MOIDIEFIED VERSION OF ADJUSTED STEP SIZE LMS ALGORITHIM (MASSLMS) FOR ADAPTIVE LINEAR FIR EQUALIZER

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ABSTRACT

In this paper a Modified version of Adjusted Step Size Least Mean Square algorithm (MASSLMS) is proposed which overcome and avoid one of the drawback of the standard LMS and our previous proposed algorithm Adjusted Step Size Least Mean Square algorithm (ASSLMS). This drawback is the requirement of a statistical knowledge of the input signal prior to the starting training of the algorithm which is necessary to determine the fixed value of the maximum step size (i.e. the upper bound value) in the initialization stage of the ASSLMS algorithm. In this proposed algorithm an appropriate time varying value of the maximum step size was calculated based on inversely proportional of the instantaneous energy of the input signal vector. Then this time varying upper bound value of the step size is used to guarantee the stability of adjusted step size of the algorithm which is a recursively adjusted based on rough estimate of the performance surface gradient square . The proposed algorithm does not need trial and error for choosing the value of the maximum step size (μ_{MAX}) compared with ASSLMS and standard LMS algorithms. The proposed algorithm shows through computer simulation results faster and low level of miss-adjustment in the steady state compared with LMS and ASSLMS for three different types of channel in adaptive linear equalizer system.

KEYWORS: Linear Adaptive Equalizer, LMS Adaptive algorithm, Variable Step Size LMS algorithm.

نسخة معدلة من خوارزمية اقل معدل للتربيع ذات معامل الخطوة المتغيرة زمنيا لمنظومة المكافىء الخطية

الخلاصة

هذا البحث يركز على اقتراح نسخة معدلة من خوارزمية اقل معدل للتربيع ذات معامل الخطوة المتغيرة زمنيا لمنظومة المكافىء الخطية وسميت الخوارزمية المقترحة الجديدة باسم (MASSLMS) وهي نسخة مطورة من اصل الخوارزمية التي سبق ان تم اقتراحها مسبقا من قبل الباحث نفسه وسميت في حينها باسم (ASSLMS) . والهدف الاساسي من هذه الخوارزمية الجديدة هو حل مشكلة اختيار اعظم قيمة لمعامل الخطوة المتغيرة زمنيا (μ_{MAX}) حيث كان سابقا يتم اختيار قيمة ثابتة لها وعن طريق التجربة والخطاء . أما الان في هذه الخوارزمية المقترحة الجديدة فانه يتم تغيريها زمنيا وحسابها لكل عينة عن طريق حساب معكوس القدرة الكهربائية للاشارة الداخلة للمرشح المتكيف . وبعد ذلك يتم استخدام هذه القيمة المتغيرة زمنيا للـ(μ_{MAX}) في تنفيذ بقية الخطوات اللازمة للخوارزمية . تعتبر عملية حساب الـ(μ_{MAX}) بهذه الطريقة مناسبة ومفضلة لانها ستتمكن من تعقب اي تغيير قد يحصل بالاشارة الداخلة مقارنة بالطريقة السابقة . اثبتت الخوارزمية المقترحة الجديدة من خلال برنامج المحكاة كفاءة بالاداء

وخصوصا السرعة في التعلم افضل من خوارزمية اقل معدل للتربيع التقليدية المسماة بـــ (LMS) وكذلك الخوارزمية المساة بـــ (ASSLMS) لمنظومة المكافىء الخطية وباستخدام ثلاثة انواع مختلفة من قنوات الاتصال .

INTRODUCTION

Adaptive equalizer was widely used in digital communication systems in order to reduce or eliminate the channel distortions or intersymbol interference ISI before demodulation at the receiver. The simple structure for adaptive equalizer was the Finite Impulse Response filter (FIR) which can be trained by the Least Mean Square adaptive algorithm (LMS). This LMS algorithm, which was first proposed by Widrow and Hoff at Stanford University, Stanford, CA in 1960 [B.Farhang Boranjrncy, 1999]. This LMS algorithm is regarded as special case of the Gradient Search algorithm and is regarded as one of the most popular algorithms in adaptive signal processing due to the simplicity in the number of calculations required for its update. Furthermore, it does not require matrix inversion, nor does it require measurements of the pertinent correlation functions [B.Farhang Boranjrncy, 1999]. But this algorithm suffers from slow convergence adaptation process since the convergence time of LMS algorithm is inversely proportional to the step size [B. Widrow and S. Stearns, 1985]. Also it suffers from trade off between low level of miss-adjustment and fast convergence i.e. If large step size is selected, then fast convergence will be obtained but this selection results in deterioration of the steady state performance (i.e. increased the miss-adjustment (excess error). Also small value of the step size will cause slow convergence but will enhance or decrease the steady state error level [B. Widrow and S. Stearns, 1985].

