THE DEFLECTION CONTROL OF A SIMPLY SUPPORTED THIN BEAM BY USING A PIEZOELECTRIC ACTUATOR / SENSOR

Waleed Kh. Al-Ashtari
Mech. Eng. Dept.
College of Engineering
University of Baghdad
Baghdad-Iraq

ABSTRACT

Piezoelectric transducers have become increasingly popular in vibration control applications. They are used as sensors and as actuators in structural vibration control systems. They provide excellent actuation and sensing capabilities. In this paper, the term smart beam refers to a beam with a finite number of collocated piezoelectric actuator / sensor pairs. The proposed thin smart beam governing equation was derived by the same procedure that the Bernoulli-Euler equation derived but with some additional mathematical terms to be valid for describing the smart beam. The engineering control techniques were used to obtain the solution of the proposed differential equation for the simply supported beam where with some auxiliary equations and modifications a block diagram for any type of applied load (static, or cyclic) as the input and the beam deflection as the output was constructed. For insuring an efficient reduction in the beam deflection an integrated system with a high voltage amplifier and lead controller was designed. Many cases were studied and simulated including the variation of load nature and its frequency, and the number of collocated piezoelectric actuator/sensor pairs and in all cases a valuable deflection reductions were obtained.

الخلاصة:

أصبحت محولات البيزو شائعة الاستخدام في تطبيقات السيطرة على الإهتزازات. فهذه المحولات ممكن ان تستعمل كمتحسسات اوكمحفزات في منظومات السيطرة على الإهتزاز الهيكلي حيث توفر هذه المحولات تحفيزاً وتحسساً ممتازين. في هذا البحث، يطلق تعبير العتبة الذكية على العتبة التي تحتوي على عدد محدد من ازواج المحفز / متحسسٍ. لقد اشتقت المعادلة الرياضية المقترحة في هذا البحث للعتبة الذكية بنفس طريقة اشتقاق معادلة برنولي - اولر مع اضافة بعض الحدود الرياضية التي تميز العتبة الذكية. لقد تم حل المعادلة التفاضلية المقترحة بأستخدام تقنيات هندسة السيطرة حيث تم انشاء بواسطة هذه المعادلة وبعض المعادلات الاخرى مخطط صندوقي فيه القوة الخارجية المسلطةعلى العتبة (سكوني أو دوري) كمدخل و ازاحة اهتزاز العتبة كمخرج. لضمان فاعلية اداء المنظومة المقترحة والحصول على اكبر نسبة تقليص ممكنة لازاحة العتبة لقد تم اضافة مسيطر مع مضخم اشارة للمنظومة. لقد تم دراسة ومحاكاة حالات كثيرة منها تغير طبيعة الحمل و تردده عدد ازواج المحفز / متحسسٍ وفي كل الحالات تم الحصول نسب تقليص مقبولة.

KEYWORDs: Smart Beam, Ordinary Beam, Bernoulli-Euler Equation, Lead Network Controller, and High Voltage Amplifier.

INTRODUCTION

Piezoelectric transducers have become increasingly popular in vibration control applications. They are used as sensors and as actuators in structural vibration control systems. They provide excellent actuation and sensing capabilities. The ability of piezoelectric materials to transform mechanical energy into electrical energy and vice versa was discovered over a century ago by Pierre and Jacque Curie. These French scientists discovered a class of materials that when pressured, generate electrical charge, and when placed inside an electric field, strain mechanically.

Piezoelectricity, which literally means “electricity generated from pressure” is found naturally in many monocrystalline materials, such as quartz, tourmaline, topaz and Rochelle salt. However, these materials are generally not suitable as actuators for vibration control applications. Instead, man-made polycrystalline ceramic materials, such as lead zirconate titanate **(PZT)**, can be processed to exhibit significant piezoelectric properties. PZT ceramics are relatively easy to produce, and exhibit strong coupling between mechanical and electrical domains. This enables them to produce comparatively large forces or displacements from relatively small applied voltages, or vice versa. Consequently, they are the most widely utilized material in manufacturing of piezoelectric transducers.

Piezoelectric transducers are available in many forms and shapes. The most widely used piezoelectric transducers are in the form of thin sheets that can be bonded to or embedded in composite structures. As actuators they are mainly used to generate moment in flexible structures, while as sensors they are used to measure strain.

Piezoelectric transducers are used in many applications such as structural vibration control, precision positioning, aerospace systems, and more recently they have been critical in advancing researches in nanotechnology. **(Moheimani and Fleming, 2006)**

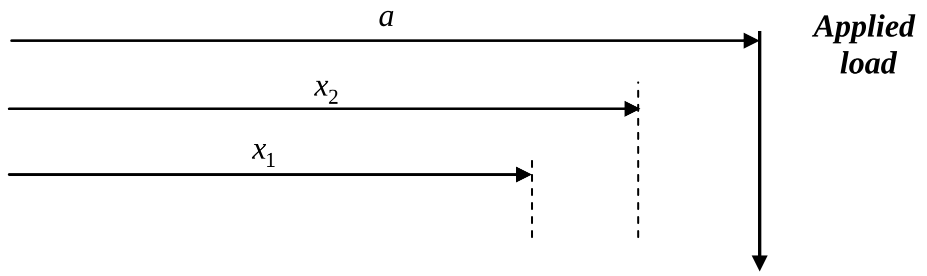
To this end, many researchers have concentrated on dynamic modeling of piezoelectric materials as elements of intelligent (smart) structures (**Crawley and Luis, 1987; Clark, Saunders, and Gibbs, 1998; Smits and Choi, 1991; Wang and Cross, 1999, Kermani, Moallem, and Patel, 2004)**, while a number of others have focused on control methods of piezoelectric actuators for suppressing vibrations and noise reduction **(Bailey and Hubbard, 1986; Sun and Mills, 1999a; Halim and Moheimani, 2002)**. **(Sun and Mills 1999b)** conducted studies on the application of segmented piezoelectric transducers PZT ceramics and poly vinylidene fluoride **(PVDF)** materials for this purpose, **(Choi, Park, and Fukuda 1998)** investigated active vibration control by utilizing hybrid smart actuators constructed from PZT and shape memory alloy. **(Patnaik, Heppler, and Wang 1992)** studied stability issues in controlling a flexible beam. A quite comprehensive literature review has been given in **(Smits and Choi 1991)**.In selecting a PZT actuator for vibration control; it is useful to know how the physical parameters of a PZT can affect system performance. This issue is of paramount importance if one notes that a PZT actuator has the major drawback of limited capability to produce high torques. This fact reduces the effectiveness of the PZT usage for suppressing vibrations. There are two ways to remedy this problem. One of these calls for the use of stronger PZT actuators such as the one developed at NASA’s Langley research center for alleviating the buffet load in the tail fin of the fuselage **(Moses, 1997)**.The other solution involves finding optimum values of the physical parameters to make use of the maximum strength of the actuator. Previous work has tried to address this issue, in an attempt to obtain the optimum size and location of the actuator **(Lim and Gawronski, 1993; Moore, 1981)**. **(Hamdan and Nayef 1989)** proposed a measure of modal controllability based on the angle between the normalized left eigenvectors of the system and the control input matrix. **(Kim and Junkins 1991)** presented a modal cost to rank each mode’s participation in system output. **(Aldraihem, Singh, and Wetherhold, 1997)** reported a weighted controllability measure by modifying Hamdan’s controllability index. They also considered a penalty for the length of the actuator in formulating the optimization problem. **(Yaghoobi and Abed, 1999)** defined a participation factor to address the participation of a mode in a state, specified as output. **(Moheimani and Ryall, 1999)** introduced the idea of spatial controllability in order to include the effect of actuator location in the optimization problem. **(Yong, Onada, and Minesugi, 2002)** considered the effect of the adding an actuator/sensor on the mass and stiffness of the structure and combined that with the control performance index to obtain the optimum values for the location, size and feedback gains, simultaneously. However, a clear description of the actuator performance with respect to each individual mode of vibration needs to be given more attention. The degree by which a certain parameter can affect each resonance mode motivates further investigations. The use of the controllability Grammian and singular value decomposition of the system dynamics can provide practical guidelines for selecting the optimal values of the aforementioned parameters. **(Vasques, and Dias Rodrigues, 2006)** introduce an analysis and comparison of the classical and optimal feedback control strategies on the active control of vibrations of smart piezoelectric beams. **(Belouettar, Azrar, Daya, Laptev, and Potier-Ferry, 2007)** they developed a simplified and consistent theory to actively control sandwich beams (the upper and bottom surfaces are covered entirely with a piezoelectric layer) at small and large amplitude.

