

# DIFFERENCE BETWEEN OF PWM STRATEGIES FOR INVERTER FED INDUCTION MOTOR

محمد حسين معهد اعداد المدربين التقنيين

## ABSTRACT

Several sophisticated or "optimum" modulation strategies have been suggested for voltage source pulse with modulated (PWM) inverters for ac motor control. These modulation strategies may suppress specific low – order harmonics or minimize total harmonic.

The effectiveness of these (PWM) techniques in minimizing harmonic and reducing torque pulsation is investigated analytically, and their performance is compared with that of the usual sinusoidal or sub-harmonic PWM approach. The influence of skin effect on motor (I<sup>2</sup>R copper losses) is taken into consideration, and harmonic core losses are compared. Peak current is also an important factor in inverter design, and the various modulation strategies are again compared on this basis. Fourier analysis techniques are used in order to allow skin effect phenomena to be taken into consideration and performance criteria are developed to allow comparisons of waveform quality with respect of harmonic copper and iron losses.

#### الخلاصة

في هذا البحث تم اقتراح اساليب تضمين لمصدر فولتية تضمين عرض النبضة (PWM) باستخدام المبدل للسيطرة على المحرك (A.C) .ان طرق التضمين المستخدمة تعمل على تشتيت التوافقيات او تقليلها الى اقل مايمكن . ان فعالية تقنية عرض النبضة (PWM) هو تقليل التوافقيات وعزم النبضات ويتحقق هذا من خلال التحليل والعمل بمقارنة الموجة الجيبية الاعتيادية والتوافقيات لعرض النبضة التريدية.

هنالك تاثيرات للقشرة الجلدية (Skin effect) في المحركات من خلال (I<sup>2</sup>R) والتي تمثل المفاقيد النحاسية وبهذا الاعتبار يتم مقارنتها مع المفاقيد الحديدية .

ان القيمة العظمى للتيار تاخذ بالاهمية من خلال عامل تصميم الميدل (Inverter) وإن اختلاف علم التضمين وبالمقارنة بالتحليل باستخدام (Basis Fourier) تم ادراك تاثيرات القشرة الجلدية (Skin effect) التي تؤخذ بنظر الاعتبارعند المقارنة وتؤخذاعتبارات العمل لتطوير جودة الموجة على اعتبار المفاقيد النحاسية والحديدية.

#### **INTRODUCTION**

Voltage source pulses with modulated (PWM) inverters for a.c motor control have conventionally employed square wave or sinusoidal PWM strategies. In recent years, more sophisticated techniques have been suggested in which specific low-order harmonics are suppressed or total harmonic content is minimized [1]-[2]-[3]

These optimized PWM strategies are extremely difficult to realize with conventional analog circuitry, but they can be effectively implemented with modern microprocessor – based control techniques [4]-[5]. Conventional modulation strategies which have been implemented by means of complex analog circuits may now be more effectively realized using a look – up table accessed by a microprocessor or digital hardware. In an ac motor drive, the modulation strategy which is most appropriate to a particular portion of the speed range is readily selected. It is therefore, of interest to compare the different modulation techniques available with regard to the additional harmonic losses in the motor and developed pulsating torque.

For the purposes of comparison, it is assumed that standard 50 or 60-Hz induction motor is fed from a standard ac supply network by a frequency converter circuit as shown in fig. (1).



### Fig. (1) Variable frequency induction motor drive

The ac supply is rectified to a fixed dc voltage by the converter and converted to variable – frequency ac by the PWM inverter which also controls the amplitude of the fundamental output voltage. If the inverter operates on un-modulated six-step voltage waveform, motor operation at rated voltage and frequency is possible. Constant-torque operation is obtained below based frequency by modulating the output voltage waveform from the PWM inverter so that the fundamental component of the output voltage is reduced proportionally with frequency, giving the usual constant volts/ hertz mode of operation.

In controlling the fundamental voltage output, the PWM strategy may introduce additional harmonic components, the presence of which detrimental to motor performance and efficiency.

