R=0.25, here the velocity decrease by 35.6% as M increases from 0.0 to 1.0. **Fig.(11c)** shows that the velocity profile is influenced considerably and increases when the value of heat radiation parameter N increases. But near the surface of the cylinder velocity increases significantly and then decreases slowly to the end of the cylinder. The maximum values of the velocity are 0.652, 0.252, and 0.154 for N=0.0, 5.0, and 10.0 respectively which occur at R=0.249, here it is observed that the velocity increases by 89.52% as N increases from 0.0 to 10.0.

A correlation has been set up to give the average Nusselt number variation with Ra, M and N and Q. This correlation is made by using the computer program (DGA v1.00).

$$Nu = 4.775\, Ra^{0.0575} + 0.0046\, M^{0.973} + 0.0346\, N^{1.487} +$$

For $Ra=10^5$ the correlation equation become.

$$Nu = 4.775\, Ra^{0.0575} + 0.013\, M + 0.0745\, N^{1.42} + 0.04\, Q$$

COMPARISON OF THE RESULTS

To the best of our knowledge there is no previous work studied the case of external flow on a cylinder taking the effect of MHD. A comparison was done between the result of the present study for the variation of Nu with Ra_l and the correlation of **(Mc Adams, 1954)** which is shown in **Fig.(12)** with a deviation of 5% for the case of no MHD, radiation and heat generation.

Fig.(13) shows a comparison between the present study and the work of **[Ganesan, 2002]** with and without radiation and for no MHD and no heat generation which shows an increase of Nu with Ra_l for a deviation of 1.7% with radiation and 1.5% without radiation. The results for all the previous figures show good agreement with other researches.

CONCLUSIONS

From the results of the present work and for the cylinder that described previously, the following conclusions can be obtained:

1. The totals mean Nusselt number (Nu) increases by 53.5% with the increase of Rayleigh number (Ra_l) from 10^2 to 10^5.
2. The totals mean Nusselt number (Nu) increases by 18 % with the increase of radiation parameter (N) from 0 to 10.
3. The total mean Nusselt number Nu increases by 0.2 % with increase MHD parameter M from 0 to 1.
4. The totals mean Nusselt number (Nu) increases by 2.68% with the increase of heat generation parameter (Q) from 0 to 2.
5. Dimensionless temperature θ increases by 18.48 % with increase heat generation parameter Q from 0 to 2 at R=0.5.
6. Dimensionless temperature (θ) increases by 11.83 % with the increase of (MHD) parameter (M) from 0 to 1 at R=0.5.
7. Dimensionless temperature (θ) increases by 32.4 % with the increase of radiation parameter (N) from 0 to 10 at R=0.5.
8. When Rayleigh number (Ra) increase the effect of MHD, radiation and heat generation decrease on total mean Nusselt number (Nu).

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Table (1) Boundary Conditions

Line	Θ	ψ	ω	W,U
AB	$\frac{\partial \theta}{\partial R}=0$	0.0	0.0	0.0
BC	1.0	0.0	$\omega=-\frac{1}{Z}\frac{\partial^2\psi}{\partial R^2}$	0.0
CD	1.0	0.0	$\omega=-\frac{1}{R}\frac{\partial^2\psi}{\partial Z^2}$	0.0
DE	1.0	0.0	$\omega=-\frac{1}{Z}\frac{\partial^2\psi}{\partial R^2}$	0.0
EF	$\frac{\partial \theta}{\partial R}=0$	0.0	0.0	0.0
FG	0.0	$\frac{\partial \psi}{\partial Z}=0$	0.0	0.0
GH	0.0	$\frac{\partial \psi}{\partial R}=0$	0.0	0.0
HA	$\frac{\partial \theta}{\partial Z}=0$	$\frac{\partial \psi}{\partial Z}=0$	0.0	0.0