In this research, the term smart beam will refer to a beam with a finite number of collocated piezoelectric actuator/sensor pairs, while ordinary beam will refer simply to the beam itself without any actuators or sensors. A smart beam differential equation had been derived by the same procedure that the Bernoulli-Euler equation derived with some mathematical modifications to be applicable for the smart beam actuating by any type of applied load such as static or cyclic. The engineering control techniques was used to obtain the solution of proposed differential equation where with some auxiliary equations and modifications a block diagram as the applied load be the input and as the smart beam deflection be the output was constructed as shown later. Also in this research, a beam deflection reduction system with a lead network controller and a high voltage amplifier has been designed mainly for two reasons: the first was to amplify the voltage generated by the sensor to be able to handle and transmit it efficiently to the actuator. And the second was to enhance the system response.

THE THIN SMART BEAMS GOVERNING EQUATION

Now, the derivation of the smart beam differential equation actuated by an external load (static or cyclic) will be accomplished. Let us consider a setup as shown in **Fig.(1)**, where *m* of identical collocated piezoelectric actuator/sensor pairs are bonded to a beam. The assumption that all piezoelectric transducers are identical is only adopted to simplify the derivations, and can be removed if necessary. The i^{th} actuator is exposed to a voltage of $e_{ai}(t)$ and the voltage induced in the i^{th} sensor is $e_{si}(t)$ .We assume that the beam has a length of l_b, width of w_b, and thickness of t_b Corresponding dimensions of each piezoelectric transducer are l_p, w_p, and t_p. Furthermore, we denote the transverse deflection of the beam at point *x* and time *t* by $v(x, t)$.

It is well known that Bernoulli-Euler equation governs the transverse vibration of beams. Therefore, the derivation of the smart beam equation will follow the same procedure that used in derivation of Bernoulli-Euler equation but with changing the applying load condition.



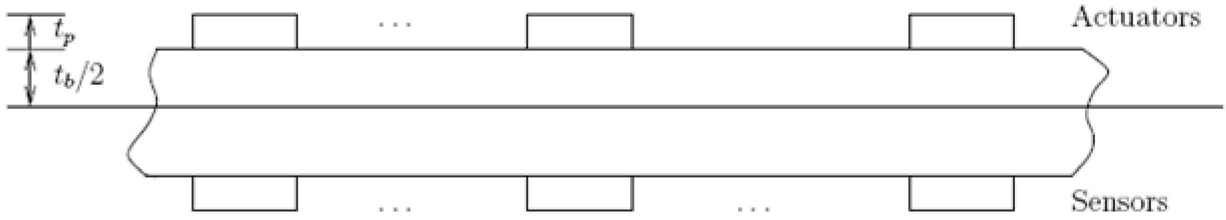


Fig.(1) A Beam with A Finite Number of Collocated Piezoelectric Actuator/Sensor Pairs and Applied Load

Consider a beam in bending, in the *x-y* plane, with *x* as the longitudinal axis and *y* as the transverse axis, of bending deflection, as shown in **Fig.(2)**. The required equation is developed by considering the bending moment– deflection relation, rotational equilibrium, and transverse dynamics of a smart beam element.

Rotatory Dynamics (Equilibrium)

Consider the beam element δx, as shown in **Fig.(2)**, where forces and moments acting on the element are indicated.

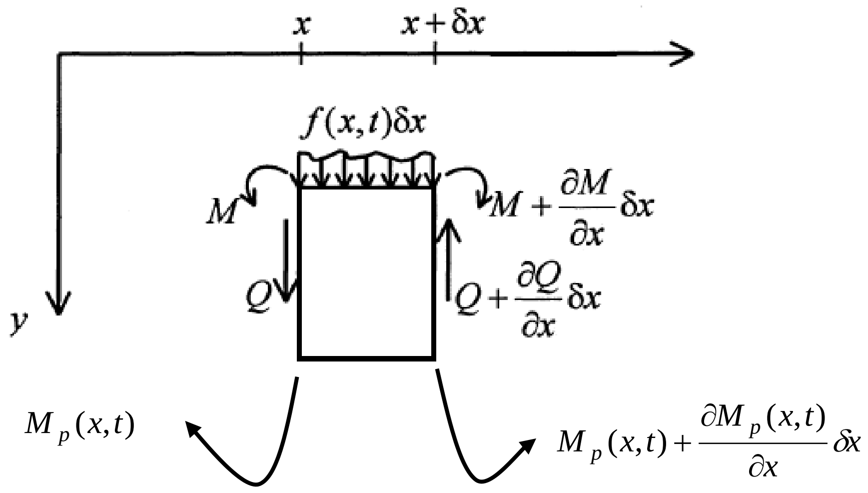


Fig.(2) Dynamic of a Beam Element in Bending

Here, $f(x,t)$ is the excitation force per unit length acting on the beam in the transverse direction at location *x*, and $M_p(x,t)$ is the total moment that generated by all the actuators and can expressed by

$$M_p(x,t)=\sum_{i=1}^{m} M_{pi}(x,t) \tag{1}$$

Where *m* is the number of identical collocated piezoelectric actuator / sensor pairs which are bounded to the beam. The equation of the angular motion is given by the equilibrium condition of moments:

$$M-M_p(x,t)+Q\delta x-\left(M+\frac{\partial M}{\partial x}\partial x\right)+\left(M_p(x,t)+\frac{\partial M_p(x,t)}{\partial x}\partial x\right)=0 \tag{2}$$

where the moment deflection relation can expressed as

$$M = E_b I_b \frac{\partial^2 v}{\partial x^2} \tag{3}$$

simplifying **Eq.(2)** and substituting **Eq.(3)** into it will give

$$Q = \frac{\partial}{\partial x}\left(E_b I_b \frac{\partial^2 v}{\partial x^2} \right) - \frac{\partial M_p(x,t)}{\partial x} \tag{4}$$

Transverse Dynamics

The equation of transverse motion (Newton's second law) for element δx is

$$\left(\rho_b A_b \delta x\right)\frac{\partial^2 v}{\partial t^2} = f(x,t)\delta x + Q - \left(Q + \frac{\delta Q}{\partial x}\delta x \right) \tag{5}$$

Here, ρ is the mass density of the beam material, more simplifying will give

$$\rho_b A_b \frac{\partial^2 v}{\partial t^2} + \frac{\partial Q}{\partial x} = f(x,t) \tag{6}$$

Now, substituting **Eq.(4)** into **Eq.(6)**, one will obtain the governing equation of forced transverse vibration of smart beam with finite number of actuators

$$\rho_b A_b \frac{\partial^2 v}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(E_b I_b \frac{\partial^2 v}{\partial x^2} \right) = \frac{\partial^2 M_p(x,t)}{\partial x^2} + f\left(x,t\right) \tag{7}$$

