# GENERATION AND SIMULATION OF MESHING FOR INVOLUTE HELICAL GEARS

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## ABSTRACT

A new method for generation and simulation of meshing for involute helical gears is presented. The approaches proposed for generation are based on the imaginary application of two rack-cutters one for pinion and the other for gear generation. The proposed simulation of meshing of aligned and misaligned helical gears gives (i) the shift of the bearing contact (real contact path) on pinion and gear tooth surfaces, (ii) the function of transmission errors.

## الخلاصة

طريقة جديدة لتوليد و محاكاة التعشيق للتروس الحلزونية ذات الجانبية العدلة قد تم عرضها التقريبات المقترحة للتوليد أستندت على تطبيق أداتي قطع تخيليتين واحدة للترس الصغير و الأخرى للترس الكبير محاكاة التعشيق المقترحة للتروس الحلزونية المنتظمة و غير المنتظمة تعطي (أ) الأنتقال في موقع التماس (مسار التماس الحقيقي) على سطح سن الترس الصغير و الترس الكبير (ب) دالة الأخطاء المنقولة.

## KEYWORDS

Helical gear; Gear generation; Gears Meshing; Simulation of Gears; Bearing Contact.

## INTRODUCTION

A review of the history of the development of involute gears shows that the involute was introduced as a profile for gear tooth on account of the simplicity in deign and manufacture of gears having these profiles, also the involute teeth not sensitive to slight errors in profile and center distance (**Avinash et. al., 2003). (Kubo**, **1988)** presented a general calculation method of load sharing to every tooth pair in meshing, load distribution, and contact pattern on tooth flank of helical gears with manufacturing and alignment error, for which some parts of tooth flanks on the geometrical line of contact can separate from each other due to the errors. For such gears, stiffness of meshing tooth pair, exciting force of gear vibration, and total composite error under loaded conditions was derived. **(Litvin et. al, 1995)** described the design and generation of modified involute helical gears that have a localized and stable bearing contact, and reduced noise and vibration characteristics.

**(Litvin and Fan, 2001)** covered the data on involute helical gears manufactured by shaving. Modification of geometry of helical gear with parallel axis was proposed. The finishing process of gear generation is shaving. **(Litvin and Ignacio, 2003)** covered design, generation, simulation of meshing and stress analysis of modified involute helical gears. The approach developed for modification of the conventional involute helical gears was based on conjugation of double-crowned pinion with a conventional helical involute gear. The bearing contact was localized and oriented longitudinally, and edge contact was avoided. Also, the influence of misalignment on the shift of bearing contact, noise and vibration were reduced.

In order to drive in a given direction and to transmit power or motion smoothly and with a minimum loss of energy, the contacting surfaces on the mating gears must have the following properties **(Dudley, 1962)**:-

1. The height and the lengthwise shape of the active profiles of the teeth must be such that, before one pair of teeth goes out of contact during meshing, a second pair will have picked up its share of load.
2. The shape of the contacting surfaces of the teeth (active profile) must be such that the angular velocity of the driving member of the pair is smoothly imparted to the driven member in the proper ratio.
3. The spacing between the successive teeth must be such that a second pair of tooth-contacting surfaces (active profiles) is in the proper positions to receive the load before the first leave mesh.

## BASIC PRINCIPLES OF GENERATION

The process for gear generation is based on the followings **(Litvin et. al, 1997)**:-

1. The generation of tooth surfaces is based on the imaginary derivation of conjugate surfaces by application of two rack-cutters. The surfaces of the two rack-cutters are rigidly connected each to other in the process of the imaginary generation, and they are in tangency along one straight line as shown in **Fig. (1).** This line and axes of the gears form a helix angle (β).

2. In the process for the generation, the two rigidly connected rack cutters perform translational motion, while the pinion and gear perform rotational motions about their axes, $O_1$ and $O_2$.
3. Surfaces of gear and pinion ($\Sigma_1$ and $\Sigma_2$) determined as the envelope to the family of rack cutter surfaces.
4. When the generated surfaces $\Sigma_1$ and $\Sigma_2$ are in mesh, their will be one contact point at every instant. The path of contact on surfaces $\Sigma_i$ (i =1, 2) is the set of points of $\Sigma i$ where $\Sigma_1$ and $\Sigma_2$ contact each other. Such a path of contact is a helix, and the contact point moves in the process of meshing along the helix on $\Sigma_i$.

