ANALYSIS OF CONSOLIDATION BEHAVIOR OF SUBSOILS UNDER CYCLIC TRAPEZOIDAL LOADING

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ABSTRACT:

This paper is aiming to develop a numerical procedure using the finite element method for the analysis of soil consolidation by taking into consideration the loading and unloading cycles with particular emphases on the pore pressure built up. The study is concerned with trapezoidal cyclic loading. The shear stresses that are developed in the subsoil due to cyclic loading cause shear strains and change in the mean normal stresses due to distortion of soil element. Biot’s consolidation theory that adopted in this study may be able to take these changes into consideration. The results indicated that the excess pore water pressure increases with load cycles and finally, a steady state conditions are reached. This may be attributed to the undrained cyclic shearing stresses to the origin initial cyclic shear stress ratio. Also it can be explained that when the load is applied very fast the collapse load approaches that undrained conditions, whereas when the load is applied very slowly the collapse load approaches that under drained conditions. When the elastoplastic model is considered the pore pressure are considerably larger than those predicted by the elastic consolidation analysis. This can be attributed to the plastic volume strain that have been taken into consideration in the elastoplastic model.

الخلاصة:

الهدف من هذه الدراسة هو لايجاد طريقة عددية باستخدام العناصر المحددة لتحليل انظمام التربة مع الاخذ بنظر الاعتبار دورات التحميل واللاتحميل مع التركيز على تولد ضغط الماء المسامي. هذه الدراسة تتعامل مع دورات الحمل شبة المنحرفة. ان اجهادات القص المتولدة في التربة التحتية نتيجة دورات التحميل تسبب انفعالات قص وتغيير في معدل الاجهادات العمودية بسبب التشوه الحاصل بالتربة. نظرية Biot’sلظاهرة الانضمام تم تبنيها في هذه الدراسة والتي بامكانها اخذ بنظر الاعتبار التغيرات الحاصلة في الانضمام . وقد اظهرت النتائج ان ضغط الماء المسامي يزداد مع دورات التحميل الى ان تصل الى حالة الثبوت. وهذا يعزى الى نسبة اجهادات القص المتكررة الي اجهادات القص الاولية. كذلك ان تسليط الحمل بسرعة يؤدي الي وصول الفشل في الحالة الغير مبزولة وعندما يسلط الحمل ببطء فان الفشل يصل الى الحالة المبزولة وعند استخدام نموذج المرونة-اللدونة فان ضغط الماء المسامي يكون اكبر منه في حالة استخدام نموذج المرونة لظاهرة الانضمام. وهذا يعزي الى انفعالات اللدونة الحجمية التي تاخذ بنظر الاعتبار في نموذج المرونة-اللدونة.

KEY WORDS:
Finite element; Biot's consolidation; trapezoidal loading; unload cycles; plasticity; pore pressure; subsoil; collapse load.

INTRODUCTION

An important subject in soil mechanics and geotechnical engineering is to analyze the deformation behavior of soils, which may be fully or partially saturated with water. The deformation is coupled with the generation of pore water pressure. This kind of behavior is generally called consolidation.

It is obvious that both quality and performance of roads and other geotechnical structures will largely depend on the mechanical behavior of subsoils, especially the time-dependent response under cyclic loading. Computer modeling with finite element method is one of the most popular numerical methods currently used in modeling response of road subsoil. In this paper the term subsoil means the natural soil below a road embankment, and the term embankment includes all soil layers above natural ground surface.

A large number of constitutive models have been proposed to describe the deformation behavior of soils. Most of these models can be classified into one of the following categories: elastic, elastoplastic, elasto-viscoplastic models. Among the elastoplastic models there are perfectly plastic, isotropic hardening and kinematic hardening models. Since the constitutive models presented in this study will be used mainly for analysis of soils under monotonic trapezoidal cyclic loading, the isotropic hardening models will suffice for the purpose.

Schiffman (1958) first obtained a general solution of soil consolidation considering loading increase linearly with time. ***Wilson and Elgohary (1974)*** presented an analytical solution of one-dimensional consolidation of saturated soil subjected to cyclic loading based on Terzighi's linear consolidation theory.

Alonso and Krizek (1974) considered the settlement of elastic soft soil under stochastic loading. ***Baligh and Levadoux (1978)*** developed a simple prediction method for one-dimensional consolidation of clay layer subjected to cyclic rectangular loading with the superposition principle.

More recently, ***Favaretti and Soranzo (1995)***, ***Guan et.al. (2003)*** derived some solution for different types of cyclic loading.

