MASS AND MOMENTUM TRANSFER TO ELLIPSOIDAL BUBBLES AND DROPS

Dr.Abdullah Abbas Kendoush
Department of Heat Transfer
Iraqi Atomic Energy commission

Dr. Abdul Nafi Shaker **Kaswar Ali Jamil**
Department of Mechanical Engineering
College of Engineering
University of Baghdad

ABSTRACT
Semi-analytical solution for the resistance of the flow and convective heat and mass transfer over the surface of ellipsoidal bubble and a drop were obtained. The fluid flow solution utilized the viscous dissipation and the heat transfer solution was based on the integral method. New relations for the drag force and the convective heat and mass transfer coefficients were derived and compared with the available theoretical solutions and experimental correlations. The range of Reynolds number was from 1 to 100 and Weber number from 0 to 3.23.

الخلاصة
تم الحصول على حل شبه تحليلي لمقاومة الجريان و انتقال الحمل الحراري و الكتلي من على السطح البيضوي للفقاعة و القطرة. اعتمد حل جريان المائع على حل طريقة التبدد اللزوجة بينما حل الانتقال الحراري على طريقة الحل التكاملي و تم مقارنتها بشكل جيد مع الحلول النظرية المتوفرة و العلاقات التجريبية المقاومة. مدى عدد رينولز للحلول المستحصلة بين الصفر و المائة بينما مدى عدد وبير كان بين الصفر وقيمة(3, 23)

KEY WORDS
Mass Transfer, Momentum Transfer, Ellipsoidal Bubble, Ellipsoidal Drop.

INTRODUCTION
Engineers, metallurgist, geologists, and industrialists all are trying to understand processes in which bubbles and drops move through liquid. Until recent decades there was not much theoretical analysis to help them, but satisfactory theories now exist for a number of important special cases.
This study considers viscous and thermal effects of the deformation of ellipsoidal gas bubble and drop. The potential velocity field of (Mieron 1989) and the equation of Frankel and Weihs (1983) for the ellipsoidal deformation bubble will be utilized for the bubble case. For drop case,the equation of Taylor and Acrivos (1964)is utilized to express drop surface in the solution. The dissipation energy method was used to get the drag on the surface of the distorted bubble and drop. The energy equation was applied for heat transfer analysis using the method of Baird and (Hamielec 1962).

THEORETICAL ANALYSIS
Consider a freely rising bubble or drop in an infinite medium under the influence of gravity, at steady state velocity, U, and under the following assumptionss:

1- Incompressible isothermal flow.
2- Axisymmetric uniform flow.
3- Constant surface tension, σ, around the bubble.
4- Neglectingthe boundary layer separation at the bubble surface.
5- Neglecting internal circulation inside the bubble.

(Meiron 1989) gave the velocity potential,ϕ, for the gas bubbles rising in an inviscid fluid as follows:

$$\phi = Ure[-\frac{r}{re}\cos\theta - \frac{1}{2}\frac{P_1(\cos\theta)re^2}{r^2}] \tag{1}$$

In spherical coordinates, the radial and angular velocity components are

$$V_r = -\frac{\partial\phi}{\partial r} = U\cos\theta(1-\frac{r_e^3}{r^3}) \tag{2}$$

$$V_\theta = -\frac{1}{r}\frac{\partial\phi}{\partial\theta} = -U\sin\theta(1+\frac{r_e^3}{r^3}) \tag{3}$$

Frankel and Weihs (1983) gave the radius of ellipsoidal bubble as:

$$r(\theta) = re.Z = re\left[1-\frac{3}{64}We(\cos 2\theta+1)\right] \tag{4}$$

Taylor and Acrivos (1964) derived the radius of ellipsoidal drop as:

$$r(\theta) = re.Z_d = re[1-\lambda..We.P_2(\cos\theta)] \tag{5}$$

where

$$\lambda = \frac{1}{4(k+1)^3}\left\{(\frac{81}{80}k^3+\frac{57}{20}k^2+\frac{103}{40}+\frac{3}{4}) - \tfrac{\gamma-1}{12}(k=1)\right\} \tag{6}$$

