STEADY-STATE INJECTION INTO A LIGHTLY DOPED REGION

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ABSTRACT
The steady-state injection into a semiconductor region was studied by many authors and various solutions were achieved.
The solutions gave the distribution of carriers inside such a region to aid in understanding devices operation. But extending this problem to a lightly doped semiconductor region was tackled by only few authors, and the solutions achieved were either numerical or analytical however, the analytical solution were suffering from crude approximations,. This paper gives a realistic analytical solution, which considers both monomolecular and bimolecular recombination factors.

الخلاصة
مشكلة حقن الناقلات في أشباه الموصلات درست من قبل باحثين عديدين و اقترحت عدة حلول . أعطت هذه الحلول توزيع الشحنات داخل منطقة أشباه الموصلات لفهم عمل هذه المكونات. ولكن دراسة هذه المشكلة داخل أشباه الموصلات قليلة التركيز قد عولجت من قبل قليل و الحلول المعطاة كانت رقمية او خطية و كانت الحلول الخطية او التحليلية من تقريبات قاسية يعطي هذا البحث حل تحليلي واقعي لهذه المشكلة آخذا بنظر الاعتبار عوامل الالتحام الأحادية و الثنائية.

KEY WORDS
physics of semiconductors, bipolar transistor

INTRODUCTION
Many textbooks and research papers have studied the problem of injection from one side into a semiconductor region, but only few [Lampert and Rose;1961,1959] have tackled this problem for a lightly doped region due to the formation of plasma layer and the accompanying bimolecular recombination kinetics. The solutions achieved were either numerical or very crudely approximated analytical solutions where the distribution of carriers inside the plasma were assumed to be straight lines.
This paper gives an analytical solution to this problem taking into account the formation of plasma layer and the bimolecular recombination for a long region ($>> L_n$) and short region ($<< L_n$).

FORMULATION OF PROBLEM

The injection of electrons into a lightly doped p-type region will be accompanied by a gathering of holes to neutralize the electron charge, since the injected electron charge will modulate the lightly doped base region even at low current densities.

Modeling, this process could be achieved by many approaches. Here , we are dealing with the analytical solution of the continuity equation for the problem.

The continuity equation could be written as eq. (3)

$$\frac{1}{q}\nabla J_n + (G - U_n) = 0 \qquad (1)$$

for steady-state operation. G will be small since there is no any light generation. Then eq. (1) in one dimension will be :

$$\frac{1}{q}\frac{d}{dx}J_n - U_n = 0 \qquad (2)$$

for a lightly doped region acting as base region for a transistor, or a biased two contact device (i.e. n^+-π-p^+) , the injected electrons will move towards the collecting end, holes will be attracted to neutralize the electron charge, thus plasma layer is formed as a result of this charge accumulation where the concetrations of holes and electron are equal. The applied voltage in such devices (e. g. a transistor) will prevent the motion of holes, and the holes will be held in their places against their concentration gradient, thus, the hole current density will be effectively zero, i.e. [Senhouse 1973, Abu Nuilah 1980]

$$J_p = {}_q\, p\mu_p E - {}_q Dp\frac{dp}{dx} = 0$$

and the electric field value is found to be

$$E = \frac{1}{q}\quad\frac{Dp}{\mu_p}\frac{dp}{dx}$$

which if substituted in the electron current density equation will give:

$$J_n = 2\,q\,D_n\frac{dn}{dx} \qquad (3)$$

Differentiating equation (3) gives:

$$\frac{d}{dx}\quad J_n = 2\,q\,D_n\frac{d^2 n}{dx^2} \qquad (4)$$

substituting this value in eq. (2) and rearranging result in:

$$2\,D_n \frac{d^2 n}{dx^2} = U_n \qquad (5)$$

where U_n is the electrons recombination factor. There are mainly two kinds of recombination namely monomolecular and bimolecular [Poon et al 1969].

THE ANALYTIC SOLUTION

Monomolecular Recombination

In the case of monomolecular recombination, U_n can be defined as:

$$U_n = R - G = \frac{-n_p - n_{po}}{\tau_n} \qquad (6)$$

substituting this value of U_n in the equation (5) and rearranging yields:

$$\frac{d^2 n}{dx^2} + \frac{1}{2D_n \tau_n}(n_p - n_{po}) = 0 \qquad (7)$$

Integrating the last equation gives [Sze 1969]

$$\mathbf{n(x)} = \mathbf{n_{po}} + (n_p(o) - n_{po})e^{-x/L_n} \qquad (8)$$

for a lightly doped region larger than L_n where $L_n=\sqrt{2D\tau}$. Equation (7) becomes [Groue 1967]

$$\mathbf{n(x) = n_{po} + (n_p(o) - n_{po})(1 - \frac{x}{w_B})} \qquad (9)$$

where W_B is the length of lightly doped region if W_B is much less than L_n.

