HEAT AND MASS TRANSFER DURING AIR DRYING OF SWEET POTATO

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ABSTRACT

This study included the convective air drying of a single sweet potato sample which is taken as an ellipsoid with (40mm) in the longitudinal direction and (20mm) in the thickness direction. Convective heat and mass transfer takes place between the sample surface and its drying environment; while, unsteady heat conduction and moisture diffusion take place within the drying body without phase change for liquid (evaporation occurs at the surface only). The numerical solution of the mass, energy conservation equations was used; by applying the finite difference technique after using the body fitted coordinate system with grid generation techniques. A set of empirical correlations have been employed to determine the product properties and the important affecting factors on the drying process were studied. The results showed that the product temperature is increased and its moisture content is decreased with time and the increase in air velocity caused an increase in the heat transfer coefficient and as a result moisture content will decrease and this accelerates the drying process. The numerical results were compared with experimental results and showed good agreement.

الخلاصة:

تضمنت هذه الدراسة استخدام الهواء كوسط تجفيف لعينة مفردة من البطاطا الحلوة ذات شكل بيضوي ذات (40) ملم طول و(20) ملم سمك. يحصل ا نتقال الحرارة بالحمل بين سطح العينة ومحيط تجفيفها بينما يحصل انتقال الحرارة بالتوصيل وانتقال الرطوبة بالانتشار داخل العينة بدون حدوث تغير بالطور للسائل الرطوبي (التبخر يحصل على السطح فقط). تم استخدام الحل العددي لمعادلات حفظ الطاقة والكتلة باستخدام طريقة الفروقات المحددة بعد استخدام نظام مطابقة احداثيات الجسم مع تقنيات التوليد الشبكي. الحسابات العملية استخدمت في هذا البحث مجموعة من المعادلات التجريبية لحساب خواص المنتج مع دراسة اهم العوامل المؤثرة على عملية التجفيف بينت النتائج زيادة درجة حرارة المنتج ونقصان محتواه الرطوبي مع الزمن وان زيادة سرعة هواء التجفيف تسبب زيادة معامل انتقال الحرارة بالحمل وكنتيجة لذلك يتناقص المحتوى الرطوبي وهذا يعجل من عملية التجفيف.قورنت النتلئج العددية مع النتائج العملية وبينت توافق جيد.

KEY WORDS:
Heat and mass transfer, Convective drying, Conduction, Diffusion, Potato

INTRODUCTION:

Drying, in general, usually means removal of relatively small amounts of water from material. The purpose of drying food products is to allow longer periods of storage with minimized packaging requirements and reduced shipping weights .In the chemical industries, drying or dehydration is one of the most important processes used in the processing of food and in the storage of grains [Mulet, 1994].
Drying is a complex operation involving transient transfer of heat and mass along with several rate processes, such as physical or chemical transformations, which, in turn, may cause changes in product quality as well as the mechanisms of heat and mass transfer. Physical changes that may occur include: shrinkage, puffing, crystallization, glass transitions. In some cases, desirable or undesirable chemical or biochemical reactions may occur leading to changes in color, texture, odor or other properties of the solid products, many of these changes are functions of temperature, moisture content, and time. Therefore, undesirable effect could be better controlled, if temperature and moisture distributions in food as a function of drying time could be accurately predicted [Arun and Devahastin, 2004].

Food materials such as grains, fruits, and vegetables have microscopic capillaries and pores which cause a mixture of transfer mechanisms to occur simultaneously when subjected to heating or cooling. The complex interactions of various phenomena occurring within a material undergoing heating, solution dependent properties and the strong coupling between processes make modeling the transient moisture and temperature within the material a difficult task, so some simplified assumptions should be taken, like ignoring capillary action, all physical changes, chemical or biochemical reactions.
[Hassini and Azzouz, 2004], performed two models of diffusion to evaluate the moisture diffusion coefficient of potato during convective drying by using perforated tray. The first model was analytical solution based on Fick′s law. The diffusion coefficient was found to vary with air temperature and also increase with the thickness of the slab at a constant temperature level. The second model consist of solving numerically the equation of conservation of mass of both solid and liquid phase which was more adequate because it take into account the shrinkage phenomena.
In modeling drying, the most widely used mass transport model in Fick′s second law of diffusion using the moisture concentration difference as the driving force and Fourier’s law for heat transport model. The heat and moisture transfer within individual particles of the material should be understood and accurately represented by a mathematical model .In the most general case, the transfer of heat and moisture must be considered simultaneously in order to accurately describe the transport processes within the material.