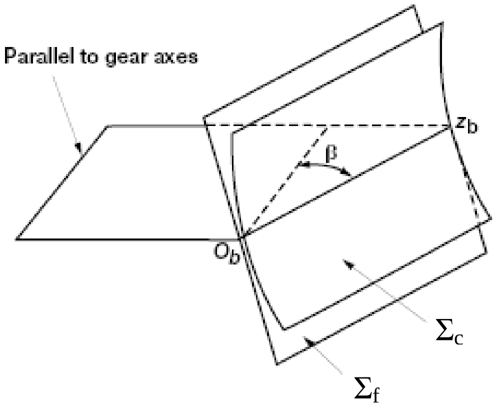


**Fig. (1):-** Rack cutter surfaces for involute helical gears.

**DERIVATION OF PINION TOOTH SURFACE:-**

The normal section of pinion rack cutter is shown in **Fig. (2)** The profile of the basic tooth of the rack cutter in the normal section is symmetric about $x_{cp}$.

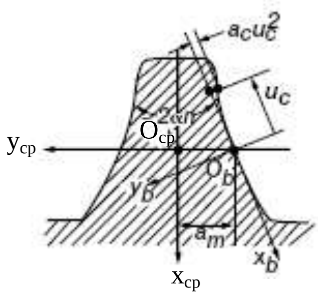


**Fig.(2):-** Normal section of pinion rack cutter.

The normal section of the pinion rack cutter is represented in the coordinate system $S_{cp}$ by the equations **(Litvin et. al, 1997)**:-

$$r_{cp}(u_c) = \begin{bmatrix} x_{cp} \\ y_{cp} \\ z_{cp} \\ 1 \end{bmatrix} = \begin{bmatrix} -u_c \cos\alpha_n - a_c u_c^2 \sin\alpha_n \\ u_c \sin\alpha_n - a_c u_c^2 \cos\alpha_n - a_m \\ 0 \\ 1 \end{bmatrix} = M_{cpb} r_b(u_c) \quad (1)$$

where $r_b(u_c) = [-u_c \quad -a_c u_c^2 \quad 0 \quad 1]^T$, $a_m = \dfrac{\pi}{4p_n}$, $(a_c)$ is the parabolic coefficient

and

$$M_{cpb} = \begin{bmatrix} \cos\alpha_n & \sin\alpha_n & 0 & 0 \\ -\sin\alpha_n & \cos\alpha_n & 0 & -a_m \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The rack cutter tooth in the three dimensional system can be defined by $S_c$ coordinate system as

$$r_c(u_c, \iota_c) = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} -u_c \cos\alpha_n - a_c u_c^2 \sin\alpha_n \\ (u_c \sin\alpha_n - a_c u_c^2 \cos\alpha_n - a_m)\cos\beta - \iota_c \sin\beta \\ (-u_c \sin\alpha_n + a_c u_c^2 \cos\alpha_n + a_m)\sin\beta + \iota_c \cos\beta \\ 1 \end{bmatrix} \quad (2)$$

where

$$M_{ccp} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & -\iota_c \sin\beta \\ 0 & -\sin\beta & \cos\beta & \iota_c \cos\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The unit normal to the pinion rack cutter surface is represented as:

$$n_c = \frac{N_c}{|N_c|} \quad , \quad N_c = \frac{\partial r_c}{\partial \iota_c} \times \frac{\partial r_c}{\partial u_c} \quad (3)$$

Thus,

$$n_c(u_c) = \begin{bmatrix} n_{xc} \\ n_{yc} \\ n_{zc} \end{bmatrix} = \frac{1}{\sqrt{1 + 4a_c^2 u_c^2}} \begin{bmatrix} -\sin\alpha_n + 2a_c u_c \cos\alpha_n \\ -(\cos\alpha_n + 2a_c u_c \sin\alpha_n)\cos\beta \\ -(\cos\alpha_n + 2a_c u_c \sin\alpha_n)\sin\beta \end{bmatrix} \quad (4)$$

To derive the equation of meshing between pinion rack cutter and pinion tooth surfaces, it is considered that the movable coordinate systems $S_c$ and $S_1$ are rigidly connected to the tool (pinion rack cutter) and the pinion, respectively. The fixed coordinate system $S_n$ is rigidly connected to the frame of the cutting machine. The derivation of equation of meshing is based on the theorem that the common normal to $\Sigma_c$ and $\Sigma_1$ must pass through the instantaneous axis of rotation **(Litvin, 1989)**. Thus

$$\frac{X_c - x_c}{n_{xc}} = \frac{Y_c - y_c}{n_{yc}} = \frac{Z_c - z_c}{n_{zc}} \qquad (5)$$

where $X_c = 0$ and $Y_c = -R_{p1}\psi_1$

Using **Eq. (5)** the equation of meshing can be represented as:

$$f(u_c, \iota_c, \psi_1) = 0 \qquad (6)$$

where $\psi_1$ is the angle of rotation of the pinion in the process for generation.