* BIOT CONSOLIDATION THEORY

Consolidation theory is the mathematical theory describing the dissipation of excess pore pressure and associated deformation of the soil ***(Lambe and Whitman, 1969)***. Most of the consolidation problems in two and three dimensions are analyzed based on two basic approaches. The first was developed from diffusion theory by Terzighi and Rendulic. The second was developed directly from elastic theory by Biot ***(Scott, 1978)***. In Biot consolidation theory the soil skeleton is treated as a porous elastic solid and the laminar porefluid is coupled to the solid by the conditions of compressibility and continuity ***(Smith and Griffithes ,1998)***.

Biot's covering equations for two or three dimensional consolidations theory is given by:

$$\frac{\partial u}{\partial t} = C_v \nabla^2 u + \frac{\partial \sigma_m}{\partial t} \qquad (1)$$

Where:

u : excess pore water pressure.

σ : mean normal stress = $\frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$

$\frac{\partial u}{\partial t}$: variation of pore pressure with time.

∇^2: Laplace's operator.

C_v: coefficient of consolidation.

Mandel (1953), showed the same behavior by solving Biot equations for a rectangular section with drainage permitted along both sides only ***(Scott, 1978)***.

Smith and Hoobbs(1976), implemented Biot's theory of consolidation by means of finite elements in the solution of field consolidation problems. The method allows for simultaneous changes in geometry, loading and material properties during and after construction and the calculated results are compared with field measurements.The shear stresses that are developed in the subgrade soil or subsoft soil due to cyclic trapezoidal loading cause shear strains and change in the mean normal stresses due to distortion of soil element. Biot's consolidation theory may be able to take these changes into consideration. For this reason this theory is adopted in the present study.

*** FINITE ELEMENT SOLUTION OF COUPLED PROBLEM:**

The finite element method is the most powerful technique for numerical analysis. It can be used to solve many classes of problems and also it can take into account both linear and non-linear material properties which may be formulated in terms of either total or effective stress ***(Scott, 1978)***.

In the analysis of Biot consolidation theory, the soil is treated as a porous elastic solid and the laminar pore fluid is coupled to the solid by the conditions of compressibility and continuity ***(Smith and Griffithes, 1998)***.

Thus the Biot's governing equation is given by:

$$\frac{K'}{\gamma_w}\left[k_x \frac{\partial^2 u_w}{\partial x^2} + k_y \frac{\partial^2 u_w}{\partial y^2} + k_z \frac{\partial^2 u_w}{\partial z^2}\right] = \frac{\partial u_w}{\partial t} - \frac{\partial p}{\partial t} \qquad (2)$$

Where:

K' : soil bulk modulus.

p : mean total stress.

Kx: permeability in the x-direction.

Ky: permeability in the y-direction.

Kz: permeability in the z-direction.

Due to coupling of fluid and solid phases the applied total stress (σ_t) are divided between a portion carried by the soil skeleton called the effective stress (σ') and a portion carried by the pore water called the pore pressure and denoted by (u_w).

In the absence of body forces, and for z- dimension problems, the equations to be solved are the following:

1) EQUILIBRIUM

The gradient of effective stress is given in equation below ***(Timoshinko and Goodier, 1970)***:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + F_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + F_y = 0 \qquad (3)$$

The gradients of the fluid pressure (u_w) are given as:

$$\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial u_w}{\partial x} = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma'_r}{\partial y} + \frac{\partial u_w}{\partial y} = 0 \qquad (4)$$

2) Constitutive Relation

The constitutive laws for the solid and fluid respectively, are shown below for plane strain **(Smith and Griffiths, 1998)**:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\upsilon} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad (5)$$

$$\begin{bmatrix} q_x \\ q_y \end{bmatrix} = \frac{1}{\gamma_w} \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \begin{Bmatrix} \frac{\partial u_w}{\partial x} \\ \frac{\partial u_w}{\partial y} \end{Bmatrix} \qquad (6)$$

Where:

q_x, q_y : the volumetric flow rate per unit area into and out of the element.

γ_w : the unit weight of water.

In the case of incompressibility the out flow from an element of soil equals the reduction in volume of the element:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial z} = -\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial v}{\partial \theta}\right) \qquad (7)$$

From (Eq.6), the third differential equation is given by:

$$\frac{k_x}{\gamma_w}\frac{\partial^2 u_w}{\partial x^2} + \frac{k_y}{\gamma_w}\frac{\partial^2 u_w}{\partial y^2} + \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial v}{\partial \theta}\right) = 0 \qquad (8)$$

The equilibrium and continuity equations for Biot consolidation are given by:

$$[KM]x + [C]u_w = f \qquad (9)$$

$$[C]^T \frac{\partial x}{\partial t} - [KP]u_w = 0$$

Where:

(KM) and (KP) are the solid and fluid stiffness respectively.