Therefore, a lot of modifications of the LMS algorithm have been reported. One technique of these modifications is using time varying step size i.e. the step size will be adjusted in each iteration according to the specific rules. Several time varying step size LMS algorithm were reported [,R.W.Harris, D.M. Chadries, 1986 , Long Le, Ozgu Ozun, and Phiipp Steurer, 2002, Charles Q. Hoang, 2000, J.J. Chen, R.R. Priemer, Feb.1995, Bozo K. ,Zdravko U. , and Ljubisa S., April 2003, S.K., G. Zeng. July 1989, R.H.Kang, E.W.Johnstone, July 1992, R.W. Wies, A. Balasubramanian, J. W. Pierre, 2006 and Yonggang Zhang, Ning Li, Jonathon A. Chambers, and Yanling Hao, 2008]. In this paper time varying step size is chosen due to its powerful effect on the performance of the system also the structure of the adaptive equalizer will not be changed and this technique require less overhead in computations which is an important factor for hardware implementation. The proposed algorithm in this paper is called MASSLMS algorithm (Modified Adjusted Step Size LMS) which is regard as modified version of previous ASSLME algorithm [Thamer M.Jamel, 2007]. This new proposed algorithm shows good performance and also gets rid of the main drawback of the previous algorithm which is the trial and error in selection of the maximum value of the step size (μ_{MAX}). The value of the maximum of the step size in this paper is adjusted according to the input power of the signal instead of the fixed value. This step size is proportional to the inverse of the total expected energy of the instantaneous values of the coefficients of the input vector.

ADAPTIVE LINEAR EQUALIZER WITH LMS ALGORITHM

Linear Equalizer LE is one type of adaptive equalization techniques which use only received signal symbols in their calculations and do not use any previously detected symbols. Fig.1 shows the classical model of the LE .As shown in this figure there are two modes of operations, namely, the training mode and decision-directed mode [Simon Haykin , 1983]. During the training mode, the transmitter generates a data symbol sequence known to the receiver.

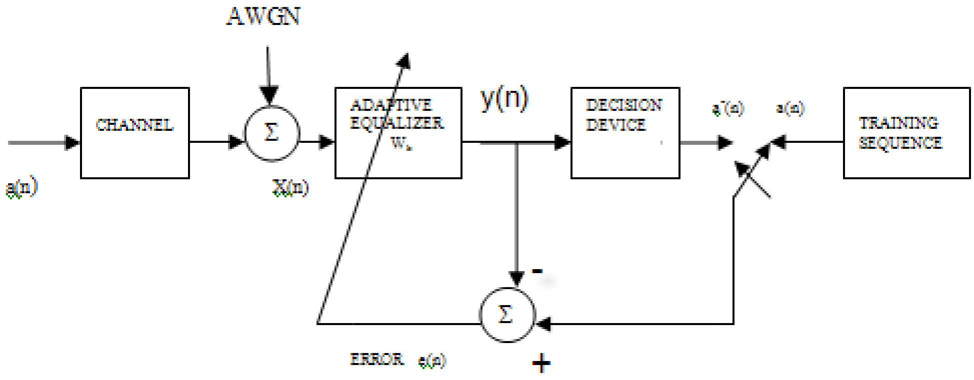


Fig. 1 Classical Model of LE

The receiver therefore, substitutes this known training signal in place of the decision device output. Once an agreed time has elapsed, the decision device output is substituted and the actual data transmission begins. When the training process is completed, the adaptive equalizer is switched to its second mode of operation: the decision-directed mode. In this mode of operation, the error signal is defined by [R.W. Wies, A. Balasubramanian, J. W. Pierre, 2006]:

$$e(n) = a^{\wedge}(n) - y(n) \qquad (1)$$

Where y (n) is the equalizer output and $a^{\wedge}(n)$ is the final correct estimate of the transmitted symbol $a(n)$. The linear transversal equalizer(i.e. FIR) Fig.2 is the simplest equalization techniqe available. It is made up of tapped-delay line with tap spacing equal to the symbol time. The equalizer input consists of sampled output of the matched filter that preceds the equalizer. These samples are placed in shift register and shifited once every sample period. The contents of each register is multiplied by a tap gain and added togother to form the output of the equalizer. This output is the estimate of the current symbol, this operation can be described by the following equation [John M. Morton, 1998].

$$\hat{d}_k = \sum_{k=-N_1}^{N_2-1} w_k y_{n-k} \qquad (2)$$

In this equation, y_n is the input sequance to the equalizer, w_k is the set of tunable complex multipliers called tap weights, N_1 is the number of the non-caus equalizer taps, N_2 is the number of causal taps, the total nunmer of equalizer taps is therfore $N_1+N_2=N$. The Ts blocks indicate a delay of one symbol period