After transformation, the following equation of meshing can be obtained

$$f(u_c, \iota_c, \psi_1) = R_{p1}\psi_1 - \iota_c \sin\beta - a_m \cos\beta + \frac{u_c(1 + 2a_c u_c^2)\cos\beta}{\sin\beta - 2a_c u_c \cos\alpha_n} = 0 \qquad (7)$$

Finally, the generated surface of the pinion $\Sigma_1$ is represented by the family of lines of contact between the rack cutter surface $\Sigma_c$ and the pinion tooth surface $\Sigma_1$ being generated. Surface $\Sigma_1$ is represented in coordinate system $S_1$ by the equations:

$$r_1(u_c, l_c, \psi_1) = M_{1n} M_{nc} r_c(u_c, l_c), \qquad f(u_c, l_c, \psi_1) = 0 \qquad (8)$$

where

$$M_{nc} = \begin{bmatrix} 1 & 0 & 0 & R_{P1} \\ 0 & 1 & 0 & R_{p1}\psi_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad M_{1n} = \begin{bmatrix} \cos\psi_1 & \sin\psi_1 & 0 & 0 \\ -\sin\psi_1 & \cos\psi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus

$$r_1 = \begin{bmatrix} (-u_c \cos\alpha_n a_c u_c^2 \sin\alpha_n)\cos\psi_1 + \{(u_c \sin\alpha_n - a_c u_c^2 \cos\alpha_n - a_m)\cos\beta \\ -l_c \sin\beta\}\sin\psi_1 + (\cos\psi_1 + \psi_1 \sin\psi_1)R_{p1} \\ \\ (u_c \cos\alpha_n + a_c u_c^2 \sin\alpha_n)\sin\psi_1 + \{(u_c \sin\alpha_n - a_c u_c^2 \cos\alpha_n - a_m)\cos\beta \\ -l_c \sin\beta\}\cos\psi_1 + (-\sin\psi_1 + \psi_1 \cos\psi_1)R_{p1} \\ \\ (-u_c \sin\alpha_n + a_c u_c^2 \cos\alpha_n + a_m)\sin\beta + l_c \cos\beta \end{bmatrix} \tag{9}$$

## DERIVATION OF GEAR TOOTH SURFACE:

The normal section of gear rack-cutter is shown in **Fig. (3)**. The profile of the basic tooth of the rack-cutter in the normal section is symmetric about $x_{cf}$.

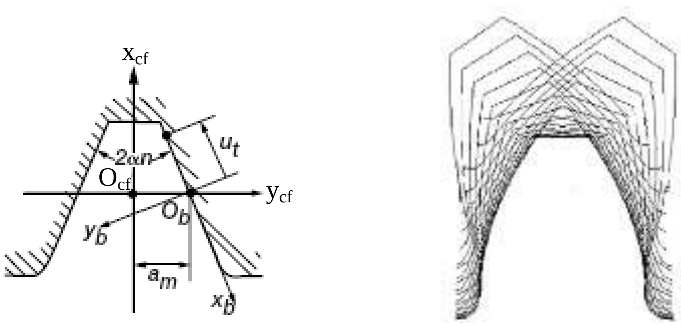


**Fig. (3):-** Normal sections of gear rack cutter.

By using coordinate system $S_{cf}$, the normal section of gear rack-cutter can be represented by the following equations **(Litvin et. al, 1997)**:-

$$r_{cf} = M_{cfb} r_b(u_f)$$

Or
$$r_{cf}(u_f) = \begin{bmatrix} x_{cf} \\ y_{cf} \\ z_{cf} \\ 1 \end{bmatrix} = \begin{bmatrix} u_f \cos\alpha_n \\ a_m - u_f \sin\alpha_n \\ 0 \\ 1 \end{bmatrix} \tag{10}$$

Hence, $r_b(u_f) = \lfloor -u_f \quad 0 \quad 0 \quad 1 \rfloor^T$, $u_f$ is the variable parameter and $M_{cfb}$ is the matrix of coordinate transformation from $S_b$ to $S_{cf}$ that is

$$M_{cfb}=\begin{bmatrix}-\cos\alpha_n & \sin\alpha_n & 0 & 0\\ \sin\alpha_n & \cos\alpha_n & 0 & 0\\ 0 & 0 & 1 & a_m\\ 0 & 0 & 0 & 1\end{bmatrix}$$