For the same linear shape functions govern the variation of pore water pressure and displacements within an element, the connection coupling matrix C is given as shown **(Smith and Griffiths,1998):**

$$C = \iint vol * FUNxdxdy \qquad (10)$$

Where:
FUN : array held the shape function in terms of local coordinates.
Vol : volumetric strain vector of an element.
The matrix product is called VOLF and C is numerically integrated as:

$$C = \sum_{I=1}^{NIP} DET_I * weights(i) * VOLF_I * R \qquad (11)$$

Where:
NIP : is the number of integrated points.
DET : is the determinant of Jacobian matrix.
Weights(I) : is the weighting coefficient corresponding to the particular integrating point.
R : the radial coordinate of each Gauss point.
After assembly into global matrices, equation (9) must be integrated in time using finite differences method as shown by:

$$\theta KMx_1 - \theta Cu_{w1} = (\theta - 1)KMx_0 + (\theta - 1)Cu_{w0} + f \qquad (12)$$
$$\theta C^T x_1 - \theta^2 \Delta tKPu_{w1} = \theta C^T x_0 - \theta(\theta - 1)\Delta tKPu_{w0}$$

Where:
f : is the total force applied
θ : parameter in implicit time-stepping $(\frac{1}{2} \le \theta \le 1)$.

* EXPERIMENTAL WORK

Undrained Triaxial Test

In undrained triaxial test the vertical and horizontal stresses are kept constant, after an initial shear stress, q has been applied. The properties of soil specimen are shown in Table (1), Under relatively small applied shear stresses (about 30% of the shear strength as determined in conventional tests) the creep movements are small while the excess pore water pressure increases. Under shear stresses of still higher intensity, an acceleration of the creep rate takes place followed by complete failure of the specimen (creep rupture).

As for the constant strain rate tests, the triaxial test had been completed by initially consolidating the samples under an effective confining pressure of 525 kPa and the test were performed at the constant shear stress of q=278.3 kPa. The results of the triaxial test were obtained and used for verification of the finite element program ***(ZCONS)***.

A comparison is made between the results obtained by the present finite element analysis and the triaxial test results. A good agreement is indicated as shown in Fig. (1).

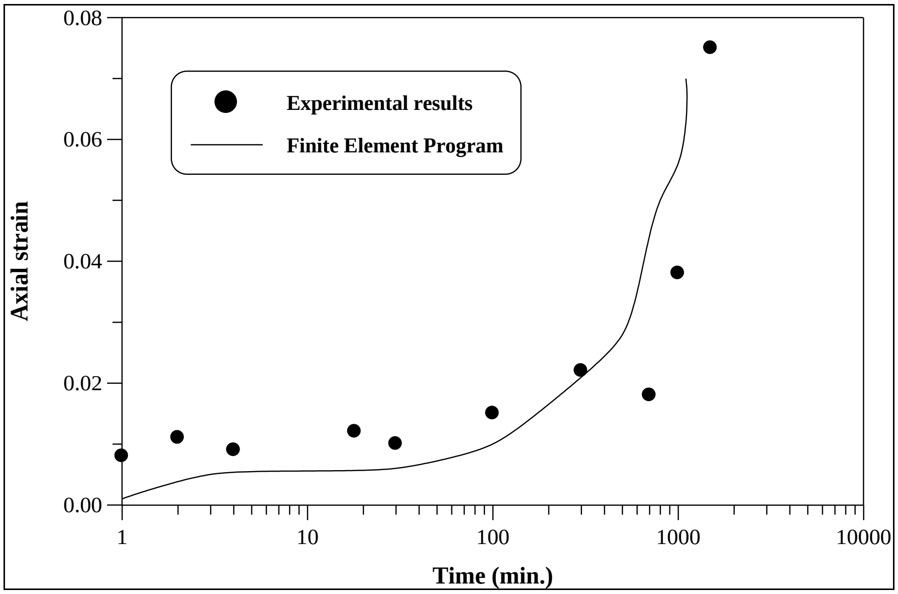


Fig. 1 Verification of Finite Element Computer Program ZCONS.

* RESULTS AND DISCUSSIONS

Finite Element Program

The computer oriented finite element method has become one of the most powerful tools in the analysis of the engineering problems. It has unified the analysis of any type of structures under boundary and loading conditions to one basic fundamental procedure. To carry out the analysis of this study, the finite element program ZCONS written in Fortran 90 language is developed. Which is primarily based on the program presented by (AL-Kaissi, 2001) for the analysis of elasto plastic constitutive relations. Extensive modifications and newly added subroutines are found necessary to incorporate the case study. ID program is divided into ten major parts with subroutine hierarchy ,as shown in Appendix (A). The basic finite steps are performed by primary subroutines, which rely on auxiliary subroutines to carry out secondary operations. An auxiliary subroutine may be required by more than subroutine, as shown in Appendix (A). The order of calling of the primary subroutine is controlled by a main or master routine.