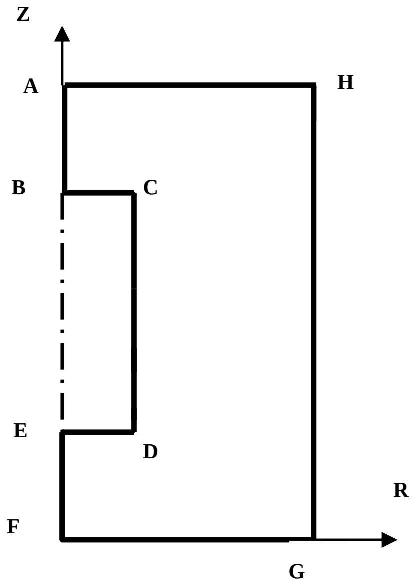


Fig. (1) Boundary condition of the problem

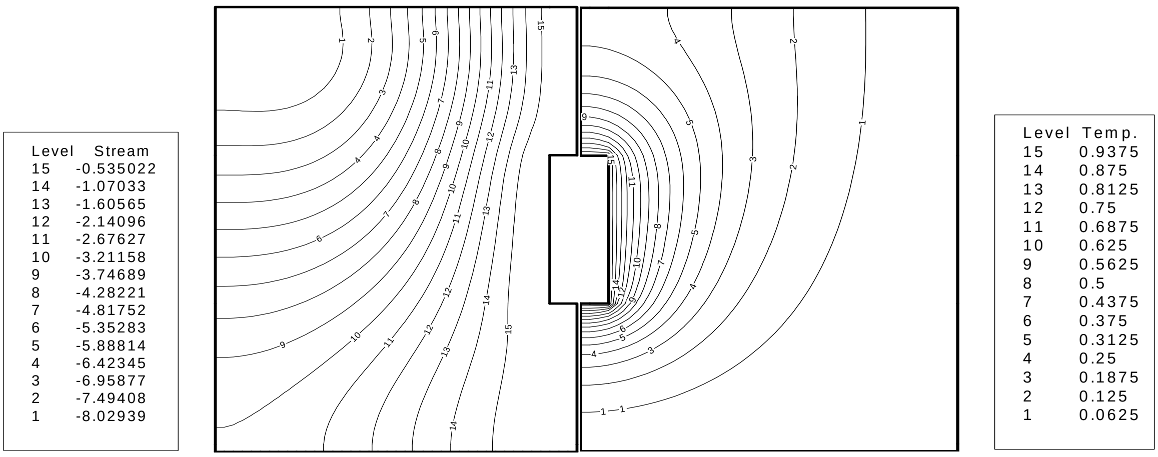


Fig. (2a) Streamlines and isotherm for $Ra_l = 10^2$, M=0 , N=0 , Q=0