Using eq.(4)and eq.(5) to get the aspect ratio of bubble and drop, yield:

$$E = \frac{b}{a} = \frac{64-12We}{64+6We} \tag{7}$$

$$E = \frac{b_d}{a_d} = \frac{2[1-\lambda We]}{[2+\lambda We]} \tag{8}$$

The tangential stress, $\tau_{r\theta}$,at the bubble surface and drop surface is given as follows, Chao(1962).

$$\left[(\tau_{r\theta})_{r=r\theta} \doteq \mu\left[r\frac{\partial}{\partial r}(\frac{V_\theta}{r})+\frac{1}{r}\frac{\partial V_r}{\partial\theta}\right]_{r=r(\theta)}\right] \tag{9}$$

So, the tangential stress for bubble and drop respectively will be:

$$(\tau_{r_\theta})_{r=r(\theta)} = \frac{3U\mu\sin\theta}{re.Z^4} \tag{10}$$

$$(\tau_{r_\theta})_{r=r(\theta)} = \frac{3U\mu\sin\theta}{re.Z_d^{\,4}} \tag{11}$$

Kendoush (2000) reported the following equation for the dissipation function of spherical particle:

$$\phi = \int_0^\pi (\tau_{r\theta}.V_\theta)_{r=r(\theta)}.dA \tag{12}$$

Since dA of the ellipsoidal deformable bubble or drop is:

$$dA = (2\pi a^2 \sin\theta\cos^2\theta + 2\pi b^2 \sin^3\theta)d\theta \tag{13}$$

Therefore

$$\phi_{bubble} = -\frac{6\pi a^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^3\theta\cos^2\theta}{Z^4}d\theta - \frac{3\pi a^2 U^2 \mu}{Z^7}d\theta - \frac{6\pi b^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^5\theta}{Z^4}d\theta - \frac{3\pi b^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^5\theta}{Z^7}d\theta d \tag{14}$$

$$\phi_{drop} = -\frac{6\pi a_d^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^3\theta\cos^2\theta}{Z_d^4}d\theta - \frac{3\pi a_d^2 U^2 \mu}{Z_d^7}d\theta - \frac{6\pi b_d^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^5\theta}{Z_d^4}d\theta - \frac{3\pi b_d^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^5\theta}{Z_d^7}d\theta \tag{15}$$

Kendoush (2000) derived the drag force on the rising bubble or drop as follows

$$D = \frac{\phi}{U} \tag{16}$$

So,

$$D_{bubble} = -\frac{6\pi a^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^3\theta\cos^2\theta}{Z^4}d\theta - \frac{3\pi a^2 U^2 \mu}{Z^7}d\theta - \frac{6\pi b^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^5\theta}{Z^4}d\theta - \frac{3\pi b^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^5\theta}{Z^7}d\theta \tag{17}$$

$$D_{drop} = -\frac{6\pi a_d^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^3\theta\cos^2\theta}{Z_d^4}d\theta - \frac{3\pi a_d^2 U^2 \mu}{Z_d^7}d\theta - \frac{6\pi b_d^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^5\theta}{Z_d^4}d\theta - \frac{3\pi b_d^2 U^2 \mu}{re}\int_0^\pi \frac{\sin^5\theta}{Z_d^7}d\theta \tag{18}$$

and the coefficient C_D is the following

$$(C_D)_{bubble} = -\frac{D}{1/2\rho U^2 \pi a^2} = \frac{24}{\mathrm{Re}}\int_0^\pi \frac{\sin^3\theta\cos^2\theta}{Z^4}d\theta + \frac{12}{\mathrm{Re}}\int_0^\pi \frac{\sin^3\theta\cos^2\theta}{Z^7}d\theta - \frac{24E^2}{e}\int_0^\pi \frac{\sin^5\theta}{Z^4}d\theta + \frac{12E^2}{\mathrm{Re}}\int_0^\pi \frac{\sin^5\theta}{Z^7}d\theta \tag{19}$$

