FREE VIBRATION ANALYSIS OF NOTCHED COMPOSITE LAMINATED CANTILEVER BEAMS

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ABSTRACT

The present work divided into two parts, first the experimental side which included the measuring of the first natural frequency for the notched and unnotched cantilever composite beams which consisted of four symmetrical layers and made of Kevlar- epoxy reinforced. A numerical study covers the effect of notches on the natural frequencies of the same specimen used in the experimental part. The mathematical model for the beam contains two open edges on the upper surface. The effect of the location of cracks relative to the restricted end, depth of cracks, volume fraction of fibers and orientation of the fiber on the natural frequencies are explored. The results were calculated using the known engineering program (ANSYS), the results obtained has been compared with those calculated analytically by (Sierakowski RL.), which have expressed the closest well also the comparison between the experimental results with that calculated by (ANSYS) has very well. The study shows that the highest difference in frequencies occur when the value of the fiber orientation equal to 0^{o}degree, the effect of location of the cracks decrease when the cracks moving toward the free end and also shows that an increase of the depth of the cracks leads to a decrease in the values of natural frequencies.

الخلاصة:

يقسم البحث الى جزئين ، الأول الجانب العملي والذي يتضمن قياس التردد الطبيعي الاول لعتبة ناتئة من الألواح المركبة محززة و غير محززة مؤلفة من أربعة طبقات متناظرة مصنوعة من مادة (Kevlar- epoxy). الدراسة العددية تبين تأثير الحزوز على الترددات الطبيعية لنفس النموذج المستخدم في الجانب العملي. النموذج الرياضي للعتبة يحتوي على شقيق على السطح العلوي. تم بحث تأثير موقع الحزوز با النسبة الى النهاية المقيدة، تأثير العمق، الكسر ألحجمي للألياف وكذلك زاوية الليف على الترددات الطبيعية. تم حساب النتائج باستخدام البرنامج الهندسي المعروف (ANSYS)، أن النتائج المحسوبة تمت مقارنتها مع تلك المحسوبة بالطريقة التحليلية لـ (Sierakowski RL.) والتي أبدت تقارباً جيدا و كذلك المقارنة بين النتائج العملية و تلك المحسوبة بالبرنامج كانت جيدة ايضاً. أن البحث يظهر أن أعظم اختلاف في الترددات يحدث عندما تكون زاوية اتجاه الليف مساوية 0^{o} وأن تأثير موقع الحزوز يقل بتحرك الحزوز باتجاه النهاية الحرة للعتبة و كذلك تظهر الدراسة أن زيادة عمق الحزوز يؤدي إلى النقصان في قيم الترددات الطبيعية.

KEY WORDS
Vibration, Finite element, Cantilever beams, Laminate, Natural frequencies.

1. INTRODUCTION

During operation, all structures are subjected to degenerative effects that may cause initiation of structural defects such as cracks which, as time progresses, lead to the catastrophic failure or breakdown of the structure. Thus, the importance of inspection in the quality assurance of manufactured products is well understood. Cracks or other defects in a structural element influence its dynamical behavior and change its stiffness and damping properties. Consequently, the natural frequencies of the structure contain information about the location and dimensions of the damage.

The first group of studies have been performed for long times and the most concepts related to the crack detection have been well established from mathematical theory **(Chondros TG,1998)** to impact echo method**(Cam E,2005, Ratcliffe CP1997)** . When a structure suffers from damages, its dynamic properties can change, especially, crack damage can cause a stiffness reduction, with an inherent reduction in natural frequencies, an increase in modal damping, and a change of the mode shapes. Consequently, there would also be a change in the dynamic response of the structure **(Matveev VV, 2002 Kim M-B,2005).**

Over the past decade, several techniques have been explored for detecting and monitoring of the defects in the composite materials. **(Adams RD,1978)** showed that any defect in fibre-reinforced plastics could be detected by reduction in natural frequencies and increase in damping. **(Nikpour K, Dimarogonas AD,1988)** studied the variation of the mixed term in the energy release rate for various angles of inclination of the material axes of symmetry and they derived the local compliance matrix of a prismatic beam with a central crack.

(Nikpour K,1990) studied the buckling of cracked composite columns and showed that the instability increases with the column slenderness and the crack depth. **(Oral S,1991)** developed a shear flexible finite element for non-uniform laminated composite beams. He tested the performance of the element with isotropic and composite materials, constant and variable cross-sections, and straight and curved geometries.

In the last years, the effort is focused on the vibration analysis of structural members using breathing crack models to simulate real fatigue cracks. Several researchers have studied the problem of beams having a breathing crack by employing different approaches. In that **(Saavedra PN,2001)** proposed a new modeling approach for cracked beam structures. They used finite elements for the beam, while for the cracked element a new finite element matrix based on an energy density function was employed. Recently, **(Sinha JK,2002)** developed an alternative finite element approach, where the phase relationship between the first and second response components is simulated correctly. **(Pugno N,2000)** studied the dynamic response of a beam with several breathing cracks subjected to harmonic excitation. Assuming that cracks open and close continuously and using finite elements to model the beam, a system of non-line a algebraic equations was obtained and solved by numerical integration.