The finite element program is utilized in conjunction with Biot's consolidation theory and the soil is treated as an elasto-plastic material basing on Mohr-Coulomb failure criterion. A quadrilateral hybrid element eight nodded regarding displacement four nodded for pore water pressure is adopted in the present analysis. The finite element method has been used together with Crank-Nicolson time discrietization and constant time steps. The displacement, rate of consolidation and pore pressure are evaluated by means of computer program which is modified to suit the objectives of this study.

CASE STUDY

Application of the proposed finite element program to a case study, the values of soil parameters used in the analysis where obtained from previous study presented by (Mannesmanne, 1974) regarding the soil investigation report about the Fao region in the far south part of Iraq. Table (1) displays the parameters regarding the clay layer and the finite element mesh used for analysis is shown in Fig. 2.

Table 1. Material Properties of the Soil.

Soil Parameters	Silty Clay
E(kN/m^2)	2108
ν	0.35
k_x (m/day)	4.33x10 $^{-4}$
k_y (m/day)	8.67x10 $^{-6}$
C (kN/m^2)	9.2
ϕ'	30.0
ψ'	0.0

This study deals with the generation and dissipation of pore water pressure and the rate of consolidation under loading and unloading cycle and their influences in the subsoils layer. This paper presents the finite element numerical procedure to study the consolidation problem by taking into consideration the loading and unloading cycles with particular emphases on the pore pressure built up. The model used to simulate this problem assumes a soil stratum with thickness H, vertical and radial permeability coefficient k_x, k_y as shown in Table (1) with the others parameters regarding the clay layer that have been used in the finite element program ***ZCONS***.