for the bubble and

$$(C_D)_{drop} = -\frac{D}{1/2\rho U^2 \pi a_d^2} = \frac{24}{\text{Re}}\int_0^{\pi}\frac{\sin^3\theta\cos^2\theta}{Z_d^4}d\theta + \frac{12}{\text{Re}}\int_0^{\pi}\frac{\sin^3\theta\cos^2\theta}{Z_d^7}d\theta$$
$$-\frac{24E_d^2}{\text{Re}}\int_0^{\pi}\frac{\sin^5\theta}{Z_d^4}d\theta + \frac{12E_d^2}{\text{Re}}\int_0^{\pi}\frac{\sin^5\theta}{Z_d^7}d\theta \qquad (20)$$

for the drop.

(Baird and Hamielec 1962) gave the following equation for mass transfer as a result of solving the diffusion equation for a fluid sphere

$$Sh = \left[\frac{2}{\pi}.Pe.\int_0^{\pi} -\frac{V_\theta}{U}\sin^2\theta d\theta\right]^{1/2} \qquad (21)$$

Substituting eq.(3) in eq.(21), yields

$$Sh = \left(\frac{2}{\pi}.Pe\right)^{1/2}\left[\int_0^{\pi}\sin^3\theta\left(1+\frac{r^3_e}{2r^3}\right)d\theta\right]^{1/2} \qquad (22)$$

therefore

$$(Sh)_{bubble} = \left(\frac{2}{\pi}.Pe\right)^{1/2}\left[\int_0^{\pi}\sin^3\theta\left(1+\frac{r^3_e}{2Z^3}\right)d\theta\right]^{1/2} \qquad (23)$$

for the bubble and

$$(Sh)_{drop} = \left(\frac{2}{\pi}.Pe\right)^{1/2}\left[\int_0^{\pi}\sin^3\theta\left(1+\frac{r^3_e}{2Z^3}\right)d\theta\right]^{1/2} \qquad (24)$$

for the drop.

RESULTS AND DISCUSSION

Flow resistance and convective heat and mass transfer to a spheroidal bubble or drop were solved numerically using Simpson's rule.

Fig.(1) illustrates graphicly the derived eqs. (19) and (20) where the drag coefficients of bubbles and drops are shown to be functions of Reynolds number at differnt aspect ratios.

Fig (2) shows the dependence of the drag coefficients on Weber number at different Reynolds numbers for bubbles (eq. 19) and drops (eq. 20).

For the case of ellipsoidal bubble and drop (i.e. for $0<We<3.23$), the present results for the drag coefficient (eq. (19)) for the bubble, and eq. (20) for drop, were compared with the analytical solution of (Moore 1965) and the numerical solution of (Masliyah and Epstein 1970) in **Figs.(3-11)**. The comparison with analytical solution of (Moore 1965) shows that there is good agreement between the result of this study and(Moor's 1965) results only for aspect ratio greater than 0.5 as shown in **Figs.(3) and (4)** for 0.9 and 0.7 aspect ratios, respectively. For aspect ratio less than or equal to 0.5, **Fig.(5)**, the two solution diverge. The exactly what (Moore 1965) obtained when he compared his results with the experimental data. He found that the agreement between the experiments and his analytical solution remains fair for the aspect ratio less than 0.5.

The comparison of the present study with (Masliyah and Epstein 1970) shows that when ($\text{Re} \leq 10$) there is good agreement between the two studies and for all aspect ratios **Figs.(9), (10)** and **(11).**When Re number increases above 10, one can see that the deviation between the(Masliyah and Epstein 1970) and this study increases as shown in **Fig . (6), (7) and (8)** for various aspect ratio.

Note that the numerical solution of Masliyah and Epstein suffers from the errors inherent in numerical results, which arises from discretization and stability (Masliyah and Epstein 1970). For mass transfer of the ellipsoidal bubble case **Fig.(12)** represents the curve of the influence of eccentricity on mass transfer around solid spheroid and shows that the results of this study are nearer to Lochiel and Calderbank (1964). **Fig.(13)** shows the relation between eccentricity and mass transfer around spheroids moving in within a potential flow regime, and compaes the present result with theoretical results of (Lochiel and Calderbank 1964) where the agreement is good.