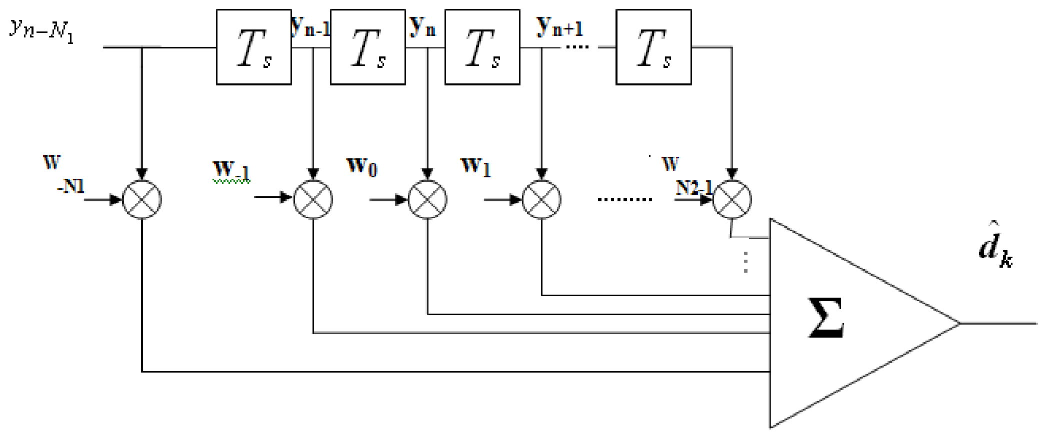


Fig. 2 The linear transversal equalizer structure of the LTE

Under the mean square error criterion, the tap weights of the equalizer are adjusted to minimize the mean-square error between the original data symbol and the output of the equalizer. This error includes both ISI as well as the additive noise. It follows that when the desired equalizer output is known (i.e., $d_k = x_k$) the error signal e_k is given by [Rappaport T.S. 2002, B. Widrow and S. Stearns, 1985]

$$e_k = d_k - \hat{d}_k = x_k - \hat{d}_k \tag{3}$$

The squared error is defined as [R.W.Harris, D.M. Chadries, 1986]

$$|e_k|^2 = |d_k - \hat{d}_k|^2 = |x_k - \hat{d}_k|^2 \tag{4}$$

To compute the mean square error $|e_k|^2$ at time instant k, from eq. (3) the following obtained

$$|e_k|^2 = x_k^2 + w_k^T y_k y_k^T w_k - 2 x_k y_k^T w_k \tag{5}$$

Taking the expected value of $|e_k|^2$ over *k* (which in practice amounts to computing a mean squared) yields [B. Widrow and S. Stearns, 1985]

$$E\left[|e_k|^2\right] = E\left[x_k^2\right] + w_k^T E\left[y_k y_k^T\right] w_k - 2E\left[x_k y_k^T\right] w_k \tag{6}$$

Where *E* is the expectation operator. To find the set of equalizer coefficients those minimize the mean squared error for this linear equalizer. The following sets of computations are made. Let *R* be defined as the (N+1)×(N+1) square matrix

$$R = E\left[y_k y_k^T\right] = E\begin{bmatrix} y_k^2 & y_k y_{k-1} & y_k y_{k-2} & \cdots & y_k y_{k-N} \\ y_{k-1} y_k & y_{k-1}^2 & y_{k-1} y_{k-2} & \cdots & y_{k-1} y_{k-N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ y_{k-N} y_k & y_{k-N} y_{K-1} & y_{k-N} y_{k-2} & \cdots & y_{k-N}^2 \end{bmatrix} \qquad (7)$$

Where $(.)^T$ denotes the transpose operation. This matrix is designated the "input correlation matrix." The main diagonal terms are the mean squares of the input signal, and the cross terms are the cross correlations among the input signal. Let P be similarly defined as the column vector

$$P = E[\ x_k y_k\ ] = E\ [\ x_k y_k \quad x_k y_{k-1} \quad x_k y_{k-2} \quad \ldots \quad x_k y_{k-N}\ ]^T \qquad (8)$$

This vector is the set of cross correlations between the desired response and the input signal. Using eq. (7) and eq.(8), equation (6) may be written as [R.W.Harris, D.M. Chadries, 1986]

$$MSE = \xi = E\left[x_k^2\right] + w^T R w - 2P^T w \qquad (9)$$

By minimizing eq. (9) in terms of the weight vector w_k, it becomes possible to adaptively tune the equalizer to provide a flat spectral response (minimal ISI) in the received signal. This is due to the fact that when the input signal y_k and the desired response x_k are stationary, the mean square error (MSE) is quadratic on w_k, and minimizing the MSE leads to optimal solutions for w_k.