The derivation of gear rack-cutter tooth surface in the three dimensional system may be accomplish similarly to that for pinion rack-cutter, and the following vector equations in $S_f$ coordinate system can be obtained:

$$r_f(u_f,l_f)=M_{fcf}r_{cf}(u_f)$$

Where $$M_{fcf}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\beta & \sin\beta & l_f\sin\beta\\ 0 & -\sin\beta & \cos\beta & l_f\cos\beta\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Thus,

$$r_f(u_f,l_f)=\begin{bmatrix}x_f\\ y_f\\ z_f\\ 1\end{bmatrix}=\begin{bmatrix}u_f\cos\alpha_n\\ (a_m-u_f\sin\alpha_n)\cos\beta+l_f\sin\beta\\ (u_f\sin\alpha_n-a_m)\sin\beta+l_f\cos\beta\\ 1\end{bmatrix} \tag{11}$$

The unit normal to the gear rack-cutter surface $\Sigma_f$ is:

$$n_f(u_f)=\begin{bmatrix}n_{xf}\\ n_{yf}\\ n_{zf}\end{bmatrix}=\begin{bmatrix}\sin\alpha_n\\ \cos\alpha_n\cos\beta\\ -\cos\alpha_n\sin\beta\end{bmatrix} \tag{12}$$

The equation of meshing can be derived in similar manner to that of **Eqs. (5) and (7)**, thus

$$\frac{X_f-x_f}{n_{xf}}=\frac{Y_f-y_f}{n_{yf}}=\frac{Z_f-z_f}{n_{zf}} \tag{13}$$

Where $X_f=0$ and $Y_f=R_{p1}\psi_1$

After transformations, the following equation of meshing between $\Sigma_f$ and $\Sigma_2$ can be obtained

$$f(u_f,l_f,\psi_2)=R_{p2}\psi_2-l_f\sin\beta-a_m\cos\beta+\frac{u_f\cos\beta}{\sin\alpha_n}=0 \tag{14}$$

Finally ,the generated gear tooth surface $\Sigma_2$ is determined as the envelope to the family of rack-cutter surfaces $\Sigma_f$ and is represented in coordinate system $S_2$ as

$$r_2(u_f, l_f, \psi_2) = M_{2m} M_{mf} r_f(u_f, l_f)$$

where
$$M_{2m} = \begin{bmatrix} \cos\psi_2 & -\sin\psi_2 & 0 & 0 \\ \sin\psi_2 & \cos\psi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and
$$M_{mf} = \begin{bmatrix} 1 & 0 & 0 & R_{p2} \\ 0 & 1 & 0 & -R_{p2}\psi_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus

$$r_2 = \begin{bmatrix} u_f \cos\alpha_n \cos\psi_2 - \{a_m - u_f \sin\alpha_n)\cos\beta + l_f \sin\beta\}\sin\psi_2 \\ +(\cos\psi_2 + \psi_2 \sin\psi_2)R_{p2} \\ \\ u_f \cos\alpha_n \sin\psi_2 - \{(a_m - u_f \sin\alpha_n)\cos\beta + l_f \sin\beta\}\cos\psi_2 \\ +(\sin\psi_2 - \psi_2 \cos\psi_2)R_{p2} \\ \\ (u_f \sin\alpha_n - a_m)\sin\beta + l_f \cos\beta \\ \\ \end{bmatrix} \quad (15)$$

**Fig. (4)** shows a sample of three teeth generated by the equations mentioned above using Ansys computer program.

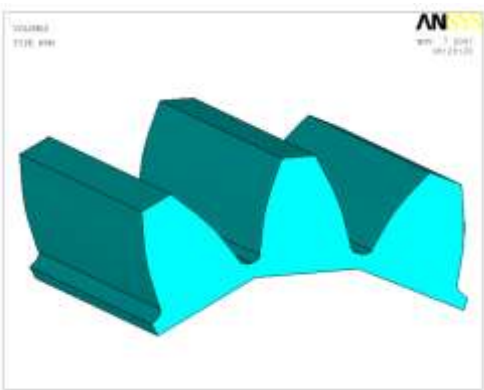

**Fig. (4):-** Sample of three teeth generated.

## SIMULATION OF MESHING:

Computerized simulation of meshing is applied to discover the influence of misalignment on the shift of the bearing contact and transmission errors. The misalignment of the gear drive is simulated by the error of installation and orientation of the gear with respect to the pinion.