Recently, **(Song O,2003)** investigated the dynamics of anisotropic composite cantilevers. They presented an exact solution methodology utilising Laplace traform technique to study the bending free vibration of cantilever composite beams with multiple open cracks.

The present search presented numerical and experimental study for the effect of the cracks on the natural frequencies of cantilever composite beams. The lamination angle of the fiber, volume fraction of fibers and the location of notches relative constrain end are studied.

2. MATHMATICAL MODEL

The model chosen is a cantilever composite beam of uniform cross-section area

A, having two open – edge transverse cracks at a variable positions $L1$ and $L2$. The width , length and height of the beam are B, L and H , respectively as shown in Fig.(1).The beam consist of four symmetric layers.

2.1. The Stiffness Matrix For Crack

According to the St. Venant's principle, the stress field is influenced only in the region near to the crack. The additional strain energy due to crack leads to flexibility coefficients expressed by stress intensity factors derived by means of Castigliano's theorem in the linear elastic range. The compliance coefficients C_{ij} induced by crack are derived from the strain energy release rate, J, developed**(Tada H, Paris PC, 1985)**theory. J can be given as:

$$J = \frac{\partial U(P_i, A)}{\partial A} \tag{1}$$

Where A is the area of the crack section, P_i are the corresponding loads, U is the strain energy of the beam due to crack and can be expressed as **(Nikpour K, Dimarogonas AD,1988)** :

$$U = \int_A \left(D_1 \sum_{i=1}^{i=N} K_{Ii}^2 + D_{12} \sum_{i=1}^{i=N} K_{Ii} \sum_{j=1}^{j=N} K_{IIi} + D_2 \sum_{i=1}^{i=N} K_{IIi}^2 \right) dA, \tag{2}$$

Where K_I and K_{II} are the stress intensity factors for fracture modes of I and II. D_1, D_{12} and D_2 are the coefficients depending on the materials parameters**(Nikpour K, Dimarogonas AD,1988)** :

$$D_1 = -0.5\bar{b}_{22} \operatorname{Im} \frac{s_1 + s_2}{s_1 s_2} \tag{3}$$

$$D_{12} = \bar{b}_{11} \operatorname{Im}(s_1 s_2) \tag{4}$$

$$D_2 = 0.5\bar{b}_{11} \operatorname{Im}(s_1 + s_2) \tag{5}$$

The coefficients s_1, s_2 are complex constant and $\bar{b}_{ij}$ are constant. The mode I and II stress intensity factors, K_I and K_{II}, for a composite beam with a crack are expressed as **(Nikpour K,1990)**.

$$K_{ij} = \sigma_i \sqrt{\pi a} Y_j(\zeta) F_{ji}(a/H) \tag{6}$$

Where σi is the stress for the corresponding facture mode, is the correction factor for the finite specimen size, $Y_j(\zeta)$ is the correction factor for the anisotropic materia**(Nikpour K, Dimarogonas AD,1988)**, a is the crack depth and H is the element height. Castigliano's theorem [**Tada H,Paris PC, Irwin GR**.**1985**] implies that the additional displacement due to crack, according to the direction of the P_i, is:

$$u_i = \frac{\partial U(P_i, A)}{\partial P_i} \tag{7}$$

Substitution of this energy rate J into Eq. (7), the relation between displacement and strain energy release rate J can be written as follows:

$$u_i = \frac{\partial}{\partial P_i} \int_A J(P_i, A) dA. \tag{8}$$

The flexibility coefficients, which are the functions of the crack shape and the stress intensity factors, can be introduced as follows**(Tada H, Paris PC, 1985)**:

$$c_{ij} = \frac{\partial u_i}{\partial P_j} = \frac{\partial^2}{\partial P_i \partial P_j} \int_A J(P_i, A) dA = \frac{\partial^2 U}{\partial P_i \partial P_j} \quad (9)$$

The compliance coefficients matrix, after being derived from above equation, can be given according to the displacement vector $\delta = \{u, v, \theta\}$ as

$$C = \lfloor c_{ij} \rfloor_{(3\times3)} \quad (10)$$

Where c_{ij} (i, j = 1, 2, 3) are derived by

using Eqs. (1-9).

The inverse of the compliance coefficients matrix, C^{-1}, is the stiffness matrix due to crack. Considering the cracked node as a cracked element of zero length and zero mass(**Ratcliffe CP1997)**, the crack stiffness can be represented as equivalent compliance coefficients. Finally, resulting stiffness matrix for the crack can be given as:

$$K_c = \begin{bmatrix} [C]^{-1} & -[C]^{-1} \\ -[C]^{-1} & [C]^{-1} \end{bmatrix}_{(6\times6)} \quad (11)$$

2.2. Component Mode Analysis

The equation of motion of a mid-plane symmetrical composite beam is **(Vinson JR, 1991)**:

$$IS_{11}\partial^4 y(x,t)/\partial x^4 + \rho A \partial^2 y(x,t)/\partial t^2 = f(t) \quad (12)$$