MATHEMATICAL MODEL:

The goal of the present model is to describe the drying process for a single sample of a sweet potato. This model is based on the fact that during the single-particle drying processes, moisture diffusion and heat conduction dominate inside that particle and convective heat and moisture transfer take place on the surface.

Governing Equations:

The assumptions are as follows:

1. the potato sample was taken as an ellipsoid having a known initial temperature and moisture distribution and subjected to a uniform convective environment of hot air as in **Fig1**
2. No phase changes occurs within the drying material i.e. evaporation occurs only at the surface.
3. Shrinkage phenomenon is not considered.

Based on the above assumptions, the 2-D governing equations in Cartesian coordinate system(x, y) can be expressed as [Yang, 2004]:

$$\rho \ c_p \frac{\partial \ T}{\partial \ t} = k \left(\frac{\partial^2 T}{\partial \ x^2} + \frac{\partial^2 T}{\partial \ y^2} \right) \tag{1}$$

$$\frac{\partial \ M}{\partial \ t} = D \left(\frac{\partial^2 M}{\partial \ x^2} + \frac{\partial^2 M}{\partial \ y^2} \right) \tag{2}$$

<u>Initial and Boundary Conditions:</u>

$$T(x, y, 0) = T_{\circ} \tag{3}$$

$$M(x, y, 0) = M_{\circ} \tag{4}$$

$$K \left(\frac{\partial \ T}{\partial \ x} + \frac{\partial \ T}{\partial \ y} \right) = h \ (T_{air} - T) + h_{fg} \ h_m \ (M \ \rho - M_{air} \ \rho_{air}) \tag{5}$$

$$D \ \rho \left(\frac{\partial \ M}{\partial \ x} + \frac{\partial \ M}{\partial \ y} \right) = h_m \ (\rho_{air} M_{air} - M\rho) \tag{6}$$

<u>Material Properties and Methods of Calculation:</u>

The sweet potato was the food material used in this study because the material properties calculation of the potato is available in different literatures. The calculation of these properties was as follows:

<u>Specific Heat Calculation:</u>

An empirical equation proposed by [Chemkhi and Zagroba, 2005] to calculate specific heat which takes into account the composition of food:

$$C_P = 4184\,(0.406 + 0.00146\ T + 0.203\ M - 0.0249\, M^2) \tag{7}$$

<u>Thermal Conductivity Calculation</u>:

An empirical equation developed by [Raisul and Mujumdar, 2005] for solid and liquid foods to calculate thermal conductivity:

$$K = \frac{0.049}{1 + M} \exp\left[- \frac{47}{8.3143 \times 10^{-3}} \left(\frac{1}{T} - \frac{1}{333.15} \right) \right] + \frac{0.611 \ M}{1 + M} \tag{8}$$

Moisture Diffusivity Calculation:

An empirical equation proposed by [Chemkhi and Zagroba, 2005] and [Kiranoudis and Maroulis, 1995] to calculate moisture diffusivity as follow:

$$D = a \exp\left(-\frac{b}{M}\right) \exp\left(-\frac{c}{T}\right) \quad (9)$$

Where:

a = 1.29×10^{-6}
b = 0.0725
c = 2044

Heat Transfer Coefficient Calculation:

The heat transfer coefficient is given by the following empirical equation [Tang and Cenkowski, 2000]:

$$h = 16.09 + 8.87 \times V^{0.53} \quad (10)$$

Mass Transfer Coefficient Calculation:

The mass transfer coefficient is given by the following empirical equation [Tang and Cenkowski, 2000]:

$$h_m = 0.01959 + 0.08073 \times V^{0.553} \quad (11)$$

Material Density Calculation:

[Raisul and Mujumdar, 2005] proposed an empirical equation to calculate the density of potato sample and as follow:

$$\rho = -10^{-3} M^4 - 6.4M^3 + 55M^2 - 154M + 1253 \quad (12)$$

The thermal and physical properties for the heating media (air) are calculated according to the equations that are mentioned in different researches and as shown in **Table1**.