The theoretically correct tooth form is defined by the parameters tooth alignment, tooth profile, tooth spacing and center distance.

## SIMULATION OF MESHING FOR INVOLUTE HELICAL GEARS:-

Consider that surfaces $\Sigma_1$ and $\Sigma_2$ and their unit normals $n_1$ and $n_2$ are represented in coordinate systems $S_1$ and $S_2$, which are connected to the pinion and gear, respectively. The meshing of the gear tooth surfaces is considered in the fixed coordinate system $S_f$ that is rigidly connected to the frame. The auxiliary coordinate systems $S_q$ and $S_p$ are used for simplification of coordinate transformation.

The simulation of meshing is based on the following procedure **(Litvin and Ignacio, 2003) and (Litvin et. al, 1997)**:-

**Step1:** Using coordinate transformation, we represent $\Sigma_1$, $\Sigma 2$, $n_1$ and $n_2$ in the fixed coordinate system $S_f$ as following **Fig. (5).**

$$r_f^{(1)}(u_c,\psi_1,\phi_1) = M_{f1} r_1(u_c,\psi_1) \tag{16}$$

$$r_f^{(2)}(u_f,\psi_2,\phi_2) = M_{fp} M_{pq} M_{q2} r_2(u_f,\psi_2) \tag{17}$$

Where

$$M_{f1} = \begin{bmatrix} \cos\phi_1 & \sin\phi_1 & 0 & 0 \\ -\sin\phi_1 & \cos\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{q2} = \begin{bmatrix} \cos\phi_2 & \sin\phi_2 & 0 & 0 \\ -\sin\phi_2 & \cos\phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{pq} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Delta\gamma & \sin\Delta\gamma & 0 \\ 0 & -\sin\Delta\gamma & \cos\Delta\gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{fp} = \begin{bmatrix} -1 & 0 & 0 & E' \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E' = R_{p1} + R_{p2} + \Delta E$$

$$n_f^{(1)}(u_c, \psi_1, \phi_1) = L_{f1} n_1(u_c, \psi_1) \quad (18)$$

$$n_f^{(2)}(u_f, \psi_2, \phi_2) = L_{fq} L_{pq} L_{q2} n_2(u_f, \psi_2) \quad (19)$$

$$L_{f1} = \begin{bmatrix} \cos\phi_1 & \sin\phi_1 & 0 \\ -\sin\phi_1 & \cos\phi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L_{q2} = \begin{bmatrix} \cos\phi_2 & \sin\phi_2 & 0 \\ -\sin\phi_2 & \cos\phi_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L_{pq} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Delta\gamma & \sin\Delta\gamma \\ 0 & -\sin\Delta\gamma & \cos\Delta\gamma \end{bmatrix}$$

$$L_{fp} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

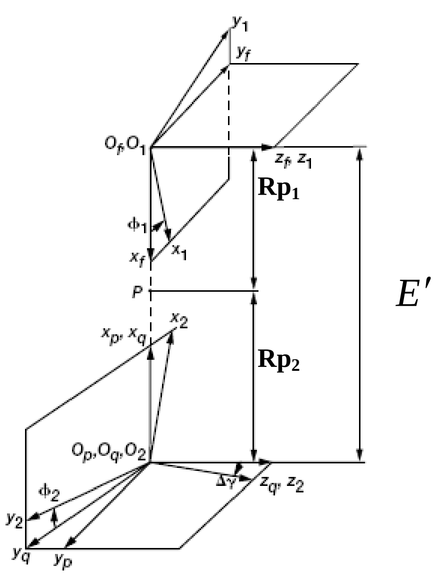


**Fig. (5):-** Applied coordinate system for simulation of meshing.

**Step2:** By using $S_f$ coordinate system, the condition of continuous tangency of gear tooth surfaces can be represented by the following vector equations **(see Fig. (6)):-**

$$r_f^{(1)}(u_c,\psi_1,\phi_1)=r_f^{(2)}(u_f,\psi_2,\phi_2) \quad (20)$$

$$n_f^{(1)}(u_c,\psi_1,\phi_1)=n_f^{(2)}(u_f,\psi_2,\phi_2) \quad (21)$$

**Step3:** Vector **Eq. (21)** yields only two independent equations since $\left|n_f^{(1)}\right|=\left|n_f^{(2)}\right|=1$. So, **Eqs. (20) and (21)** provide a system of five independent equations represented as

$$f_i(u_c,\psi_1,\phi_1,u_f,\psi_2,\phi_2)=0 \quad (22)$$

**Eq. (22)** contains six unknowns but the theorem of Implicit Function System Existence yields that one parameter, say $\phi_1$, may be chosen as input and obtain the solution to equation system (22) by functions

$$\{u_c(\phi_1),\psi_1(\phi_1),u_f(\phi_1),\psi_2(\phi_1),\phi_2(\phi_1)\}\in c^1 \quad (23)$$

