THERMAL BUCKLING OF RECTANGULAR PLATES WITH DIFFERENT TEMPERATURE DISTRIBUTION USING STRAIN ENERGY METHOD

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ABSTRACT:

By using governing differential equation and the Rayleigh-Ritz method of minimizing the total potential energy of a thermoelastic structural system of isotropic thermoelastic thin plates, thermal buckling equations were established for rectangular plate with different fixing edge conditions and with different aspect ratio. The strain energy stored in a plate element due to bending, mid-plane thermal force and thermal bending was obtained. Three types of thermal distribution have been considered these are: uniform temperature, linear distribution and non-linear thermal distribution across thickness. It is observed that the buckling strength enhanced considerably by additional clamping of edges. Also, the thermal buckling temperatures and thermal buckling load have lowest values at first mode of buckling for all types of ends condition and with all values of aspect ratios.

Key words: **Thermal bulking, Strain energy, Thin plate, Ends condition, Buckling mode**

الخلاصة :

باستخدام المعادلات التفاضلية الرئيسية الخاصة وطريقة ريلي-رتز لخفض الطاقة الكامنة تم استحداث معادلات خاصة بالانبعاج الحراري الخاص بالصفائح المستطيلة المتجانسة الرقيقة متضمنة مختلف حالات النهايات مع مختلف نسب الأبعاد. تم إيجاد طاقة الانفعال المخزونة الناتجة من الانحناء في عنصر الصفيحة والقوى والعزوم الحرارية في المستوي الوسطي للصفيح بالإضافة إلى درجة الحرارة الخاصة بالانبعاج القلق .تم اعتماد ثلاثة أنواع مختلفة من التوزيع الحراري وهي:درجة حرارة منتظمة، وتوزيع درجة حرارة خطية عبر السمك وتوزيع درجة حرارة لا خطية عبر السمك. لوحظ أن مقاومة الانبعاج تزداد مع تثبيت النهايات كما أن قوى الانبعاج الحراري تملك اقل قيم لها في الطور الأول للانبعاج لكل أنواع تثبيت النهايات ومع كل نسب الأبعاد.

INTRODUCTION:

Thermoelastic buckling of beam-plates and plates has long been of vivid interest to researchers. Perfectly isotropic beams and plates, which are fixed from motion in their plane, are found to exhibit bifurcation buckling at a critical temperature when they are exposed to a homogeneous temperature field (i.e., the plate will remain flat during increasing temperature until a critical temperature is reached at which point the magnitude of transverse deflection becomes indeterminate [Shariat, & Eslami 2006]

When a plate is compressed in its midplane, it becomes unstable and begins to buckle at ascertain critical *value* of the in-plane force. Buckling of plates is qualitatively similar to column buckling [Chen& Virgin 2006]. However; a buckling analysis of the former case is not performed as readily as for the latter. Plate-buckling solutions usually involve considerable difficulty and subtlety [Matsunaga 2006] and the condition that result in the lowest eigenvalue, or the actual buckling load, are not at all obvious in many situations. This is especially true in plates having other than simply supported edges.

For a plate, the in-plane load that results in an elastic instability, as in the case of a beam-column, is independent of the lateral loading. Thin plates or sheets, although quite capable of carrying tensile loadings, are poor in resisting compression. Usually, buckling or wrinkling phenomena observed in compressed plates (and shells) takes place rather suddenly and are very dangerous. However, a change in temperature may also induce instability of a thin Structure, such as bifurcation buckling, snap-through buckling, or 'just' unacceptable large out-of-plane deflections of the structure [Timoshenko & Krieger 1959]. The present work takes theoretical approach to determine the buckling temperature and buckling mode for a flat, rectangular plate with various types of thermal loads and edge conditions, buckling eigenvalue problem

ANALYTICAL STUDY:

The plate analyzed usually has been assumed to be composed of a single homogeneous and isotropic material with shape and dimensions as in **Fig. (1)** [Ko 95].

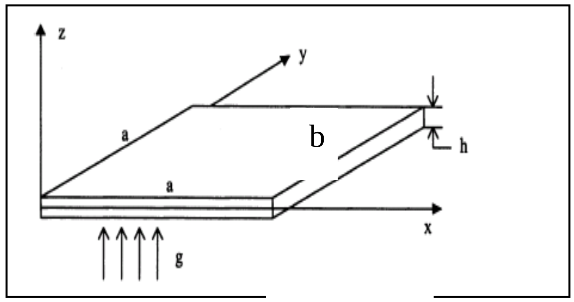

Fig. (1) Schematic Diagram of Thin Plate

Thermal Distribuation Types:

Three types of thermal distribution on plate have been considered

A- Uniform heating of plate ,the whole plate been warmed up to specificic temperature then $\Delta T = Tc$

B- Linear temperature distribution across the plate thickness h then

$$\Delta T = T_{0l} + zT_{1l} \tag{1}$$

C-Temperature field across the plate thickness is assumed in nonlinear form as

$$\Delta T = T_{0n} + zT_{1n} + z^2T_{2n} \tag{2}$$

Boundary Conditions:

General closed – form solutions are given of a thermoelastiuc rectangular plate with different aspect ratio (a/b) and various elementary boundary conditions on each of the four edges. appendix A collect some important combinations of end boundary conditions. [Let the plate be placed in a coordinate system with the origin at it center and the edge width (a) be parallel to x – axis and and the edge width (b) be parallel to y as in **Fig. (1)**

Strain Energy Methods:

As an alternative to the equilibrium methods, the analysis of deformation and stress in an elastic body can be accomplished by employing energy methods. These two techniques are respectively, the newtonian and lagrangian approaches to mechanics. The latter is predicted upon the fact that the governing equation of a deformed elastic body is derivable by minimizing the energy associated with deformation and loading. Applications of energy methods are effective in situations involving irregular shapes, non-uniform loads, variable cross sections, and anisotropic materials [Langhaar 1962]. We shall begin our discussion of energy techniques by treating the case of loaded thin plates. The strain energy stored in an elastic body, for a general state of stress, is given by [Lee 2002]

$$\Pi_b = \frac{1}{2}\iiint_V (\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz})dxdydz \tag{3}$$