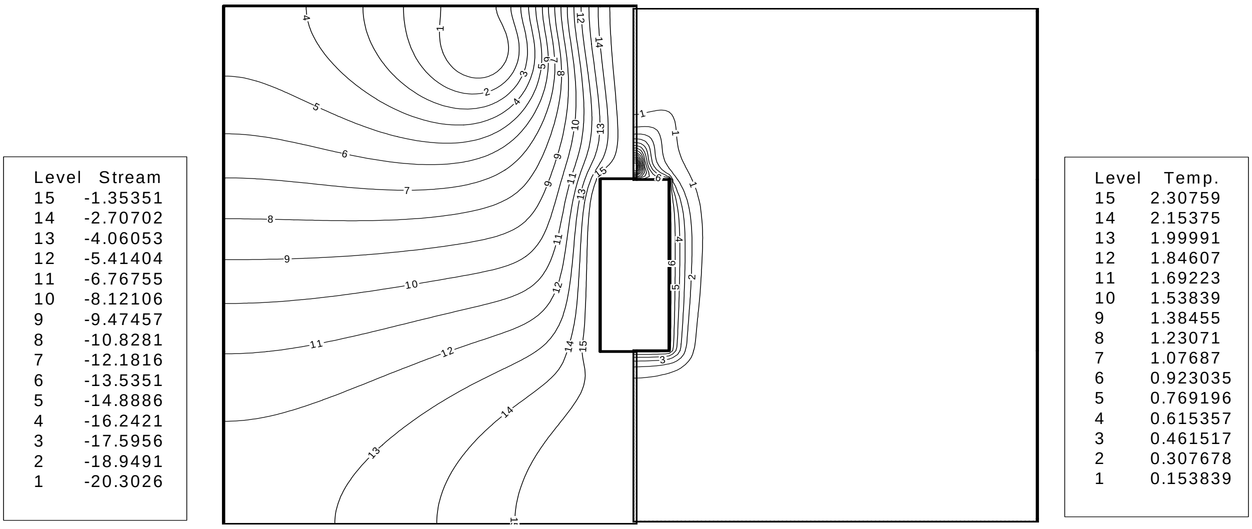


Fig. (2b) Streamlines and isotherm for $Ra_l = 10^5$, M=0 , N=0 , Q=0

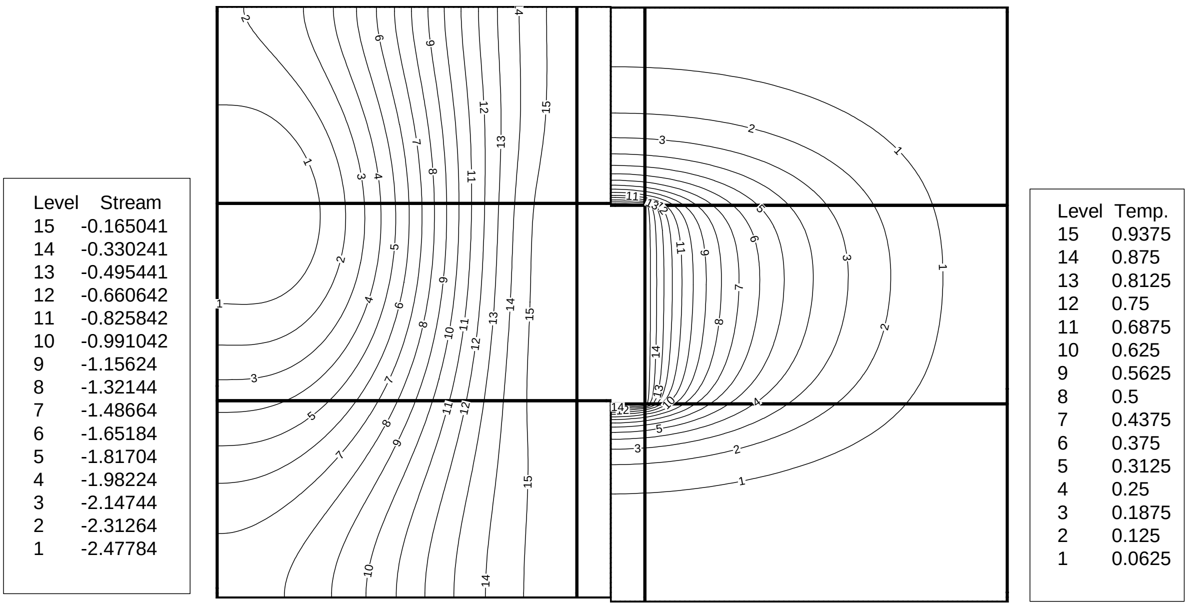


Fig. (3) Streamlines and isotherm