Transformation of Coordinate System:

The determination of the coordinate transformation is called grid generation. Once the coordinate transformation has been determined differential equations must be transformed from physical space (x,y) as shown in **Fig.(3)** to computational space (ξ, η) which is shown in **Fig. (4)**. The transformation relating the physical space and the computational space is specified by the direct transformation:

$$\zeta = \zeta(x, y), \eta = \eta(x, y) \quad (13)$$

Equation (13) can now be solved on a uniform grid in computational plane. The derivatives of ξ_x, η_x, ξ_y and η_y are called the metrics of the direct transformation. The Jacobean determinant J of the direct transformation is defined as

$$J = \frac{\partial(x,y)}{\partial(\xi,\eta)}$$

(14)

Transformation of the Governing Equations:

Equation (1) will now be used for transform the conduction equation from Cartesian (x, y) coordinate to (ξ, η) coordinate

$$\alpha\frac{\partial\psi}{\partial t} = \left(\lambda.\frac{\partial\psi}{\partial\xi} + \sigma\frac{\partial\psi}{\partial\eta} + \alpha\frac{\partial^2\psi}{\partial\xi^2} - 2.\beta\frac{\partial^2\psi}{\partial\xi\eta} + \gamma\frac{\partial^2\psi}{\partial\eta^2}\right)\Big/ J^2 \quad (15)$$

$$D\frac{\partial\psi}{\partial t} = \left(\lambda.\frac{\partial\psi}{\partial\xi} + \sigma\frac{\partial\psi}{\partial\eta} + \alpha\frac{\partial^2\psi}{\partial\xi^2} - 2.\beta\frac{\partial^2\psi}{\partial\xi\eta} + \gamma\frac{\partial^2\psi}{\partial\eta^2}\right)\Big/ J^2 \quad (16)$$

Where:

$$\sigma = \left[\left(\frac{\partial y}{\partial\xi}\right)Dx - \left(\frac{\partial x}{\partial\xi}\right)Dy\right]\Big/ J \quad (17)$$

$$\lambda = \left[\left(\frac{\partial x}{\partial\eta}\right)Dy - \left(\frac{\partial y}{\partial\eta}\right)Dx\right]\Big/ J \quad (18)$$

$$Dy = \alpha\frac{\partial^2 y}{\partial\xi^2} - 2\beta\frac{\partial^2 y}{\partial\xi\eta} + \gamma\frac{\partial^2 y}{\partial\eta^2} \quad (19)$$

$$Dx = \alpha\frac{\partial^2 x}{\partial\xi^2} - 2\beta\frac{\partial^2 x}{\partial\xi\eta} + \gamma\frac{\partial^2 x}{\partial\eta^2}$$

(20)

$$\alpha = \left(\frac{\partial x}{\partial\eta}\right)^2 + \left(\frac{\partial y}{\partial\eta}\right)^2 \quad (21)$$

$$\beta = \left(\frac{\partial x}{\partial\xi}\right)\left(\frac{\partial x}{\partial\eta}\right) + \left(\frac{\partial y}{\partial\xi}\right)\left(\frac{\partial y}{\partial\eta}\right) \quad (22)$$

$$\gamma = \left(\frac{\partial x}{\partial \xi} \right)^2 + \left(\frac{\partial y}{\partial \xi} \right)^2 \qquad (23)$$

Finite-Difference Form of the Governing Equations:

The line successive over relaxation form of the finite-difference equation for the interior node i, j will be as follow:

$$\frac{\psi_{i,j}^{n+1} - \psi_{i,j}^{n}}{\Delta t} = \Gamma_\phi \left(\lambda \frac{\psi_{i+1,j}^{n} - \psi_{i-1,j}^{n}}{2.\Delta\xi} + \sigma \frac{\psi_{i,j+1}^{n+1} - \psi_{i,j-1}^{n+1}}{2.\Delta\eta} + \alpha \frac{\psi_{i+1,j}^{n} - 2\psi_{i,j}^{n} + \psi_{i-1,j}^{n}}{\Delta\xi^2} \right. \qquad (24)$$

$$\left. - 2.\beta \frac{\psi_{i+1,j}^{n} - \psi_{i,j-1}^{n+1} - \psi_{i,j+1}^{n+1} + \psi_{i-1,j}^{n}}{4.\Delta\xi.\Delta\eta} + \gamma \frac{\psi_{i,j+1}^{n+1} - 2\psi_{i,j}^{n+1} + \psi_{i,j-1}^{n+1}}{\Delta\eta^2} \right)$$

$$\psi_{i,j}^{n+1}\left(\frac{1}{\Delta t} + 2\Gamma_\psi \gamma / J^2 \right) = \psi_{i+1,j}^{n}\left(\Gamma_\psi \lambda / J^2 / 2 + \Gamma_\psi \alpha / J^2 \right) + \psi_{i-1,j}^{n}\left(-\Gamma_\psi \lambda / J^2 / 2 + \Gamma_\psi \alpha / J^2 \right)$$

$$+ \psi_{i,j+1}^{n+1}\left(\Gamma_\psi \sigma / J^2 / 2 + \Gamma_\psi \gamma / J^2 \right) + \psi_{i,j-1}^{n+1}\left(-\Gamma_\psi \sigma / J^2 / 2 + \Gamma_\psi \gamma / J^2 \right) \qquad (25)$$

$$+ \psi_{i,j}^{n}\left(\frac{1}{\Delta t} - 2\Gamma_\psi \gamma / J^2 \right) - 2\beta\ \Gamma_\psi \psi_{\eta\xi} / J^2$$

$$AP(i,j) = \left(\frac{1}{\Delta t} + 2\Gamma_\psi \gamma / J^2 \right)$$
$$AE(i,j) = \left(\Gamma_\psi \lambda / J^2 / 2 + \Gamma_\psi \alpha / J^2 \right)$$
$$AW(i,j) = \left(-\Gamma_{\alpha\psi} \lambda / J^2 / 2 + \Gamma_\psi \alpha / J^2 \right)$$
$$AN(i,j) = \left(\Gamma_\psi \sigma / J^2 / 2 + \Gamma_\psi \gamma / J^2 \right) \qquad (26)$$
$$AS(i,j) = \left(-\Gamma_\psi \sigma / J^2 / 2 + \Gamma_\psi \gamma / J^2 \right)$$
$$SU(i,j) = \psi^n(i,j)\left(\frac{1}{\Delta t} - 2\Gamma_\psi \gamma / J^2 \right)$$

Where Γ_ψ denotes ($k/\rho\ c_p$) or D.

$$AP(i,j)\psi_{i,j}^{n+1} = AE(i,j)\psi_{i+1,j}^{n} + \psi_{i-1,j}^{n} AW(i,j) + AN(i,j)\psi_{i,j+1}^{n+1}$$
$$+ AS(i,j)\psi_{i,j-1}^{n+1} + Su\,\psi_{i,j}^{n} \qquad (27)$$

RESULTS AND DISCUSSION

Fig (4) shows the temperature distribution as a function of location. It can be seen that the temperature of the product drops sharply at the beginning of the drying process .This drop indicates that the heat convected from the drying air to the product surface can not sustain the higher evaporation rate of moisture during the initial period of drying (initial cooling period of the product). Consequently, product surface temperature increases rapidly after fifteen minutes which is clearly seen in **Fig (4)** but the temperature inside remains at low values. Also **Fig (4)** clearly shows that the product temperature in thickness direction increased more quickly than that in the longitudinal direction.