A correct choice of modulation strategy is necessary for optimum drive performance.

#### **PWM Strategies**

The basic three-phase bridge inverter configuration develops an output voltage waveform as in fig. (2) which shows the inverter phase voltage ( $V_b$ ) relative to the center-point of the dc supply – the amplitude of the kth harmonic voltage, assuming quarter – wave and half – wave symmetry, is[5].

$$V_{k} = \frac{2V_{b}}{K\pi} \left[ 1 - 2\sum_{i=1}^{m} (-1)^{i} \cos k \,\alpha_{i} \right] \qquad \dots \dots (1)$$

The switching angles  $\alpha_1, \alpha_2, ..., \alpha_m$  can be determined in a number of ways, [6].

Many commercial PWM inverters have employed sinusoidal or sub-harmonic PWM in which the switching instants are determined by the intersection of a high frequency triangular carrier wave with a sine wave reference signal, which has the desired fundamental output frequency.

The triangular carrier wave usually has fixed amplitude, and the ratio of sine-wave amplitude to carrier amplitude is termed the modulation index.

The ratio of the carrier and reference frequencies is termed the carrier ratio.

In a three-phase PWM inverter, it is uses to generate a three-phase set of reference voltages, each phase of which is compared with a common triangular carrier wave.



Fig. (2): Inverter phase voltage relative to center point of dc supply

It has been pointed out that harmonic elimination PWM technique can be used to determine the switching angles necessary to set the fundamental voltage at some magnitude and suppress specific harmonics. But numerical techniques are necessary to solve the non linear equations of the problem [1].

If  $\mathbf{m}$  switching occurs per quarter cycle the fundamental can be controlled and (m-1) harmonics suppressed.

An alternative approach is to define a performance index related to the undesirable effects of the voltage harmonics and to select the switching angles so that the fundamental voltage is controlled, and the performance index is minimized [2]-[3]. This can be classified as a distortion minimization PWM technique.

Overall drive efficiency is the product of inverter efficiency and motor efficiency. Inverter losses are a function of the number of commutation per second, and in order to compare drive performance with different PWM strategies, it is desirable that the number of commutation per cycle should be the same in each case. For this reason, the following (PWM) techniques were selected:

- a) Harmonic elimination PWM with fifth, seventh, eleventh, and thirteenth harmonics suppressed (control of fundamental voltage and elimination of these four harmonics require 22 commutation per phase per cycle, including the commutations at (0° and 180°).
- b) Distortion minimization PWM with five switching angles per quarter-cycle, also requiring 22 commutations per phase per cycle.
- c) Sinusoidal PWM with a carrier ratio of nine, requiring 18 commutations per phase per cycle.
- d) Sinusoidal PWM with carrier ratio of 12, requiring 24 commutations per phase per cycle. Waveform c) has fewer commutations per cycle than waveform a) and b), whereas waveform d) has too many commutations.

Exact correspondence in the number of commutation is not possible, since sinusoidal PWM must have a carrier ratio which is a multiple of three.

This ensure that identical phase outputs are obtained in a three-phase system and also eliminates the dominant harmonic which is the carrier frequency, since all harmonic multiples of three are suppressed in a three-phase three wire load.

The study of sinusoidal PWM strategies is confined to the region, where the modulation index is less than unity, and pulse dropping does not occur.

Comparison of waveforms a)-d) is performed over the constant volts/hertz range of operation below base frequency.

In practice, of bourse, each modulation strategy would have an increased number of commutations per cycle at low fundamental frequencies to minimize motor losses and torque pulsations.

As base frequency is reached, the number of commutations per cycle is reduced to minimize inverter switching losses and to allow a gradual transition to six-step operation, it is possible to draw general conclusions regarding the relative merits of the PWM strategies under consideration by confining the comparison to the particular number of switching per cycle specified above.