Integration extends over the entire body volume. Based upon the assumptions of thin plates $\sigma_x, \gamma_{xz}, \gamma_{yz}$ can be omitted. Thus, introducing Hook's law, the above expression reduces to the following form involving only stresses and elastic constants:

$$\Pi_b = \frac{1}{2}\iiint_V \{(\sigma_x^2 + \sigma_y^2 - 2v\sigma_x\sigma_y)/E + \tau_{xy}^2/G)\}dxdydz \tag{4}$$

For a plate of uniform thickness, Eq. (4) may be written in terms of deflection w as follows

$$\Pi_b = \frac{1}{2}\iint_A D\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-v)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\}dxdy \tag{5}$$

Where A is the area of the plate surface .The strain energy associated with in plane forces is given by

$$\Pi_N = -\frac{1}{2}\iint_A \left[N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial x}\right)\right]dxdy \tag{6}$$

Also the strain energy of thermal moments will have the form

$$\Pi_M = \iint_A \frac{M_t}{(1-v)}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)dxdy \tag{7}$$

The total potential energy will equal to

$$\Pi_{strain} = \Pi_b + \Pi_N + \Pi_M \tag{8}$$

The expression $\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2$ represents the Gaussian curvature of the deformed surface and for the plate with general three types of ends conditions will have been studied; all edges are simply supported, all edges are clamped and two opposite edges are simply supported and the another's are clamped, then the Gaussian curvature becomes zero[Bhat 1985] . As a result the bending strain energy expression simplified to

$$\Pi_b = \frac{1}{2}\iint_A D\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2\right\}dxdy$$

Where $D = \frac{Eh^3}{12(1-\upsilon^2)}$, here the quantities

$$N_t = \alpha E \int_{-h/2}^{h/2} (\Delta T)dz$$

$$M_t = \alpha E \int_{-h/2}^{h/2} (\Delta T) z dz \qquad (9)$$

Are termed the thermal stress resultants.

Uniform Heating of Plate:

The whole plate been warmed up to specificic temperature then $\Delta T = Tc$ then there is no thermal moments developed in plate $\Pi_M = 0$ then the total strain will become $\Pi_{strain} = \Pi_b + \Pi_N$

Assuming all edges are restrained then

$$N_x = N_y = -\frac{N_t}{1-\upsilon}$$

$$N_{xy} = 0 \qquad (10)$$

In Ritz method we minimize the Eq. (8) with respect to the arbitrary parameter w_{ij} i.e.

$$w(x,y) = \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij} X_i(x) Y_j(y)$$

Assuming $\frac{\partial \Pi_{strain}}{\partial A_{ij}} = 0$ (11)

Substituting w(x, y) into Ritz formula with in plane thermal force of Eq. (11) then

$$\sum_{k=1}^{m}\sum_{j=1}^{n}\left[\begin{array}{l} D\int_0^a\int_0^b \left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy \\ +\frac{N_t}{(1-\nu)}\int_0^a\int_0^b \left((X')^2Y^2 + X^2(Y')^2\right)\ dxdy \end{array}\right] A_{ij} = 0$$

(12)

Assume that $A_{ij} \neq 0$ then

$$N_{tcr} = -\frac{(1-\nu)D\int_0^a\int_0^b \left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\ \right)dxdy}{\int_0^a\int_0^b \left((X')^2Y^2 + X^2(Y')^2\right)dxdy}$$

(13)

In terms of temperature critical thermal buckling temperature Tc_{cr}

$$Tc_{cr} = -\frac{h^2\int_0^a\int_0^b \left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\ \right)dxdy}{12(1+\nu)\alpha\int_0^a\int_0^b \left((X')^2Y^2 + X^2(Y')^2\right)dxdy}$$

(14)

If edges at x=0, a are restrained and edges at y=0, b are unrestrained .Then Eq. (8) will have the form

$$\Pi_N = \frac{N_t}{2(1-\nu)}\int_0^a\int_0^b \left(\frac{\partial w}{\partial x}\right)^2 dxdy$$

(15)

Assume the same w(x, y) as in Eqs. (11) taking the same procedure of the previous example then the critical thermal buckling force will introduce as

$$N_{tcr} = -\frac{(1-\nu)D\int_0^a\int_0^b \left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\ \right)dxdy}{\int_0^a\int_0^b \left((X')^2Y^2\right)dxdy}$$

(16)

In terms of critical buckling temperature

$$Tc_{cr} = -\frac{h^2\int_0^a\int_0^b \left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\ \right)dxdy}{12(1+\nu)\alpha\int_0^a\int_0^b \left((X')^2Y^2\right)dxdy}$$

(17)

Linear Temperature Distribution :

Assume the temperature across the plate thickness h have the form $\Delta T = T_{0l} + zT_{1l}$ .In Ritz method we minimize the Eq. (8) with respect to the arbitrary parameter w_{mn}

$$\frac{\partial \Pi_b}{\partial A_{ij}} + \frac{\partial \Pi_N}{\partial A_{ij}} + \frac{\partial \Pi_M}{\partial A_{ij}} = 0 \tag{18}$$

Assuming all edges are restrained taking the form of deflection as in Eq. (11) and substituting w(x, y) into Ritz formula with in plane thermal force and thermal moments then Eq. (18) will be

$$\sum_{k=1}^{m}\sum_{j=1}^{n}\begin{bmatrix} D\int_0^a\int_0^b \left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy \\ +\dfrac{N_t}{(1-\nu)}\int_0^a\int_0^b ((X')^2Y^2 + X^2(Y')^2)\ dxdy \end{bmatrix} A_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{M_t}{(1-\nu)}\int_0^a\int_0^b (X''Y + XY'')dxdy = 0 \tag{19}$$

Finding A_{ij} from Eq. (19)

$$A_{ij} = -\frac{M_t\int\int (X''Y + XY'')dxdy}{(1-\nu)D\int\int\left((X'')^2Y^2 + 2XXYY + X^2(Y'')^2\right)dxdy + N_t\int\int\left((X')^2Y^2 + X^2(Y')^2\right)dxdy} \tag{20}$$