To determine the minimum MSE (MMSE), the gradient of (9) can be used. As long as R is nonsingular (has an inverse), the MMSE occurs when w_k are such that the gradient is zero. The gradient of ξ is defined as [Rappaport T.S. 2002, B. Widrow and S. Stearns, 1985]

$$\nabla \cong \frac{\partial \xi}{\partial w} \cong \left[\frac{\partial \xi}{\partial w_0} \frac{\partial \xi}{\partial w_1} \cdots \frac{\partial \xi}{\partial w_L}\right]^T \qquad (10)$$

Where L is number of weight coefficients. By expanding (9) and differentiating with respect to each signal in the weight vector, it can be shown that eq.(10) yields [Rappaport T.S. 2002, B. Widrow and S. Stearns, 1985]

$$\nabla = 2RW - 2P \qquad (11)$$

Setting $\nabla = 0$ in eq. (11), the optimum weight vector w_{opt} for MMSE is given by [Rappaport T.S. 2002, B. Widrow and S. Stearns, 1985]

$$w_{opt} = R^{-1} P \qquad (12)$$

Using equation (12) to substitute w_{opt} for w in eq. (9) $\xi_{\min}$ is found to be [Rappaport T.S. 2002, B. Widrow and S. Stearns, 1985]

$$\xi_{\min} = MMSE = \mathrm{E}[\ x_k^2\ ] - P^T R^{-1} P = \mathrm{E}[\ x_k^2\ ] - P^T w_{opt} \qquad (13)$$

Eq. (13) solves the MMSE for optimal tap weights w_{opt}.

The LMS algorithm is an iterative procedure that continuously updates a vector of equalizer coefficients. It updates these coefficients based on the mean-square error cost function given in eq. (9). This cost function is dependant on the output of the equalizer which is dependant on the tap coefficients. Each vector of equalizer coefficients will have a certain mean square error associated with it. One such vector will produce the minimum mean-square error. The LMS algorithm attempts to find the desired vector [B. Widrow and S. Stearns, 1985, John M. Morton, 1998]. The change in weights vector is represented as [B. Widrow and S. Stearns, 1985]:-

$$W_{k+1} = W_k + \mu(-\nabla_k) \qquad (14)$$

Where μ is constant called the step size that regulates the stability and convergence time of the adaptive process. To develop the LMS algorithm, e_k^2 itself is taken as an estimate of ξ_k. Then, at each iteration in the adaptive process, a gradient estimate of the following form has been obtained [B. Widrow and S. Stearns, 1985],

$$\hat{\nabla}_k = \begin{bmatrix} \dfrac{\partial e_k^2}{\partial w_0} \\ \vdots \\ \dfrac{\partial e_k^2}{\partial w_L} \end{bmatrix} = 2e_k \begin{bmatrix} \dfrac{\partial e_k}{\partial w_0} \\ \vdots \\ \dfrac{\partial e_k}{\partial w_L} \end{bmatrix} = -2e_k y_k \qquad (15)$$

Put eq. (15) into eq. (14) then the updating weights vector became [B. Widrow and S. Stearns, 1985]:-

$$W_{k+1} = W_k - \mu\ \hat{\nabla}_k$$
$$W_{k+1} = W_k + 2\mu e_k y_k \qquad (16)$$

This is the LMS algorithm and it is also known as the "stochastic gradient algorithm", and μ is the step size that regulates the speed and stability of adaptation. Since the weight changes at each iteration are based on imperfect gradient estimates, one would expect the adaptive process to be noisy, also the iterative procedure start with initial guess which may be a null vector [Qureshi S.U. 1985, B. Widrow and S. Stearns, 1985]. If the step size is made too large, the algorithm can become unstable and will not converge to the optimal tap vector. The main drawback of the LMS algorithm is the slow convergence rate. To overcome this limit, a modified version of the LMS algorithm is presented which used time varying step size instead of the fixed step size as shown in the next section.

MODIFIED ADJUSTED STEP SIZE LMS (MASSLMS) ALGORITHM:-

As explained previously this paper propose algorithm which is called Modified Adjusted Step Size LMS (MASSLMS) algorithm. MASSLMS regards as modified version of the ASSLMS algorithm

[Thamer M.Jamel, 2007]. ASSLMS algorithm used variable step size that will be adjusted according to the square of the gradient of the performance surface (i.e. $e_k y_k)^2$ as follows:-

$$\mu_{k+1} = \alpha\mu_k + \delta(e_k . y_k)^2 \qquad (17)$$

Where $0\langle\alpha\langle 1$ and $\delta\rangle 0$, then:-

$$\mu_{k+1} = \mu_{MAX} \quad \text{if} \quad \mu_{k+1}\rangle\mu_{MAX} \quad , \text{ or } \quad \mu_{k+1} = \mu_{\min} \quad \text{if} \quad \mu_{k+1}\langle\mu_{\min}, \qquad (18)$$
$$\text{Otherwise} \quad \mu_{k+1} = \mu_{k+1}$$