Evaporation of liquid moisture takes place from the product surface by absorbing the heat of vaporization as well as heat of desorption when removing bound moisture. As the initial moisture content at the drying surface of the product is high, it can evaporate rapidly at the beginning of the drying process. In **Fig (5)** Product moisture in thickness direction decreased more quickly than that in the longitudinal direction starting from the center to the surface due to the small diffusion distance of the product in thickness direction (20mm) relative to the distance in longitudinal direction (40mm). Also **Fig (5)** shows profiles of moisture content which have a parabolic shape before flatting at end of drying.
The convective heat transfer coefficient is directly related to the air flow velocity by **eq.(10).**An increase in air velocity directly influences this coefficient, which promotes a higher product temperature and larger moisture loss for a higher air velocity and this higher temperature increases the diffusivity of mass transfer**.** **Fig. (6) & (7)** is clearly shown moisture will be decreased and temperature will be increased more fast when the heat transfer coefficient value is high and that will accelerate the drying process .This can be explained by the fact that the air flow is responsible for decreasing moisture content; while increasing this velocity this favors the transport phenomenon.

Table (1) The physical and transport properties for the drying air [Raisul Islam and Mujumdar, (2005].

Property	Expression
ρ_{air}	$\rho_{air} = -3.5101\times10^{-8}T_{air}^{\ 3} + 1.58398\times10^{-5}T_{air}^2 - 4.6995\times10^{-3}T_{air} + 1.2921$
μ_{air}	$\mu_{air} = 1.7676\times10^{-13}T_{air}^{\ 3} - 5.541110^{-11}T_{air}^2 + 4.9832\times10^{-8}T_{air} + 17.1964\times10^{-6}$
$\Pr_{air}$	$pr_{air} = -2.2727\times10^{-8}T_{air}^3 + 4.1991\times10^{-6}T^{2air} - 3.5335\times10^{-4}T_{air} + 0.719$
K_{air}	$K_{air} = 6.8181\times10^{-10}T_{air}^3 - 1.474\times10^{-7}T_{air}^2 + 8.0291\times10^{-5}T_{air} - 0.024$

