VIBRATION ANALYSIS OF ROTATING PRE-TWISTED CANTILEVER PLATE BY USING THE FINITE ELEMENT METHOD

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Abstract:

In this paper the finite element method has been used to determine the fundamental natural frequencies of a pre-twisted plate mounted on the periphery of a rotating disc. Three dimensional, finite element programs was built using three noded triangular shell element as a discretization element for cantilever plate, this element has six degrees of freedom at each node. All formulations and computations are coded in (FORTRAN-77). The investigation covers the effect of speed of rotation, disc radius, aspect ratio, pre-twist angle and skew angle on the vibration characteristics of rotating cantilever plate. For this analysis, the initial stress effect (geometric stiffness) and other rotational effects except the corioles acceleration effect have been included. The eigenvalues have been extracted by using simultaneous iteration technique. Results shown that the natural frequencies increase when; angular speed and disc radius are increases.

الخلاصة:

استخدمت في هذا البحث تقنية العناصر المحددة لحساب الترددات الطبيعية الاساسية لصفيحة ملتوية مثبته بقرص دوار. تم تمثيل هذه الصفيحة كقشرة دوارة, باستعمال العنصر المثلث كعنصر تجزئة لهيكل الصفيحة, حيث يحتوي هذا العنصر على ثلاث عقد لكل عقدة ست درجات من الحرية. تم الحصول على كل النتائج باستخدام برنامج حاسوب بلغة (FORTRAN-77). البحث درس تأثير عدة متغيرات مثل سرعة الدوران و نصف قطر القرص و نسبة الطول إلى العرض و زاوية الالتواء و زاوية التثبيت على خصائص الاهتزاز في الصفائح الدوارة. في هذا التحليل تمت دراسة تأثير الإجهاد الابتدائي (الجسأة الهندسية) و التأثير ألدوراني بدون الأخذ بنظر الاعتبار وجود تأثير تعجيل كيريولس (Coriolis acceleration), وباستخدام تقنية التكرار المتزامن لحساب الترددات الطبيعية. تبين النَتائِجَ بأنّ التردداتَ الطبيعيةَ تَزِيدُ بزيادة السرعةِ الزاويةِ ونصفِ قطر القرصِ.

KEY WORDS
rotating pre-twist plate, natural frequency, geometric stiffness, FEM.

INTRODUCTION:

The natural frequencies of rotating turbomachinery blades are known to be significantly higher than those of the non- rotating blades. For reliable and economic designs of the structures, it is necessary to estimate the dynamic characteristics of those structures accurately and efficiently. Since the blades are generally idealized as cantilever beams (a few investigations assumed the blade as a cantilever plate), the vibrations of rotating cantilever beams have been studied in several investigations. An early analytical model to calculate natural frequencies of a rotating cantilever beam was suggested by (**Southwell** 1921). Based on the Rayleigh energy theorem, a simple equation that relates the natural frequency to the rotating frequency of a beam was suggested. This equation is known as the Southwell equation, and widely used by many engineers nowadays. Later, to obtain more accurate natural frequencies, a linear partial differential equation that governs bending vibration of a rotating beam was derived by **(Schilhansl** 1958). Applying the Ritz method to the equation, more accurate coefficients for the Southwell equation could be obtained. Since the early 1970s, due to the progress of computing technologies, a large number of papers based on numerical approaches have been published. For instance, in references (**Bauer** 1980), approximation methods for the modal analysis of rotating beams were employed. More complex shapes and the effects of beams were also considered. The effects of tip mass (**Wright** 1982), elastic foundation and cross-sectional variation (**Kuo** 1994), shear deformation (**Yokoyama** 1988), pre-twist and orientation of a blade (**Subrahmanyam** 1987), and gyroscopic damping effect (**Yoo** 1998) on the modal characteristics of rotating cantilever beams were studied. Survey papers for the vibration analysis of rotating structures are available (**RAO** 1987). The most widely used modeling method for the transient analysis of structures is the classical linear modeling method (**Bodley** 1978). This modeling method employs the Cartesian deformation variables and the linear Cauchy strain measures. It has several merits such as simplicity of formulation, ease of implementation in finite elements methods, and efficiency of computation, which results from the use of co-ordinate reduction techniques (**Hale** 1980). This modeling method, however, often provides erroneous results when structures undergo overall motion such as rotation. To resolve the problem of the classical linear modeling method, several non-linear modeling methods (**Christensen** 1986) have been developed. With these non-linear modeling methods, the problem of accuracy can be resolved. However, serious computational inefficiency results from the non-linearity that disables the co-ordinate reduction techniques. More recently, a new linear modeling method for the dynamic analysis of a flexible beam undergoing overall motion was introduced (**Yoo** 1995).