for $Ra_l = 10^2$, M=0 , N=5 , Q=0

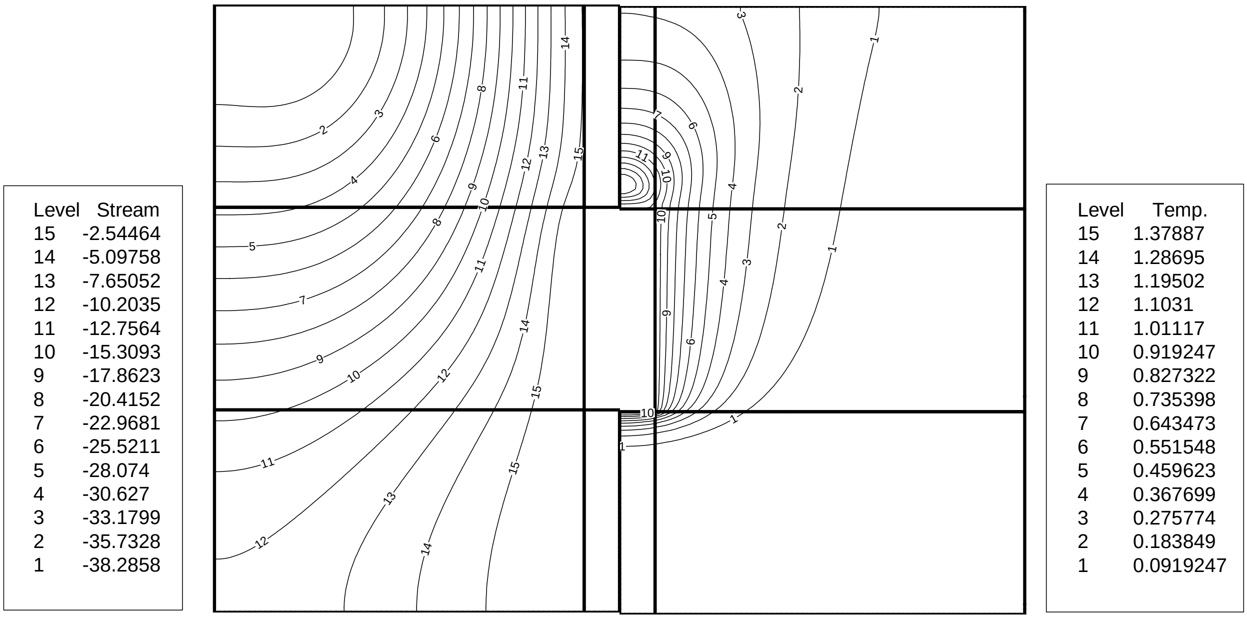


Fig. (4) Streamlines and isotherm for $Ra_l = 10^4$, M=0 , N=5 , Q=0

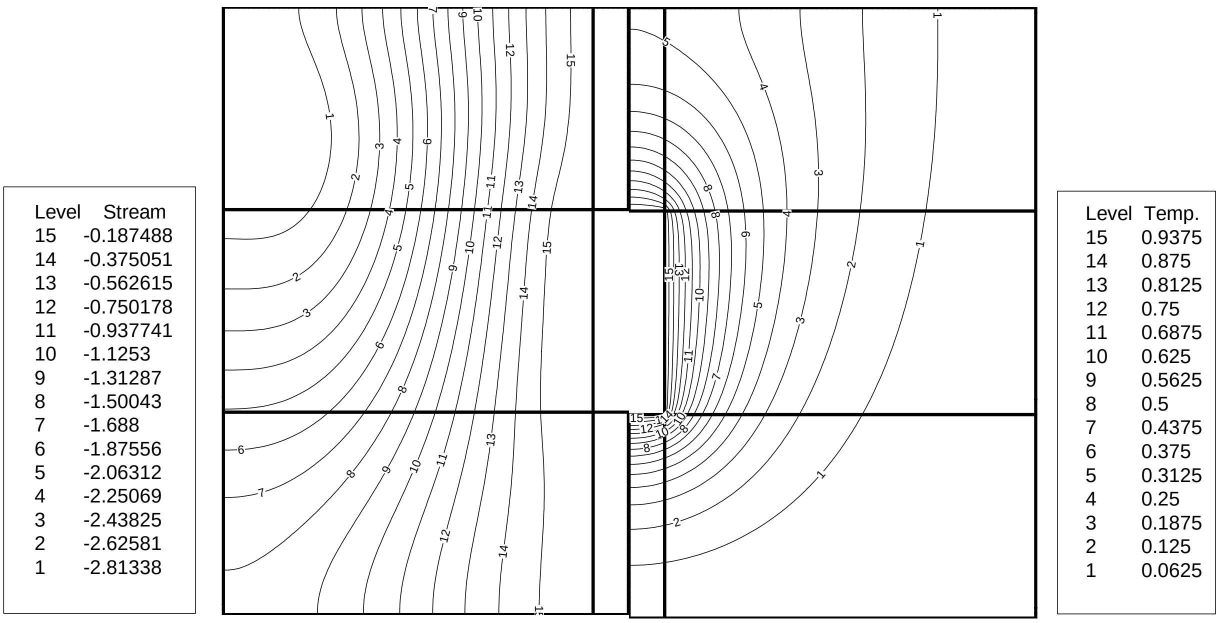