CONCLUSIONS

The present investigation demonstrates that the dissipation method in momentum and integral method for heat or mass transfer in bubble and drops can be used to give good results for momentum and heat transfer. The accuracy expected to be improved further if one can cast actual radius equation for oblate spheroidal bubble or drop or drop using experimental data.

REFERENCES

Baird, m. H. I. Hamielec, A.E. (1962), Forced convection transfer around spheres at intermediate Reynolds numbers. Can.J. Chem. Eng. 40,119-121.

Chao, B.T. (1962) . Motion of spherical gas bubble in a viscous liquid at large Reynolds number.Phys. Fluids. 5, 69-79.

Francisco , S.Y. , and Lightfoot , E.N. (1968), The effect of distortion on mass transfer to spheroidal drops. AIChE Journal. 14, 835-836.

Frankel, I., Applied Scientific Research .40, 279-294.

Haberman , W.L. and Morton , R.K.(1953), An experimental investigation of the drag and shape of air bubbles rising in various liquid. David Taylor Model Basin Rep. No. 802.

Harper, J.F. (1972), The motion of bubbles and drops through liquid .Adv. Appl. Mech. 12.59-129.

Kendoush,A.A.(2000), Calculation of flow resistance from a spherical particle. Chemical Engineering and Processing.39,81-86.

Lochiel, A.C., and Calderbank, P.H.(1964), Mass transfer in the continuous phase around axisymmetric bodies of revolution. Chem. Eng. Sci. 19.471-484.

Masliyah, J.H., and Epstein, N. (1970), Numerical study of steady flow past spheroids.J. Fluid Mech.44.493-512.

Meiron, D.I (1989), On the stability of gas bubbles rising in an inviscid fluid. J. Fluid Mech. 198, 101-114.

Moore, D. W. (1965), The velocity of rise of distorted gas bubble in a liquid of small viscosity J. Fluid. Mech. 32, 749-766

NOMENCLATURE

a: Semi major axes of the ellipsoidal spheroid bubble
a_d: Semi major axes of the ellipsoidal spheroid drop
b: Semi minor axes of ellipsoidal spheroid bubble
b_d: Semi minor axes of the ellipsoidal spheroid drop

C_d: Drag coefficient
D:Equivalent diameter
E: Bubble aspect ratio
E_d: Drop aspect ratio
g : Acceleration due to gravity
K: Ratio of viscosity of the continuos phase to that of the dispersed phase
M_o: Morton number
Nu :Nusselt number
$P_1(\cos\theta)$:Legendre polynomial $[P_1(\cos\theta)=\cos\theta]$
$P_2(\cos\theta)$: Legendre polynomial $[P_1(\cos\theta)=\cos\theta]$
Pe :Peclet number
Pr: Prandtl number
r: Variable bubble surface radius
re: Spherical equivalent radius of the bubble
Re: Reynolds number
Sc: Schmidt number
Sh: Sherwood number
U: Main upstream velocity
We: Weber number
Z: Bubble deformation factor
Z_d: Drop deformation factor

Greek Symbols
μ : Dynamic viscosity
σ: Surface tension
ρ: Fluid density
Φ: Dissipation function
Ø: Velocity potential
V_r: Radial surface velocity component
V_θ: Tangential surface velocity component
Φ : Dissipation function
$\tau_{r\theta}$: Tangential bubble surface stress