THE ENGINEERING CONTROL SOLUTION

Now, **Eq.(7)** will be solved by using the linear engineering control techniques, but this deferential equation must be linearized to be able for handling it and this done firstly by specifying the case of study where a simply supported beam has been chosen, and follow the below procedure

Beam and Actuator Equation

If young's modules and second moment of area about the neutral axis are constant then

$$\rho_b A_b \frac{\partial^2 v}{\partial t^2} + E_b I_b \frac{\partial^4 v}{\partial x^4} = \frac{\partial^2 M_p(x,t)}{\partial x^2} + f\left(x,t\right) \tag{8}$$

As shown in **Fig.(1)** where the actuating force is applied at $x = a$, then by using Dirac delta **Eq.(8)** can be rewrite as

$$\rho_b A_b \frac{\partial^2 v}{\partial t^2} + E_b I_b \frac{\partial^4 v}{\partial x^4} = \frac{\partial^2 M_p(x,t)}{\partial x^2} + f(t)\,\delta\left(x-a\right) \tag{9}$$

And the moment exerted on the beam by the i^{th} actuator can expressed **(Moheimani and Fleming, 2006)** as

$$M_{pi}(x,t) = \bar{k}\, e_{ai}(t)\left[u(x - x_{1i}) - u(x - x_{2i})\right] \tag{10}$$

where $u(x)$ is a unit step input and $\bar{k}$ was formulated by **(Zhang, Meng, and Li, 2006**) as

$$\bar{k} = \frac{E_b\, E_p t_b^2 w_b\, w_p d_{31}}{E_b\, t_b\, w_b + E_p t_p w_p} \tag{11}$$

Substitute that

$$v(x,t) = \sum_{k=1}^{\infty} Y_k(x) q_k(t) \tag{12}$$

where normalized mode shapes for the simply supported beam are **(De Silva, 2000)**

$$Y_k(x) = \sin k\pi x / l_b \tag{13}$$

where $k = 1, 2, 3, \cdots$

multiply by $Y_j(x)$; integrate over $x = [0, l_b]$ and use the orthogonality of mode shapes to obtain

$$E_b I_b \left(\frac{i\pi}{l}\right)^4 \frac{l}{2} q_j(t) + \rho_b A_b \frac{l}{2} \ddot{q}_j = \bar{k}\, \psi_{ji}\, e_{ai}(t) + f(t) \sin \frac{j\pi a}{l} \tag{14}$$

where

$$\psi_{ji} = \int_0^l Y_j(x)\left[\delta'(x - x_{1i}) - \delta'(x - x_{2i})\right] dx \tag{15}$$

and using the derivative Dirac delta function property stated by **(Moheimani and Fleming, 2006)**, will have

$$\psi_{ji} = Y_j'(x_{2i}) - Y_j'(x_{1i}) \tag{16}$$

now **Eq.(14)** can be rewrite as

$$\ddot{q}_j + \omega_j^2\, q_j(t) = \gamma\, \psi_{ji}\, v_{ai}(t) + \alpha_j\; f(t) \tag{17}$$

Where $$\omega_1 = \left(\frac{\pi}{l_b}\right)^2 \sqrt{\frac{E_b I_b}{\rho_b A_b}} \tag{18}$$

$$\omega_j = j^2 \omega_1 \tag{19}$$

$$\alpha_j = \frac{2}{\rho_b A_b l_b} \sin j \frac{\pi a}{l_b} \tag{20}$$

$$\gamma = \frac{2\bar{k}}{\rho_b A_b l_b} \tag{21}$$

To this end we point out that the differential equation **Eq.(17)** dose not contain a term to a count for the natural damping associated with beam. The presence of damping can be incorporated into **Eq.(17)** by adding the term $2\zeta_j \omega_j \dot{q}_j$ to **Eq.(17)**. This results in the differential equation

$$\ddot{q}_j + 2\zeta_j \omega_j q_j + \omega_j^2 q_j(t) = \gamma v_{ai}(t) + \alpha_j \ f(t) \tag{22}$$

Applying the Laplace transform to **Eq.(22)**, assuming zero initial conditions and solving for beam deflection and for N mode shapes, will get

$$V(s) = \gamma \sum_{j=1}^{N} \frac{\psi_{ji} Y_j(x)}{s^2 + 2\zeta_j \omega_j s + \omega_j^2} E_{ai}(s) + \sum_{j=1}^{N} \frac{\alpha_j Y_j(x)}{s^2 + 2\zeta_j \omega_j s + \omega_j^2} F(s) \tag{23}$$

<u>Sensor Equation</u>

The voltage generated by the i^{th} piezoelectric sensor e_{si} can be expressed as **(Moheimani and Fleming, 2006)**

$$e_{si}(t) = \frac{d_{31} E_p W_p}{C_p} \int_{x_{1i}}^{x_{2i}} \varepsilon_{si} \, dx \tag{24}$$

The expression of the mechanical strain in sensor patch can be obtained from **(Moheimani and Fleming, 2006)**

$$\varepsilon_{si} = -\left(\frac{t_b}{2} + t_p \right) \frac{\partial^2 v_i}{\partial x^2} \tag{25}$$

Now $e_{si}(t)$ will be

$$e_{si}(t) = -\left[\frac{d_{13} E_p W_p}{C_p} \left(\frac{t_b}{2} + t_p \right) \sum_{j=1}^{N} \frac{\psi_{ji}}{Y_j} \right] v_i(t) \tag{26}$$

Applying Laplace transform to result equation, assuming zero initial condition we get

$$\frac{E_{si}(s)}{V(s)} = -\frac{d_{13} E_p W_p}{C_p} \left(\frac{t_b}{2} + t_p \right) \sum_{j=1}^{N} \frac{\psi_{ji}}{Y_j} = -\sum_{j=1}^{N} k_{ji} \tag{27}$$

ORDINARY BEAMS GOVERNING EQUATION

Starting from Bernoulli-Euler equation for constant young's modules and second moment of area about the neutral axis and for the actuating force is applied at $x = a$, and following the previous derivation procedure and for N mode shape, will have

$$\frac{V(s)}{F(s)} = \sum_{j=1}^{N} \frac{\alpha_j Y_j(x)}{\left[s^2 + 2\zeta_j \omega_j s + \omega_j^2 \right]} \tag{28}$$

SMART BEAM BLOCK DIAGRAM

A complete block diagram representing the smart beam had been constructed, where Form the previous derived equations and with some block diagram modification, the block diagram that shown in **Fig.(4)** had obtained

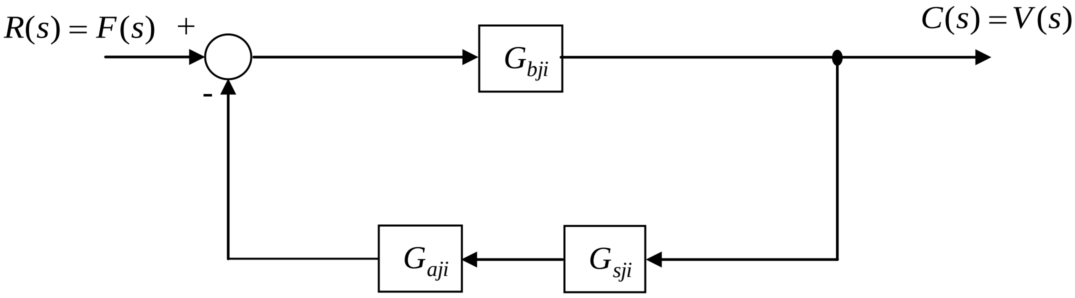


Fig.(4) The Block Diagram for Smart Beam with *m* Collocated Piezoelectric Actuator / Sensor Pairs and Infinite Beam Vibration Mode Shapes