Fig. (5) Streamlines and isotherm for $Ra_l = 10^2$, M=1, N=0 , Q=0

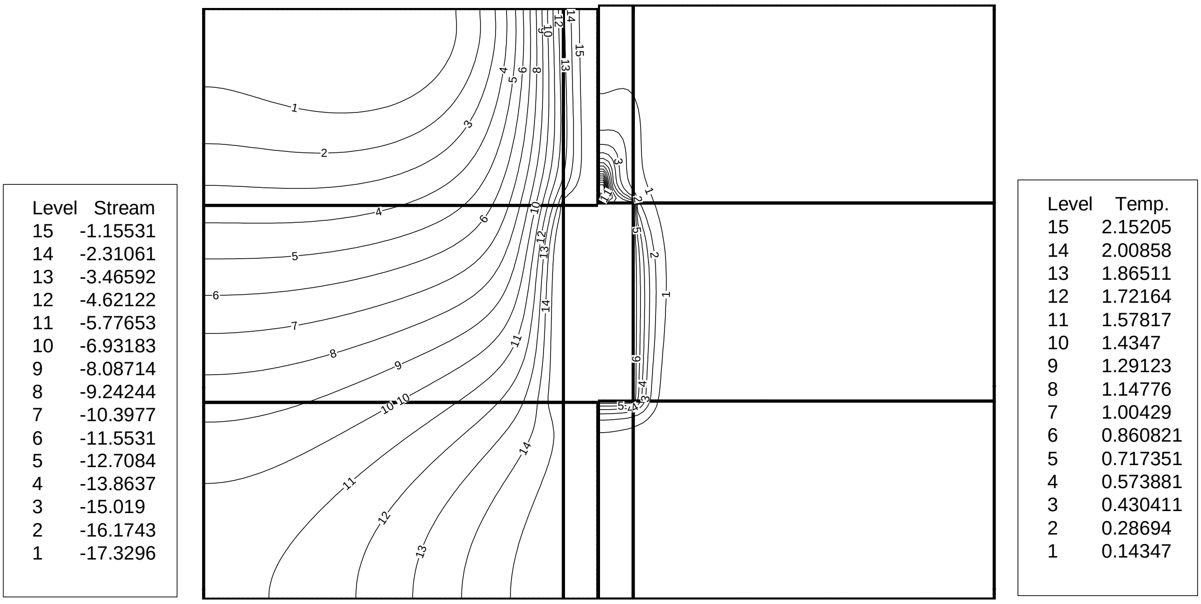
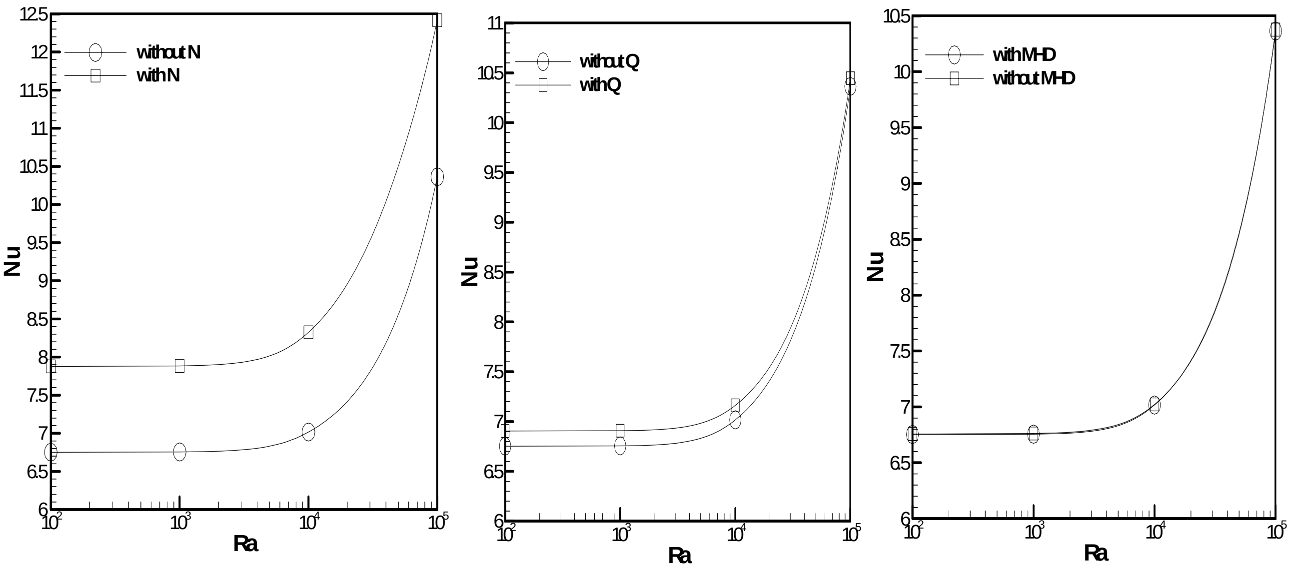
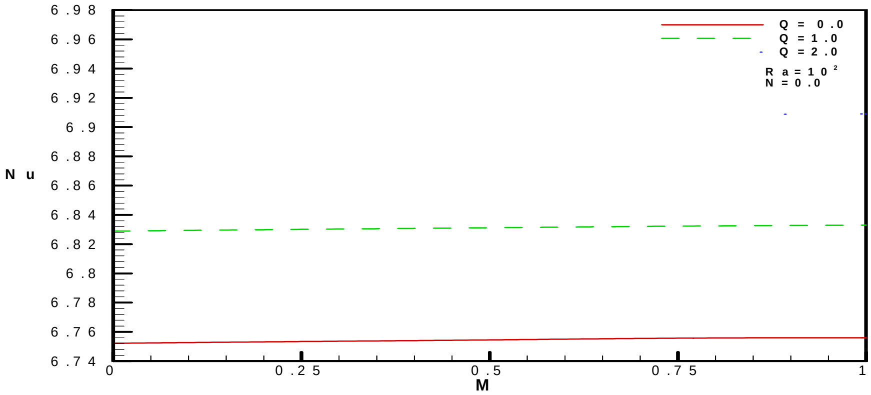