THEORETICAL BACK GROUND:

The formulation follows a pattern similar to that in (**Henry** 1974). Two Cartesian co-ordinate systems are used, an absolute fixed system $R_0\,(X_0Y_0Z_0)$ and a local system $R_1\,(X\ Y\ Z\ )$ (see **Fig. 1**) attached to the rotating disc.

The potential strain energy U and kinetic T are, respectively,

$$U = \frac{1}{2}\int_{vol} \varepsilon^t\, \sigma\, d(vol) \qquad (1)$$

$$T = \frac{1}{2}\int_{vol} \rho \vec{V}^2\, d(vol) \qquad (2)$$

For plate bending problems, according to **(Zienkiewicz** 1979), the strains are given by:

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon_p \\ \varepsilon_f \end{Bmatrix} + \begin{Bmatrix} \varepsilon_g \\ 0 \end{Bmatrix} \qquad (3)$$

ε_p and ε_f are strains due to in-plane and bending displacements respectively and ε_g is the effect of bending displacements on mid-surface strains.

The stresses are given by,

$$\{\sigma\} = \begin{Bmatrix} \sigma_p \\ \sigma_f \end{Bmatrix} \quad (4)$$

Where $\{\sigma_p\}$ and $\{\sigma_f\}$ are in – plane stress resultants and bending and twisting moments, respectively.

With the definitions for stresses and strains U is given by,

$$U = P_1 + P_2 + P_3 \quad (5)$$

Where P_1, P_2 plane stress, bending strain energy and P_3 supplementary strain energy due to the effect of bending displacement on mid-surface strains, more expressions for P_1, P_2 and P_3 are standard (**Timoshenko** 1959).

At rest the co-ordinates of a typical Point M on the mid-surface are $(x, y, 0)$. Due to the displacement,

$$\{d\} = [u, v, w]^t \quad (6)$$

The instantaneous co-ordinates of M are $(x+u, y+v, z+w)$, and then

$$\overrightarrow{OM}^{R_1} = \overrightarrow{OI}^{R_1} + \overrightarrow{IM}^{R_1} = \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix} \begin{Bmatrix} x+u \\ y+v \\ w \end{Bmatrix} = \begin{Bmatrix} x_i + x + u \\ y_i + y + v \\ z_i + w \end{Bmatrix} \quad (7)$$

The angular velocity in the R_1 system is

$$\vec{\Omega}^{R_1} = [\Omega_1 \; \Omega_2 \; \Omega_3]^t \quad (8)$$

And the absolute velocity of the point M is given by,

$$\vec{V} = \frac{\partial \overrightarrow{OM}^{R_1}}{\partial t_{R_0}} = \begin{Bmatrix} u^\bullet \\ v^\bullet \\ w^\bullet \end{Bmatrix} + \begin{Bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{Bmatrix} \times \begin{Bmatrix} x_i + x + u \\ y_i + y + v \\ z_i + w \end{Bmatrix} = \begin{bmatrix} u^\bullet + \Omega_1 (z_i + w) - \Omega_3 (y_i + y + v) \\ v^\bullet + \Omega_3 (x_i + x + u) - \Omega_1 (z_i + w) \\ w^\bullet + \Omega_1 (y_i + y + v) - \Omega_2 (x_i + x + u) \end{bmatrix} \quad (9)$$

Computing $\vec{V}^2 (i.e., \vec{V}^t \vec{V})$, canceling the terms like those proportional to z_i^2 which give no contribution when Lagrange's equations are applied, and substituting the result in **Eq.(2)** one can write the kinetic energy as (**Timoshenko** 1959),

$$T=\frac{1}{2}\int_{vol}\rho\begin{Bmatrix}u^{\bullet}\\v^{\bullet}\\w^{\bullet}\end{Bmatrix}^{t}\begin{Bmatrix}u^{\bullet}\\v^{\bullet}\\w^{\bullet}\end{Bmatrix}d(vol)+\frac{1}{2}\int_{vol}\rho\begin{Bmatrix}u^{\bullet}\\v^{\bullet}\\w^{\bullet}\end{Bmatrix}^{t}[A_1]\begin{Bmatrix}u\\v\\w\end{Bmatrix}d(vol)+$$
$$\frac{1}{2}\int_{vol}\rho\begin{Bmatrix}u^{\bullet}\\v^{\bullet}\\w^{\bullet}\end{Bmatrix}^{t}[A_2]\begin{Bmatrix}u^{\bullet}\\v^{\bullet}\\w^{\bullet}\end{Bmatrix}d(vol)+\frac{1}{2}\int_{vol}\rho\begin{Bmatrix}x_i+x\\y_i+y\\z_i\end{Bmatrix}^{t}[A_2]\begin{Bmatrix}u\\v\\w\end{Bmatrix}d(vol) \qquad (10)$$

$$[A_1]=\begin{bmatrix}0 & -2\Omega_3 & 2\Omega_2\\ 2\Omega_3 & 0 & -2\Omega_1\\ -2\Omega_2 & 2\Omega_1 & 0\end{bmatrix} \qquad (11)$$

$$[A_2]=\begin{bmatrix}\Omega_2^2+\Omega_3^2 & -\Omega_1\Omega_2 & -\Omega_1\Omega_3\\ -\Omega_1\Omega_2 & \Omega_1^2+\Omega_3^2 & -\Omega_3\Omega_2\\ -\Omega_1\Omega_3 & -\Omega_3\Omega_2 & \Omega_2^2+\Omega_2^2\end{bmatrix} \qquad (12)$$