Where I, ρ, A and y (x, t) are geometrical moment of inertia of the beam cross-section, material density, cross sectional area of the beam and transverse deflection of the beam, respectively. Now, consider the component A_1, for undamped vibration analysis, Eq. (12), in matrix notation, can be given as:

$$M_{A1}\ddot{q}_{A1} + K_{A1}q_{A1} = f_{A1}(t) \quad (13)$$

where M_{A1} and K_{A1} are the mass and stiffness matrices of the component A_1, respectively, q_{A1} and f_{A1} (t) are the generalized displacement and external force vectors, respectively. Assuming that:

$$\{q_{A1}\} = \{\phi_{A1}\}\sin(\omega_{A1}t + \beta) \quad (14)$$
$$\{\ddot{q}_{A1}\} = -\omega_{A1}^2\{\phi_{A1}\}\sin(\omega_{A1}t + \beta)$$

and substituting them into Eq. (13), one ends up with the Standard free vibration equation for the component A_1 as,

$$\omega_{A1}^2 M_{A1}\phi_{A1} = K_{A1}\phi_{A1} \quad (15)$$

Which gives eigen values $\omega_{A1}^2, ..., \omega_{A1n}^2$ and modal matrix ϕ_{A1} for the component A_1. Making the transformation

$$q_{A1} = \phi_{A1}p_{A1} \quad (16)$$

where P_{A1} is the principal coordinate vector. By premultiplying ϕ_{A1}^T and substituting Eq. (16), Eq. (13) becomes:

$$(\phi_{A1}^T M_{A1}\phi_{A1})\ddot{p}_{A1} + (\phi_{A1}^T K_{A1}\phi_{A1})p_{A1} = \phi_{A1}^T f_{A1}(t) \quad (17)$$

Where

$$\phi_{A1}^T M_{A1}\phi_{A1} = [m_m]$$
$$\phi_{A1}^T K_{A1}\phi_{A1} = [K_m] \quad (18)$$

where [m_m] and [Kn] are modal mass and stiffness matrices, respectively. Mass normalising the modal matrix by:

$$\psi_{ij} = \frac{\phi_{ij}}{\sqrt{m_{ij}}} \tag{19}$$

where ψ_{ij} is mass normalized mode vector. By using the transformation

$$q_{A1} = \psi_{A1} s_{A1} \tag{20}$$

by premultiplying ψ_{A1}^T and substituting Eq. (20), Eq. (13) becomes

$$I\ddot{s}_{A1} + \omega_{A1}^2 s_{A1} = \psi_{A1}^T f_{A1}(t) \tag{21}$$

where ω_{A1}^2 is. a diagonal matrix comprising the eigenvalues of A_1.

Consider components. A_1, A_2,…, A_N, Joined together by means of springs capable of carrying axial, shearing and bending effects. The kinetic and-strain energy of the components, in terms of principal modal coordinates, can be given as:

$$T = \frac{1}{2}\dot{s}^T M \dot{s}$$

$$U = \frac{1}{2}s^T K s \tag{22}$$

Where T and U are kinetic and strain energy, respectively. M and K in Eq. (22) are:

$$M = \begin{bmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & 1 \end{bmatrix},$$

$$K = \begin{bmatrix} \omega_{A1}^2 & 0 & ... & 0 \\ 0 & \omega_{A2}^2 & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \omega_{AN}^2 \end{bmatrix} \tag{23}$$

The strain energy of the connectors, in terms of principal modal coordinates, is:

$$Uc = \frac{1}{2}s^T \psi^T Kc \psi s \tag{24}$$

Where Kc is the stiffness matrix of the cracked nodal element and can be calculated by using Eq. (9) ψ in Eq. (24) can be written as:

$$\psi = \begin{bmatrix} \psi_{A1} & 0 & ... & 0 \\ 0 & \psi_{A2} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & \psi_{AN} \end{bmatrix} \tag{25}$$

The total strain energy of the system is, therefore:

$$U_T = \frac{1}{2}s^T \left(K + \psi^T Kc \psi\right)s, \tag{26}$$

Where K has been given by Eq. (23). The equation motion of the complete structure is:

$$\ddot{s} + \left(K + \psi^T Kc \psi\right)s = \psi^T f(t) \tag{27}$$

Where ψ has been given by Eq. (25), f (t) is the global force vector for the system. From Eq. (27), the eigenvalues and mode shapes of the cracked system can be determined. After solving these equations, the displacement for each component are calculated by using Eq. (20).

3.RESULTS AND DISCUSSION

3.1. Validation Of The Current Work

In order to check the accuracy of the present method , the results found by using the finite element method (ANSYS 5.4) are compared with the analytical solution of (cracked unidirectional beam), found by Sierakowski RL. **(Vinson JR, Sierakowski RL, 1991)**, as shown in Fig.(1). The beam assumed to be made of graphite fiber reinforced polyamide which contains four notches of triangle shape. The numerical results show a good agreement compared with analytical solution.

3.2. Vibration Of The Laminate Beam

The geometrical characteristics of the beam used in experimental and theoretical analysis are the length (L)= 0.75 m, height (H) =0.03 m and width(B)=0.03 m as shown in Fig.(2) . The material properties of the kevlar- epoxy are**(kawczuk M, 1997)**:

$E1$ =221 Gpa, $E2$ = 23 Gpa ,

$G\,12$ =8.6 Gpa,

$G\,23$ = 6.5 Gpa,

$\upsilon\,12$ = 0.2,

$\upsilon\,23$ = 0.3.