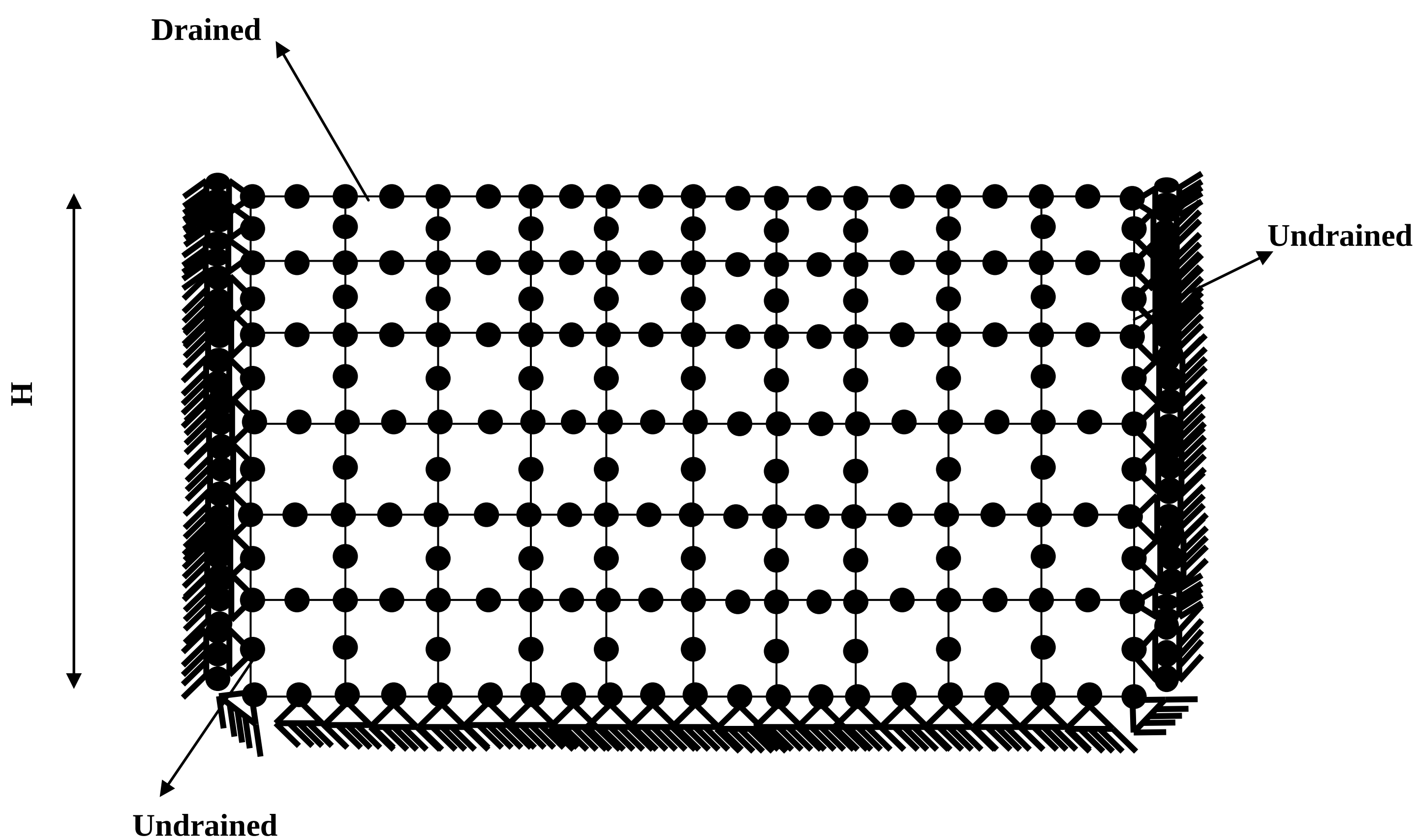


Fig. 2 The Finite Element Mesh.

The soil layer is subjected to a vertical trapezoidal loading as shown in Fig. (3), also the rectangular cyclic loading also considered in this study. In which q_0 is the maximum load, t_L is cyclic loading time. Fig. (4) shows the variation of consolidation with load cycles (loading and unloading). It can be seen from this figure, that for a given load cycle, the degree of consolidation for each cycle reaches the maximum value at the end of the constant loading phase and the minimum value at the beginning of the next cycle (i.e. at the end of the unloading cycle).

In Fig. (5) the effect of loading type is shown. The influence of loading type on consolidation process, from which it can be seen that it more significant for trapezoidal loading. (Since it simulate the actual traffic loading on soil). Fig. (6) was plotted to investigate the influence of construction time on consolidation, here Tv time factor is varying, it can be seen that the longer the construction time which means the greater time factor, the slower the rate of consolidation.

Also Fig. (7) presents the excess pore water pressure distribution with depth at maximum applied load of one cycle for different number of load cycles. We obtain from this figure that pore pressures build up increase. Additionally, a steady state conditions is reached between cycles (4) to (7). Since the excess pore water pressure is affected by changes in mean total stress, it may continue to increase for some time after the application of load cycle. This may be attributed to the undrained cyclic shearing stresses with the origin initial cyclic shear stress ratio, where shear stress ratio is defined as the cyclic shear stress on the horizontal plane over the initial vertical stress.

Figs. (8) and (9) show the analysis of collapse load response with the axial deformation. For the applied stress or load on the soil, the load is assumed to be as shown in Fig. (3).

At some given rate of loading, this is defined as:

$$\omega = \frac{dq}{dTv} \qquad (13)$$

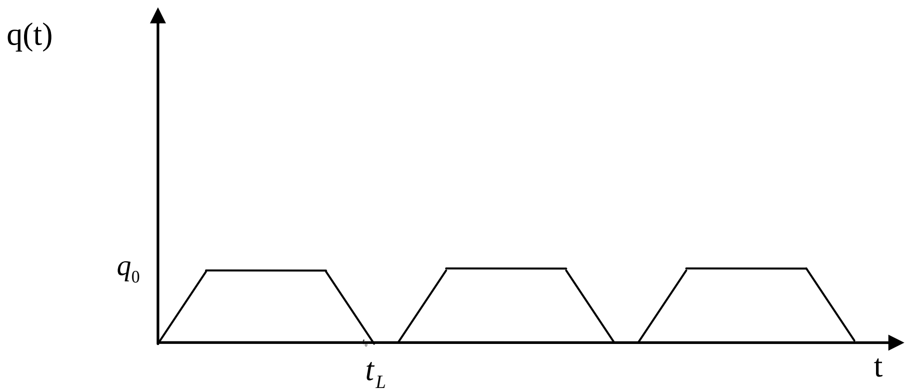
Where Tv is the non-dimensional time factor, with H (one way drainage conditions) being the drainage length. And different values are used to obtain different rates of loading.

The collapse load under drained and undrained conditions can be estimated. The load deformation curves obtained for two different loading rates are shown in Fig. (8) and (9). From these figures it can obtained that when the load is applied very fast the collapse load approaches that undrained conditions, where as the load is applied very slowly the collapse load approaches that under drained conditions.