DERIVATION OF THE STIFFNESS MATRIX:

The polynomials for the displacements u and v are linear in $L_1, L_2\ and\ L_3$ while for the displacements w the polynomial assumed is cubic (**Zienkiewicz** 1979).
The in – plane nodal displacements are defined by,

$$\{q_1\}=[u_1 \quad v_1 \quad u_2 \quad v_2 \quad u_3 \quad v_3]^t \qquad (13)$$

And the bending nodal displacements are defined by,

$$\{q_2\}=[w_1,\theta_{x_1},\theta_{y_1},w_2,\theta_{x_2},\theta_{y_2},w_3,\theta_{x_3},\theta_{y_3}]^t \qquad (14)$$

$$\theta_{x_i}=-\left(\partial w/\partial y\right)_i,\ \theta_{y_i}=-\left(\partial w/\partial x\right)_i \qquad (15)$$

After standard finite elements procedure one arrives at,

$$\begin{Bmatrix}u\\v\end{Bmatrix}=[N_1]\{q_1\},[N_2]=\begin{bmatrix}L_1 & 0 & L_2 & 0 & L_3 & 0\\ 0 & L_1 & 0 & L_2 & 0 & L_3\end{bmatrix} \qquad (16, 17)$$

$$\{w\}=[N_2]\{q_2\},[N_2]=[N_{b_1}\ N_{b_2}\ ....N_{b_9}]^t \qquad (18, 19)$$

$$N_{b_1}=L_1+L_1^2L_2+L_1^2L_3-L_1L_2^2-L_1L_3^2 \qquad (20)$$

$$N_{b_2}=b_3(L_1^2L_2+\frac{1}{2}L_1L_2L_3)-b_2(L_3L_1^2+\frac{1}{2}L_1L_2L_3) \qquad (21)$$

$$N_{b_3} = a_3 (L_1^2 L_2 + \frac{1}{2} L_1 L_2 L_3) - a_2 (L_3 L_1^2 + \frac{1}{2} L_1 L_2 L_3) \qquad (22)$$

The other shape functions for nodes 2 or 3, can be written down by a cyclic permutation of the suffixes 1, 2, 3.

L_1, L_2, L_3 and the area co–ordinates, and a_i and b_i, are defined in **Fig.2**.

Once one knows the expression for the strain and the shape functions, then k_p the in–plane stiffness matrix, and k_f, the bending stiffness matrix can be easily derived. The integration is performed by using numerical three-point integration (**Cowper** 1973) over the triangular area.

DERIVATION OF GEOMETRIC STIFFNESS MATRIX:

Owing to the presence of the in–plane stresses σ_x^0, σ_y^0 and τ_{xy}^0 in the middle surface caused by rotation, the additional strain energy stored in the element is given by P_3. This additional strain energy results in an increase in the stiffness of the elements by an amount,

$$\{k_g\} = \int\int_A [G]^t \begin{bmatrix} \sigma_x^0 & \tau_{xy}^0 \\ \tau_{xy}^0 & \sigma_y^0 \end{bmatrix} [G]\, h\, dA \qquad (23)$$

Where $[G]$ is defined by

$$\begin{Bmatrix} \dfrac{\partial w}{ax} \\ \dfrac{\partial w}{ax} \end{Bmatrix} = [G]\{q_2\} \qquad (24)$$

For details, see (**Zienkiewicz** 1979). It is easy to show that,

$$U = \frac{1}{2} q^{\,t} [k + k_f] q + \frac{1}{2} q^t\, k_G\, q, \qquad \{q\} = [q_1,\, q_2]^t \qquad (25, 26)$$

Therefore,

$$\partial U / \partial q = [k + k_f] q + k_G\, q \qquad (27)$$

DERIVATION OF MASS, CORIOLES AND SUPPLEMENTARY MATRICES, AND THE LOAD VECTOR:

By using **Eq. (16 and 18)** in **Eq. (10)** and then applying Lagrange's equations one obtain,

$$(d / dt)[\partial T / \partial q^{\bullet}] - [\partial T / \partial q] = m_E\, q^{\bullet\bullet} + c\, q^{\bullet} + k_R\, q - F \qquad (28)$$