Fig. (6) Streamlines and isotherm for $Ra_l = 10^5$, M=1, N=0 , Q=0

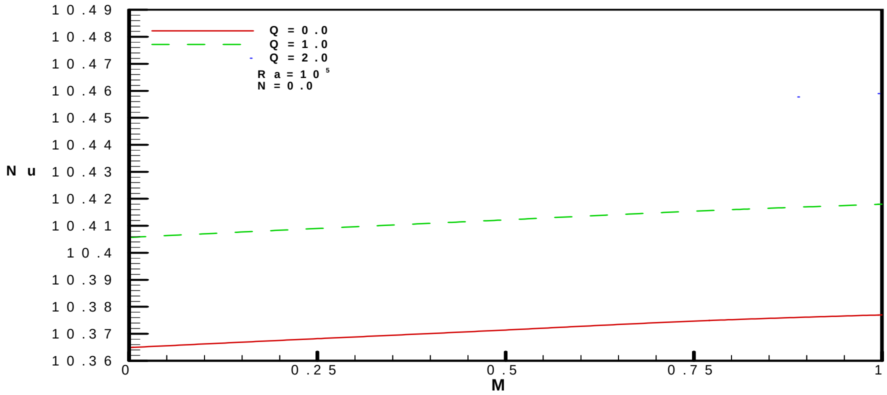


(a) (b) (c)

Fig. (7) Variation of total mean Nusselt number with Rayleigh number

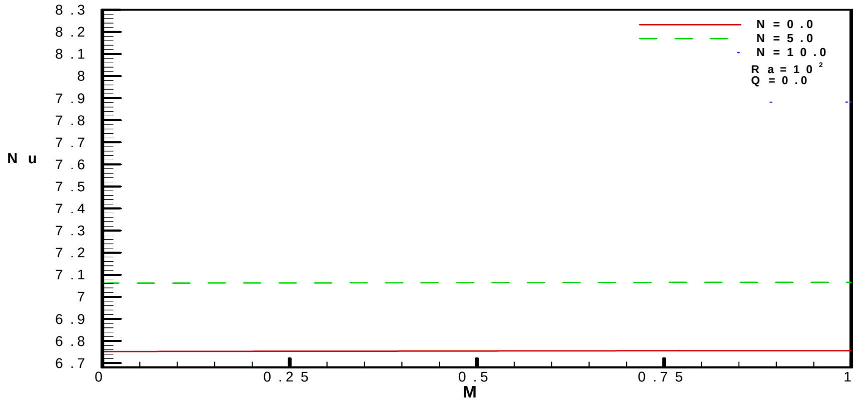


a. ($Ra_l = 10^2$)

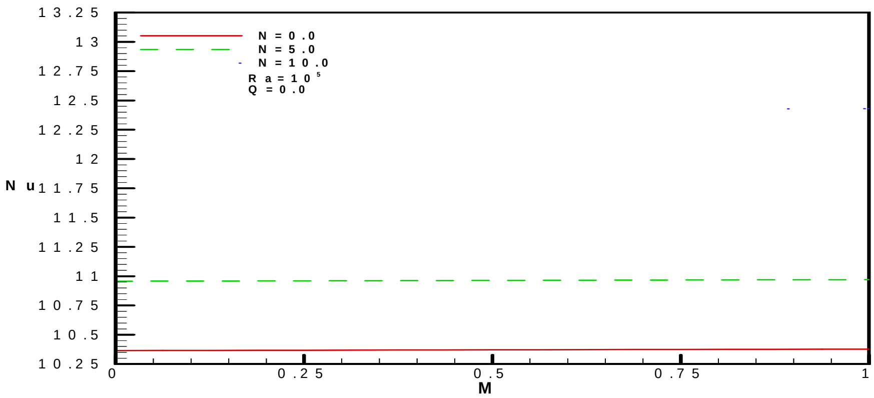


b. ($Ra_l = 10^5$)