Eq. (17) is a formula to adjust the step size in each iteration and it is modified from the original equation of in [R.H.Kang, E.W.Johnstone, July 1992]. In this equation the step size will be adjusted according to the square of the gradient of the performance surface (i.e. $e_k y_k)^2$ as shown in eq. (17). To ensure stability, the variable step size μ(*n*) is constrained to the pre-determined maximum and minimum step size values while α and δ are the parameters controlling the recursion . 0< α<1, and δ>0 , and μ(n+1) is set to μmin or μmax when it falls below or above these lower and upper bounds, respectively. The constant μmax is normally selected near the point of instability of the conventional LMS to provide the maximum possible convergence speed. The value of μmin is chosen as a compromise between the desired level of steady state misadjustment and the required tracking capabilities of the algorithm. The parameter δ controls the convergence time as well as the level of misadjustment of the algorithm at steady state. However there is no any formula or equation to calculate α and δ in all papers including the original paper [R.W. Wies, A. Balasubramanian, J. W. Pierre, 2006] but usually they assigned high value for α which is very close to 1 (i.e. 0.97-to-0.99) and very small value for δ.
Then the update eq. (16) for the weight vector will be:-

$$w_{k+1} = w_k + 2\mu_k e_k y_k \qquad (19)$$

Where $\mu_{\min}$ is chosen to provide minimum level of miss-adjustment at steady state, and μ_{MAX} ensures the stability of this algorithm []R.H.Kang, E.W.Johnstone, July 1992]. This proposed algorithm (ASSLMS) algorithm regard as modified version of the VSSLMS algorithm [Thamer M.Jamel, 2007]. Involving the term (y_k) which represents the input signal in the updating step size formula in addition to error factor is favorite choice in order to speed up the estimation and adaptation process. The main drawback of the ASSLMS algorithm is how to select the value of the upper bound of step size i.e. μ_{MAX} . In other words this drawback is the requirement of a statistical knowledge of the input signal prior to the starting training of the algorithm which is necessary to determine the fixed value of the maximum step size μ_{MAX} (i.e. the upper bound value) in the initialization stage of the ASSLMS algorithm.
In this proposed algorithm an appropriate time varying value of the maximum step size is calculated based upon inversely proportional of the instantaneous energy of the input signal vector .

$$\mu_{MAX} = \frac{1}{2y_k^T y_k} \qquad (20)$$

This sum of the expected energies of the input samples is also equivalent to the dot product of the input vector with itself. Then this time varying upper bound value of the step size is used to

guarantee the stability of adjusted step size of the algorithm eq. (17) which is a recursively adjusted based on rough estimate of the performance surface gradient square.
Eq. (20) is used in Normalized LMS (NLMS) algorithm which is an extension of the LMS algorithm that overcomes the drawback of the LMS algorithm by selecting a different step size value, $\mu(k)$, for each samples of the input signal. [Scott C. Douglas, march 1994]. Eq. (20) is implemented as follows:-

$$\mu_{MAX} = \frac{\beta}{2y_k^T y_k + \Psi} \qquad (21)$$

Where the value of ψ is a small positive constant in order to avoid division by zero when the values of the input vector are zero and β is within the range of $0<\beta>2$, usually it is equal to 1. In the MASSLMS algorithm the upper bound available to each element of the step size vector, μ_{MAX}, is calculated for each iteration.

SIMULATION RESULTS

Case 1:- In this case LE was simulated with different algorithms. The channel used here is called channel 1 which is raised cosine function. The order of FIR adaptive filter for all simulation was 11 taps and signal to noise ratio was 26 dB, the additive noise was Gaussian noise with zero mean, and variance $\sigma^2 = 0.001$. The training samples were 1000 samples then the adaptive process is switched to decision mode. Fig.3 shows the learning curves for this case with different algorithms. The optimum step size for LMS algorithm was chosen by trial and error to be 0.03. The optimum values (by trial and error) of μ_{max} and μ_{min} was chosen to be 0.05 and 0.0001 respectively for ASSLMS algorithm. The values of α and δ was chosen to be 0.97 and 0.001 respectively for all algorithms. The β is equal 1 and ψ is equal 0.1 for MASSLMS algorithm.
As shown in Fig. 3 the proposed algorithm has fast convergence time than LMS and ASSLMS algorithms. The convergence time from Fig.3 is equal to 1000, 600 and 500 iterations for LMS, ASSLMS and MASSLMA algorithms respectively. Also the proposed algorithm has smooth descending towards the minimum point compared with the LMS and ASSLMS algorithms. This is because the upper bound of the step size is time varying value which can track any change in the input signal as shown in eq. (21).