With $[N]$ defined by,

$$\begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} = [N]$$

The element mass matrix is,

$$[m_E] = \rho \int_{vol} [N]^t \, [N] \, d\,(vol) \qquad (29)$$

The element Coriolis matrix is

$$[C] = \rho \int_{vol} [N]^t \, [A_1][N] \, d\,(vol) \qquad (30)$$

The element supplementary or rotational stiffness matrix is,

$$[k_R] = \rho \int_{vol} [N]^t \, [A_2][N] \, d\,(vol) \qquad (31)$$

Moreover, the force vector is,

$$\{F\} = \rho \int_{vol} [N]^t \, [A_2] \begin{Bmatrix} x_i + x \\ y_i + y \\ z \end{Bmatrix} d\,(vol) \qquad (32)$$

<u>FINAL MATRICES AFTER ASSEMBLY:</u>

Adding expressions **Eq. (27** and **28)** and equating the result to zero gives the final differential equation of the structure after assembly in from,

$$M_E \, q^{\bullet\bullet} + C \, q^{\bullet} + [K_E + K_G + K_R] q = \vec{F}(\Omega^2) \qquad (33)$$

The matrix K_G depends on the initial stress distribution. Initially the stresses are taken as zero and the equation.

$$[K_E + K_R] q = \vec{F}(\Omega^2) \qquad (34)$$

Is solved for the initial stress distribution σ_0. Then the solution of,

$$(K_E + K_G(\sigma_0) + K_R) \, q = \vec{F}(\Omega^2) \qquad (35)$$

Gives a new stress distribution σ. The stress values were found to converge within two iterations. Finally the frequencies and eigenvectors are found for the deformed configuration. The equation of motion of the structure, with the cariolis matrix neglected, is

$$M_E \, q^{\bullet\bullet} + [K_E + K_G(\sigma) + K_R] q = 0 \qquad (36)$$

Assuming harmonic vibrations, $q = q_o \; e^{iwt}$, one has

$$[K_E + K_G(\sigma) + K_R - \omega^2 \, M_E] q_o = 0 \qquad (37)$$

In which M_E and $K_E + K_G + K_R$ are symmetric and positive definite matrices **Eq. (37)** is standard eigenvalue problem and is solved for the eigenvalues and eigenvectors by using a simultaneous iteration technique (**Jennings** 1977).
The tapered and skewed plate can be also be modeled by triangular shell elements, the variation in thickness being accounted for by defining the thickness of the element at the three nodes. For formulating all the matrices the element thickness can by taken as the mean of the nodal thicknesses.

VERFICATION TEST:

The present works were comparing with the numerical results in (**Rao** 1999) to find the fundamental non-dimensional frequency of vibration for rotating cantilever plate. **Table. (1)** explains the current results with numerical results in (**Rao** 1999), and the values of percentage error with numerical results. In this table, the maximum error not exceeds (2%). The data for the verification case are:

$$E = 217\,\mathbf{G}pa,\ \rho = 7850\,Kg/m^3, \upsilon = 0.3,\ b = 35\,mm, t = 3\,mm, l/b = 2, t/b = 0.0625, r/b = 4, \alpha = \theta = 30^\circ$$

RESULTS AND DISCUSSIONS:

The fundamental bending frequencies with out Coriolis effect are computed for pre- twisted cantilevers plates of two different aspect ratios (1 and 2), and for various values of non-dimensional speed of rotation $\left(\overline{\Omega}\ from\, 0\, to\, 1\right)$, of pre-twist angle $\left(\alpha = 0^\circ, 45^\circ\ and\ 90^\circ\right)$, of skew angle $\left(\theta = 0^\circ, 45^\circ\ and\ 90^\circ\right)$ and non-dimensional disc radius $\left(\overline{r} = r/l,\ from\, 0\ to\ 1.5\right)$. **Fig. 3** show the suitable mesh sizes where chosen for aspect ratios (1 and 2) in this analysis (The suitable mesh for the aspect ratio 1 and 2, which obtained after convergence test). In all computations, Poisson's ratio was taken as (0.3), and the material of the plate has been assumed homogeneous and isotropic $(E = 200*10^9\ N/m^2, \rho = 7850\,Kg/m^3)$ and the dimensions of the cantilever plate were $(b = 40\,mm,\ t = 2\,mm)$.

Figs. (4 and 5) show the variation of non-dimensional frequency of vibration (β) with non-dimensional speed of rotation $\left(\overline{\Omega}\right)$ for different twist and skew angles corresponding to the aspect ratios (1 and 2) respectively, for plate having $\left(\overline{r} = 0\right)$. The second set of results will initiate the tendency of change of (β) with $\left(\overline{r}\right)$ for different twist and skew angles were show in **Figs. (6 and** 7**)** corresponding for aspect ratios (1 and 2) respectively, for plate having $\left(\overline{\Omega} = 0.5\right)$.