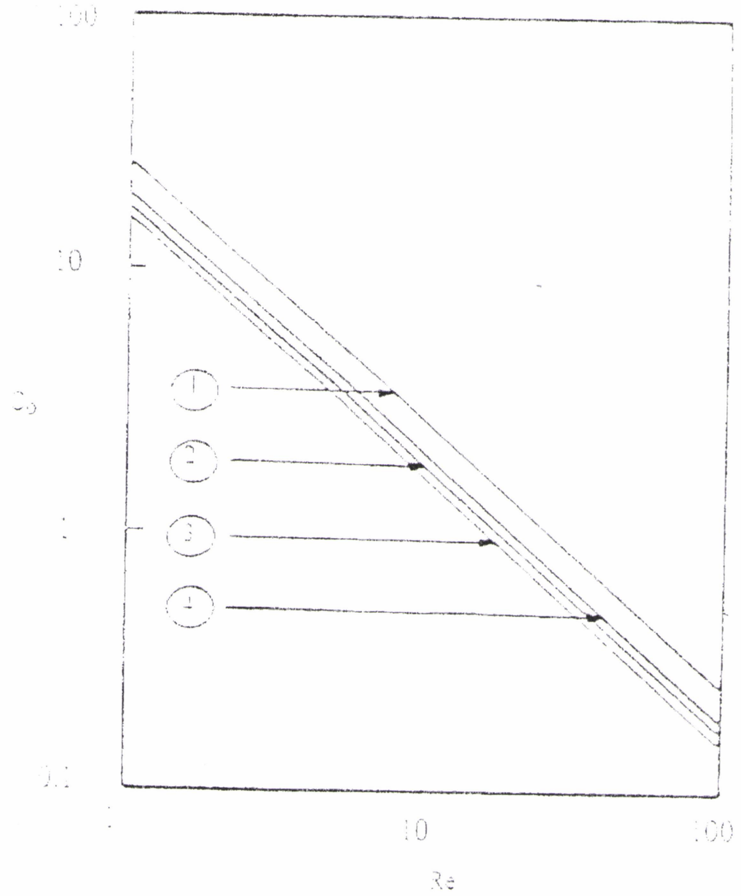

Figure (1) Drag coefficient of the bubble and drop versus Reynolds number for various aspect ratios according to Eq. (2-19) for bubble and Eq. (2-20) for carbon tetrachloride dropin water.

1- Aspect ratio = 1 , We = o for both spherical bubble and drop.

2- Aspect ratio = 0.74. We = 1 for bubble. We=0.878 for carbon tetrachloride drop.

3- Aspect ratio = 0.52, We = 2 for bubble, We = 1.762 for carbon tetrachloride drop

4-Aspect ratio = 0.30, We = 3.23 for bubble. We = 2.816 for carbon tetrachloride drop.

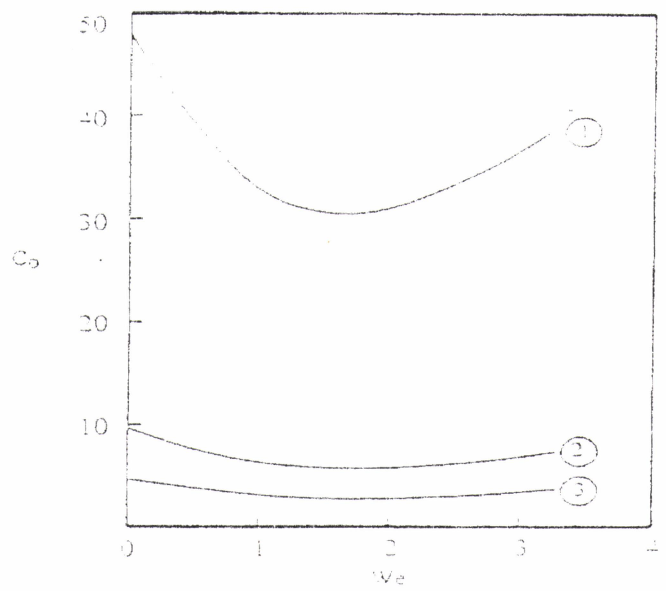

Figure (2) Variation of the drag coefficient of the bubble and drop versus Weber number for various Reynolds numbers according to Eq. (2-19) for bubble and Eq. (2-20) for carbon tetrachloride drop in water.