Fig (8) The variation of Nu with MHD for different values of Ra and Q

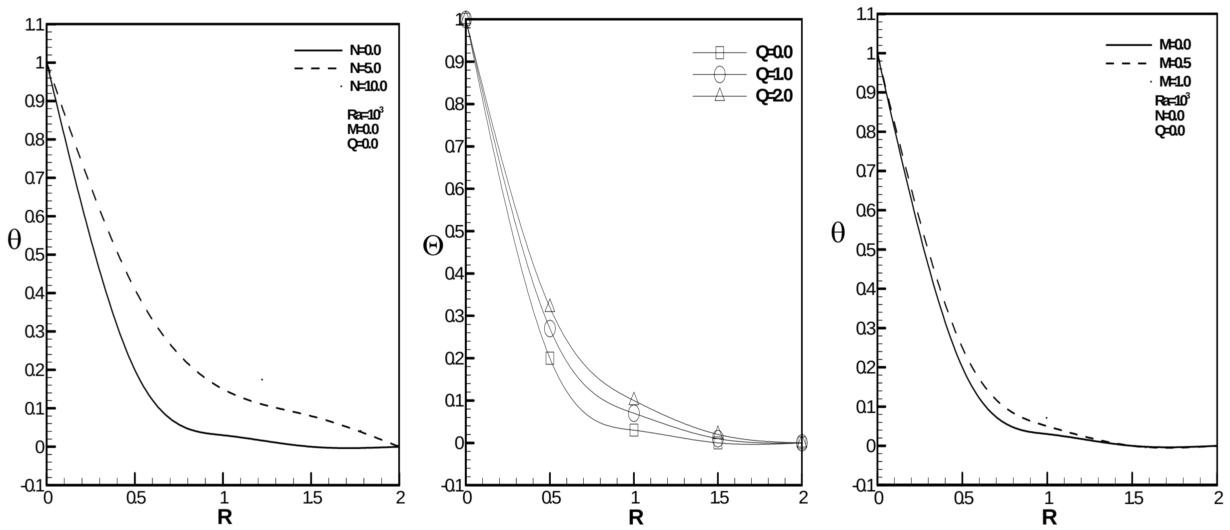


a. ($Ra_l=10^2$)



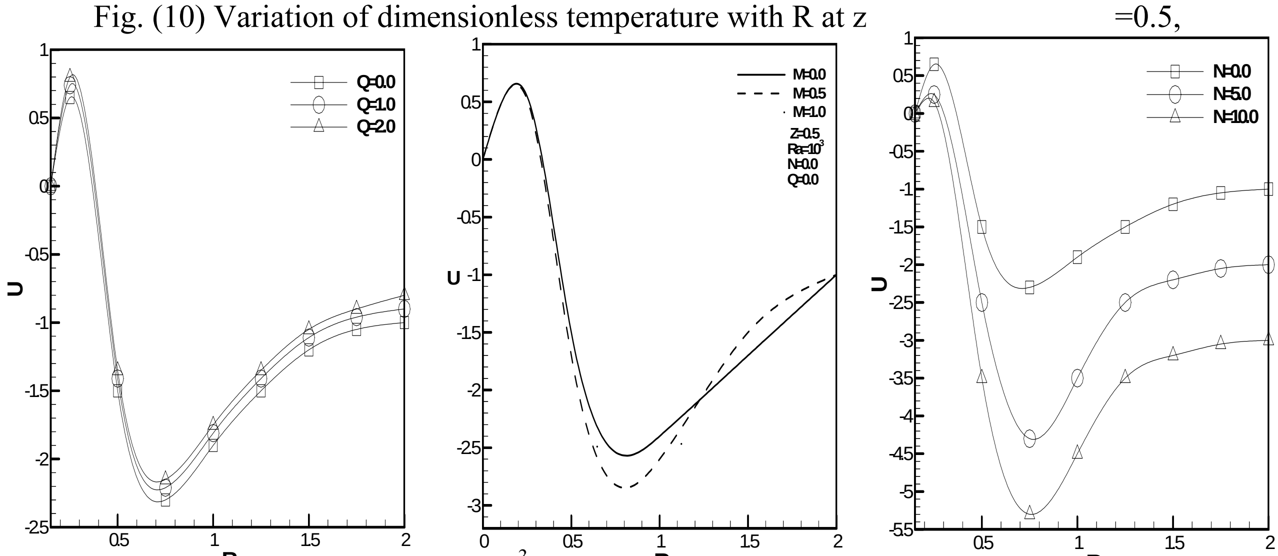
b. ($Ra_l=10^5$)

Fig (9) The variation of Nu with MHD for different values of Ra_l and N



(a) (b) (c)

Fig. (10) Variation of dimensionless temperature with R at z =0.5, $Ra_l=10^3$



(a) (b) (c)