The frequencies of all results are independent of skew angle and disc radius but dependent on pre-twist when the structure is stationary. The all-natural frequencies increase with increase in non-dimensional speed of rotation $\left(\overline{\Omega}\right)$ for all combinations of three skews $\left(\theta = 0^\circ, 45^\circ\ and\ 90^\circ\right)$ and three twists $\left(\alpha = 0^\circ, 45^\circ\ and\ 90^\circ\right)$, and for both cases of aspect ratios (1 and 2). The rate of increase is maximum for the combination skew=$\left(0^\circ\right)$ and twist=$\left(0^\circ\right)$, and is minimum for the combination skew= $\left(90^\circ\right)$ and twist=$\left(90^\circ\right)$. In general it has been observed that the rate of increase decrease with increase in pre-twist as well as with the increase in skew angle. At any given speed (including stationary case) and skew angle the frequency decreases with the increase in pre-twist angle. The maximum and minimum percentage increases in the frequency value, with the increase in speed $\left(\overline{\Omega}\ from\,0\,to\,1\right)$ are roughly about 48% (when the skew is $\left(0^\circ\right)$, the twist is $\left(0^\circ\right)$ and $\left(\bar{r}=0\right)$) and about 6% (when the skew is $\left(90^\circ\right)$, the twist is $\left(90^\circ\right)$ and $\left(\bar{r}=0\right)$).

From **Fig. (6 and 7)** it can be seen that all the frequencies increase with increase in disc radius $\left(\bar{r}=r/l\right)$ for all cases. When the non-dimensional disc radius is increased from (0 to 1.5) the value of (β) is maximum for the combination skew= $\left(90^\circ\right)$ and twist= $\left(90^\circ\right)$, and is minimum for the combination skew= $\left(0^\circ\right)$ and twist= $\left(0^\circ\right)$. The maximum and minimum percentage increases in the frequency value, with the increase in non-dimensional disc radius (0 to 1.5) are roughly about 25% (when the skew is $\left(90^\circ\right)$, the twist is $\left(90^\circ\right)$) and about 20% (when the skew is $\left(0^\circ\right)$, the twist is $\left(0^\circ\right)$).

CONCLUSION:

A modal formulation for the free vibration of a pre-twist cantilever plate with setting angle is presented. Three dimensionless parameters are identified through a dimensional analysis: the aspect ratio of the plate, the ratio of hub radius to the plate length, and the dimensionless angular speed. The effects of the other parameters, such as well as the setting angle and twist angle on the natural frequencies of rotating cantilever plates are investigated. It is shown that the rotating plate's natural frequencies increase with the angular speed, that their increasing rates grow as the hub radius increases, and that the natural frequency decreases when the skew and twist angles increases. In addition, it can be noted that the frequencies of all results of are independent of skew angle and disc radius when the cantilever plate is stationary.

Table. 1 Values of fundamental non-dimensional frequency of vibration (β) for stationary and rotating cantilever plate.

	Present Work	(**Rao** 1999)	**Error %**
$\left(\Omega=0\right)$	3.403	3.437	1.0
(Ω =2000 rpm)	3.525	3.579	1.5
(Ω =4000 rpm)	3.902	3.974	1.8
(Ω =7000 rpm)	4.799	4.892	1.9

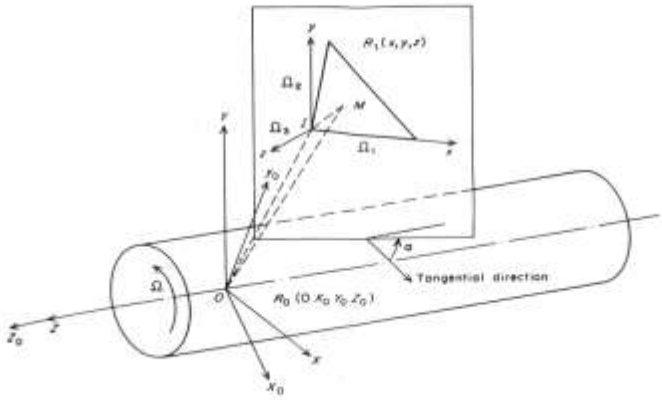


Fig.1 Cartesian co-ordinate system.