3.2.1 Experimental Side:

Experimental work presents the measurement the first natural frequency of the Specimens shown in Fig.(2). Experimental configuration shown in Fig.(3), used to measured the natural frequency. An accelerometer with a mass of 3 gm attached on the top edge of the beam using a wax at distance of about of 100mm from the clamped end. The accelerometer is connected by an amplifier. The charge amplifier is analyzed by an FFT analyzer which enable to readout the peaks from the digital analyzer.Tables (1,2) show the effect of the fibers orientation and cracks size on the first natural frequency for the cantilever composite beams which contents two cracks which were located as L_1 / L = 0.1, L_2 / L = 0.25 and the volume of fiber (V) was 0.8. It can be clearly seen from table (1) that, the high decreasing in the natural frequency occur until angle of fiber 50^o and greater than it the change is very low. Table (2) shows that the first natural frequency of the un notched beams equal to (45.2 Hz) and also shows that increasing the depth of the of cracks lead to decreasing in the value of the first natural frequency.

3.2.2 Theoretical Analysis:

To more deeply analysis, theoretical analysis using the finite element (ANSYS). Figs.(4, 5 and 6) show the first three natural frequencies as a function of the fiber orientation (α) for different crack ratios (a/H). In the model, the composite beam has two cracks which were located as L_1 / L = 0.1, L_2 / L = 0.25 and the volume of fiber (V) was 0.8. It is noticeable that a decrease in the natural frequencies become more intensive with the growth of the crack depth. The most difference in frequencies occur when the value of the fiber (α) is 0^o. When the value of the angle of fiber is greater than 55^o, the effects of the cracks on the frequencies decrease. This can be explained as the flexibility due to crack is negligible when the angle of the fiber is greater than 55^o, especially when the crack ratio is relatively low.

In Figs. (٧, ٨ and ٩), the variation of the three lowest natural frequencies of the composite beam with two cracks is shown as a function of fiber orientation (α) for different cracks locations. In these figures, three cases, labeled as E, F and G, were considered .The cracks locations (L_1/ L , L_2 /L) for the cases E, F and G, where chosen as (0.2, 0.35), (0.5, .65), (0.8,0.95) respectively. From the previous figures it can be clearly seen that , when the cracks are placed near the fixed end the decreases in the first natural frequency are highest , when the cracks are located near the free end ,This observation leads to the conclusion that, the first, second and third natural frequencies are most affected when the cracks located at the near of the fixed end, the middle of the beam and the free end .

Figs. (10, 11 and 12) present the first three natural frequencies as a function of the volume of fiber (V) for several values of the crack ratios $L_1/L = 0.2$, $L_2/L = 0.35$ and the angle of fiber (α) is 0^o. As can be seen from the figures, the natural frequencies are affected by the values of the volume of fiber (V) and the crack ratios (a/H), as expected. The flexibility due to cracks is high when the volume of fiber is between 0.2 and 0.8.

4.CONCLUSION

From the previous discussion, the following can be concluded

1. The effect of notches decrease as the notches moved toward the free end.
2. The natural frequencies decrease as the depth of notches are growth.
3. As increase the angle of fibers orientation the change on natural frequencies is negligible.
4. The high flexibility of cracks due at volume of fiber range (0.2 – 0.8).

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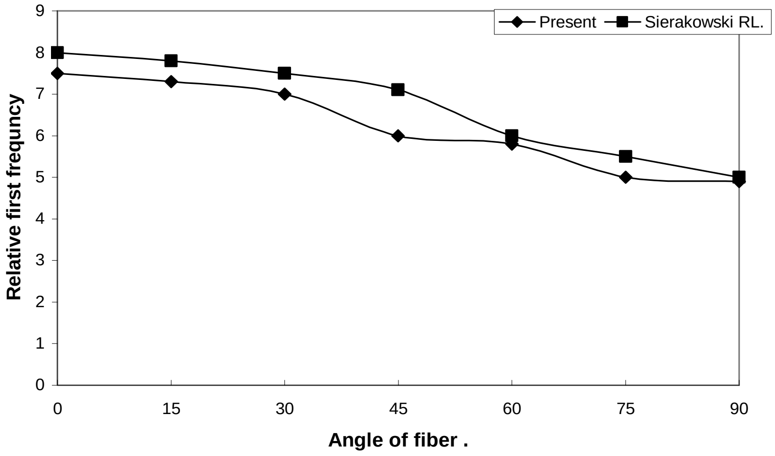

Fig. (1) The relation between the relative first natural frequany and the angle o f the fiber.