The transfer function matrix G_{bj} shown in **Fig.(4)** consists of a very large number of parallel second order terms while the transfer functions G_{sji} and G_{aji} have a m number of parallel terms, and there values can be expressed as

$$G_{bj} = \sum_{j=1}^{N} \frac{\alpha_j Y_j}{s^2 + 2\zeta_j \omega_j s + \omega_j^2} \tag{29}$$

$$G_{sji} = \sum_{i=1}^{m} \sum_{j=1}^{N} k_{ji} \tag{30}$$

$$G_{aji} = \gamma \sum_{i=1}^{m} \sum_{j=1}^{N} \frac{\psi_{ji}}{\alpha_j} \tag{31}$$

In most scenarios, only control of a limited bandwidth is of importance. Typically *N* modes of the structure would fit within this bandwidth while modes *N* + 1 and above are left uncontrolled. The uncontrolled modes, however, do exist and have the potential to destabilize the closed-loop system. Therefore, the existence of these modes should be taken into account, and a controller should be designed to ensure adequate damping performance, as well as stability in the presence of these out-of-bandwidth modes.

THE PROPOSED INSTRUMENTATION

Practically, the voltage generated by the piezoelectric sensor was very small to handle and transmitted to the piezoelectric actuator, therefore, the needs for an integrated instruments system not only for amplifying purpose but also for controlling the beam response. **Fig.(5)** shows the overall proposed system components. And the block diagram representing this system is shown in **Fig.(6)**.

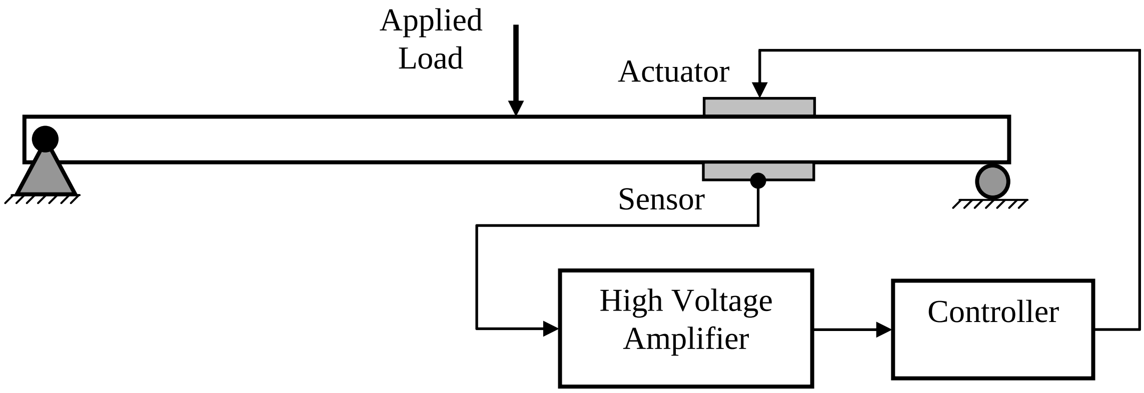


Fig.(5) The Proposed Instrumentation

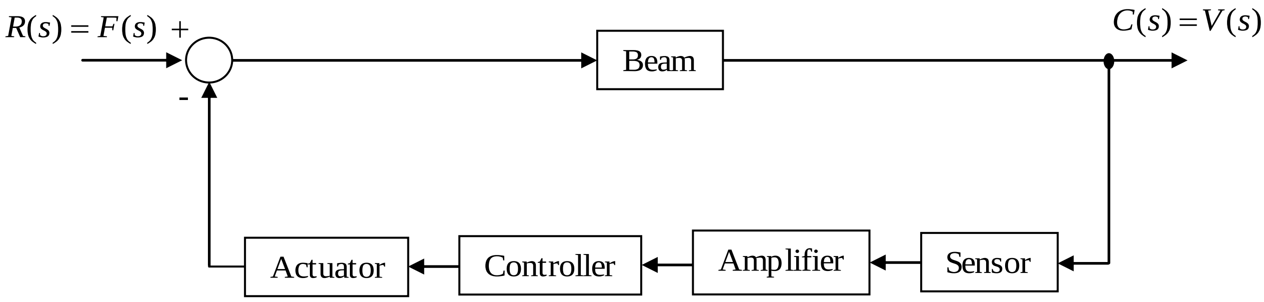


Fig.(6) Smart Beam With Controller And Amplifier Block Diagram

FIRST MODE CONTROLLER DESIGN

Because of the unique properties for the lead net work controller shown in **Fig.(7)** especially it property to accelerate the system response. For performing the controller design some physical properties for the desired system response must be assumed in order to be achieved by the controller operation.

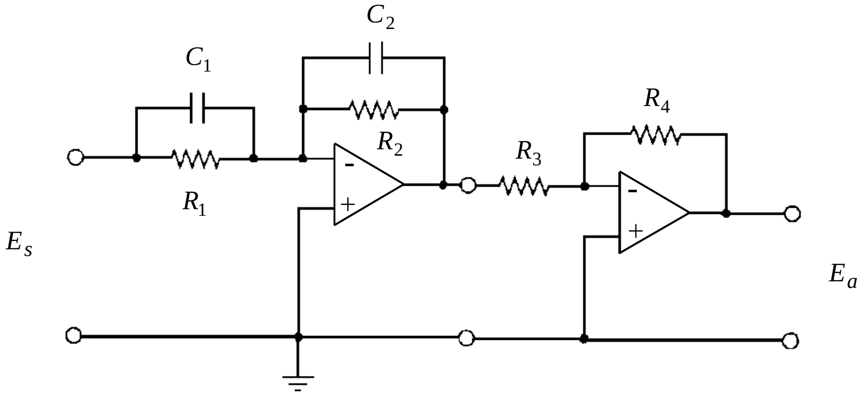


Fig.(7) Electronic Circuit of A Lead Network

The transfer function of the lead network compensator can be expressed as **(Ogata, 2002)**

$$G_c = K_c \alpha \frac{Ts+1}{\alpha Ts+1} \tag{32}$$

Referring to **Fig.(5)** , $T = R_1C_1$, $\alpha T = R_2C_2$, $K_c = \frac{R_4C_1}{R_3C_2}$, and $\alpha = \frac{R_2C_2}{R_1C_1}$

The Bode diagram for the smart beam is shown in **Fig.(8)**, and after executing the control system design by frequency response procedure as stated by **(Ogata, 2002)**, for static velocity error about $50\,s^{-1}$ and at least 100° phase margin. After calculations the following controller transfer function will obtained

$$G_c = \frac{1.4(s+47.5)}{(s+135.1)}$$

Where $K_c = 1.4$, $\alpha = 0.35$, and $T = 0.022$. The system with the controller Bode diagram is shown in **Fig.(9)**