#### Harmonic Copper Losses

An optimum PWM technique should minimize additional harmonic losses in the motor. These losses are primarily harmonic copper losses [6], [7]. At the harmonic frequencies, stator resistance and rotor resistance are usually negligible compared with the leakage reactance of the motor. If  $\mathbf{x}$  denotes the per unit (pu) leakage reactance at base frequency, the pu kth harmonic current is given by:

$$I_{k} = \frac{V_{k}}{kf_{1} x} \qquad \dots \dots (2)$$

Where  $V_k$  is the pu kth harmonic voltage, and  $f_1$  is the pu fundamental frequency. The kth harmonic copper loss is  $I_k^2 R_k$ 

Where  $R_k$  is the resistance of the motor to the kth harmonic. The total harmonic copper losses is therefore.

$$P_{loss} = \sum_{k \neq 1} I_k^2 R_k = \frac{1}{x^2} \sum_{k \neq 1} \left( \frac{V_k}{kf_1} \right)^2 R_k \qquad \dots \dots (3)$$

If  $R_k$  can be assumed constant and unaffected by frequency, the harmonic copper losses are proportional to the quantity

$$\sigma_1 = \sum_{k \neq 1} \left( \frac{V_k}{kf_1} \right)^2$$

..... (4)

This is a loss factor which ideally has a value of zero and can be used to compare the harmonic copper losses due to different PWM techniques fig. 3 compares the four PWM waveforms over the fundamental voltage and frequency range. Evidently, the harmonic elimination PWM technique:

- a- Is superior to sinusoidal PWM above 0.6 pu voltage. Despit having fewer commutations per cycle, curve
  - (a) Shows harmonic losses of less than one-third of those for curve

(d) in the region of 0.9 pu voltage. At low fundamental voltages, however, harmonic elimination PWM has large losses.

The distortion minimization curve (b) is a composite curve consisting of a number of segments and gives the absolute minimum value of loss factor which is possible with five switching angles per quarter-cycle. Harmonic losses in the region of 2.9 pu voltage are now less than one-sixth of those for sinusoidal PWM with a carrier ratio of 12 the comparison. For six step operation,  $\sigma_1$  is a horizontal straight line at a value of 2.15x10<sup>-3</sup>. The constant value of  $\sigma_1$  over the constant volts/hertz range is explained by the fact than the six-step wave shape is retained at all frequencies, and so the relative harmonic content and harmonic losses do not vary. The results indicate that sinusoidal PWM with a carrier ratio of nine, or less, is always inferior to the six-step wave.



Fig. (3) Copper loss factor as function of per unit fundamental frequency.

## **MODIFIED LOSS FACTOR**

In practice, skin effect can have a significant influence on harmonic losses, particularly if the rotor has deep-bar construction. The slot leakage component of rotor inductance decreases with frequency, but the overall reduction in the leakage inductance of the motor is less significant than the appreciable increase in rotor resistance which occurs. Because the loss factor  $\sigma_1$  as defined in equation (4), ignores skin effect, it may not be a reliable criterion for comparing PWM waveforms.

Thus a fifth harmonic voltage component of (0.2 pu) makes the same contribution to the loss factor as a 25th harmonic of (1 pu) whereas in practice, the motor will offer a significantly higher resistance to the 25th harmonic, resulting in greater copper losses. The loss factor ( $\sigma_1$ ) is therefore unduly favorable to waveforms with pronounced high-order harmonics. Statar resistance and rotor resistance increase with frequency due to skin effect, but the additional harmonic copper losses are primarily in the rotor [6]- [7].

If  $f_{2k}$  is the rotor frequency corresponding to the kth harmonic, the rotor resistance  $R_{2k}$  taking skin effect into account, is [8].

$$R_{2k} \cong \sqrt{f_{2k}}$$

Assuming that the motor operates near its synchronous speed, then

$$f_{2k} = (k \neq 1) f_1 \cong k f_1$$

And hence

$$R_{2k} \cong (kf_1)^{1/2}$$
 ..... (5)

The harmonic rotor copper losses are given by:

$$P_{2loss} = \frac{1}{x^2} \sum_{k \neq 1} \left( \frac{V_k}{kf_1} \right)^2 R_{2k}$$
 .....(6)

Substituting for  $R_{2k}$  from (5) gives a modified loss factor

$$\sigma_2 = \sum_{k \neq 1} \frac{V_k^2}{(kf_1)^{3/2}} \qquad \dots \dots (7)$$

Fig. (4) plots  $\sigma_1$  for the previous PWM waveforms. The percentage loss reduction obtained by the use of optimum PWM techniques is slightly less than in fig. (3), but their superiority over the sinusoidal PWM strategies is again quite evident.