Bimolecular Recombination

To arrive at a more accurate result for this problem, we need to consider the case of bimolecular recombination, since there is an equal concentration of holes and electrons in the plasma layer. The recombination factor, Un , resulting from such process is defined as :

$$\mathbf{U_n = f(n,p)}$$
$$= q.C.n.p \qquad (10)$$

where C is a constant which will be defined below, substituting this value into equation (2) yields:

$$\frac{d}{dx} \quad \mathbf{J_n = C.n.p} \qquad (11)$$

In order to find the constant, C, we shall define U_n as:

$$U_n = n/\tau_n \qquad (12)$$

where τ is the bimolecular recombination lifetime of electrons which depends on the concentration of holes in the plasma layer (7) as given:

$$\tau_n = 1/\alpha.p. \qquad (13)$$

where $\alpha = V_s.s$ (14)

and V_s is the thermal velocity of carriers. S is the bimolecular recombination cross sectional area. Thus eq. (13) can be written as :

$$\tau_n = 1/V_s.p.S. \qquad (15)$$

Substituting this value in eq. (12) yields:

$$U_n = V_s.s.\ n.p. \qquad (16)$$

This relation gives constant, C. in equation (10) and shows that recombination in plasma region varies with the product of electron and holes concentrations. Using the value of U_n derived above in eq.(11) gives:

$$\frac{d}{dx} J_n = q.V_s.s.n.p. \qquad (17)$$

using eq. (4) and rearranging yields:

$$\frac{d^2 n}{dx^2} = \left(\frac{V_{S.s}}{2D_n}\right).n.p. \qquad (18)$$

assuming that the bimolecular recombination cross sectional area will be constant, and the values of electron and hole concentrations are equal gives:

$$\frac{d^2 n}{dx^2} = C_n^2 \qquad ...(19)$$

this equation, when solved for a long lightly doped region under the following boundary conditions,
at x=0, n= n(0), $n^{\cdot}$ = n(0)
at x = ∞ , n=0, $n^{\cdot}$ = 0

will give the following distribution:

$$\mathbf{n}^{-\frac{1}{2}} = \mathbf{n}(0)^{-\frac{1}{2}} - \sqrt{\frac{\mathbf{c}}{6}}\ x \tag{20}$$

where C = V_s. s/2.D_n , and n(0) is the value of injected electrons at x=0, and is given by :

$$\mathbf{n}(0) = \mathbf{n}_i\ e^{\frac{qv}{2KT}} \tag{21}$$

Another analytical solution can be achieved if a solution is sought for a short lightly doped region acting as a base for a transistor for the following boundary conditions:

at x = 0 n = n(0), $n^{\cdot}$ = n (0)
x = x_1 n = K , $n^{\cdot}$ = 0

Where K represents the electrons value at the collector depletion layer edge (x = x_1). Then the following distribution will be achieved:

$$\mathbf{n} = \mathbf{K}(1 + \mathbf{F}(\mathbf{x})) \tag{22}$$

where

$$\mathbf{F}(\mathbf{x}) = (\mathbf{g}\,\frac{\dfrac{-(1/\mathbf{a})\,\mathrm{sn}(\mathrm{wx}+\mathrm{H})}{((\mathrm{sn}(\mathrm{wx}+\mathrm{H})^2 - 1^{\frac{1}{2}})^2}}{\dfrac{-(1/\mathrm{a})\,\mathrm{sn}(\mathrm{wx}+\mathrm{H})}{((\mathbf{sn}(\mathbf{wx}+\mathbf{H})^2 - 1)^{\frac{1}{2}}}} \tag{23}$$

And $g^2=\sqrt{3}$, a = 2.0333175, w = $-\sqrt{\dfrac{2C}{3}}\mathbf{A.B}/8\mathbf{g}$

A = 2.294142, B = 4.64406, H is the integration constant which can be found if K is defined, along with the boundary conditions, using the mathematical tables for Jacobean function (Sn). The distribution of carriers in lightly doped region(e. g. plasma base of a transistor) is a proven to be other than a straight line as assumed in all related published literature.

CONCLUSION

The problem of injection into a lightly doped region is solved here analytically without the need to use crude approximations. Monomolecular and bimolecular recombination were used. The solutions were made to include both cases for a region longer than diffusion length of carriers and that of a very short region. The equations derived in this paper gives the distribution of carriers as a function of distance from the source.

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