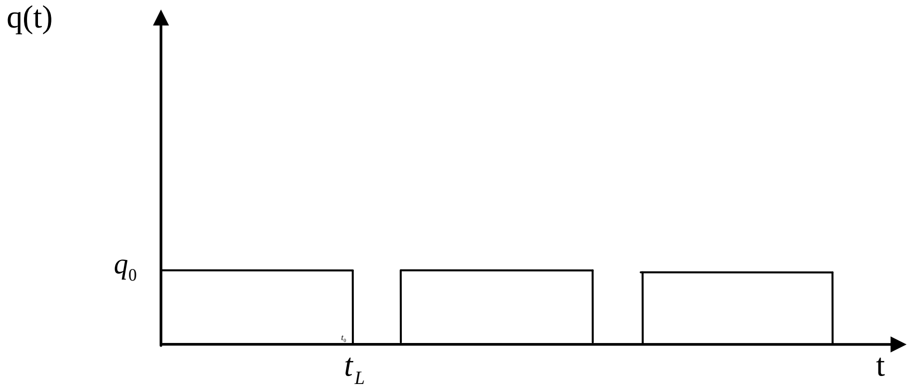
To study the effects of plasticity model of soil on consolidation analysis, elastic and elastoplastic consolidation model are compared as shown in Fig. (10) and (11) respectively.

From these figures it can be seen that when the elastoplastic model is considered the pore pressure are considerably larger than those predicted by the elastic consolidation model analysis as shown in Fig. (10), which may be attributed to the plastic strain that have been taken into consideration in the elastoplastic model.

Also Fig. (11) shows that the rate of consolidation takes a longer time to complete when the nonlinear elastoplastic model is considered.



(a) Trapezoidal, t_L :time of loading



(b) Rectangular

Fig. 3 Loading Types: a) Trapezoidal loading
b) Rectangular loading

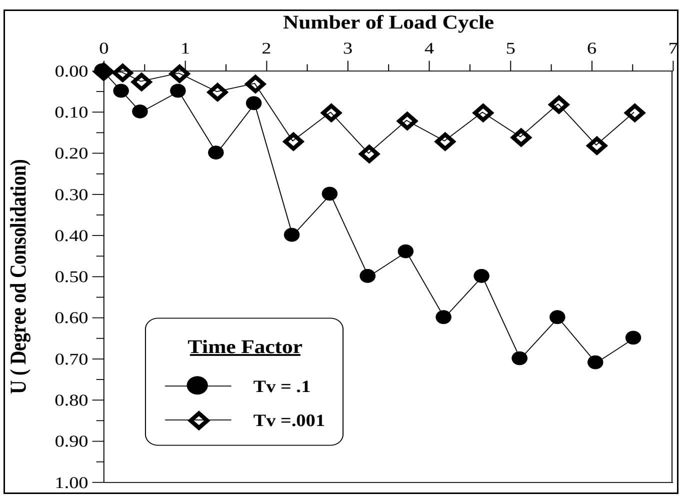


Fig. 4 Variation of Degree of Consolidation with Load Cycles.

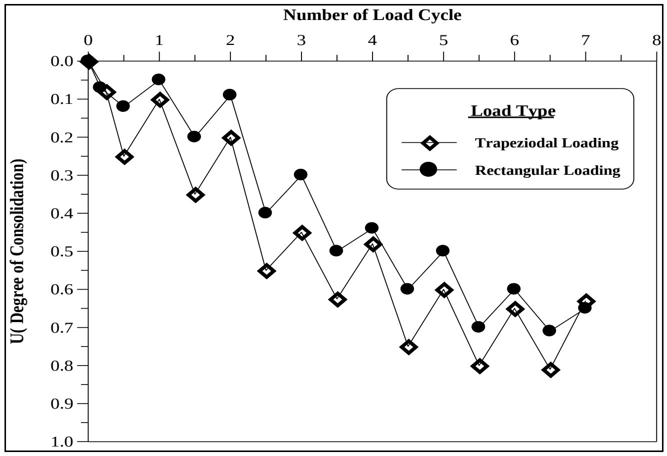


Fig. 5 Effect of Loading Type on Rate of Consolidation.

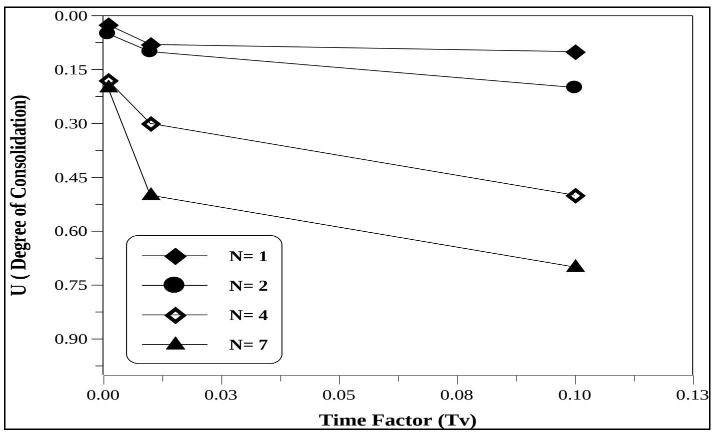


Fig. 6 Effect of Time Factor on Consolidation Rate for different number of load cycles)