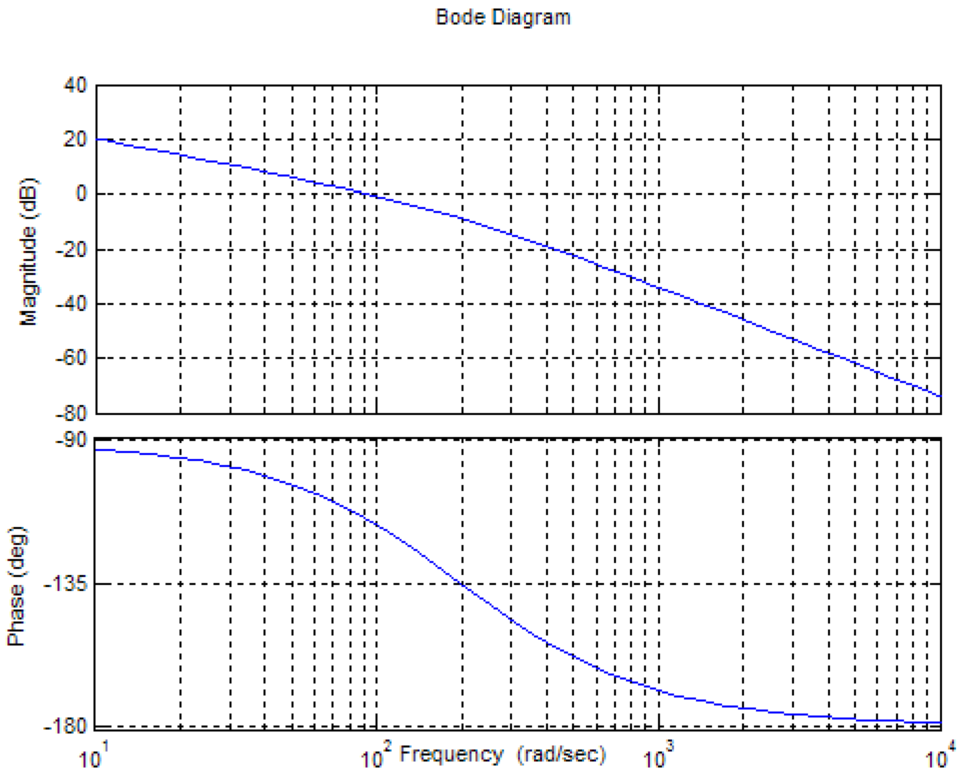


Fig.(8) Bode Diagram for First Mode Simply Supported Thin Smart Beam

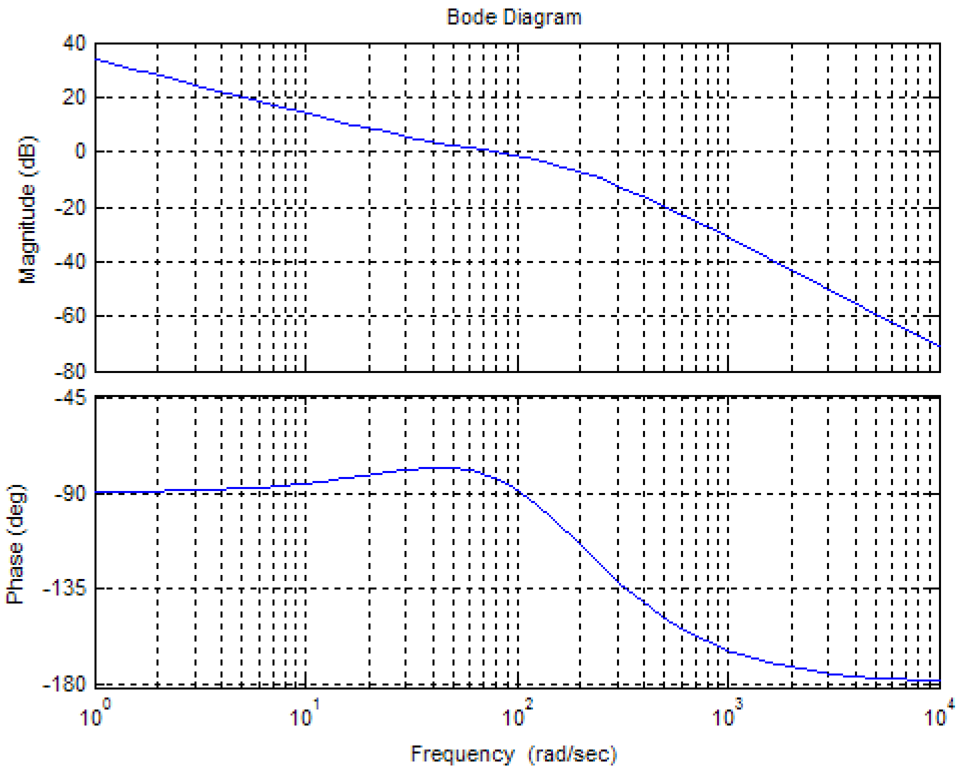


Fig.(9) Bode Diagram for Compensated First Mode Simply Supported Thin Smart Beam

FIRST MODE HIGH VOLTAGE AMPLIFIER

After studying and analyzing the block diagram of the proposed system without controller, for the first mode case. In the present paper, the voltage amplifier gain for the i^{th} collocated piezoelectric actuator / sensor pair was formulated to be

$$K_a = \frac{\omega_1^2}{\gamma \psi_{1i} k_{1i}} \tag{33}$$

RESULTS AND DISCUSSION

SIMULINK / MATLAB software was constructed to simulate the proposed system block diagram. The properties that listed in **Table (1)** which was adopted from **(Moheimani and Fleming, 2006)** had been used as numerical values for such software.

The simulation results of the ordinary beams are showing the high accuracy of the ordinary beam transfer function in comparing with the analytical solution given by any text.

Many cases had been studied for the first mode vibration and for static deflection, where the type of the applied load was changed, the number of collocated piezoelectric was varied, and the beam damping ratio also was varied. **Fig.(10)** to **Fig.(17)** show the results of the software simulations for different cases where the applied load was about 10 *N*, all the piezoelectric collocated pairs configurations exhibit a significant reduction in smart beam deflection in compare with the ordinary beam deflection, as example in the case of cyclic load with $\zeta = 0.7$ about 42% deflection reduction for single piezoelectric collocated pair at $x = l/2$, about 52% deflection reduction, for double piezoelectric collocated pairs at $x_1 = l/4$ and $x_2 = 3l/4$, and about 65% deflection reduction for three piezoelectric collocated pairs at $x_1 = l/4$, $x_2 = l/2$, and

$x_3 = 3l/4$.For the static load cases, the simulation results was decided to introduce for a very short time about 1 *s*, and this for showing the beam transient response and effect of the controller. Where with $\zeta = 0.7$ about 34% deflection reduction for single piezoelectric collocated pair at $x = l/2$, about 40% deflection reduction, for double piezoelectric collocated pairs at $x_1 = l/4$ and $x_2 = 3l/4$, and about 55% deflection reduction for three piezoelectric collocated pairs at $x_1 = l/4$, $x_2 = l/2$, and $x_3 = 3l/4$.

The simulation of the smart beam without voltage amplifier and lead controller was didn't exhibits any valuable reduction in beam deflection and this was expected because of the voltage that generated by the sensor can not initiate the actuator to be strained to the required limit, and this lead us to assemble the system shown in **Fig.(5)**, where the voltage amplifier was designed to obtain the required actuator stain while the controller was designed to obtain the desired beam response.

Beam Properties	
Length	550 *mm*
Thickness	3 *mm*
Width	50 *mm*
Density	$2.77 \times 10^3 \ kg/m^3$
Young's Modules	$7 \times 10^{10} \ N/m^2$
PZT Properties	
Length	50 *mm*
Thickness	0.25 *mm*
Width	25 *mm*
Charge constant	$-210 \times 10^{-12} \ m/V$
Young's Modules	$6.3 \times 10^{10} \ N/m^2$
Capacitance	115 *nF*

Table (1) Numerical Values

CONCOLUSIONS

The principle of operation of the proposed system is to reduce the beam deflection for any external load situation and this was done by bonded a number of finite of collocated actuator / sensor pairs working to generate an additional moment operate inversely to reduce the effect of the applied load.