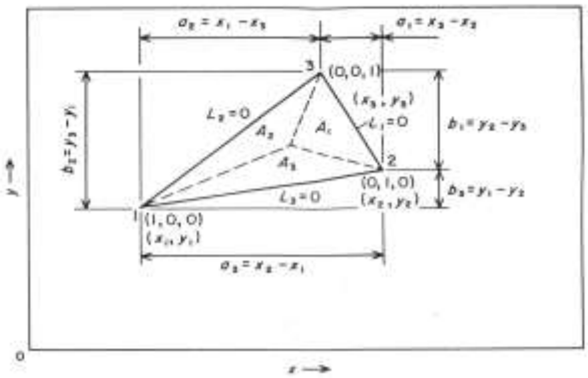
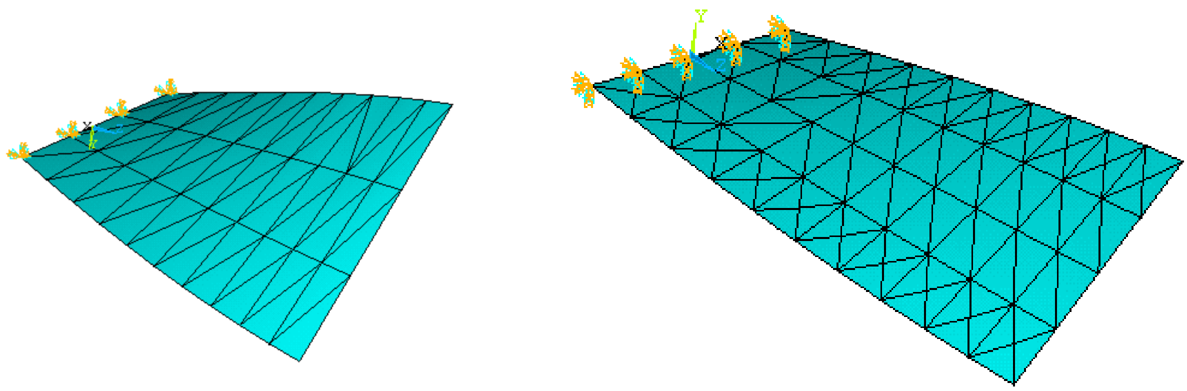


Fig. 2 Area co-ordinates.



(a/b=1, No. of elements=60) (a/b=2, No. of elements=120)

Fig.3 Suitable mesh size for pre-twisted cantilever plate.

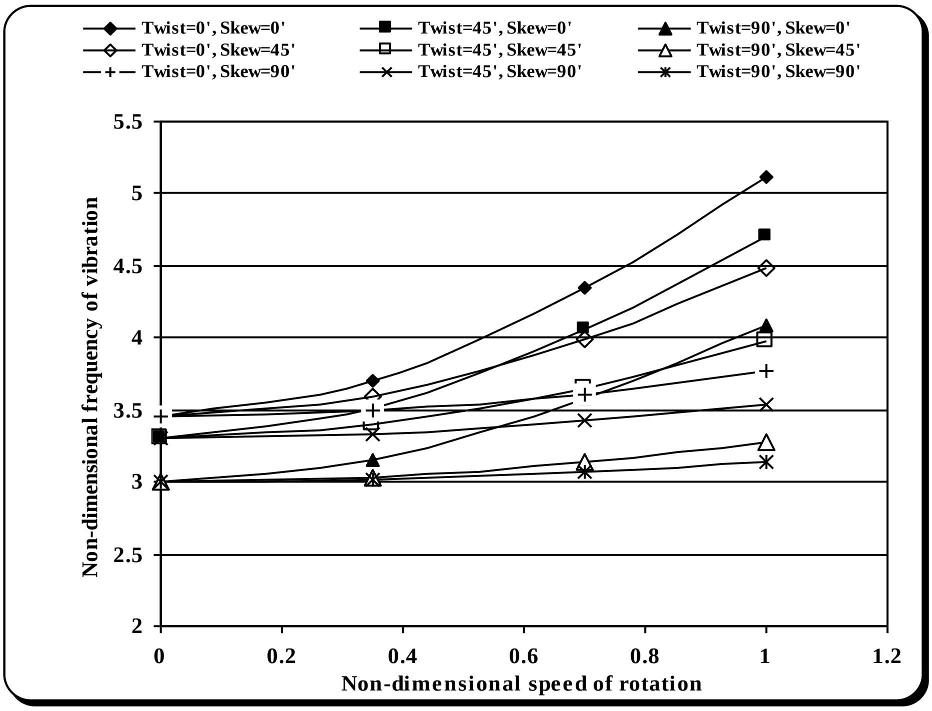


Fig. 4 Variation of non-dimensional frequency of vibration with non-dimensional speed of rotation $(a/b = 1,\ \overline{r = 0})$.