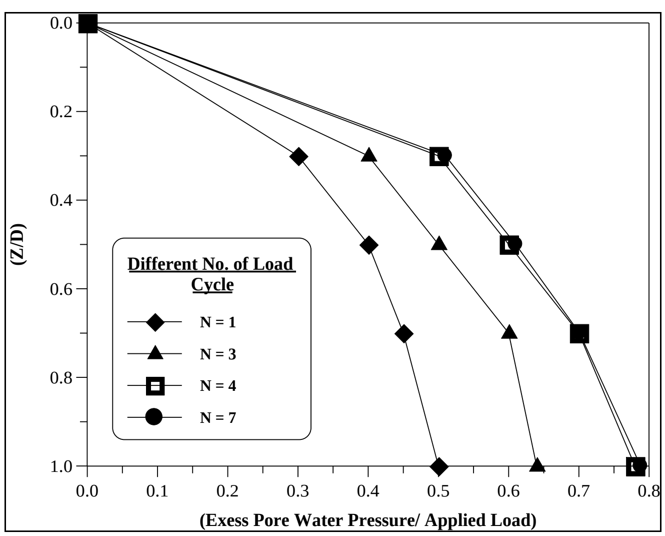


Fig. 7 Distribution of Excess Pore Water Pressure with Depth for Different Load Cycles.

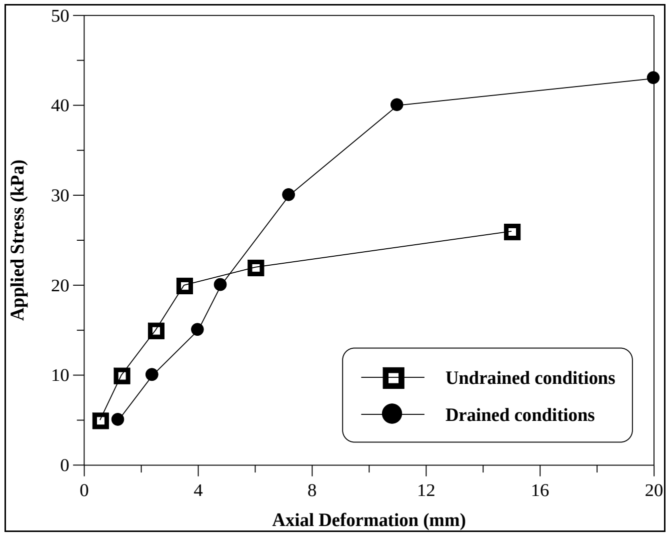


Fig. 8 Response of Collapse Load at fast rates of Loading and under the loading center.

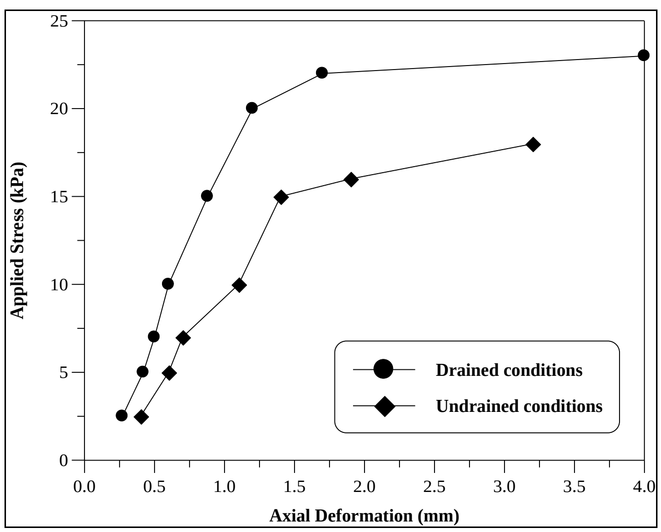


Fig. 9 Response of Collapse Load at slow rate of Loading.