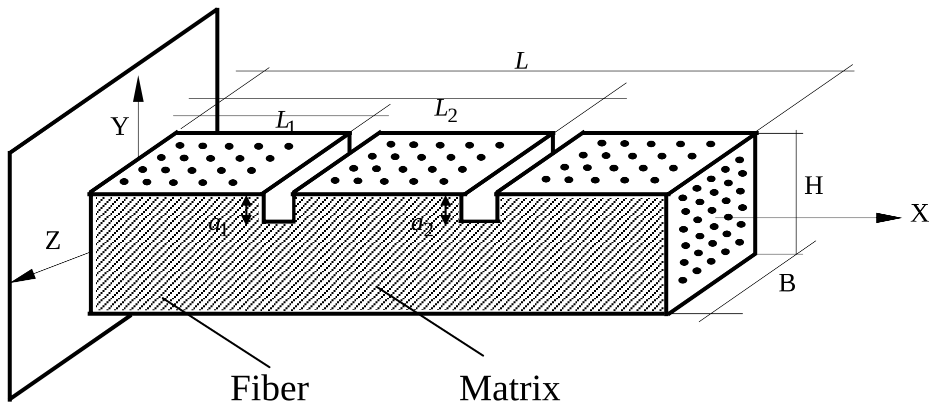

Fig (2)
Geometry of the cantilever composite beam with two cracks.

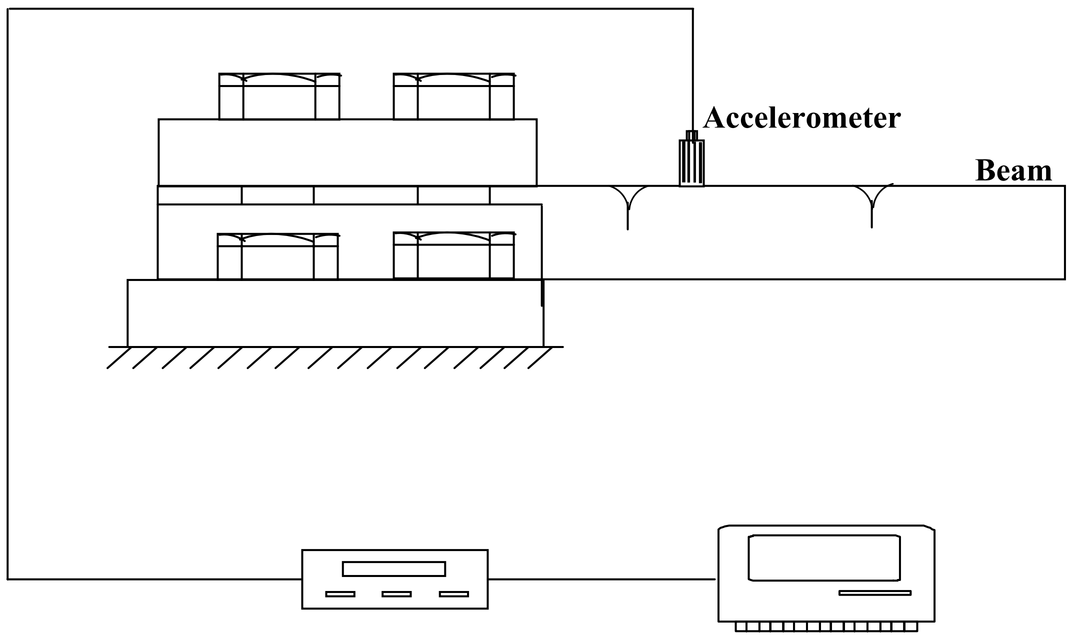

Fig.(3) Experimental configuration

Table (1): The effect of the fibers orientation on the first natural frequency for the notched cantilever plates.

θ^o (Degree)	Experimental ω_1 (HZ)	Present Work (ANSYS) ω_1 (HZ)
0	30	31.4
10	28.5	30.5
20	27	28.6
30	25	26.1
40	22	23.2
50	21.9	22.8
60	21.5	22.51
70	21.4	22.3
80	21.1	22.25
90	20	22

Table(2): The effect of the size of the notches on the first natural frequency for the notched cantilever plates. first natural frequency for the notched cantilever plates.

ratio (a/h)	Experimental First Natural Frequency (HZ)	ANSYS First Natural Frequency (HZ)
0	45.2	46.5
0.1	43	44
0.2	41	42
0.3	36	37
0.4	33	34.5
0.5	30	33
0.6	25	26
0.7	21.9	23

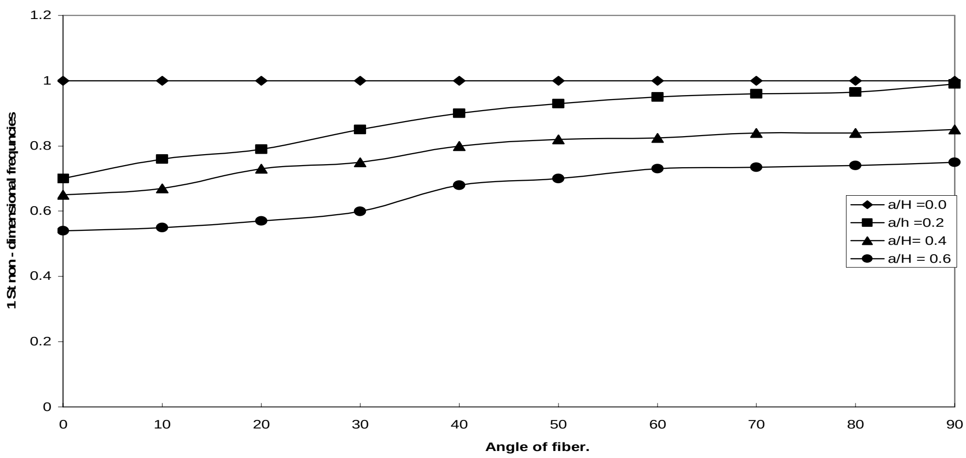

Fig. (4) First non- dimensional natural frequancies as a function of fiber orintation for different crack ratios a/H =0.2, 0.4, 0.6, and volume of the fiber V is 0.8.

