PID CONTROLLER DESIGN FOR THE SATELLITE ATTITUDE CONTROL SYSTEM

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ABSTRACT

A PID controller satellite attitude yaw-axis control system was designed to step the angle of the satellite body via actuating a precise angular position of a DC motor as quickly and accurately as most optimally possible. The (PID) controller was advantageously chosen for its quick transient response and zero steady-state error. A mathematical model has been derived for the whole satellite attitude yaw-axis control system. Then, the computation power of the MATLAB is utilized to obtain a generalized optimal controller design that enables the satellite attitude yaw-axis control system to have the quickest settling time without excessive overshoot and zero steady state error.

الخلاصة

تم تصميم مسيطر ثلاثي الانشطة (تناسبي / تكاملي / تفاضلي) لمنظومة السيطرة حول المحورالعمودي (الانعراج) لموقع القمر الاصطناعي بحيث يتم وضع جسم القمر الاصطناعي بالزاوية المطلوبة عبر محرّك مباشر التيار بامثل سرعة وافضل دقّة .تم اختيار المسيطر ثلاثي الانشطة لانفراده بسرعة اجتيازه للاستجابة العابرة وانعدام الخطأ في الحالة المستقرة.

تم اشتقاق النموذج الرياضي الكلي لمنظومة السيطرة حول المحورالعمودي (الانعراج) لموقع القمر الاصطناعي . ثم استثمرت القدرة الحاسوبية لبرامجيات **الماثلاب** للتوصّل لتصميم المسيطر ثلاثي الانشطة الامثل الذي يستطيع جعل منظومة السيطرة حول المحورالعمودي (الانعراج) لموقع القمر الاصطناعي ذات زمن استقرار اسرع بدون المرور عبر استجابات متجاوزة لنقطة الضبط وانعدام الخطأ في الحالة المستقرة.

KEY WORDS

PID controller, ACS mathematical model, satellite attitude yaw-axis control system, ACS

INTRODUCTION

The control task of the satellite attitude control system demands an optimal controller capable of precisely rotating the satellite body into the desired attitude. The researches concerning satellite attitude control system are now conducted within the premises of international space centers like NASA, The Danish Ørsted Geomagnetic Research center / ESA , and charted universities / institutions like Princeton Satellite

Systems, Inc., CTA Space Systems, Inc. [1], engineering college of swarthmore of sccs .

Although some international efforts had been recently reported in the design of attitude control system such as the Ørsted satellite attitude control system (ACS) project [5] [6] [7] [8] for its 65 kg micro-satellite, and a mutual project paper issued by Princeton Satellite Systems, Inc., together with CTA Space Systems, Inc., the development of Satellite Attitude Control System architecture, called the SPACECRAFTC ONTROL SYSTEM [1]. However this research paper reports an alternative powerful optimal controller design for the satellite ACS empowered by the computational tools of MATHLAB [3].

The satellite attitude control system manipulates, in this paper, the angular position of the satellite body around the yaw axis. A DC motor is advantageously selected [2] [5] as the actuating element to rotate the satellite body to the desired yaw angle. An amplifier is saturated by the correcting signal from the designed optimal PID controller [1] [2] [4] to obtain the quickest response with zero steady state error.

The mathematical model of the whole satellite system has been derived. A PID controller is optimally designed accordingly via a powerful MATLAB program and a Nyquist stability analysis is conducted.

SATELLITE SYSTEM MATHEMATICAL MODEL:

The satellite attitude control system can diagrammatically be represented by the following block diagram;

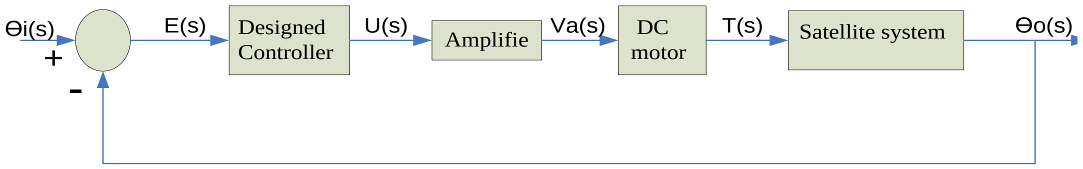

Fig.(1) The Satellite attitude control system

The moment of inertia of the entire system is J, which encompasses both the satellite body moment of inertia about the axis of rotation at the center of mass (J_f) and that's of the motors' armature (J_a). There is a viscous friction, B, as part of the load elements.

Let us first obtain the transfer function of the satellite system. The angular displacement $\theta_o(s)$ of the satellite body around the yaw-axis is the output of the satellite system and the DC motor torque T(s) is the input. Thus the transfer function of the satellite attitude system is ($\theta_o(s)/ T(s)$).