Then the deflection

$$w(x,y) = -\sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\frac{M_t\int_0^a\int_0^b (X''Y + XY'')dxdy}{(1-\nu)D\int_0^a\int_0^b\left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy + N_t\int_0^a\int_0^b\left((X')^2Y^2 + X^2(Y')^2\right)dxdy}\right\}X_iY_j \tag{21}$$

In terms of temperatures substituting Eqs. (9) in (21) then

$$w(x,y) = -\sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\frac{\alpha E h^3 T_{1l}\int_0^a\int_0^b (X''Y + XY'')dxdy}{12\left\{(1-\nu)D\int_0^a\int_0^b\left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy + \alpha E h T_{0l}\int_0^a\int_0^b\left((X')^2Y^2 + X^2(Y')^2\right)dxdy\right\}}\right\}X_iY_j \tag{22}$$

From Eqs. (21)& (22), the deflection w(x, y) tends to infinity when the critical thermal forces and critical thermal buckling temperature satisfy the following condition:

$$N_{tcr} = -\frac{(1-\nu)D\int\int\left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy}{\int\int\left((X')^2Y^2 + X^2(Y')^2\right)dxdy} \tag{23}$$

$$T_{0lcr} = \left[\left[-\frac{h^2\int\int\left[\left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy\right]}{12(1+\nu)\alpha\int\int\left((X')^2Y^2 + X^2(Y')^2\right)dxdy}\right]\right] \tag{24}$$

Assume the same w(x, y) as in Eqs. (11) Substituting it into Ritz formula with in plane thermal force and thermal moments with edges at (x=0, a) are restrained and edges at (y=0, b) are unrestrained then Eq. (19) will be

$$\sum_{k=1}^{m}\sum_{j=1}^{n}\begin{bmatrix} D\int_0^a\int_0^b \left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy \\ +\dfrac{N_t}{(1-\nu)}\int_0^a\int_0^b ((X')^2Y^2)\ dxdy \end{bmatrix} A_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{M_t}{(1-\nu)}\int_0^a\int_0^b (X''Y + XY'')dxdy = 0 \tag{25}$$

then

$$A_{ij} = -\frac{M_t \int\int (X''Y + XY'')dxdy}{(1-\nu)D\int\int\left((X'')^2Y^2 + 2XXYY + X^2(Y'')^2\right)dxdy + N_t\int\int\left((X')^2Y^2\right)dxdy} \qquad (26)$$

Then the deflection

$$w(x,y) = -\sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\frac{M_t\int_0^a\int_0^b (X''Y + XY'')dxdy}{(1-\nu)D\int_0^a\int_0^b\left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy + N_t\int_0^a\int_0^b\left((X')^2Y^2\right)dxdy}\right\}X_iY_j \qquad (27)$$

In terms of temperatures substituting Eq. (7) in Eq. (25) then

$$w(x,y) = -\sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\frac{\alpha Eh^3T_{1l}\int_0^a\int_0^b (X''Y + XY'')dxdy}{12\left\{(1-\nu)D\int_0^a\int_0^b\left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy + \alpha EhT_{0l}\int_0^a\int_0^b\left((X')^2Y^2\right)dxdy\right\}}\right\}X_iY_j \qquad (28)$$

From Eqs. (26) & (27), the critical thermal forces and critical thermal buckling temperature as the deflection tends to infinity will develop as

$$N_{tcr} = -\frac{(1-\nu)D\int\int\left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy}{\int\int\left((X')^2Y^2\right)dxdy} \qquad (29)$$

$$T_{0lcr} = -\frac{h^2\int\int\left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy}{12(1+\nu)\alpha\int\int\left((X')^2Y^2\right)dxdy} \qquad (30)$$

Non- Linear Temperature Distribution :

Temperature field across the plate thickness is assumed in nonlinear form as in Eq.(2)

$$\Delta T = T_{0n} + zT_{1n} + z^2T_{2n}$$

The total strain energy and Ritz formulas in terms of in plane thermal forces and thermal moment will have the same form as in case of linear temperature distribution Eqs.(20),(21) and(22). But they different when taking the thermal forces and moments as a functions of temperatures, therefore, when all edges are restrained then the deflection in terms of temperatures will be

$$w(x,y) = -\sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\frac{\alpha Eh^3T_{1n}\int_0^a\int_0^b (X''Y + XY'')dxdy}{12\left\{(1-\nu)D\int_0^a\int_0^b\left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy + \alpha Eh(T_{0n} + h^2T_{2n}/12)\int_0^a\int_0^b\left((X')^2Y^2 + X^2(Y')^2\right)dxdy\right\}}\right\}X_iY_j \qquad (31)$$

Then the deflection tends to infinity when the critical thermal buckling temperature have this magnitude

$$(T_{0n} + h^2T_{2n}/12)_{cr} = -\frac{h^2\int\int\left((X'')^2Y^2 + 2X''XY''Y + X^2(Y'')^2\right)dxdy}{12(1+\nu)\alpha\int\int\left((X')^2Y^2 + X^2(Y')^2\right)dxdy} \qquad (32)$$

Again when edge at(x=0, a) are restrained and edges at (y=0, b) are unrestrained then Eq. (31) will be

$$w(x,y)=$$

$$-\sum_{i=1}^{m}\sum_{j=1}^{n}\left\{\frac{\alpha E h^3 T_{1n}\int_0^a\int_0^b (X''Y+XY'')dxdy}{12\left\{(1-v)D\int_0^a\int_0^b\left((X'')^2Y^2+2X''XY''Y+X^2(Y'')^2\right)dxdy+\alpha Eh(T_{0n}+h^2T_{2n}/12)\int_0^a\int_0^b\left((X')^2Y^2\right)dxdy\right\}}\right\}X_iY_j \quad (33)$$

And the critical thermal buckling temperature will be

$$(T_{0n}+h^2T_{2n}/12)_{cr}=$$

$$-\frac{h^2\int\int\left((X'')^2Y^2+2X''XY''Y+X^2(Y'')^2\right)dxdy}{12(1+v)\alpha\int\int\left((X')^2Y^2\right)dxdy} \quad (34)$$