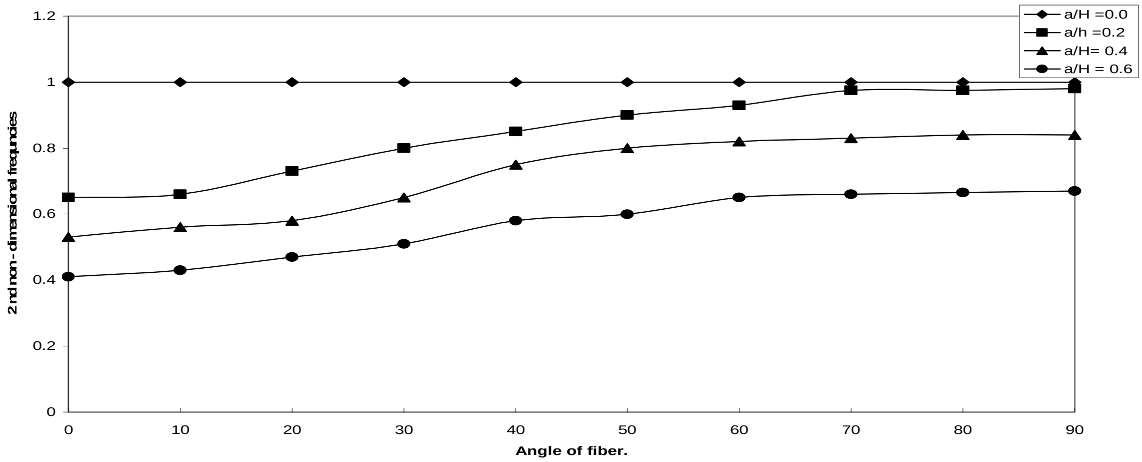

Fig. (5) second non- dimensional natural frequancies as a function of iber orintation for different crack ratios a/H =0.2, 0.4, 0.6, and volume of the fiber V is 0.8.

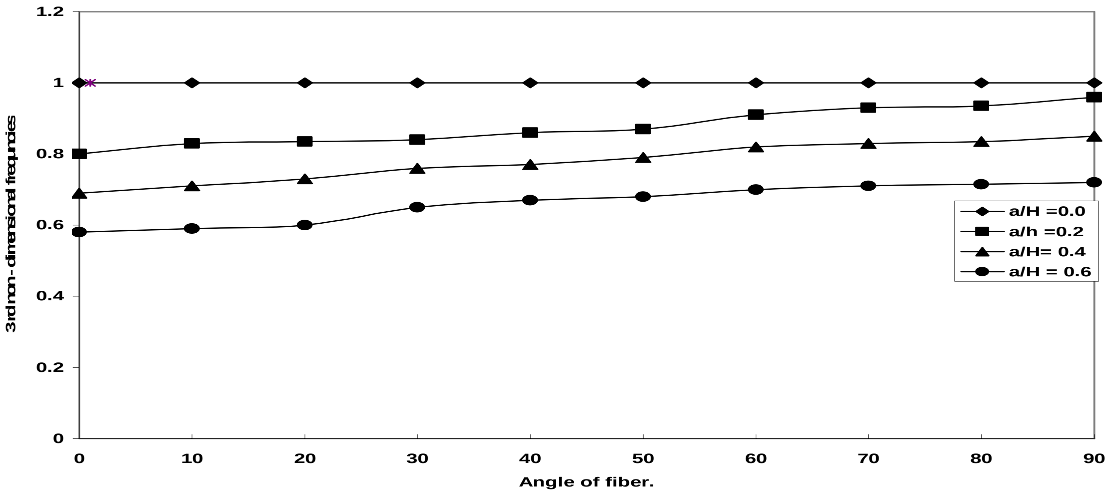

Fig. (6) Third non- dimensional natural frequancies as a function of iber orintation for different crack ratios a/H =0.2, 0.4, 0.6, and volume of the fiber V is 0.8.