The distortion minimization curve is calculated for the same switching angles as used previously, although a slightly better solution is possible.

Number 3



## Fig. (4) Modified copper loss factor as function of per unit fundamental frequency HARMONIC IRON LOSSES

Harmonic iron losses are significantly in fluencies by the machine construction and magnetic materials used and are difficult to predict accurately. Theoretical and experimental investigations have. Confirmed that the increase in core loss due to the harmonic main fluxes is negligible [9]- [10]. The core loss due to space harmonic fluxes is also small, but the end-leakage and skew-leakage fluxes, which normally contribute to the stray load loss, may produce an appreciable core loss at the harmonic frequencies. If an unskewed rotor construction is employed, end-leakage losses are the dominant component and may be calculated using the equation of Alger et al. [11] which indicates that these losses are proportional to frequency times current squared, hence the stator and rotor end losses associated with the kth harmonic are nearly proportional to  $I^2_k$  (kf<sub>1</sub>) and the total harmonic end loss is give by

$$p_{endloss} \cong \sum_{K \neq 1} (I_k)^2 k f_1$$

An end loss factor can be defined for these dominant harmonic iron losses and is

$$\sigma_3 = \sum_{k \neq 1} \frac{(V_k)^2}{k f_1} \qquad \dots \dots (8)$$

Fig. (5) plots this loss factor for the four PWM techniques under consideration. The distortion minimization strategy is again seen to be the optimum despite the fact that the switching angles are chosen to minimize harmonic copper losses the total stray load (SL) losses are given more generally by

$$P_{SLLoss} \cong \sum_{k=1}^{\infty} (I_k)^x (kf_1)^y$$

Where the (x) only coefficients depend on the machine construction. It has been determined experimentally that the total stray load losses due to harmonics are obtained with responsible accuracy by putting x=2 and y=1.5 [10].

This gives a loss factor of

$$\sigma_4 = \sum_{k \neq 1} \frac{I_k^2}{(kf_1)^{0.5}} \qquad \dots \dots (9)$$

The stray load loss factor  $\sigma_4$  is plotted in fig. (6) which confirms the concitious reached in fig. (5) regarding the superiority of distortion minimization PWM, but shows a some what less significant less reduction.

 $\bigcirc$ 



Fig. (5) End loss factor as function of per unit fundamental frequencies



Fig. (6) Stray loss load factor as function of per unit fundamental frequency.

#### PEAK CURRENT

Peak current variation as function of fundamental frequency is also investigated for each of the modulation strategies. It is assumed that the inverter delivers rated fundamental current at 0.85 power factor over the full costant volts/hertz range.

The leakage reactance of the motor is 0.15 pu at base frequency. The resulting peak current ( $I_{max}$ ) is expressed in per unit with peak fundamental current as its base under these conditions, the six-step inverter has a constant peak current of (1.32 pu) as shown in fig. (7) for the PWM strategies a small value of loss factor in general also implies a low peak current value.

It is evident that above a fundamental frequency of about 0.6 pu, the optimum PWM techniques again display improved performance as compared with conventional analog modulation strategies.



Fig. (7) Peak current variation as function of per unit fundamental frequency.

### **PULSATING TORQUES**

Low-frequency pulsating torques is detrimental to low-speed rotation of a.c motor. It is characteristic of a PWM strategy that low-frequency cogging torques can be eliminated at the expense of large-amplitude high-frequency torque harmonics [9]-[10]-[11].

()

This is advantageous if the pulsating torque frequencies lie above the shaft mechanical resonances. In order to compare the low-speed capability of various PWM techniques, the dominant pulsating torques developed by each waveform is calculated [9].