Thermal Buckling Temperature for Uniform Temperature :

Uniform heating of plate have been considerd ,the whole plate been warmed up to specificic temperature then $\Delta T = Tc$. The corresponding strain components are identically zero all over the domain. Then, the resultant forces are given as

$$N_x = N_y = -\frac{N_t}{1-\upsilon} \quad ,$$

$$N_{xy}=0 \quad N_t = \alpha E h T_c$$

$$M_t = 0 \quad (35)$$

Substituting X_i and Y_j from Eq. (11) with help of appendix C the critical thermal buckling temperature for SSSS ends condition with edges at (x=0, a) are restrained and edges at y=0,b are unrestrained will be:

$$Tc_{cr}(m,n)=\frac{D\pi^2(1-v)(m^2+r^2n^2)^2}{E\alpha h m^2 a^2} \quad (36)$$

For CCCC ends condition with edges at (x=0, a) are restrained and edges at y=0,b are unrestrained the critical thermal buckling temperature as

$$Tc_{cr}=\frac{h^2(\alpha_1^4+2r^2\alpha_2+r^4\alpha_3^4)}{12(1+v)a^2\alpha_1^2} \quad (37)$$

For CSCS ends condition with edges at (x=0, a) are restrained and edges at y=0,b are unrestrained the critical thermal buckling temperature will be

$$Tc_{cr}=\frac{h^2(\beta_1^4+2r^2\beta_2+r^4\beta_3^4)}{12(1+v)a^2\beta_1^2} \quad (38)$$

Thermal Buckling Temperature for Linear Temperature Distribution :

Assume the temperature across the plate thickness h have the form as in Eq.(1) $\Delta T = T_{0l}+zT_{1l}$ So that

$$N_t = \alpha Eh(T_{0n}+h^2T_{2n}/12) \quad \text{and}$$

$$M_t = \frac{\alpha E h^3 T_{1n}}{12} \quad (39)$$

Substituting X_i and Y_j from Eq. (11) with the help of appendix C. For SSSS ends condition when edges at (x=0, a) are restrained and edges at (y=0, b) are unrestrained the critical thermal buckling temperature will be

$$T_{0lcr}(m,n)=\frac{h^2\pi^2(m^2+n^2r^2)^2}{12(1+v)\alpha m^2 a^2} \quad (40)$$

For CCCC with edges at (x=0, a) are restrained and edges at (y=0, b) are unrestrained

$$T_{0lcr} = \frac{h^2(\alpha_1^4 + 2r^2\alpha_2 + r^4\alpha_3^4)}{12(1+v)a^2\alpha_1^2} \qquad (41)$$

And for CSCS with edges at (x=0, a) are restrained and edges at (y=0, b) are unrestrained

$$T_{0lcr} = \frac{h^2(\beta_1^4 + 2r^2\beta_2 + r^4\beta_3^4)}{12(1+v)a^2\beta_1^2} \qquad (42)$$

Thermal Buckling Temperature For Non - Linear Temperature Distribution :

Assume the temperature across the plate thickness h have the formas in Eq.(2)

$\Delta T = T_{0n} + zT_{1n} + z^2T_{2n}$ so that

$N_t = \alpha Eh(T_{0n} + h^2T_{2n}/12)$ and

$$M_t = \frac{\alpha Eh^3T_{1n}}{12} \qquad (43)$$

Substituting X_i and Y_j from Eq. (11) with the help of appendix C. with edges at(x=0, a) are restrained and edges at (y=0, b) are unrestrained

For SSSS

$$(T_{0n} + h^2T_{2n}/12)_{cr} = \frac{D\pi^2(1-v)(m^2 + r^2n^2)^2}{E\alpha hm^2a^2} \qquad (44)$$

For CCCC

$$(T_{0n} + h^2T_{2n}/12)_{cr} = \frac{h^2(\alpha_1^4 + 2r^2\alpha_2 + r^4\alpha_3^4)}{12(1+v)a^2\alpha_1^2} \qquad (45)$$

And For CSCS

$$(T_{0n} + h^2T_{2n}/12)_{cr} = \frac{h^2(\beta_1^4 + 2r^2\beta_2 + r^4\beta_3^4)}{12(1+v)a^2\beta_1^2} \qquad (46)$$

RESULTS AND DISCUSSIONS:

The sample of calculations was made on Aluminum 1060-H18 rectangular plate which has the mechanical and thermal properties given in appendix A respectively. Rectangular plate with three different aspect ratio a/b (2, 1.5, and 1.2). And three different dimensional ratio φ (16, 80 and 60) where $\varphi = a/h$ and owing constant magnitude of a=0.12 m have been studied

Tables (1) to (9) listed the first four buckling temperatures for SSSS, CCCC and CSCS plates which edges are restrained at x=0, a and unrestrained at y=0, b at different aspect ratios r (1.2, 1.5 and 2) and different φ (16, 80 and 60) from these results finding that the thermal buckling temperatures increased with increase the thickness to span ratio of the plates for all ends conditions. For each type of Ends condition the thermal buckling temperature increased when the aspect ratio (r) increased for the same end condition.

The CSCS ends condition have the lowest thermal buckling temperatures and the CCCC ends condition have the highest thermal buckling temperatures and the SSSS ends thermal buckling temperatures are between as shown in **figures (2) to (4).**

The CSCS ends thermal buckling temperatures will tends to be close to the thermal buckling temperatures of SSSS ends condition when the values of aspect ratio (r) increase, as shown in **figures (2) to (4).**

Finally thermal buckling temperatures and thermal buckling load have lowest values at first mode of buckling (1, 1) for all types of Ends condition and with all values of aspect ratios (r and φ).