The differential equation for the load elements is; $J(\frac{d^2\theta_o}{dt^2}) + B(\frac{d\theta_o}{dt}) = T$ .. (1)

Taking the Laplace transforms of both sides of equation (1) and assuming zero initial conditions, we get after rearranging; $s(s+\frac{B}{L})\theta_o(s) = \frac{T(s)}{J}$ .... (2)

Directly from equation (2), $\dfrac{\theta_o(s)}{T(s)} = \dfrac{1/J}{s(s+\frac{B}{J})}$ ………….(3)

Next, the transfer function of the DC motor can be obtained as the torque T(**s**) is the output form the motor, and the input is the amplifier voltage (Va(**s**)), which supplies current (i_a) to the resistance (R) and inductance of the armature winding, consequently, the motor creates a back-emf, v_m. Thus, by applying the kirchhoff's voltage law, the differential equation for the DC motor armature circuit is; $L(\dfrac{di_a}{dt}) + Ri_a = (v_a - v_m)$ ……………. (4)

Taking the Laplace transforms of both sides of equation (4) and assuming zero initial conditions, we get; $I_a(s) = \dfrac{(V_a(s) - V_m(s))}{L(s+\frac{R}{L})}$ …………….. (5)

Since the motor's torque (T) is proportional to the armature current, (i_a). Then, $T=K_T\ i_a$ …….. (6)

Taking the Laplace transforms of both sides and Substituting, we get;

$$T(s) = \frac{K_T}{L}\frac{(V_a(s) - V_m(s))}{(s+\frac{R}{L})} \quad \text{………….. (7)}$$

Where, $K_{T;}$ is the torque gain.

And the angular velocity from the motor is proportional to the back-emf, v_m, thus;

$$v_m = K_m \frac{d\theta}{dt} \quad \text{………… (8)}$$

Where, $K_{m;}$ is the back-emf, (v_m), gain.

Since at steady state, the angular acceleration term = zero, in equation (1), so we get;

$$T = B\frac{d\theta}{dt} \quad \text{………… (9)}$$

From equations (8) & (9) we get, $v_m = \dfrac{K_m}{B}T$ ………… (10)

Taking the Laplace transforms of both sides, substituting, and rearranging we get;

$$T(s) = \frac{K_T}{BL} \frac{(BV_a(s) - K_m T(s))}{(s + \frac{R}{L})} \quad \text{……….. (11)}$$

$$BL(s + \frac{R}{L})T(s) = (BV_a(s) - K_m T(s)) \quad \text{………….. (12)}$$

$$(BLs + BR + K_T K_m)T(s) = K_T BV_a(s) \quad \text{…………. (13)}$$

$$(s + \frac{(BR + K_T K_m)}{BL})BLT(s) = K_T BV_a(s) \quad \text{………….. (14)}$$

$$T(s) = \frac{K_T}{L} \frac{1}{(s + \frac{(BR + K_T K_m)}{BL})} V_a(s) \quad \text{………….. (15)}$$

Hence, the DC motor has a first order transfer function; $\frac{T(s)}{V_a(s)} = \frac{K_k}{(s+b)}$ ….. (16)

Where, $K_k = \frac{K_T}{L}$, and, $b = \frac{(BR + K_T K_m)}{BL}$

Now, the control signal, U(s), is the feeding signal to the isolating amplifier of the DC motor, and its output, V_a (s), equals the input signal multiplied by the amplifier gain, $\mathbf{K}_{\mathbf{amp}}$, hence the amplifier's transfer function is; $\frac{V_a(s)}{U(s)} = K_{amp}$ ... (17)

Therefore, the satellite attitude system transfer function $\frac{\theta_o(s)}{U(s)}$ can be obtained from equations. (3), (19) & (20), with the angular displacement θ_o(s) of the satellite body around the yaw axis is the output of the satellite system and the control signal, U(s), is the input as following; $\frac{\theta_o(s)}{U(s)} = \frac{K_{ak}}{s(s + \frac{B}{L})(s+b)}$ …………… (18)

Where, $K_{ak} = \frac{K_{amp} K_k}{L}$

Equation (18) represents the satellite system transfer function which is a type one system.

The gains and the constant parameters within the general transfer function, as presented in equ (21), of the satellite system may be determined by selecting the closed loop pole locations. When selecting the pole locations, it is important to consider first the numerical values of the particular satellite body inertia, the DC motor armature inertia and electrical & electronics components, and the amplifier gain value. Hence, the following numerical values will be accounted for in this paper to go forward in the optimal controller design; $\frac{\theta_o(s)}{U(s)} = \frac{1}{s(s+2(s+5)}$ ……………… (19)

PID CONTROLLER DESIGN:

The satellite attitude control problem, in this paper, is to design a PID controller, able to step the yaw-axes angle so that the satellite body can be rotated into the desired attitude with little overshoot as quickly as less than two second settling time estimated for an approximately 90 degree step. The settling time criterion was within 98% of the final value.