Fig. (11) Variation of velocity profiles with R at z =0.5, $Ra_l=10^3$

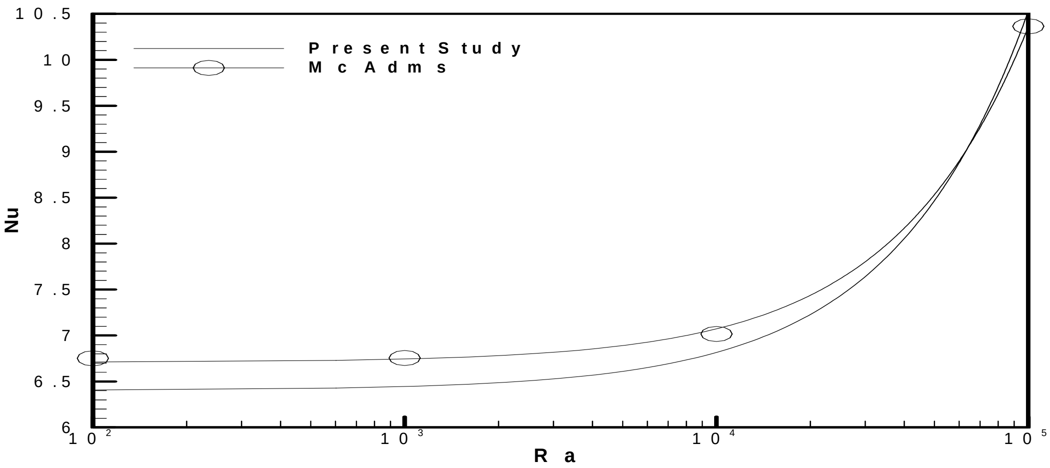


Fig. (12) Comparison of the present work with **[Mc Adam, 1954]**

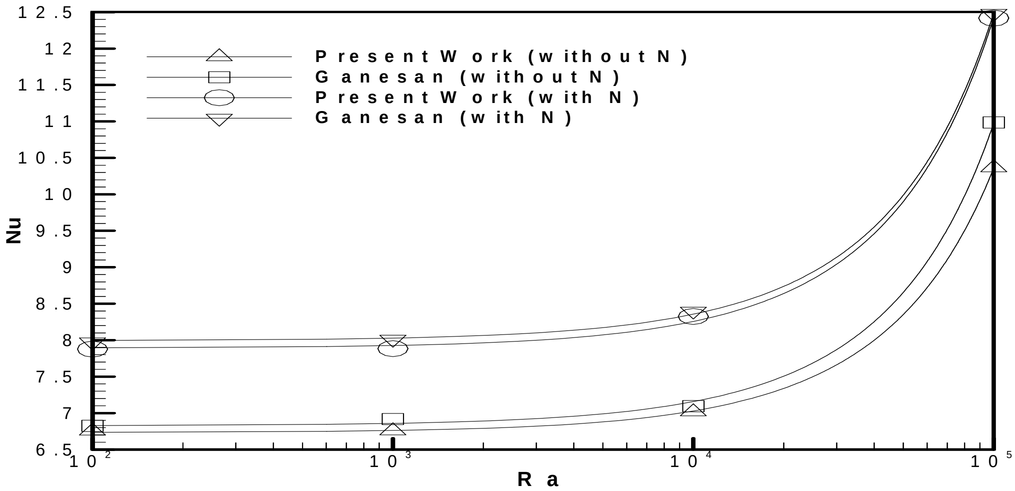


Fig. (13) Comparison of the present work with **[Ganesan, 2002]**

NOMENCLATUR

LATIN SYMBOLS

Symbol	Description	Unit
Cp	Specific heat at constant pressure	kJ/kg.K
f_r	Electromagnetic force in (r) direction	m/s^2
f_z	Electromagnetic force in (z) direction	m/s^2
g	Acceleration due to gravity	m/s^2
h	Heat transfer coefficient	$W/m^2.^oC$
i	R-direction directory	-
j	Z-direction directory	-
K	Thermal conductivity	$W/m.^oC$
K^*	Mean absorption coefficient	m^{-1}
l	Length of cylinder	m
M	Dimensionless Magneto hydrodynamic parameter	-
n	Indicate the unit vactor	m
N	Dimensionless Conduction-Radiation parameter	-
Nu	Average Nusselt number(Nu=hl/k)	-
P^*	Air pressure	N/m^2
P	Normalized air pressure	-
Pr	Prandtl number(Pr=ν/α)	-
q_r	Radiative heat flux	W
q	Overall heat transfer	W
Q	Dimensionless overall heat generation	-
Q_o	Heat generation/absorption	$W/m^3.^oC$
q'''	Volumetric heat generation	W/m^3
r	Radial direction	m
R	Dimensionless Radial direction	-
Ra_l	Rayleigh no. $\left(Ra_l = \frac{\Pr g\,\beta (T_w - T_\infty) l^3}{\nu^2} \right)$	-
T	Air temperature	K
T_∞	Ambient temperature	K
u	Radial velocity	m/s
U	Dimensionless Radial velocity	-
w	Vertical velocity	m/s
W	Dimensionless Vertical velocity	-
X	Nodes number in r-direction	-
Y	Nodes number in z-direction	-
z	Vertical direction	m
Z	Dimensionless Vertical direction	-

CREAK SYMBOLS

Symbol	Description	Unit
α	Thermal diffusivity	m^2/s
β	Coefficient of thermal expansion	K^{-1}
ε	Emissivity	-
θ	Dimensionless temperature	-
λ	Radii ratio	-
μ	Viscosity	kg/m.s
ν	Kinematics' viscosity	m^2/s
ρ	Air density	kg/m^3
σ	Stefan-Boltzmann constant	$W/m^2.K^4$
ψ	Dimensionless stream function	-
ω	Dimensionless vorticity	-

Subscript

Symbol	Description	Unit
(i,j)	Grid nodes in (r,z) direction	-