It is observed that the minimum value of Nt occur when n=1 for SSSS case. Thus, for SSSS rectangular plate panels will buckle into several half-waves (m) in the loading direction, and only one half-wave (n) in the transverse direction, when the simply supported plate buckles the buckling mode can only be one half sine wave, $\sin(\pi y/b)$, across the span, while several have waves in the direction of (compression) restrained edges can occur thus

$$N_{tcr} = F\frac{(1-v)\pi^2 D}{b^2}$$

Where $F = (\frac{m}{r} + \frac{r}{m})^2$

To ascertain the aspect ratio r at which the critical thermal load is a minimum we set $\frac{\partial N_{tcr}}{\partial r} = 0$ and the result will be r = m, this provides the following minimum value of the critical thermal load where F=4 at r=1,2,3 and 4 as shown in **Fig. (5),** therefore no thermal buckling taking place when

$$N_{tcr} < \frac{4(1-v)\pi^2 D}{b^2}$$

The intersection point for the curves $m = 1$ and $m = 2$ is given from $\left(\frac{1}{r} + \frac{r}{1}\right) = \left(\frac{2}{r} + \frac{r}{2}\right) \Rightarrow r = 1.414$ Similarly the intersection points for the curves m=2 and m=3, etc., can be obtained as 2.449, 3.464 etc. as shown in **Fig. (5)** these results are similar to mechanical buckling of SSSS plate with uniaxial load only mentioned in references [McFarland et al. 1975].

For CCCC case It is observed that the minimum value of *Nt* occur when j=1 when the CCCC plate buckles the buckling mode can only be first mode appendix C

Across the span b, while several have waves in the direction of compression can occur thus

$$N_{tcr} = F\frac{(1-v)D}{b^2}$$

Where $F = \frac{\alpha_1^4 + 2\alpha_2 r^2 + \alpha_3^4 r^4}{\alpha_1^2 r^2}$

To ascertain the aspect ratio r at which the critical thermal load is a minimum we set $\frac{\partial N_{tcr}}{\partial r} = 0$ and the result will be $r = 0.2114\alpha_1$, this provides the following minimum value of the critical thermal load where F=58.2711 at r=1 and 4 as shown in **Fig. (6)** ,therefore no thermal buckling taking place for CCCC case when

$$N_{tcr} < \frac{58.2711(1-v)D}{b^2}$$

The intersection point for the curves i = 1 and $i = 2$ is r=1.3556 and is driven as for SSSS case. similarly the intersection points for the curves i=2 and i=3 can be obtained as r=2.0228 , and for curves i=3 and i=4 the intersection point will be at r=2.694 as shown in **Fig. (6).**

For CSCS case It is observed that the minimum value of *Nt* occur when j=1 when the plate buckles the buckling mode can only be first mode, the buckling mode can only be one half sine wave , $\sin(\pi y/b)$,across the span b ,while several have waves in the direction of compression can occur thus

$$N_{tcr} = F\frac{(1-v)D}{b^2}$$

Where $F = \frac{\beta_1^4 + 2\beta_2 r^2 + \beta_3^4 r^4}{\beta_1^2 r^2}$

To ascertain the aspect ratio r at which the critical thermal load is a minimum we set $\frac{\partial N_{tcr}}{\partial r} = 0$ and the result will be $r = \alpha_1/\pi$, this provides the following minimum value of the critical thermal load where F=71.437 at r=1.5056 and as shown in **Fig. (7)**, therefore no thermal buckling taking place for CSCS case when

$$N_{tcr} < \frac{71.437(1-v)D}{b^2}$$

The intersection point for the curves *i* = 1 and *i = 2* is r=2.228 and is driven as for SSSS case. similarly the intersection points for the curves *i*=2 and *i*=3 can be obtained as *r*=3.149, and for curves *i*=3 and *i*=4 the intersection point will be at *r*=4.229 as shown in **Fig. (7)** .Therefore for SSSS and CCCC at r=1.2 the uniaxial thermal compression load buckles at mode 1 and for r= 1.5 and 2 the buckles at mode 2. In CSCS case the uniaxial thermal compression load buckles at mode 1 for r=1.2 and 1.5 and buckles at mode 2 for r=2 the intervals between two intersection points are very useful to determine the lowest thermal buckling mode .

CONCLUSIONS:

Following the main summarized conclusions raised by this paper are:

1- The buckling strength could be enhanced considerably by additional clamping of edges (i.e. from SSSS and CSCS cases to CCCC case) .
2- Thermal buckling temperatures and thermal buckling load have lowest values at first mode of buckling for all types of ends condition and with all values of aspect ratios (r and φ).
3- Thermal loads on plate induced thermal stresses when the edges are restrained. It has been found that a compression stress will developed when the temperature are uniform at the direction perpendicular to restrained edges with no deflection but at non uniform temperature thermal bending will appear and lateral deflection will occur.
4- Thermal buckling temperature and thermal stresses of thermoelastic plate affected by dimensional aspect ratios, temperature distributions and boundary conditions.

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Table (1) Four Lowest Critical Buckling Temperature for SSSS, Ends at x=0, a are Restrained and Ends at y=0, b are Unrestrained with r=1.2

r=1.2	Thermal critical temperature Tc_{cr} C^0		
Mode number	φ =16	φ =80	φ =60
1,1	11.1783	25.1512	44.729
2,1	13.8910	31.2548	55.5641
3,1	22.7382	51.1609	90.9526
4,1	35.6919	80.3068	142.7676

Table (2) Four Lowest Critical Buckling Temperature for SSSS, Ends at x=0, a are Restrained and Ends at y=0, b are Unrestrained with r=1.5

r=1.5	thermal critical temperature Tc_{cr} C^0		
Mode number	φ =16	φ =80	φ =60
1,1	19.8319	44.6217	79.3275
2,1	18.3357	41.2553	73.3427
3,1	26.4034	59.4076	105.631
4,1	39.0843	87.9397	156.3373

Table (3) Four Lowest Critical Buckling Temperature for SSSS, Ends at x=0, a are Restrained and Ends at y=0, b are Unrestrained with r=2

r=2	Thermal critical temperature Tc_{cr} C^0		
Mode number	φ =16	φ =80	φ =60
1,1	46.9393	105.631	187.7573
2,1	30.0412	67.5926	16.1647
3,1	35.2566	79.3275	141.0266
4,1	46.9393	105.631	187.7573