It has been shown practically that the best result of deflection reduction is obtained by increasing the number of the collocated actuator / sensor pairs for the case of multi collocated actuator / sensor pairs, but for single collocated actuator / sensor pair the best reduction is done if the collocated actuator / sensor pair is bonded exactly at the location of load application

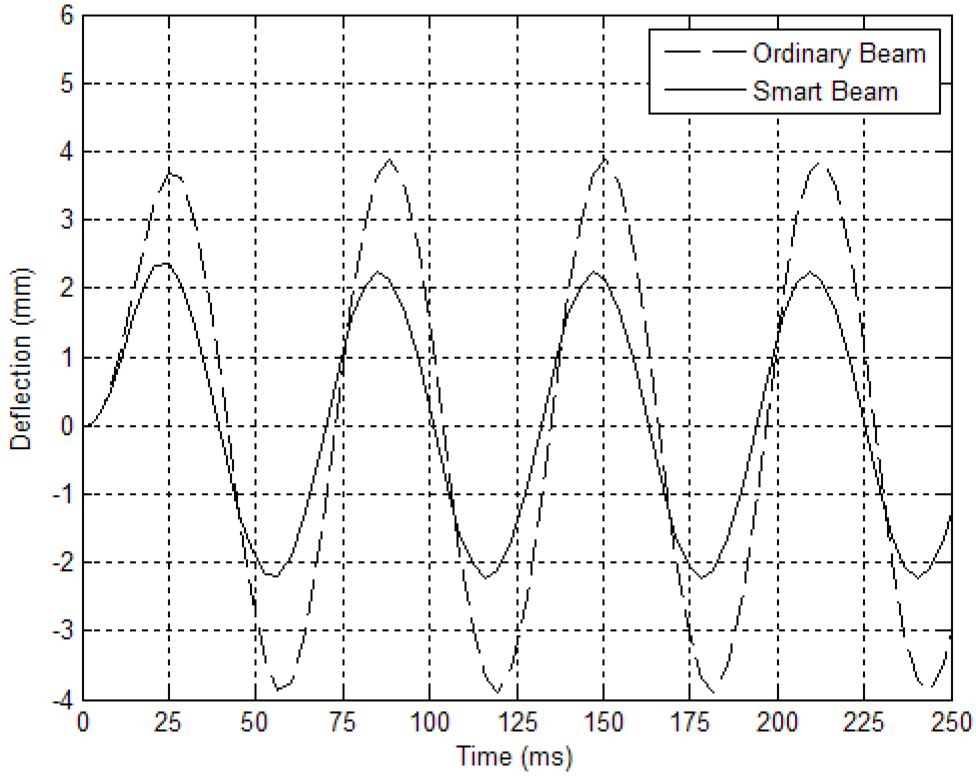


Fig.(10) First Mode Beams Vibrations with $\zeta = 0.7$. The Smart Beam with Single Collocated Piezoelectric Actuator / Sensor Pair at $x = l/2$

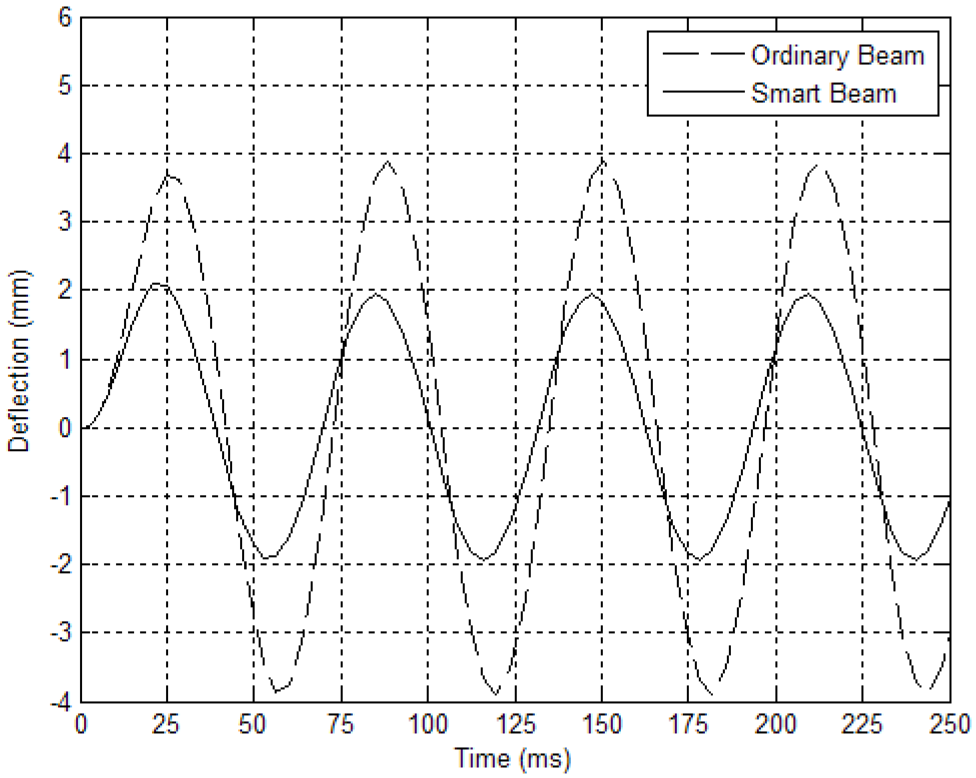


Fig.(11) First Mode Beams Vibrations with $\zeta = 0.7$. The Smart Beam with Double Collocated Piezoelectric Actuator / Sensor Pairs at $x_1 = l/4$ and $x_2 = 3l/4$.

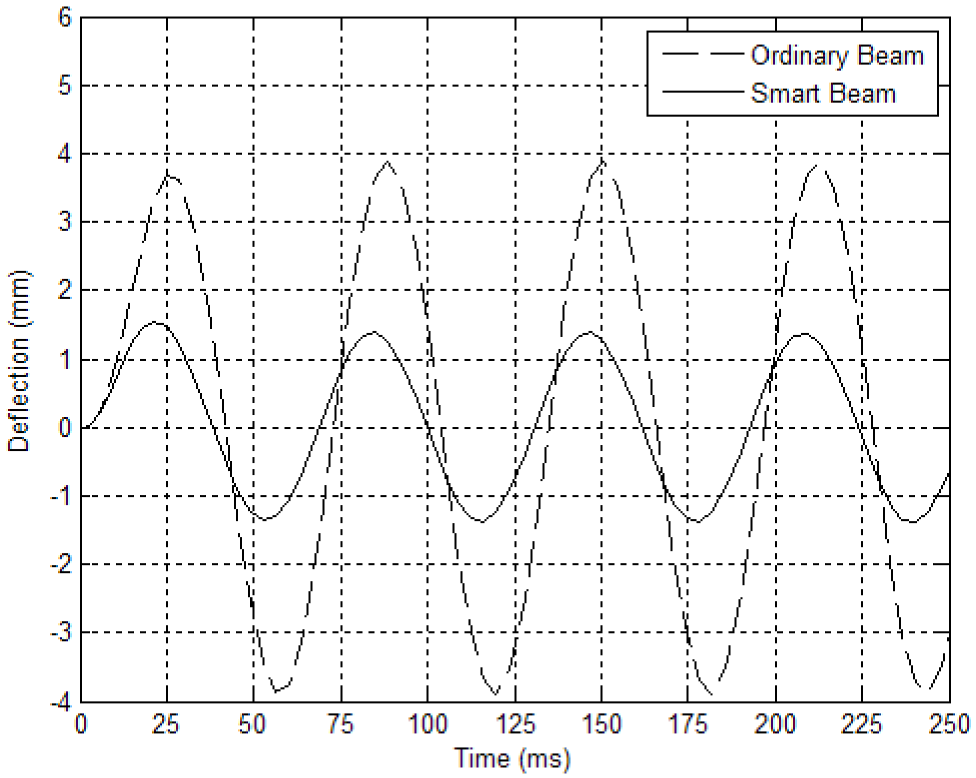


Fig.(12) First Mode Beams Vibrations with $\zeta = 0.7$. The Smart Beam with Three Collocated Piezoelectric Actuator / Sensor Pairs at $x_1 = l/4$, $x_2 = l/2$, and $x_3 = 3l/4$