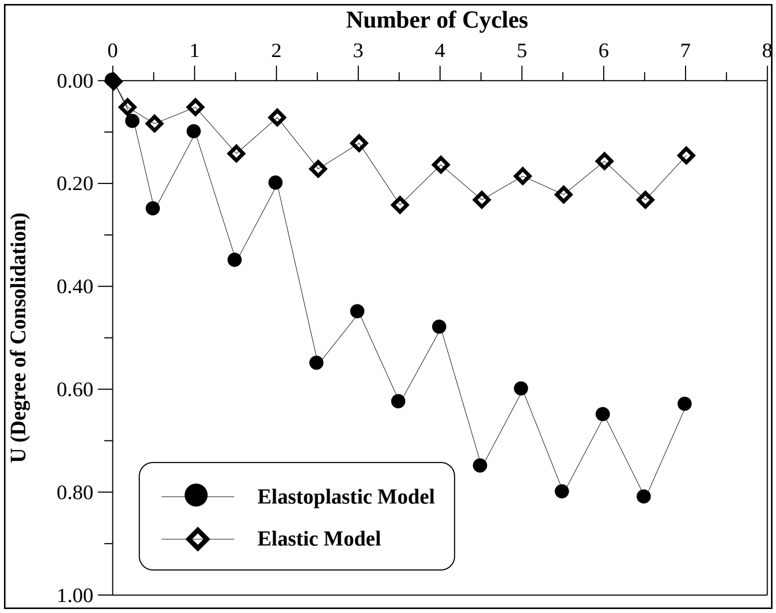


Fig. 10 Comparisons of Consolidations for Two Models.

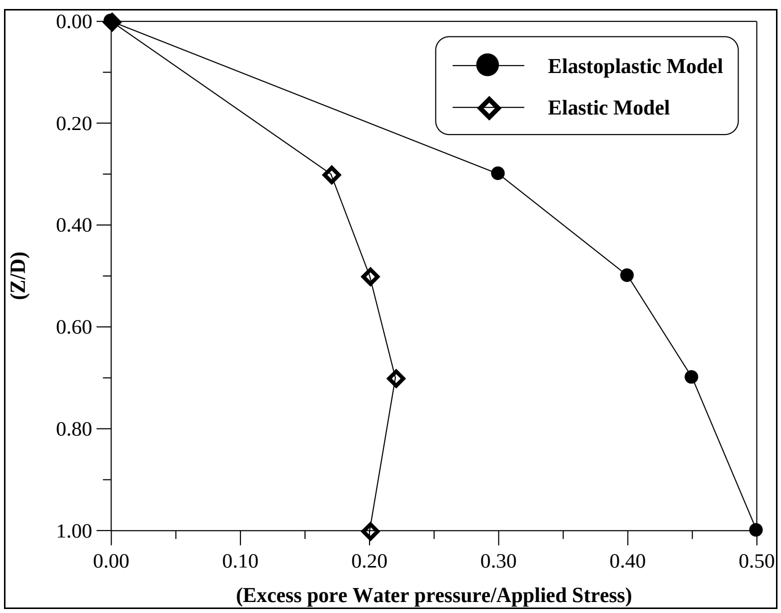


Fig. 11 Comparisons of Pore water Pressure for Two Models.

* CONCLUSIONS

The main conclusions that can be drawn from this study are the following:

1. For a given load cycle, the degree of consolidation for each cycle reaches the maximum value at the end of the constant loading phase and the minimum value at the beginning of the next cycle (i.e. at the end of the unloading cycle).
2. The excess pore water pressure increased with load cycles and finally, steady state conditions are reached between cycles 4 to 7. Since the excess pore water pressure is affected by changes in

mean total stress, it may continue to increase for some time after the application of load cycle. This may be attributed to the undrained cyclic shearing stresses with the origin initial cyclic shear stress ratio, where shear stress ratio is defined as the cyclic shear stress on the horizontal plane over the initial vertical stress.

3. The loading type affects the consolidation process, from results it can be seen that it is more significant for trapezoidal loading. (Since it simulate the actual repeated traffic loading on soil).
4. Analysis on collapse load has been carried out, it can be obtained that when the load is applied very fast the collapse load approaches that undrained conditions, where as the load is applied very slowly the collapse load approaches that under drained conditions.
5. When the elastoplastic model is considered the pore pressures are considerably larger than those predicted by the elastic consolidation model analysis. This can be attributed to the plastic volume strain that have been taken into consideration in the elastoplastic model. Also the rate of consolidation takes a longer time to complete when the nonlinear elastoplastic model is considered.

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List of Symbols

Symbols	**Description**
E	Modulus of elasticity
ν	Poisson's ratio
C	Cohesion
C_v	Coefficient of consolidation
u_w	Excess pore water pressure
σ_m	Mean normal stress
∇^2	Laplace's operator
K'	Soil bulk modulus
q_x, q_y	Volumetric flow rate per unit area
k_x, k_y	Permeability in x, y direction
γ_w	Unit weight of water
$[KM],[KP]$	Solid and fluid stiffness matrices
FUN	Array held the shape function
VOL	Volumetric strain vector of an element
DET	Determinant of Jacobian matrix
NIP	Number of Gauss
θ	Parameter in implicit time-stepping
Δt	Time interval
$\sigma_x, \sigma_y, \sigma_z$	Total normal stress in x,y and z direction
τ	Shear stress
ϕ'	Angle of shearing resistance
ψ'	Angle of dilation

Appendix A
Flow Chart of Computer Program (ZCONS)