Case 2:- The channel used here is called channel 2 which has frequency response with two spectral null in the middle region. The impulse response of this channel is (h = [0.2, -0.15, 1.0, 0.21, 0.03]) and is shown in Fig.4.
The same parameters of the case 1 are used in this case except that the optimum value of the upper bound of the μ_{MAX} of the ASSLMS algorithm was found by trial and error to be equal 0.03. Fig.5 shows the learning curves for different algorithms for this 2[nd] channel. As shown in figure (5) the proposed algorithm has fast convergence time than both LMS and ASSLMS algorithms. The convergence time from Fig.5 is equal to 250, 200 and 100 iterations for LMS, ASSLMS and MASSLMA algorithms respectively. Notice that the parameters of the MASSLMS algorithm are kept the same without any need to be changed by trial and error and this fact is also present in the next case i.e. case 3.

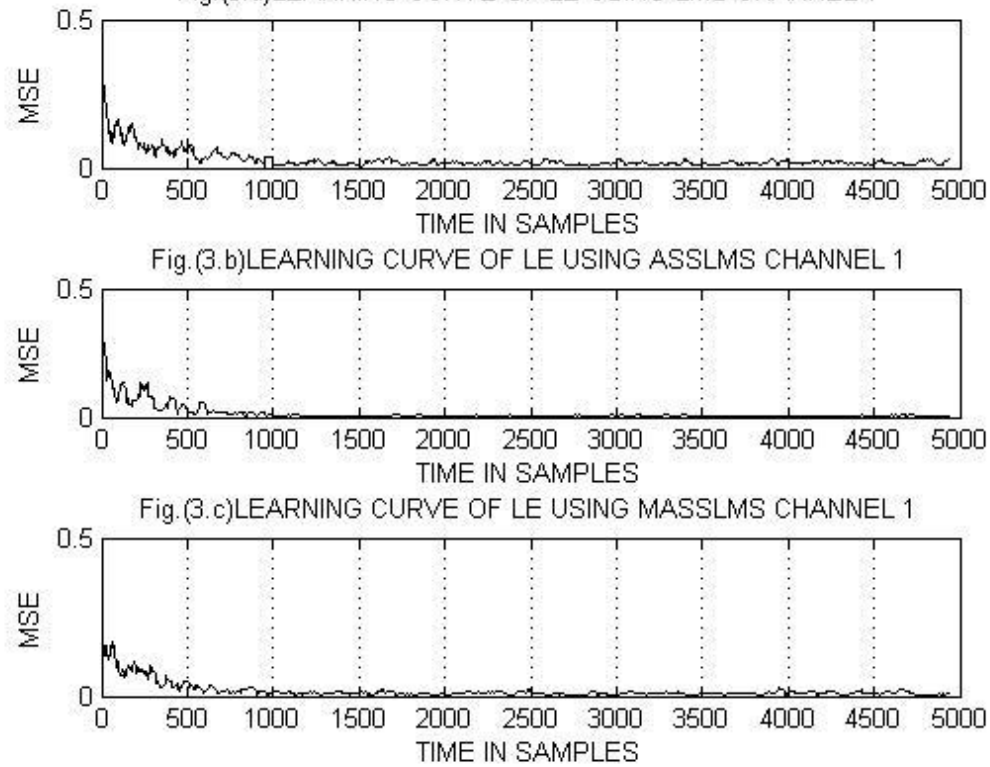


Fig.(3.a)LEARNING CURVE OF LE USING LMS CHANNEL 1

Fig.(3.b)LEARNING CURVE OF LE USING ASSLMS CHANNEL 1

Fig.(3.c)LEARNING CURVE OF LE USING MASSLMS CHANNEL 1

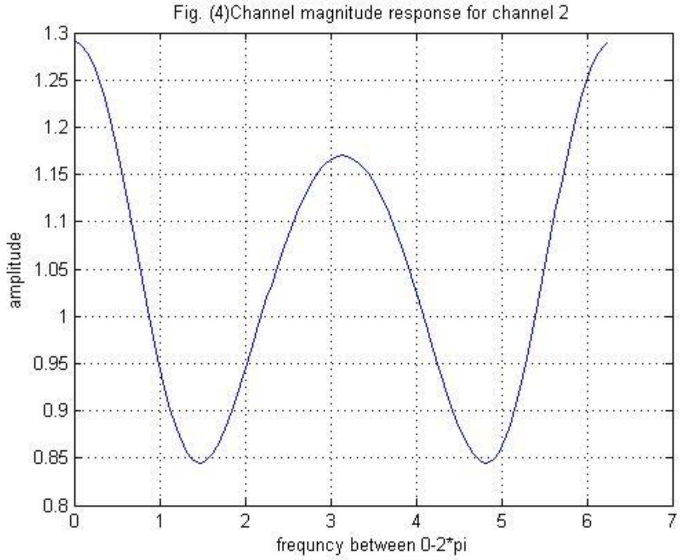


Fig. (4)Channel magnitude response for channel 2

Case 3:- The channel used here is called channel 3 which has the following impulse response h=[0.01,0.08,-0.126,-0.25,0.7047,0.25,-0.02,0.016,0.0]; and shown in Fig.6. Fig.7 shows the learning curves of different algorithms using the same parameters as in case 2 above for all algorithms.