The PID controller might be designed, as the control platform scheme for the satellite attitude control system. The controller transfer function, $G_c(s)$, is;

$$G_c(s) = \frac{E(s)}{U(s)} = K_p(1 + \frac{1}{T_i s} + T_d s) \quad \text{……………..} \ (20)$$

Where, K_p = the proportional gain, T_i = the integration time, T_d = the derivative time. Since the satellite attitude transfer system has an integrator, equ (22), hence the PID design starts with the second method of the Ziegler-Nichols tuning rule [7] [9]; and the controller transfer function become; $G_c(s) = K\frac{(s+a)^2}{s}$ …………. (21)

And the control problem is to determine the values of, K, and, a, such that the unit step response will exhibit the maximum overshoot, m, between 5% and 0%, and the settling time, ts, will be less than 2 sec.

A MATHLAB program is written to set the search region as;

$2 \le K \le 40 \qquad 0.05 \le a \le 0.5$

The step size for, K, to be 1 and that for, a, to be 0.05, so that to find the first set of variable (K) and (a) that will satisfy the satellite attitude control specifications.

The closed loop transfer function $\frac{\theta_o(s)}{\theta_i(s)}$ is given by;

$$\frac{\theta_o(s)}{\theta_i(s)} = \frac{(Ks^2 + 2Kas + Ka^2)}{(s^4 + 6s^3 + (5+K)s^2 + 2Kas + Ka^2)} \quad \text{………………..} \ (22)$$

A possible MATLAB program that produces the first optimal set of variable (K,a) and that will satisfy the given specifications is shown in appendix (1). The optimal values obtained by this program are; K = 21.0000, a = 0.3000, m% =4.71%, ts = 1.8100 sec. The resulting unit-step response curve is shown in appendix (2).

STABILITY ANALYSIS:

The designed optimal controller satellite attitude system may confidently be verified further by the stability analysis via Nyquist diagram. Thus, the open loop gain is given below: $G_c(s)G_{sat}(s) = \dfrac{K(s+a)^2}{s^2(s+1)(s+5)}$ …………… (23)

Substituting the optimal values (K, a) of the PID controller in the above equ., and re-write it into a polynomial form, we get; $G_c(s)G_{sat}(s) = \dfrac{(21s^2 + 1.26s + 1.89)}{(s^4 + 6s^3 + 5s)}$ … (24)

Consequently, the Nyquist plot is shown in appendix (4) as a result of running the MATLAB program in appendix (3).

Since the (-1) point is not encircled, the system is stable, which is confirmed by the obtained step response of the satellite attitude control system, as the unit step response exhibits the maximum overshoot, m, between 5% and 0%, the settling time, ts, is less than 2 sec, and zero steady state error. Thus the satellite did not lose control of itself when performing the real life operations in space.

COCLUSION

A PID controller is optimally designed for the satellite attitude control system so that the satellite body can be rotated around the desired Yaw-axis attitude of (90) degrees as quickly as less than (2) second settling time without excessive overshoot of not more than 4.71 %. A mathematical model for the satellite attitude control system is derived to be repeatedly utilized in analysis and design of the PID controller.

The powerful MATHLAB was the tool for the design and stability analysis stage. The outcome is a MATHLAB program which outputs an optimal PID controller parameters (K, a) to suite the control requirements of any satellite attitude control system.