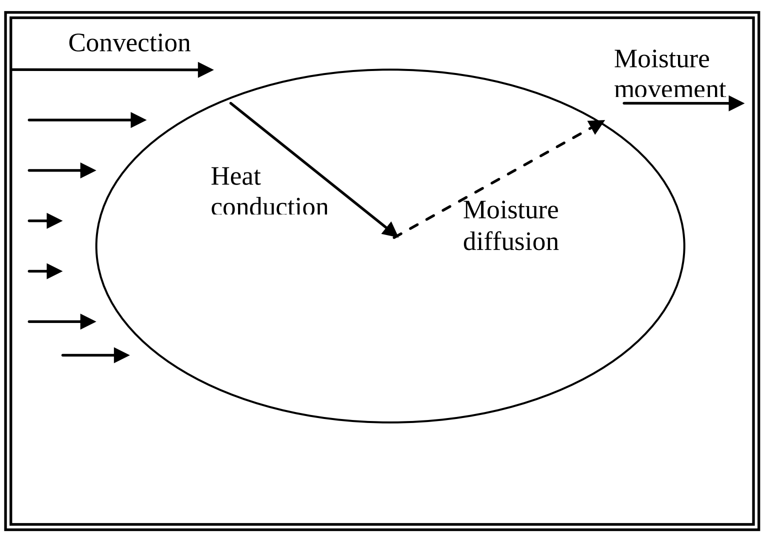

Fig 1 heat and mass transfer during drying of the product

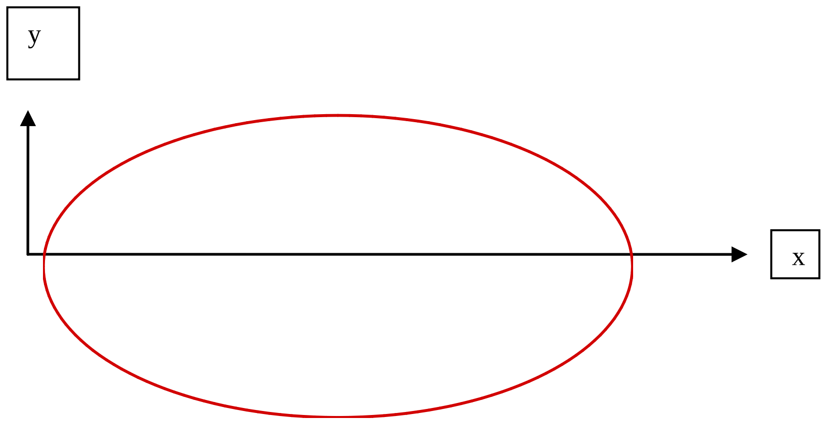

Fig. (2) Solution region in (x, y) plane

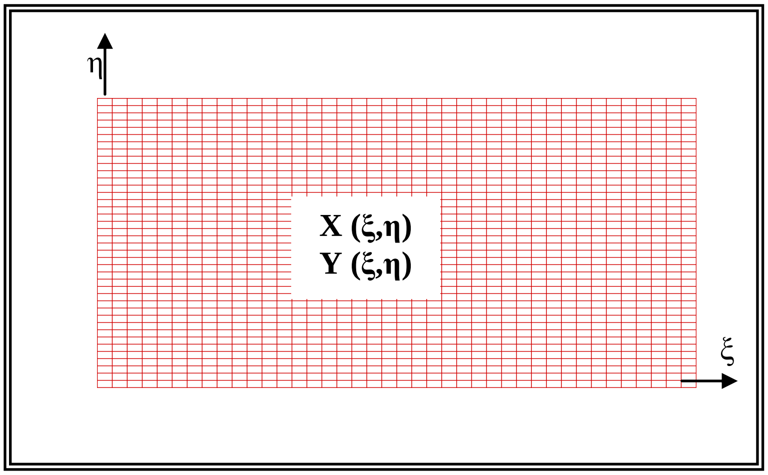

Fig (3) Transformed region in (ξ, η) plane

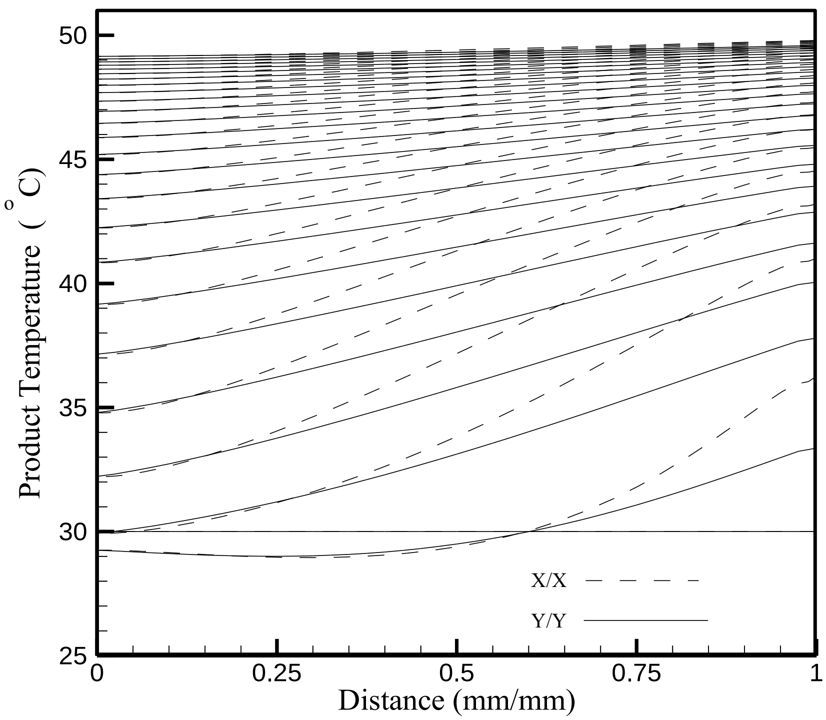

Fig (4) Temperature profiles at time intervals of 15 minutes.