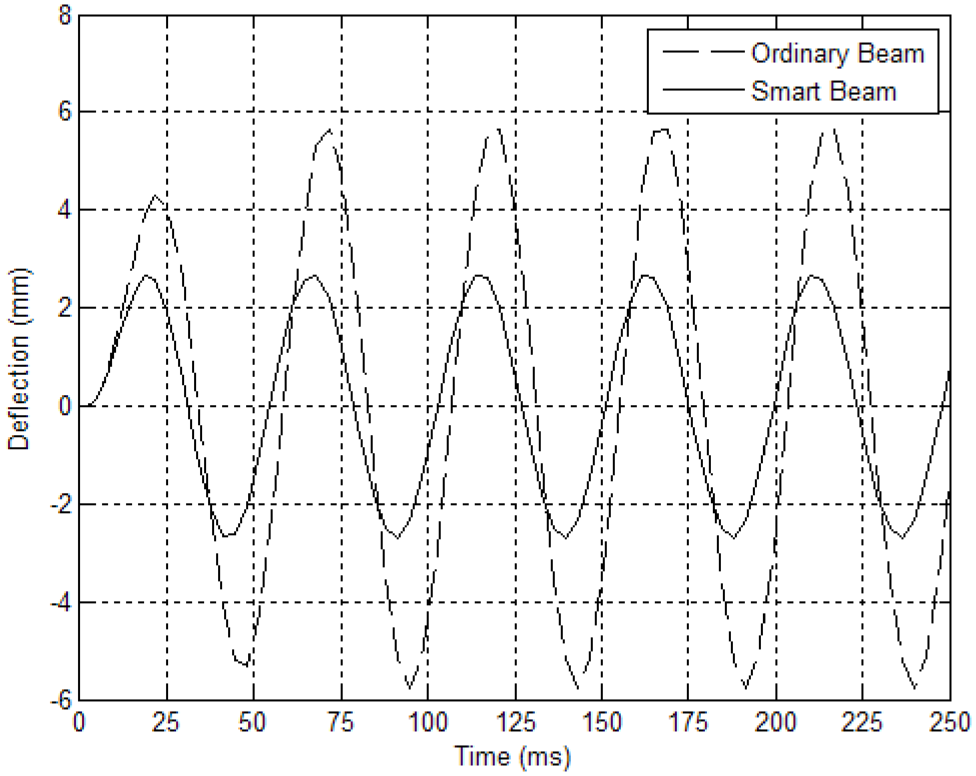


Fig.(13) First Mode Beams Vibrations with $\zeta = 0.4$. The Smart Beam with Single Collocated Piezoelectric Actuator / Sensor Pair at $x = l/2$.

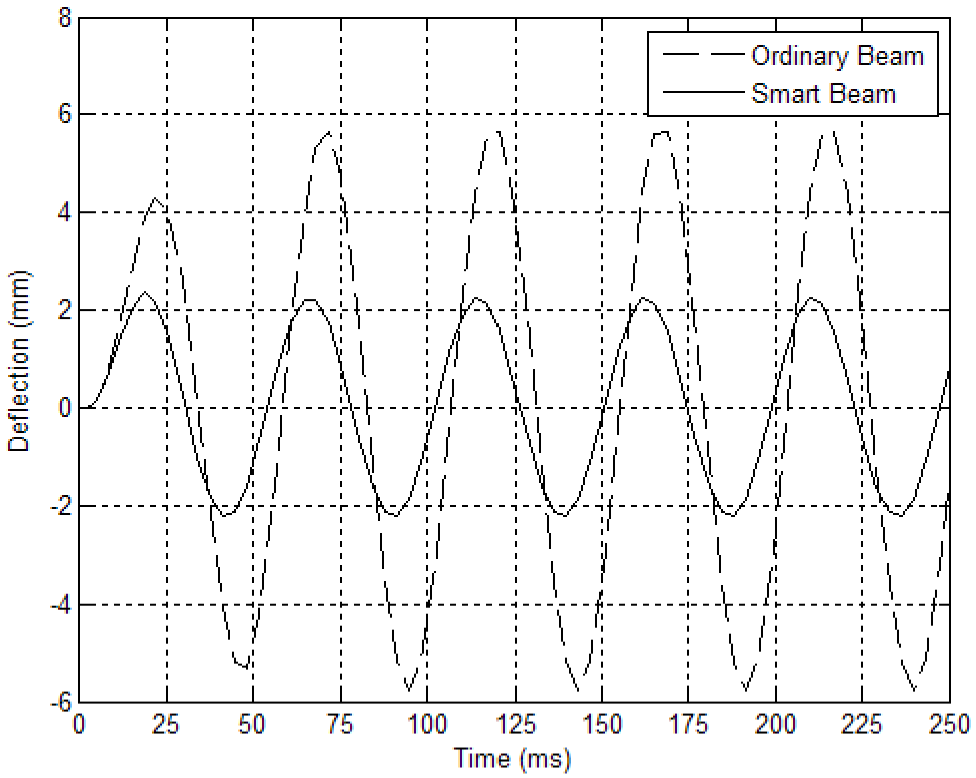


Fig.(14) First Mode Beams Vibrations with $\zeta = 0.4$. The Smart Beam with Double Collocated Piezoelectric Actuator / Sensor Pairs at $x_1 = l/4$ and $x_2 = 3l/4$

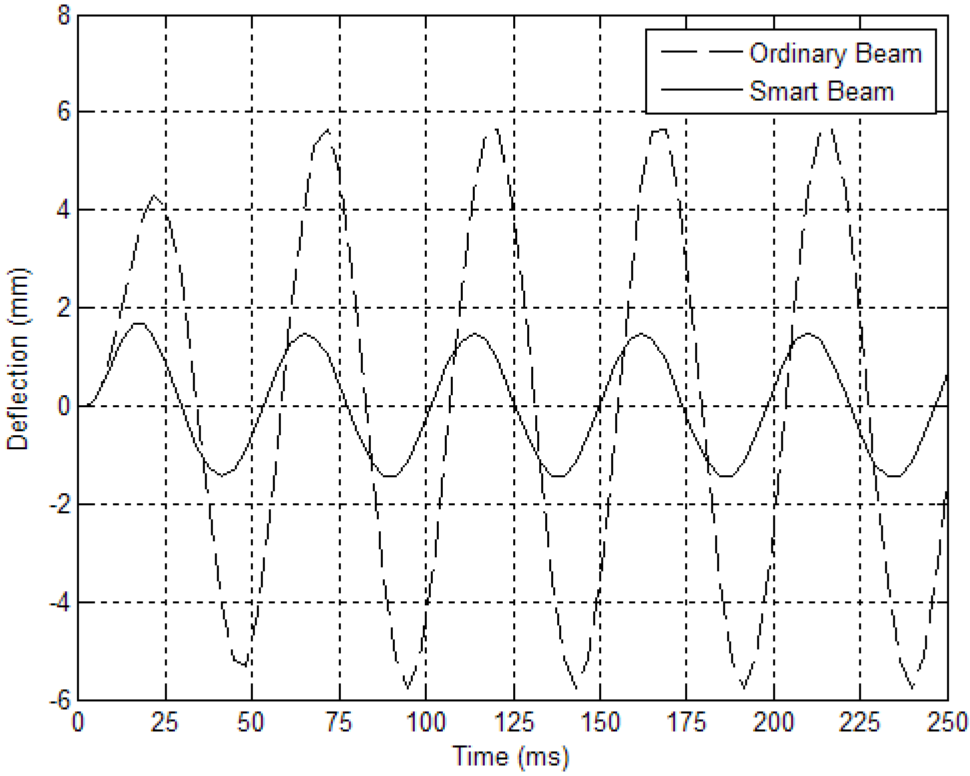


Fig.(15) First Mode Beams Vibrations with $\zeta = 0.4$. The Smart Beam with Three Collocated Piezoelectric Actuator / Sensor Pairs at $x_1 = l/4$, $x_2 = l/2$, and $x_3 = 3l/4$.