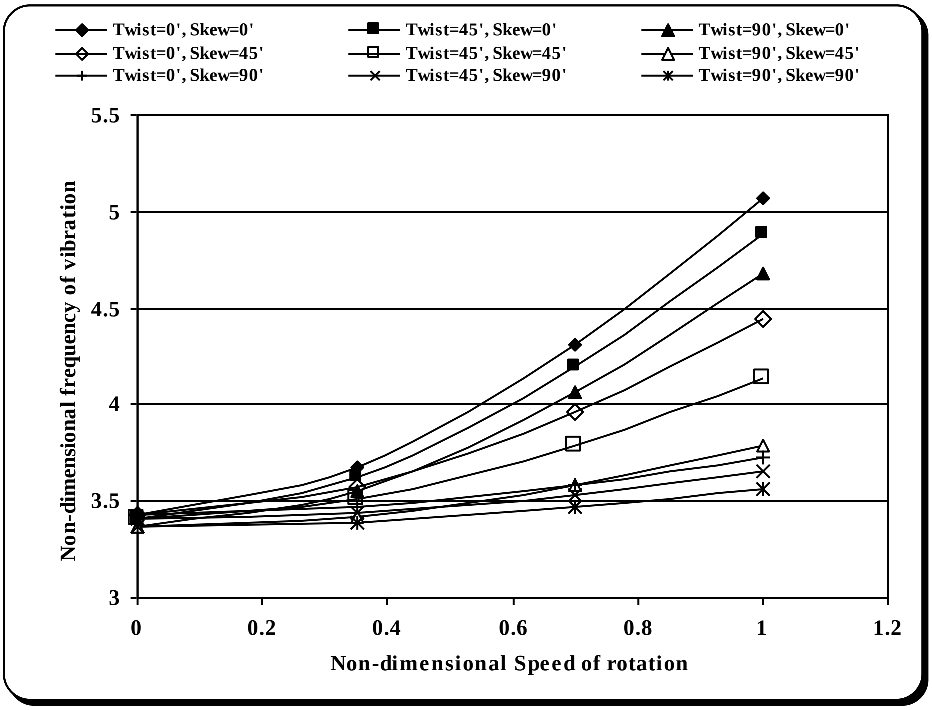


Fig. 5 Variation of non-dimensional frequency of vibration with non-dimensional speed of rotation $(a/b = 2,\ \overline{r = 0})$.

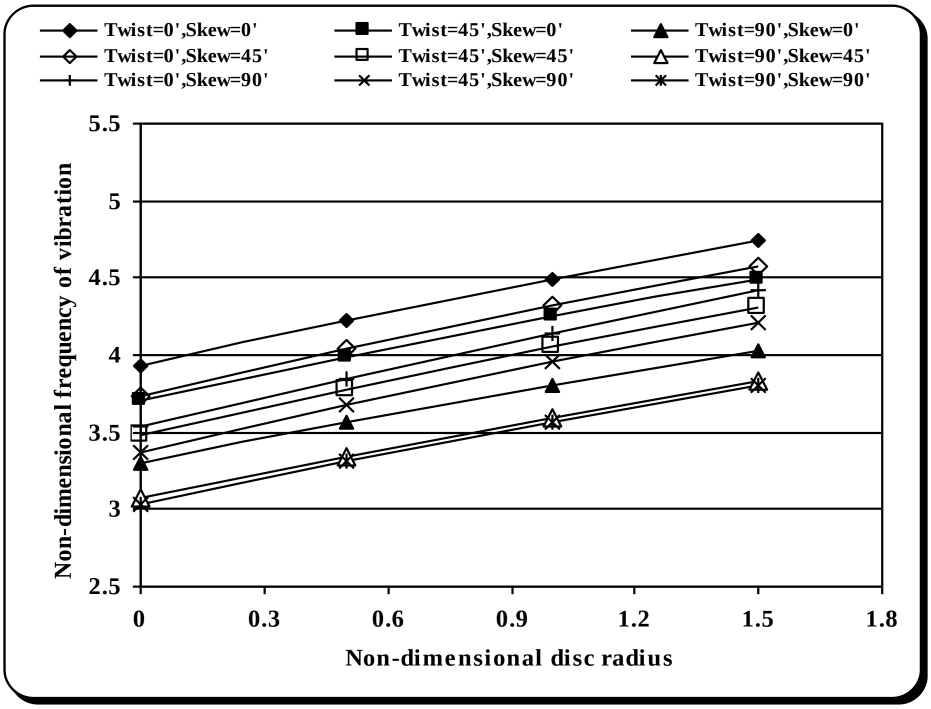


Fig. 6 Variation of non-dimensional frequency of vibration with non-dimensional disc radius $(a/b=1,\ \overline{\Omega}=0.5)$.