Table (4) Four Lowest Critical Buckling Temperature for CCCC, Ends at x=0, a are Restrained and Ends at y=0, b are Unrestrained with r=1.2

r=1.2	Thermal critical temperature Tc_{cr} C^0		
Mode number	φ =16	φ =80	φ =60
1,1	16.7869	37.7706	67.1477
2,1	19.9588	44.9072	79.8350
3,1	30.1467	67.8300	16.5866
4,1	44.7944	100.7875	179.1777

Table (5) Four Lowest Critical Buckling Temperature for CCCC, Ends at x=0, a are Restrained and Ends at y=0, b are Unrestrained with r=1.5

r=1.5	Thermal critical temperature Tc_{cr} C^0		
Mode number	φ =16	φ =80	φ =60
1,1	31.5924	71.0828	22.3694
2,1	27.3981	61.6457	5.5924
3,1	35.6019	80.1043	142.4076
4,1	49.4729	7.3140	197.8916

Table (6) Four Lowest Critical Buckling Temperature for CCCC, Ends at x=0, a are Restrained and Ends at y=0, b are Unrestrained with r=2

r=2	Thermal critical temperature Tc_{cr} C^0		
Mode No.	φ =16	φ =80	φ =60
1,1	82.6470	185.9558	330.5882
2,1	50.3865	9.3697	201.5461
3,1	50.9164	10.5619	203.6655
4,1	61.7152	34.8592	246.8608

Table (7) Four Lowest Critical Buckling Temperature for CSCS, Ends at x=0, a are Restrained and Ends at y=0, b are Unrestrained with r=1.2

r=1.2	Thermal critical temperature Tc_{cr} C^0		
Mode number	φ =16	φ =80	φ =60
1,1	8.9465	20.257	35.7861
2,1	16.3882	36.8734	65.5528
3,1	27.7419	62.4194	6.9678
4,1	42.8555	96.4249	171.4221

Table (8) Four Lowest Critical Buckling Temperature for CSCS, Ends at x=0, a are Restrained and Ends at y=0, b are Unrestrained with r=1.5

r=1.5	Thermal critical temperature Tc_{cr} C^0		
Mode number	φ =16	φ =80	φ =60
1,1	13.0944	29.4624	52.3775
2,1	19.5532	43.9947	78.224
3,1	30.6885	69.0491	18.7539
4,1	45.7440	102.9240	182.9761

Table (9) Four Lowest Critical Buckling Temperature for CSCS, Ends at x=0, a are Restrained and Ends at y=0, b are Unrestrained with r=2

r=2	Thermal critical temperature Tc_{cr} (C^0)		
Mode number	φ =16	φ =80	φ =60
1,1	25.7665	57.9745	103.0659
2,1	27.7370	62.4083	6.9482
3,1	37.7411	84.9174	150.9643
4,1	52.4000	13.8999	209.5999

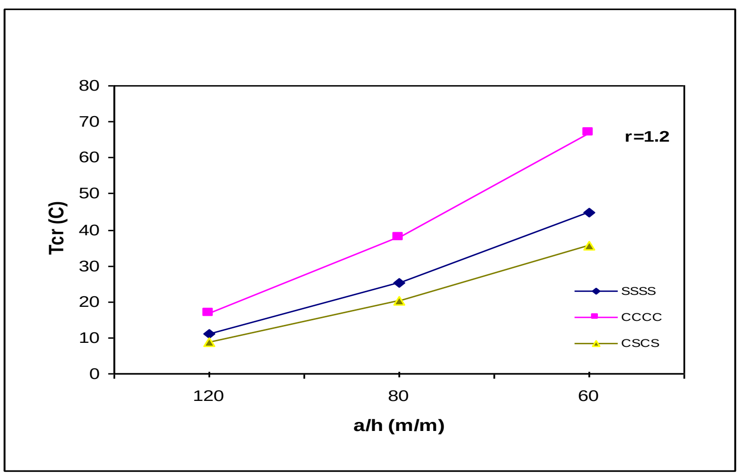

Fig. (2) Critical Thermal Buckling Temperatures at Different End Conditions, Ends are Restrained only at x=0, a. with Different φ Values at Constant Aspect Ratio (r=1. 2)

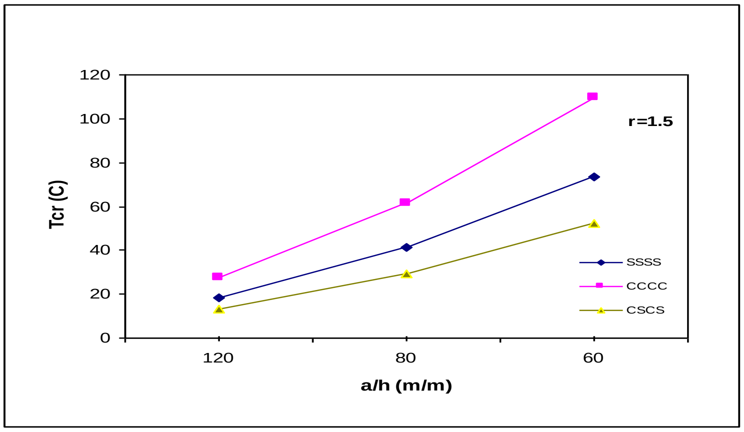

Fig. (3) Critical Thermal Buckling Temperatures at Different End Conditions, Ends are Restrained only at x=0, a with Different φ Values at Constant Aspect Ratio (r=1. 5)