Possible amplification of the harmonic torques due to rotor speed fluctuation and d.c link voltage variations is not taken into consideration [12].

It is well-known that a pulsating harmonic torque is developed by the interaction of an air gap flux harmonic with a rotor current harmonic of a different order.

Air gap flux levels at harmonic frequencies are very small, and the dominant torque fluctuations are those due to the interaction of the fundamental flux in the air gap with the harmonic rotor currents. Thus the (Kth) harmonic rotor current  $I_k$  reacts with the fundamental flux  $\phi_1$  to produce a pulsating torque component whose per unit amplitude is given by:

$$\boldsymbol{T}_{k\mp 1} = \boldsymbol{\phi}_1 \boldsymbol{I}_k \qquad \dots \dots (10)$$

The torque harmonic is of order (k+1) for negative-sequence rotor currents and of order (k-1) for positive-sequence currents Base torque corresponds to one pu fundamental rotor current at unity power factor and is therefore some what larger than the rated torque of the motor.

Subsitting from (2) into (10) for  $I_k$  gives the per unit harmonic torque amplitude as:

$$T_{k\mp 1} = \frac{\phi_1 V_k}{k f_1 X} \qquad \dots \dots (11)$$

The dominant pulsating torque component may be calculated using (11) moor operation is at 0.2 pu, fundamental frequency and voltage is assumed, as is a typical induction motor leakage reactance of 0.15 pu.

Sinusoidal PWM with a carrier ratio (p) is characterized by huge-amplitude voltage harmonics at  $(p\pm 2)$  and  $(2p\pm 1)$  times the fundamental frequency [13]. The harmonics of order (p-2) and (2p+1) have positive sequence while the harmonics of order (p+2) and (2p-1) have negative sequence.

The  $(2p \pm 1)$  harmonics both develop pulsating torques at (2p) times the fundamental frequency. There two torque component are approximately in phase so that a major hunting torque component of order (2p) is present.

Fourier analysis shows that the amplitude of the  $(2\rho\pm1)$  the harmonics are 0.18 pu and are independent of harmonic order ( $\rho$ ) for  $\rho$ >p. For a carrier ratio of (12), assuming that the fundamental flux is close to 1 pu, the dominant pulsating torque is therefore of order (24) and has an amplitude of 0.513 pu. For a carrier ratio of the amplitude is 0.684 p.u and the harmonic order is (18). The harmonics of order ( $\rho\pm2$ ) cause lower order harmonic torques. In the case of  $\rho=12$  there are additional ninth and fifteenth harmonic torques of amplitude 0.065 pu and 0.067 pu respectively. For  $\rho=9$  there are sixth and twelfth harmonic torques with amplitudes of 0.094 pu and 0,060 pu respectively [13].

Harmonic elimination of PWM seeks to suppress the specific lower order torque harmonics which cause speed fluctuation is reduced. Elimination of the fifth, seventh, eleventh and thirteenth harmonic voltages removes the sixth and twelfth harmonic pulsating torques, but higher order hunting torques may be significant. For 0.2 pu fundamental voltage harmonic analysis shows that the seventeenth and nineteenth harmonic voltages have amplitudes of 0.157 pu and 0.218 pu respectively [13].

Each of the resulting current harmonic reacts with the fundamental air gap flux to produce an eighteenth harmonic pulsating torque. The two torque components are additive, giving resulting torque amplitude of 0.69pu which is approximately the same as that for sinusoidal PWM with p=9 lower order torque are absent so that low-speed capability may be some what improved as compared with sinusoidal PWM distortion minimization PWM can not be seriously considered for low-frequency operation. The overall harmonic distortion is minimized, but no specific attention is paid to the lower order harmonic so that large low-frequency hunting torques is developed.