The gear tooth surfaces are in point contact, and so that it is suppose that the Jacobian of the fifth order differs from zero. Thus,

$$\left|\frac{\partial f_i}{\partial u_c}\frac{\partial f_i}{\partial \psi_1}\frac{\partial f_i}{\partial u_f}\frac{\partial f_i}{\partial \psi_2}\frac{\partial f_i}{\partial \phi_2}\right|\neq 0 \quad (24)$$

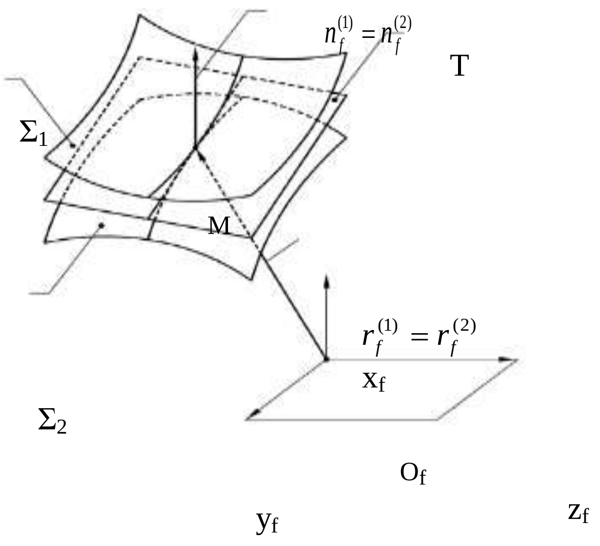


**Fig. (6):-** Tangency of contacting tooth surfaces in ideal gear.

If **Eqs. (24)** are satisfied then, using functions (23) we are able to determine:

(i) the path of contact on surfaces $\Sigma_1$ and $\Sigma_2$ represented as

$$r_i(u_t,\psi_i) \quad , u_t(\phi_1) \quad , \psi_i(\phi_1) \tag{25}$$

where (i =1, 2 and t =c, f) for pinion and gear, respectively.

(ii) the transmission errors determined as

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2}\phi_1 \tag{26}$$

**CONCLUSIONS:**

1. Generation procedure based on equations of tooth surfaces has been conducted.
2. Algorithms for simulation of meshing indicates that such gear drives are sensitive to misalignment.
3. Simulation of meshing permits one to determine the shift of the bearing contact and transmission errors.

**NOMENCLATURES:**

$a_c$ Parabolic coefficient of profile of pinion rack cutter.

$C^1$ Denote that the functions have continuous derivatives to the first order, at least.

E' Center distance between gear and pinion (m).

$f$ Equation of meshing between tooth surface and rack-cutter surface.

$l_j$ ,$u_j$ Parameters of surface (j = c, f) (m).

$m_n$ Module in normal plane (m).

$N_i$ Number of teeth on pinion (i=1) or for gear (i = 2).

$n_j$ , $n_f^j$ , $N_j$ unit normal and normal to surface j (j = c, f).

$P_n$ Normal diametral pitch (1/m).

$r_i$ Position vector of a point in coordinate system $S_i$ (mm).

$R_{pi}$ Radius of pitch circle of pinion (i = 1) or for gear (i = 2) (m).

$S_i$ Coordinate system (i = $c_p$, b, c, n, 1, 2, $c_f$, p, f).

($x_i$, $y_i$, $z_i$) Coordinates (i =$c_p$, c, 1, 2, $c_f$, f, p) (mm).

$\alpha_n$ Normal pressure angle (deg).

$\beta$ Helix angle (deg).

$\Delta\gamma$ Error of shaft angle (deg).

$\Delta\phi_2$ Function of transmission errors (deg).

$\in$ Denotes that the element belongs to the set.

$\Psi_i$ Angle of rotation of pinion (i = 1) or the gear (i = 2) in the process of generation (deg).

$\Sigma_i$ Surfaces (i = c, f, 1 , 2) .

$\phi_i$ Function of transmission errors (deg) (i = 1, 2).

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