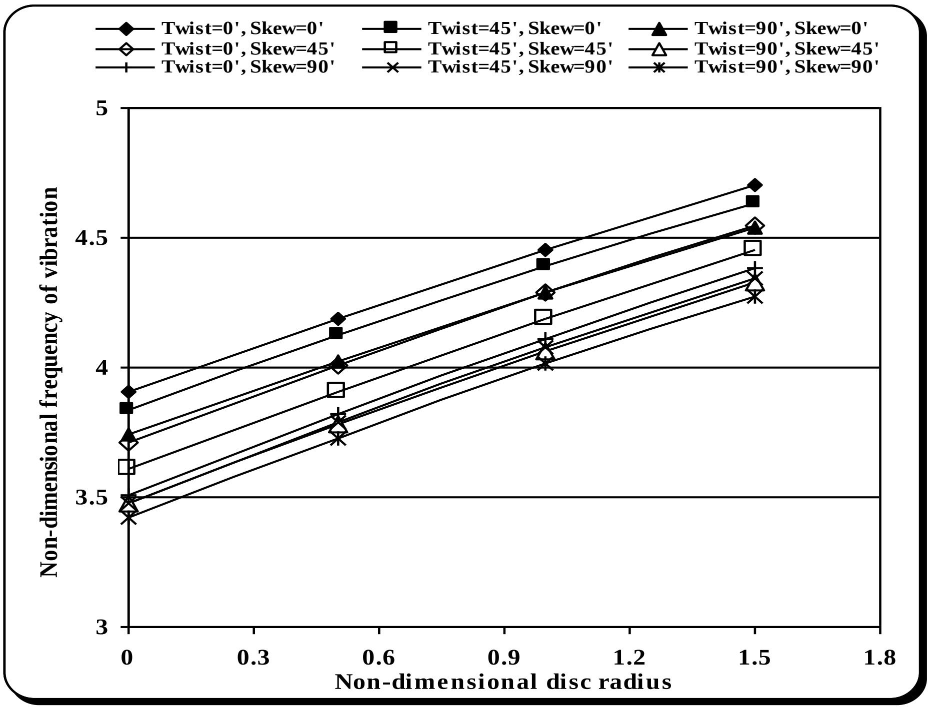


Fig. 7 Variation of non-dimensional frequency of vibration with non-dimensional disc radius $(a/b=2,\ \overline{\Omega}=0.5)$.

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NOMENCLATURES:

c	Coriolis matrix of the element
b	Width of the rotating plate m
a	Length of the rotating plate m
$\vec{d}$	Displacement of a typical point in the mid-surface
h	Thickness of a typical of an element
h_1, h_2, h_3	Thickness at the nodes 1, 2, 3
t	Thickness of the rotating plate
k_p, k_f, k_G	In-plane, bending and geometric stiffness matrices of an element
k_R	Additional stiffness of an element
m_E	Mass matrix of an element
q	Nodal displacements of the structure
u, v	In-plane components of the displacement $\vec{d}$
u_1, v_1	In-plane nodal displacements of node i
w	Components of the displacement $\vec{d}$ normal to the mid-surface
w_i	Bending nodal displacements of node i
x, y	Components of $\overrightarrow{IM}$ *in* R_i
x_1, t_i, z_i	Components of $\overrightarrow{OI}$ *in* R_i
A	Area of the triangular element
C	Coriolis matrix of the structure
D	Flexural rigidity of the plate, $= Et^3/12(1-\upsilon^2)$
E	Young's modulus
F	Nodal centrifugal force vector for the element
$\vec{F}(\Omega^2)$	Nodal centrifugal force vector for the structure
K_E, K_G	Elastic and geometric stiffness of the structure
K_R	Additional stiffness of the structure
L_1, L_2, L_3	Area co-ordinates of the triangle
M_E	Mass matrix of an structure
$N, N_1, N_2, N_{b1} - N_{b9}$	Shape function
P_1, P_2	Plane stress, bending strain energy
P_3	Supplementary strain energy due to the effect of bending displacement on mid-surface strains
$R(OXYZ)$	Global Cartesian co-ordinate system attached to the rotating disc
$R_0\,(OX_0Y_0Z_0)$	Absolute fixed Cartesian co-ordinate system
$R_1\,(IX_0Y_0Z_0)$	Local Cartesian co-ordinate system

T	Kinetic energy
U	Total potential energy
$\vec{V}$	Absolute velocity of M
β	Non-Dimensional frequency of vibration, $=\omega a^2 \sqrt{\rho t / D}$
ε	Strains
$\varepsilon_p, \varepsilon_f$	Strain due to in-plane and bending displacements
ε_g	Effect of bending displacements on mid-surface strains
θ	Skew angle, setting angle
α	Twist angle`
υ	Poisson's ratio
ρ	Mass density Kg/m^3
σ	Stress N/m^2
σ_p	In-plane stress resultant
σ_f	Bending and twisting moment
$\sigma_x^0, \sigma_Y^0, \tau_{xy}^0$	Initial in-plane stress
ω	Frequency in rotation $(rad/\sec)$
ω_o	Frequency at rest $(rad/\sec)$
$\vec{\Omega}$	Speed of rotation $(rad/\sec)$
$\Omega_1, \Omega_2, \Omega_3$	Components of $\vec{\Omega}$ in R_1
$\overline{\Omega}$	Non-dimensional speed, $=\Omega/\omega_o$
$\bar{r}$	r/a