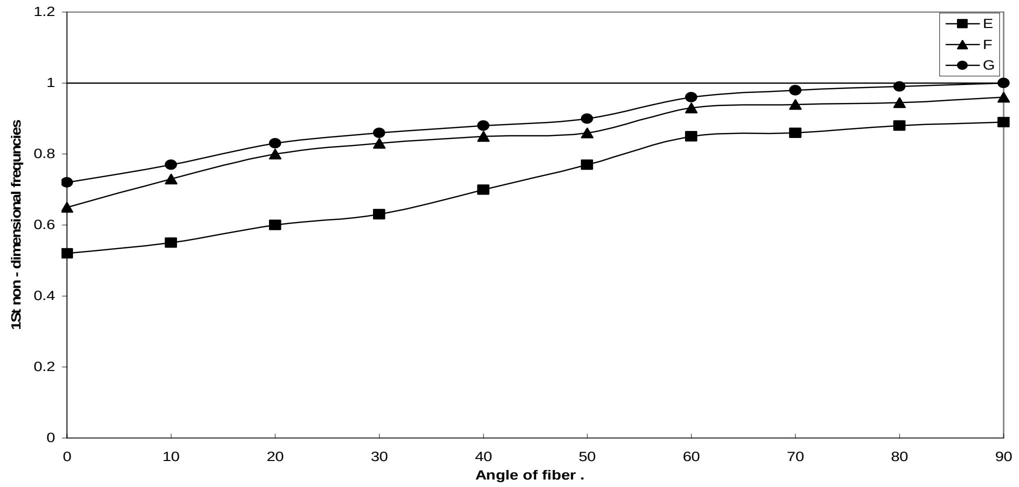

Fig. (7) First non- dimensional natural frequancies as a function of fiber orintation for different crack locations for the cases E, F and G, and volume of the fiber V is 0.8.

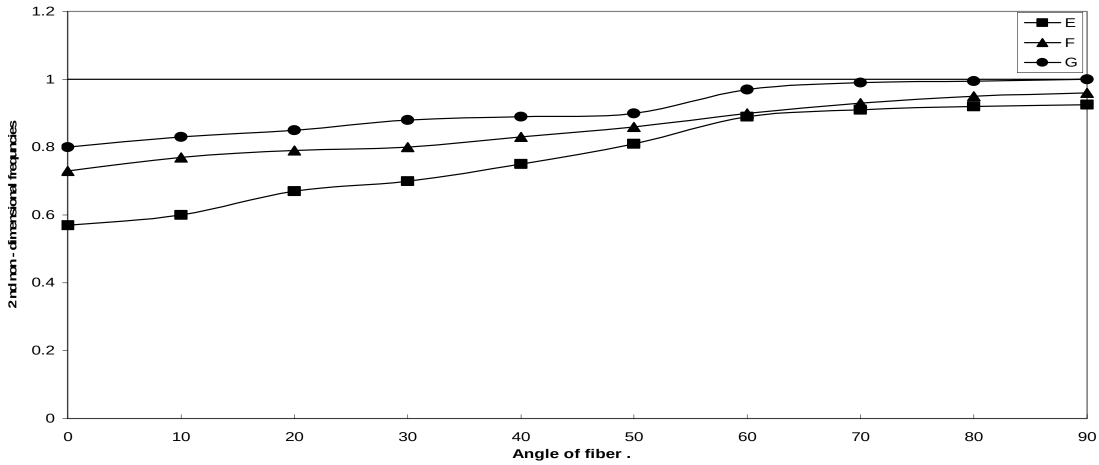

Fig. (8) Second non- dimensional natural frequancies as a function of fiber orintation for different crack locations for the cases E, F and G, and volume of the fiber V is 0.8.

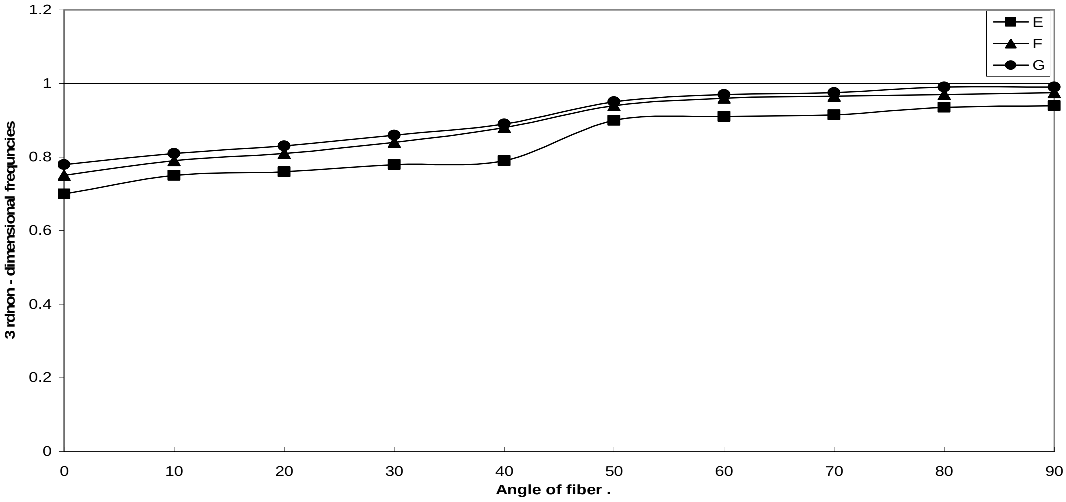

Fig. (9) Third non- dimensional natural frequancies as a function of fiber orintation for different crack locations for the cases E, F and G, and volume of the fiber V is 0.8.