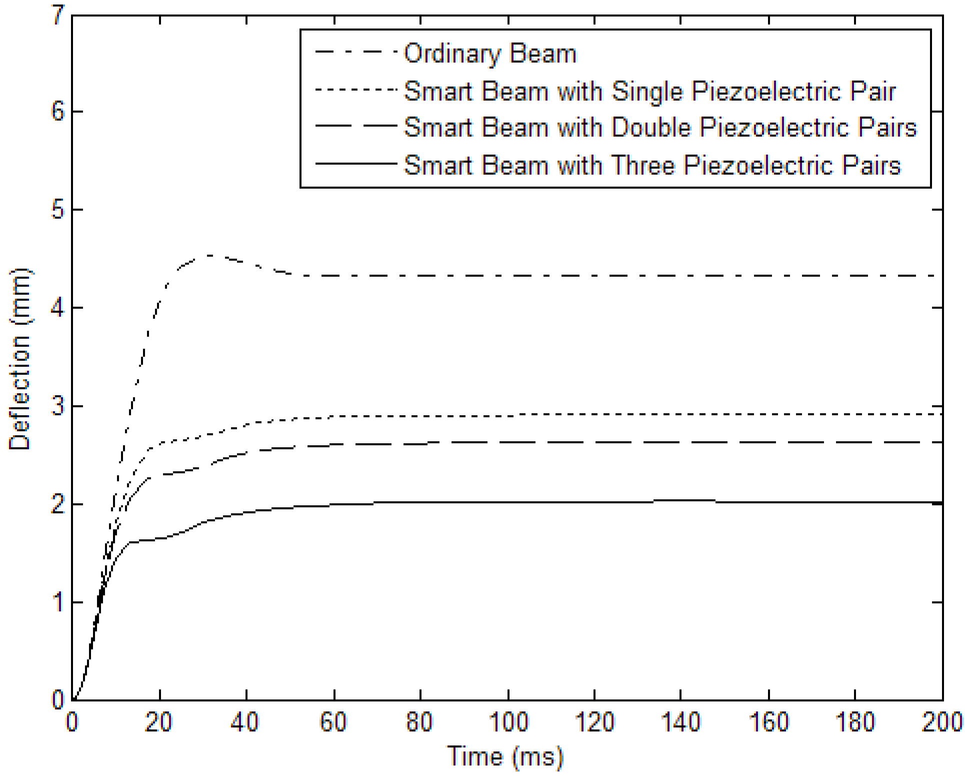


Fig.(16) Static Load Beams Deflections with $\zeta = 0.7$

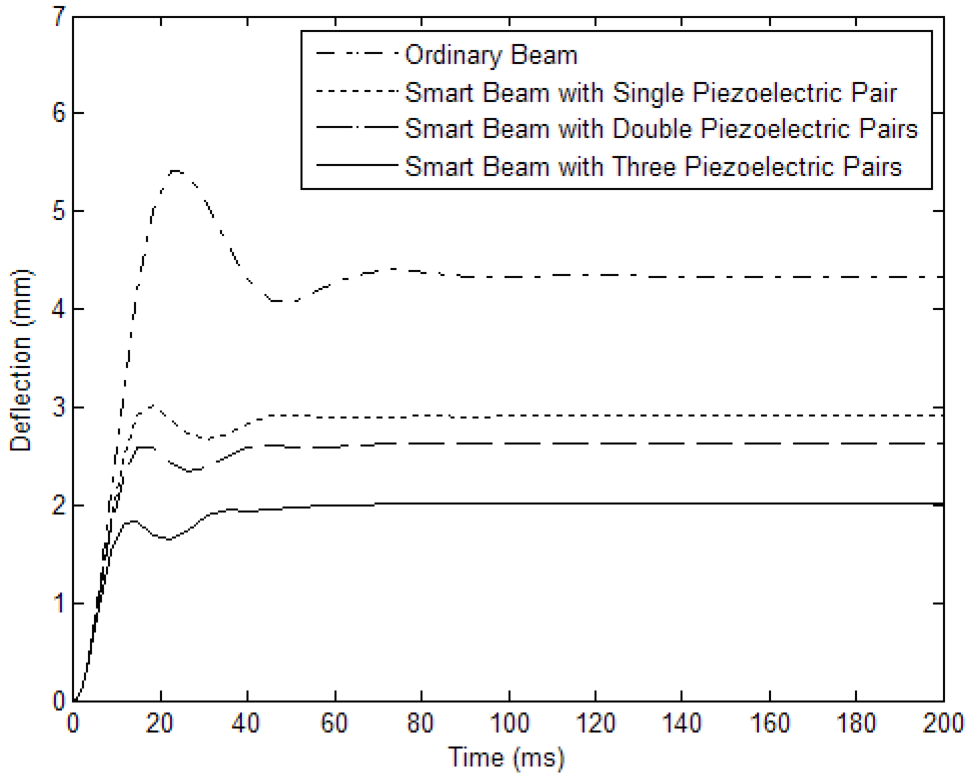


Fig.(17) Static Load Beams Deflections with $\zeta = 0.4$

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NOMENCLATURE

(SI units are used, unless otherwise stated)

A	The location of the applied load
A_b	The beam cross-sectional area
$C_1, and\, C_2$	The controller capacitances
C_p	The piezoelectric constant
d_{31}	The piezoelectric charge constant
e_{ai}	The exposed voltage to the i^{th} actuator
E_b	The beam Young's modules
E_p	The piezoelectric Young's modules
e_{si}	The voltage induced by the i^{th} sensor
f	The applied external force
I_b	The beam second moment of area
$\bar{k}$	A piezoelectric constant
K_a	The high voltage amplifier gain
K_c	The controller gain
l_b	The beam length
l_p	The piezoelectric length
m	The total number of the collocated piezoelectric actuator / sensor pairs
M_{pi}	The piezoelectric actuator control moment generated by the i^{th} actuator
M	The applied external moment
N	The controlled vibration mode number
Q	The external shear force
$R_1, R_2, R_3\, and\, R_4$	The controller resistances
T	The time
T	The controller time constant
t_b	The beam thickness
t_p	The piezoelectric thickness
$u(x)$	The unit step function
v	The beam deflection
w_p	The piezoelectric width
w_b	The beam width
x_{1i}	The location of the closer edge of the i^{th} piezoelectric
x_{2i}	The location of the further edge of the i^{th} piezoelectric
x_i	The location of the center of the i^{th} piezoelectric
$Y_k(x)$	The normalized mode shapes
α	The controller constant
α_j	A constant for j mode
ε_{si}	The mechanical strain of the i^{th} piezoelectric sensor
ψ_{ji}	Piezoelectric constant for i^{th} piezoelectric and for j^{th} mode
γ	A beam constant

ω_j	The j^{th} natural frequency
ρ_b	The beam density
ρ_p	The piezoelectric density
ζ	The beam damping ratio