## CONCLUSIONS

- Based on results the following aspects are concluded:
- General loss factors have been developed which permit a rapid comparison of PWM waveform quality with respecte to harmonic motor losses. Loss factors are derived from harmonic copper loss (with and without skin effect), harmonic end-leakage loss, and total harmonic stray load loss.
- Loss factor  $(\sigma_r)$  has been shown to be a general measure of waveform "badness" with respect to all types of harmonic motor loss. Despite the fact that it is based only on harmonic copper loss and ignores skin effect. A large value of  $\sigma_1$  is also an indicator of high peak current. Using this loss factor, an appropriate choice of

()

modulation strategy can be quickly made for each portion of the constant volts/hertz range of operation without performing detailed loss calculations for a particular machine. Conversely, if minimization of loss factor  $\sigma_1$  adopted as a criterion for the derivation of an optimum PWM waveform, the resulting solution gives near optimum results for all harmonic motor losses and also for peak current amplitude.

- For low-speed operation with a high switching frequency sinusoidal PWM is perfectly satisfactory.
- At these low frequencies computation of the numerous switching angles for the optimum PWM techniques is very tedious, and subsequent implementation does not yield a significant improvement in efficiency.
- As motor speed increases, the number of switching angles per cycle must be reduced to avoid an excessive number of commutation per second and allow a gradual transition to six-step operation at about base frequency. At these higher speeds, the optimum PWM strategies have been shown to be superior to sinusoidal PWM in respect to harmonic motor losses and peak current amplitude.
- The PWM strategies used immediately prior to the change to six-step operation have few commutations per cycle and must be carefully selected, whether an analog or digital approach is used for waveform generation. A poor choice of transition strategy can result in very light harmonic losses and rapid overheating of the motor in this region, loss factor  $\sigma_1$  is of great benefit in selecting correct transition strategies.

## REFERENCES

- H.S. Patel and R.G. Hoft, "Harmonic elimination and voltage control in thyristor inverters: Part1 – Harmonic elimination IEEE trans. 1nd. Appl., vol. IA-13. PP. 310-317 May/ June 2002.
- G.S. Baja and G.B. Indri, "Design Pulse width modulation for fed induction motor", IEEE trans. 1nd. Appl., vol. IA-13, pp.38-44 Ian/Feb. 2001.
- Haasz. "Design of control voltage source inverters feeding induction motors," in proc 2<sup>nd</sup> IFAC symp. On. Control in power electronic and electrical drives. Oct. 2002. pp. 379-385.

- j.M.O. Murphy, L.S. Howard. And R.G. Hoft, "microprocessor control of PWM inverter induction motor drive," in proc IEEE 2003 power electronic specialist conf., pp. 344-348.
- J.M.O. Murphy. R.G. Hoft and L.S. Howard, "controlled-slip operation of an induction motor with optimum PWM waveforms." In proc inst. Elect. Eng. Conf. on Electrical variable speed drives Sept. 2004. pp. 157-160.
- A.H.kadhim, "Control of an inverter drive reluctance motor," Ph.D Thesis submitted to the electrical and electronic engineering Department Bradford university,1990.
- E.A. Khistirm and H.E. Jordan. "polyphase induction motor performance and losses on nonsinusoidal voltage sources" IEEE Trans proc. App. Syst. Vol. PAS-87, pp. 624-631- Mar. 1988.
- C.W. Lander . "Induction mtors," New York: Gordon and Breach. 2004.
- B.J. Chaimes and B. R. Sarakar, "Additional losses due to nonsinusoidal supply waveforms with machines, "proc. Inst. Elect. Eng. Vol. 115, pp. 1782 Dec. 2005.
- G.K.Dubey, "Design aspects of AC machine application with supply variable static frequency," Brown Bovers Apr. 1999.
- P.L. Alger. G. Angest, and E.I Davies, "stray-load losses in polyphase noncation machines," AIEE trans. Power App. Syst. Vol. 78, pp. 349-357. 1999
- Mika Salo and Heiki Tuusa," control of the PWM current source inverter fed induction motor drive," 2005.
- J.W. A. Wilson and J.A. Yeamans. "intrinsic harmonics of idealized
- Inverter PWM system," inconf.Rec.1987. pp.973-976