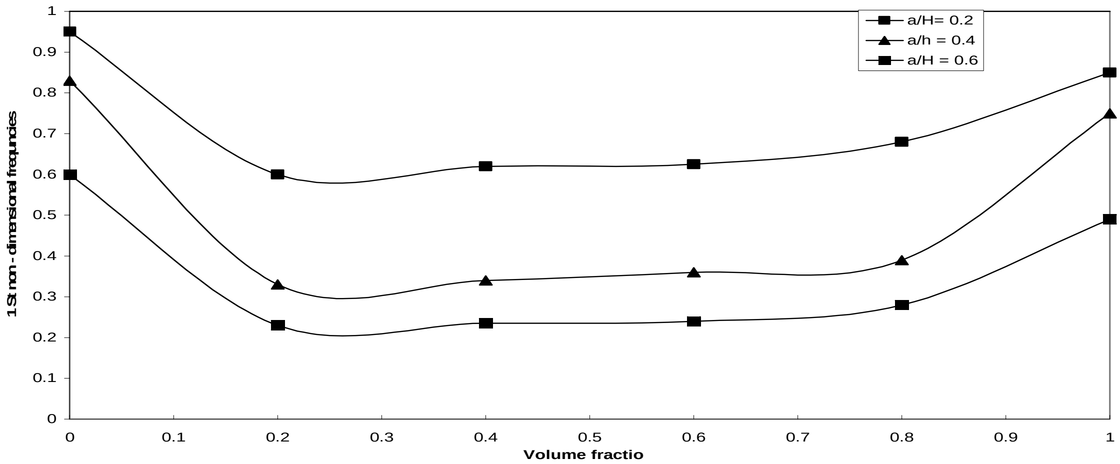

Fig. (10)First non- dimensional natural frequancies as a function of the fiber volume fraction for different crack ratios a/H = 0.2, 0.4, 0.6.

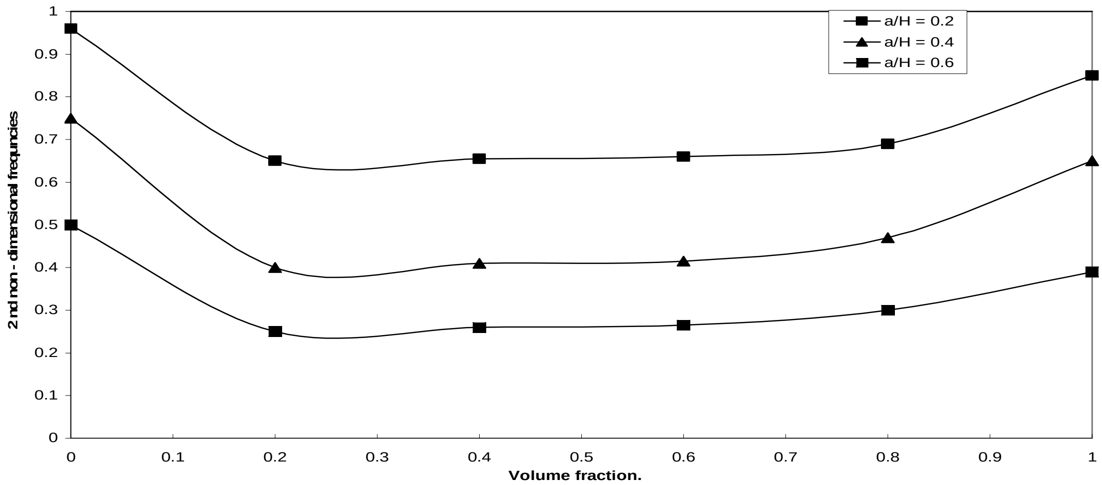

Fig. (11) Second non- dimensional natural frequancies as a function of the fiber volume fraction for different crack ratios a/H = 0.2, 0.4, 0.6.

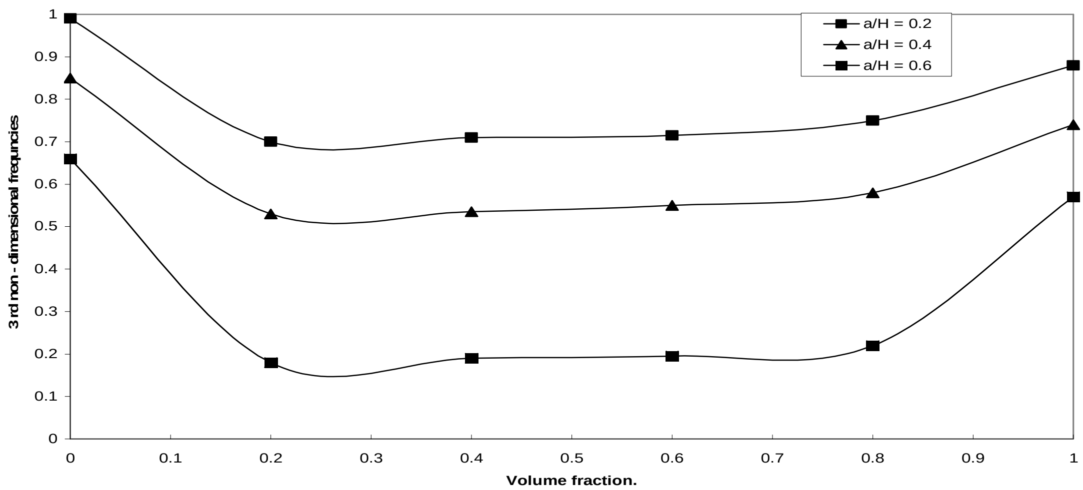

Fig. (12)Third non- dimensional natural frequancies as a function of the fiber volume fraction for different crack ratios a/H = 0.2, 0.4, 0.6.