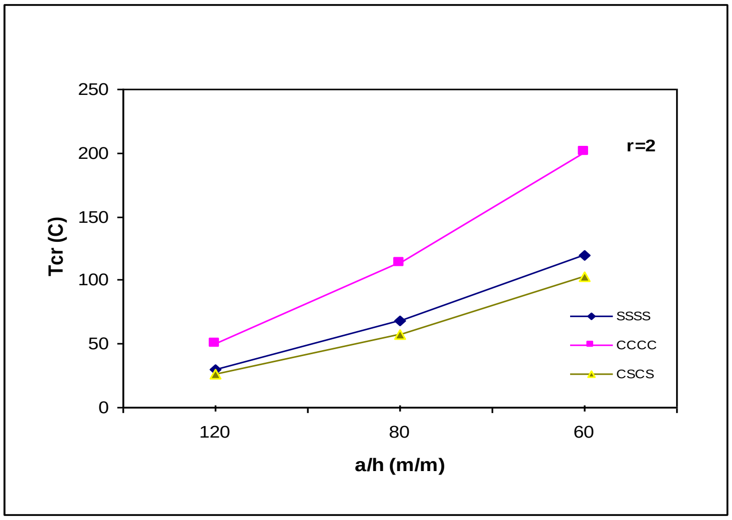

Fig. (4) Critical Thermal Buckling Temperatures at Different End Conditions, Ends are Restrained only at x=0, a with Different φ Values at Constant Aspect Ratio (r=2)

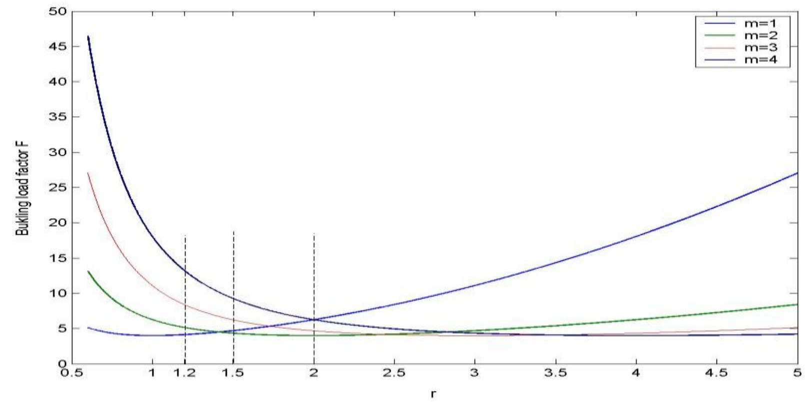

Fig. (5) Thermal Load vs. Aspect Ratio (r) for SSSS End Conditions, Ends are Restrained only at x=0, a

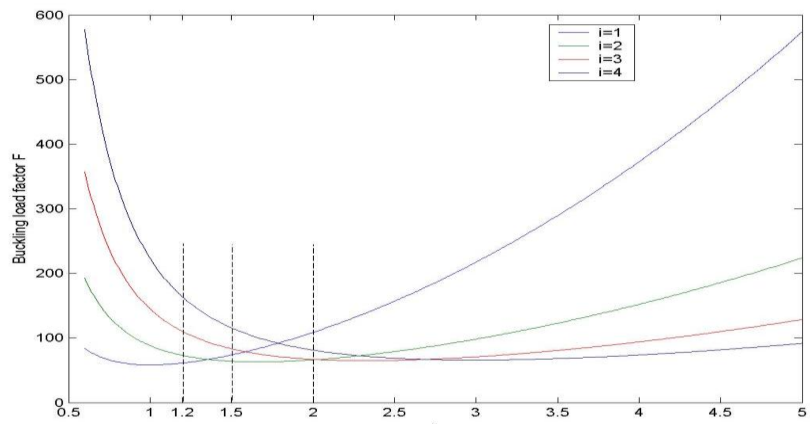

Fig. (6) Thermal Load vs. Aspect Ratio (r) for CCCC End Conditions, Ends are Restrained only at x=0, a.

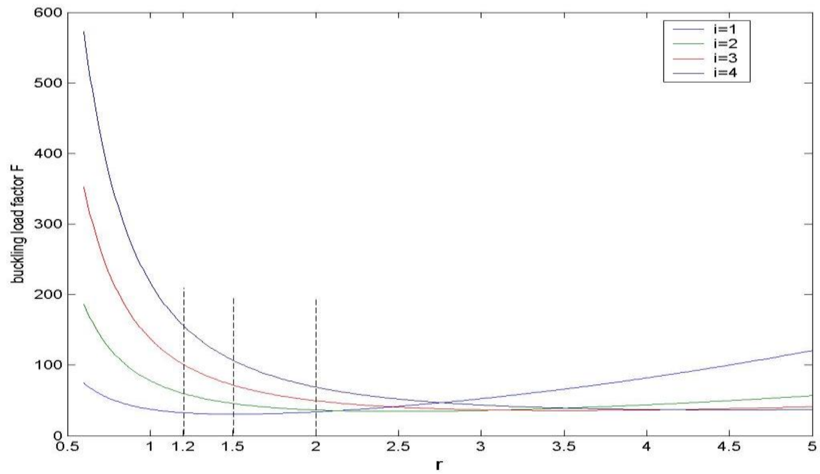

Fig. (7) Thermal Load Factor vs. Aspect Ratio (r) for CSCS End Conditions, Ends are Restrained only at x=0, a.

APPENDICES

Appendix A:

Some Combinations of End Boundary Conditions

deflection	mid—plane deformation	symbol
clamped	restrained	
	unrestrained	
supported	restrained	
	unrestrained	
free	restrained	
	unrestrained	

Appendix B :

Mechanical Properties of Aluminum 1060-H18

Density	**2705 kg/m³**
Hardness, Brinell	**35**
Ultimate Tensile Strength	**27 MPa**
Tensile Yield Strength	**20 MPa**
Elongation at Break	**6 %**
Modulus of Elasticity	**69 GPa**
Poisson's Ratio	**0.3**
Fatigue Strength	**44.8 MPa**
Machinability	**30 %**
Shear Modulus	**26 GPa**
Shear Strength	**75.8 MPa**

Thermal Properties of Aluminum 1060-H18

Heat Capacity	**0.9 J/g °C**
Thermal Conductivity	**233 W/m °C**
Coefficient of Thermal expansion	**2.34e-5/°C**
Convection Coefficient	**2.5 W/m² °C**

Appendix C:

For SSSS ends condition

$$X_i = \sin \mu_i x \quad , \quad Y_j = \sin \mu_j y$$

$$w_{x=0} = w_{x=a} = 0 \quad , \quad w_{y=0} = w_{y=b} = 0 \quad , \quad \frac{\partial^2 w_{x=0}}{\partial x^2} = \frac{\partial^2 w_{x=a}}{\partial x^2} = 0 , \quad \frac{\partial^2 w_{y=0}}{\partial y^2} = \frac{\partial^2 w_{y=b}}{\partial y^2} = 0$$

For CCCC ends condition

$$X_i = \sin \mu_i x - \sinh \mu_i x - \eta_i (\cos \mu_i x - \cosh \mu_i x)$$

$$\eta_i = (\sin \mu_i a - \sinh \mu_i a) / (\cos \mu_i a - \cosh \mu_i a)$$

$$Y_j = \sin \mu_j y - \sinh \mu_j y - \eta_j (\cos \mu_j y - \cosh \mu_j y)$$

$$\eta_j = (\sin \mu_j b - \sinh \mu_j b) / (\cos \mu_j b - \cosh \mu_j b)$$

$$w_{x=0} = w_{x=a} = 0 , \quad w_{y=0} = w_{y=b} = 0 \quad , \quad \frac{\partial\ w_{x=0}}{\partial x} = \frac{\partial\ w_{x=a}}{\partial x} = 0 , \quad \frac{\partial w_{y=0}}{\partial y} = \frac{\partial\ w_{y=b}}{\partial y} = 0$$

For SCSC ends condition

$$X_i = \sin \mu_i x - \sinh \mu_i x - \eta_i (\cos \mu_i x - \cosh \mu_i x)$$

$$\eta_i = (\sin \mu_i a - \sinh \mu_i a) / (\cos \mu_i a - \cosh \mu_i a) \quad , \quad Y_j = \sin \mu_j y$$

$$w_{x=0} = w_{x=a} = 0 \quad , \quad w_{y=0} = w_{y=b} = 0 \quad , \quad \frac{\partial w_{x=0}}{\partial x} = \frac{\partial\ w_{x=a}}{\partial x} = 0 , \quad \frac{\partial^2 w_{y=0}}{\partial y^2} = \frac{\partial^2 w_{y=b}}{\partial y^2} = 0$$

Where $\mu_i a$ and $\mu_j b$ are the roots of the above equations

The roots of SSSS ends condition are;

$$\mu_i = \frac{m\pi}{a} \quad , \quad \mu_i = \frac{n\pi}{b}$$

The roots of CCCC ends condition are;

$$\alpha_1 = \alpha_3 = 4.73 \qquad \alpha_2 = 151.3$$
For i=1 , j=1

$$\alpha_1 = 4.73 \qquad \alpha_3 = (j+0.5)\pi \qquad \alpha_2 = 12.3\alpha_3(\alpha_3 - 2)$$
For i=1 , j=2,3,4,….

$$\alpha_1 = (i+0.5)\pi \qquad \alpha_3 = 4.37 \qquad \alpha_2 = 12.3\alpha_1(\alpha_1 - 2)$$
For i=2,3,4,… j=1

$$\alpha_1 = (i+0.5)\pi \qquad \alpha_3 = (j+0.5)\pi \qquad \alpha_2 = \alpha_1(\alpha_1 - 2)\alpha_3(\alpha_3 - 2)$$
For i=2,3,4,. j=2,3,4,….\

$$\alpha_1 = (i+0.5)\pi \qquad \alpha_3 = (j+0.5)\pi \qquad \alpha_2 = \alpha_1(\alpha_1 - 2)\alpha_3(\alpha_3 - 2)$$
For i=2,3,4,. j=2,3,4,….

The roots of CSCS ends condition are

$$\beta_1 = 4.73 \qquad \beta_3 = j\pi \qquad \beta_2 = 12.3 j^2\pi^2$$
For i=1 , j=1, 2, 3,..

$$\beta_1 = (i+0.5)\pi \qquad \beta_3 = j\pi \qquad \beta_2 = \alpha_1(\alpha_1 - 2) j^2\pi^2$$
For i=2,3,4,… j=1,2,3….

NOMENCLATURE

Latin Symbols:

A	**Area (mm ^{2})**
a, b	**Plate side length (mm)**
D	**Flexural rigidity of an isotropic plate (N.mm)**
E	**Modulus of elasticity of isotropic material (N/mm^2)**
G	**Shear modulus of isotropic material (N/mm^2)**
h	**Plate thickness (mm)**
i ,j	**Integer**
Mt	**Thermal bending moment (N.m)**
m,n	**Integer**
Nx, Ny	**Edge forces per unit length (N/m)**
Nxy	**Shearing forces per unit length (N/m)**
Nt	**Thermal forces per unit length (N/m)**
r	**Dimensional aspect ratio a/b (m/m)**
T	**Temperature (C ^{0}), Kinetic energy of the element (J)**
T_0	**Initial reference temperature (C ^{0})**
u, v, w	**Displacement components in x,y,z directions**
x, y, z	**Cartesian coordinates**

Greek Symbols:

ν	**Poisson's ratio**
$\sigma_x, \sigma_y, \sigma_z$	**Normal stresses parallel to x, y, z axes (N/mm^2)**

$\tau_{xy}, \tau_{xz}, \tau_{yz}$	**Shear stresses component xy, xz, yz plain (N/mm^2)**
$\varepsilon_x, \varepsilon_y, \varepsilon_z$	**Direct strain in x, y, z directions**
$\gamma_{xy}, \gamma_{xz}, \gamma_{yz}$	**Shear strain component**
Π_{strain}	**Strain energy stored in complete plate (J)**
Π_b	**Strain energy stored due to bending (J)**
Π_N	**Strain energy stored due to mid-plane thermal forces (J)**
Π_M	**Strain energy stored due to thermal bending (J)**
φ	**Dimensional aspect ratio side / thickness (m/m)**
α	**Coefficient of thermal expansion (1/C ^{0})**
w	**Deflection (mm)**

Abbreviations Symbols:

CCCC	**Clamped-Clamped-Clamped-Clamped**
CSCS	**Clamped-Simply-Clamped-Simply**
SSSS	**Simply-Simply-Simply-Simply**