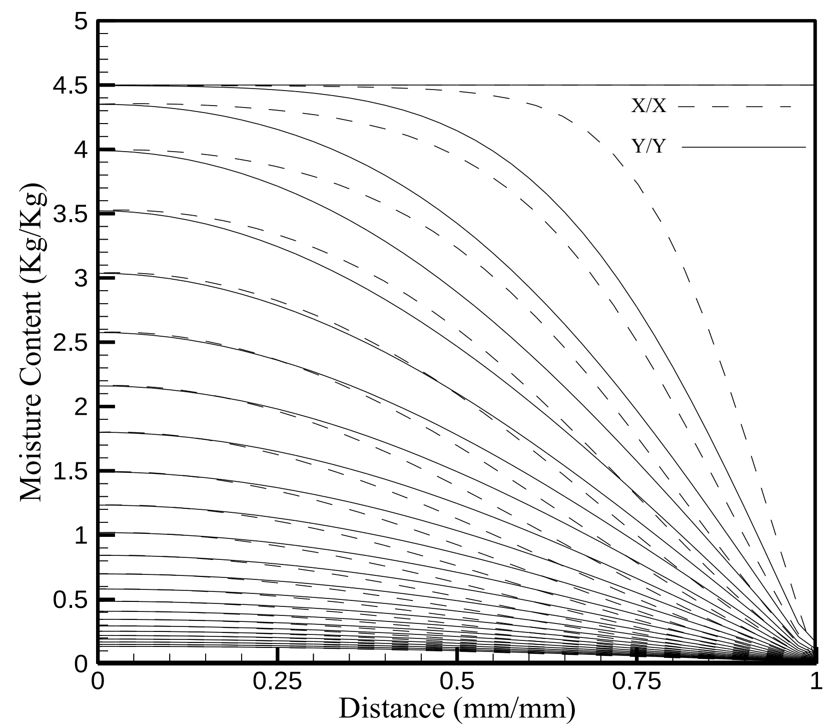

Fig. (5) Moisture content profiles at time intervals of 15 min

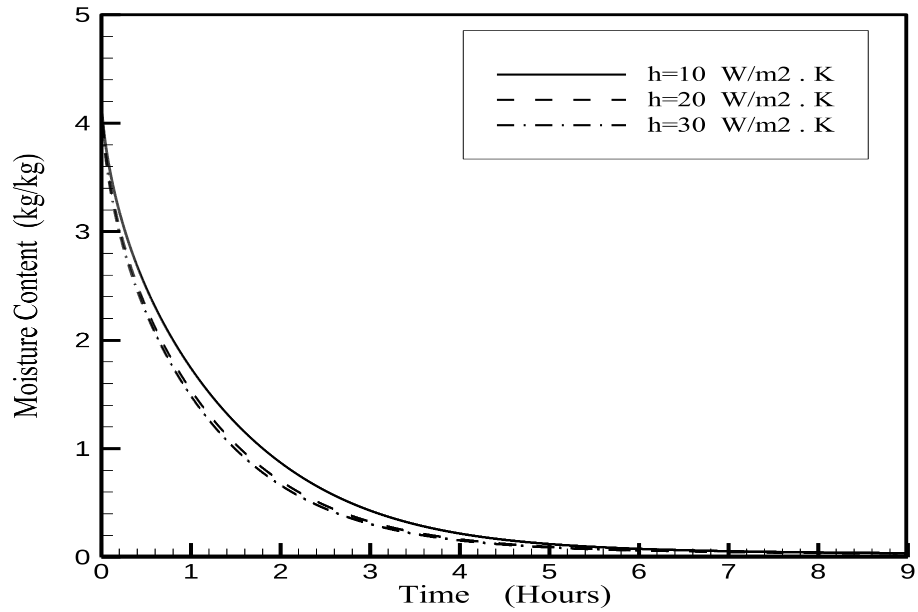

Fig. (6) The effect of convective heat transfer coefficient on
moisture loss during drying