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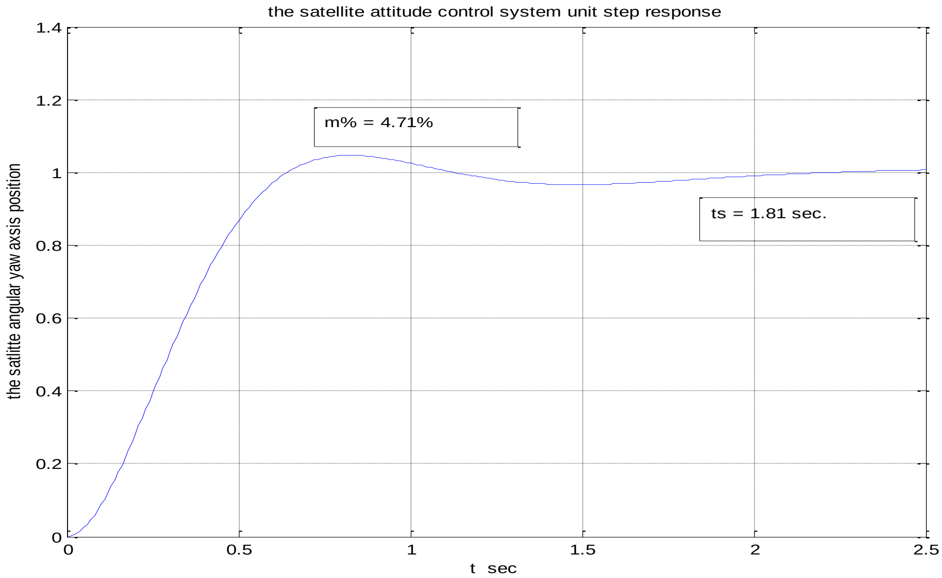
```
%%%%%%    PID controller for the SAC system %%%%%%%%%%%%%%%%%
t=0:0.01:2.5;
for k = 40:-1:2;%start outer loop to vary the k values
   for a = 0.5:-0.05:0.05;%start inner loop to vary the k values
      num = [0 0 k 2*k*a k*a^2];
      den = [1 6 5+k 2*k*a k*a^2];
        y = step(num,den,t);
        m = max(y);
        s = 251; while y(s)>0.98&y(s)< 1.02;
           s = s-1;end;
        ts = (s-1)*0.01;
        if m<1.05 & m > 1.00 & ts < 2.0
           break;% breaks the inner loop
        end
   end
   if m<1.05 & m > 1.00 & ts < 2.0
           break;% breaks the outer loop
        end
   end
   plot(t,y); grid; title( 'the satellite attitude control system unit step response')
   xlabel('t  sec'); ylabel(' the satellite angular yaw axis position')
  solution = [k;a;m;ts]

solution =

  21.0000
   0.3000
   1.0471
   1.8100
```

Appendix (1): a MATHLAB program to obtain the optimal PID controller parameters (K, a) that meets the satellite attitude control system.

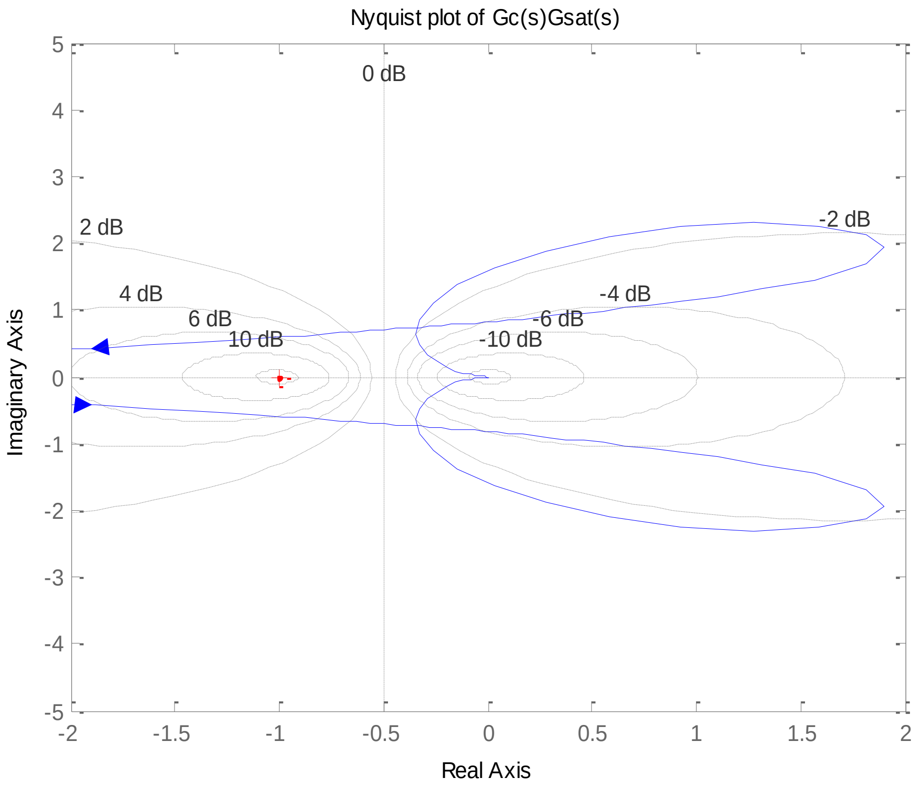


Appendix (2): The unit step response of the SAC system.

```
%%%%%%%% Nyquist plot of the SAC system open-loop transfer function %%%%%
%%%% Gc(s)Gsat(s) = (21s2+1.26s+1.89)/(s4+6s3+5s2)%%%%%%% %%%%%%%

num = [0 0 21 1.26 1.89];
den = [1 6 5 0 0];
nyquist(num,den)
v = [-2 2 -5 5];axis(v)
grid
title('Nyquist plot of Gc(s)Gsat(s)')
```

Appendix (3): MATLAB program to plot the Nyquist Stability Diagram for the PID-controlled satellite attitude system.



Appendix (4): Nyquist Stability Diagram for the PID-controlled satellite attitude system.