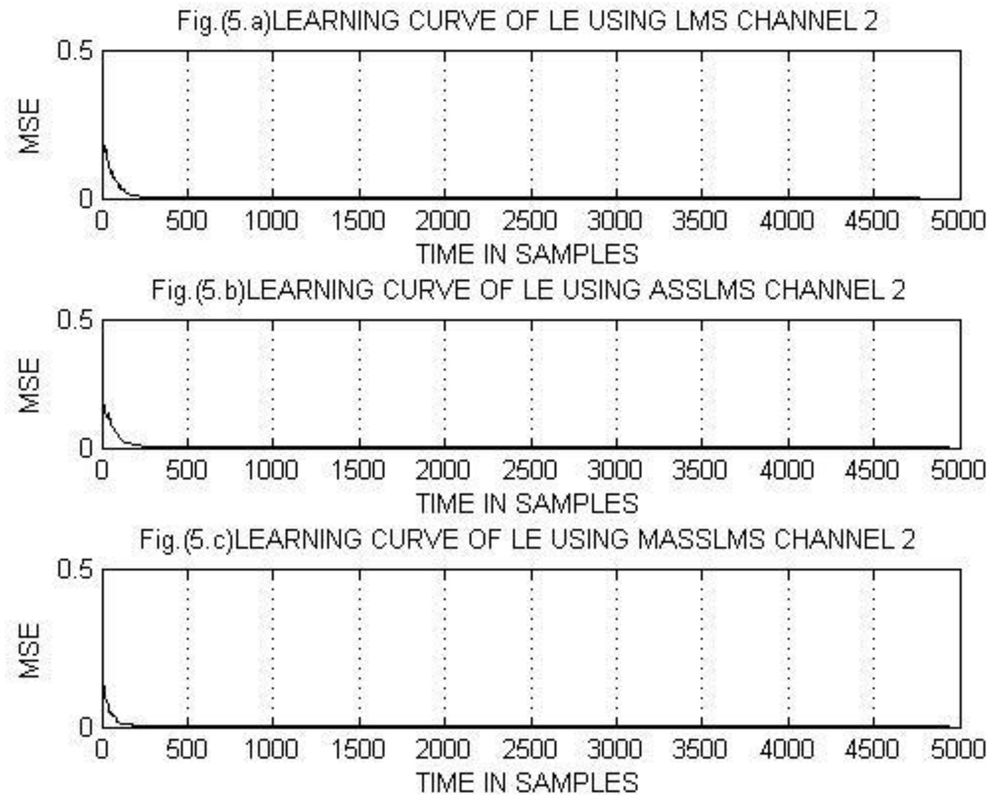


Fig.(5.a)LEARNING CURVE OF LE USING LMS CHANNEL 2

Fig.(5.b)LEARNING CURVE OF LE USING ASSLMS CHANNEL 2

Fig.(5.c)LEARNING CURVE OF LE USING MASSLMS CHANNEL 2

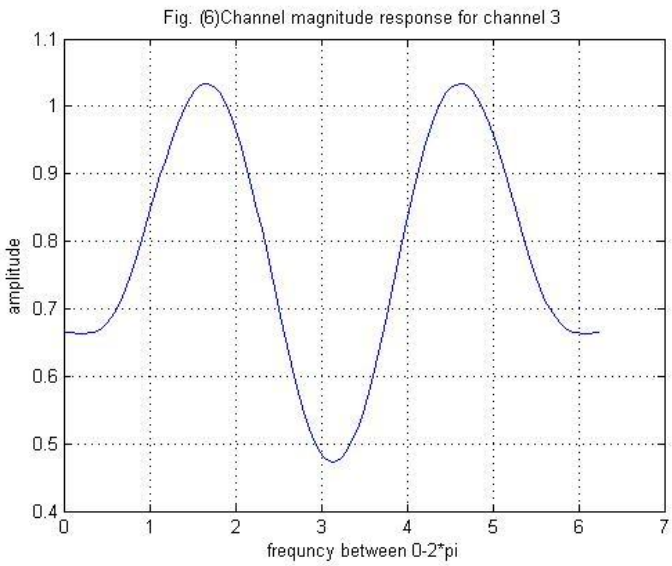
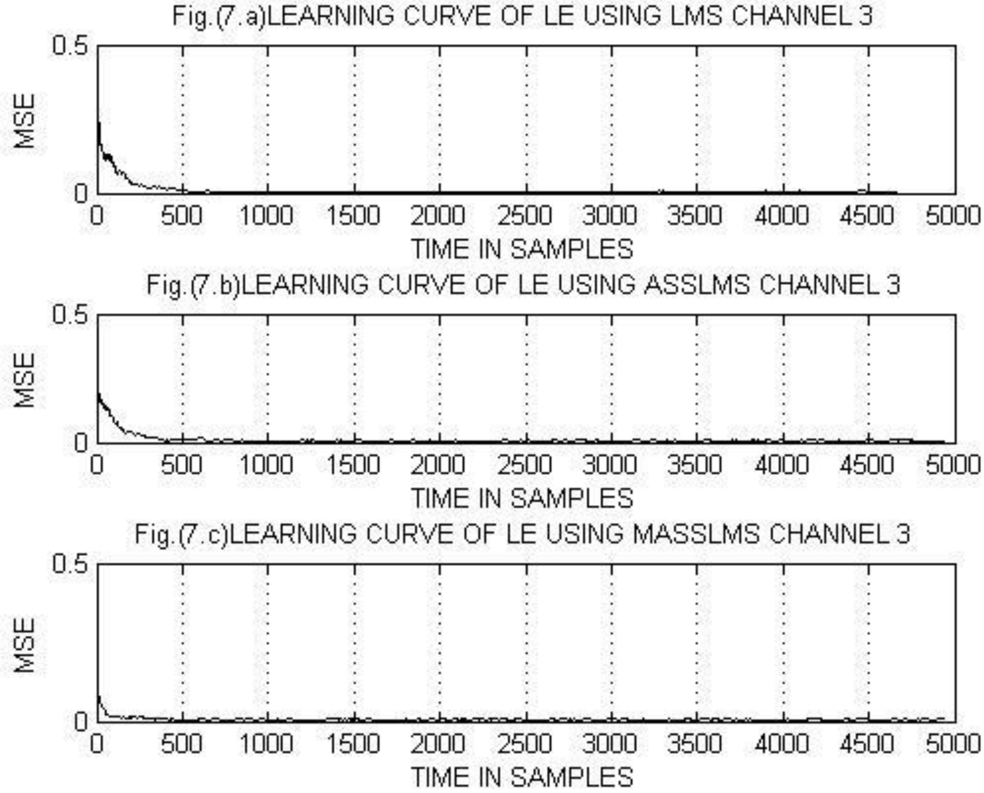


Fig. (6)Channel magnitude response for channel 3

As shown in Fig.7 the proposed algorithm has fast convergence time than LMS and ASSLMS algorithm. The convergence time from Fig.7 is equal to 500, 400 and 200 iterations for LMS, ASSLMS and MASSLMA algorithms respectively. Also as seen in Fig.7.b , the learning curve of the ASSLMS algorithm has the same performance compared with the LMS algorithm due to that , the same parameters of the ASSLMS algorithm are used as in case 2 . So in order to enhance the performance of the ASSLMS algorithm the parameters of this algorithm must be optimized by trial and error which in turns represents the main draw back point of the ASSLMS algorithm. This draw back is overcome with the proposed algorithm (i.e. MASSLMS) which does not need any optimizations of its parameters.



CONCLUSIONS

This paper focused on enhance the performance of our previous proposed algorithm (ASSLMS) which suffer from choosing the suitable value of the upper bound of the step size μ_{MAX}. The upper bound of the step size μ_{MAX} needs a statistical knowledge of the input signal prior to the starting training of the algorithm which is necessary to determine the fixed value of the maximum step size (i.e. the upper bound value) in the initialization stage of the ASSLMS algorithm. The proposed algorithm called Modified Adjusted Step Size LMS (MASSLMS) which used an appropriate time varying value of the maximum step size μ_{MAX} that is calculated based upon inversely proportional of the instantaneous energy of the input signal vector . This method is favorite choice because the time varying μ_{MAX} will track any chang in the input signal power. Then this time varying μ_{MAX} is used to guarantee the stability of adjusted step size of the algorithm which is a recursively adjusted based on rough estimate of the performance surface gradient square (i.e. $e_k y_k)^2$.

The proposed algorithm MASSLMS shows fast convergence time through the simulation of the adaptive linear equalizer using three different channels compared with the LMS and ASSLMS algorithms in spite of using the same parameters for all different cases.

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