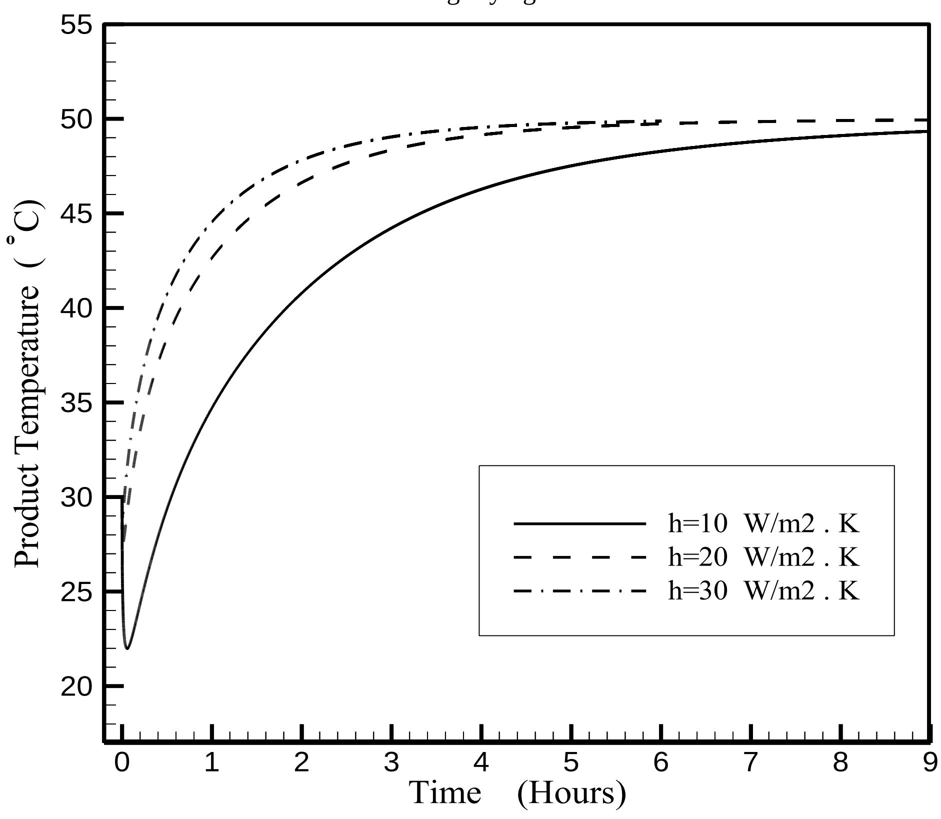

Fig. (7) The effect of convective heat transfer coefficient
on product temperature during drying

CONCLOUSIONS

From the present work results, the following conclusions can be obtained:

- The product temperature in thickness direction increased more quickly than that in the longitudinal direction.
- Product moisture in thickness direction decreased more quickly than that in the longitudinal direction starting from the center to the surface.
- Moisture decreased and temperature increased more fast when the heat transfer coefficient value is high and that will accelerate the drying process.

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NOMENCLATURE

Symbol	Definition	Units
C_P	Specific heat of product at constant pressure	J/kg.°K
D	Moisture diffusion coefficient in product	m^2/ s
h_m	Mass transfer coefficient of vapor in air	m/ s
h	Convective heat transfer coefficient in air	W/m^2. °K

h_{fg}	Latent heat of vaporization of water	J/kg
k	Thermal conductivity	W/m.°K
k_{air}	Thermal conductivity of air	W/m.°K
M	Moisture content	kg of water/kg of dry solid
M_o	Initial moisture content	kg of water/kg of dry solid
M_{air}	Air moisture content	kg of water/kg of dry solid
Pr_{air}	Prandtl number of air	-
T	Product temperature	°K
T_o	Initial temperature of the product	°K
T_{air}	Drying air temperature	°K
t	Time	sec
V	Air velocity	m/ s
X	Axis along product length	m
Y	Axis along product thickness	m

GREEK SYMBOLS

Symbol	Definition	Units
ρ	Density of product	kg/m^3
ρ_{air}	Density of air	kg/m^3
μ_{air}	Dynamic viscosity of air	kg/m. s
ξ, η	General coordinates	-

SUBSCRIPTS

Symbol	Definition
air	Drying air
o	Initial