(1) Re = 1
(2) Re = 5
(3) Re = 10

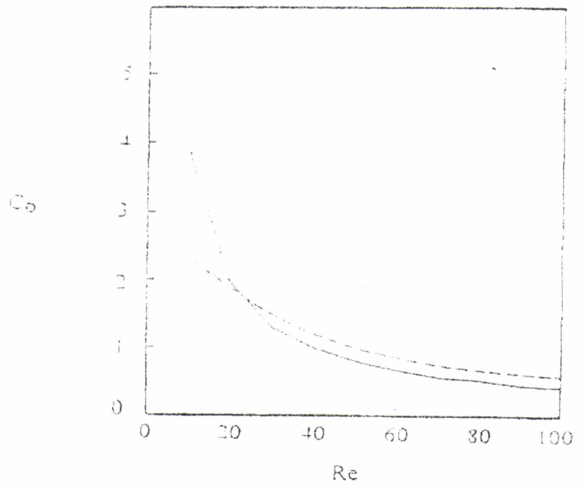

Figure (3) Drag coefficient versus Reynolds number for 0.9 aspect ratio.

______ . Eq. (2-19) for bubble and Eq. (2-20) for drop .
------- . Moore's results (1965).

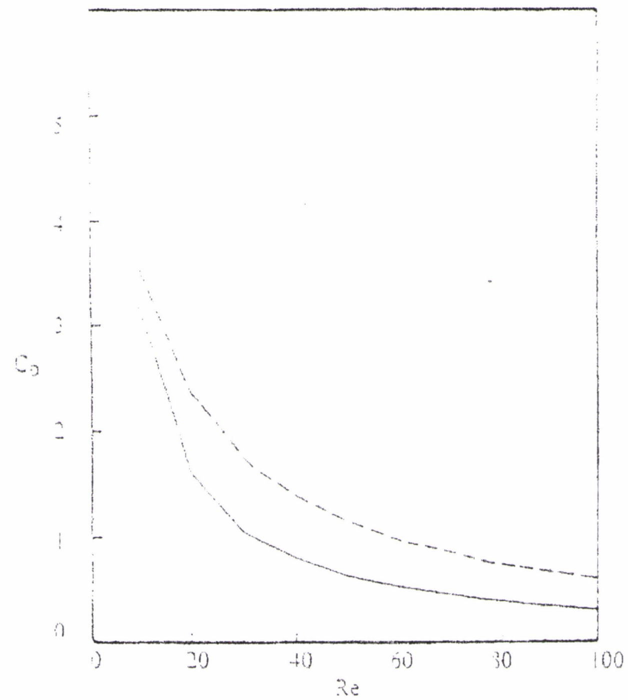

Figure (4) . Drag coefficient versus Reynolds number for 0.7 aspect ratio.

_______ . Eq. (2-19) for bubble and Eq. (2-20) for drop .
------- . Moore's results (1965).

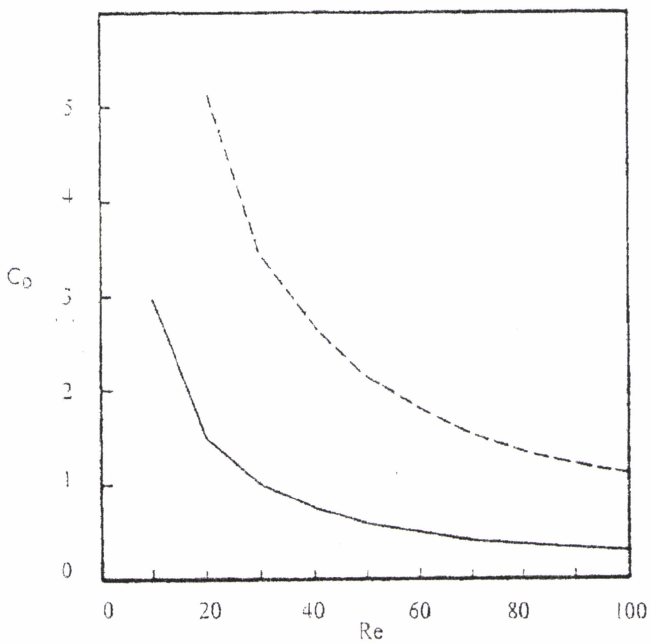

Figure (5) . Drag coefficient versus Reynolds number for 0.5 aspect ratio.

_______ , Eq. (2-19) for bubble and Eq. (2-20) for drop .
------- , Moore's results (1965).

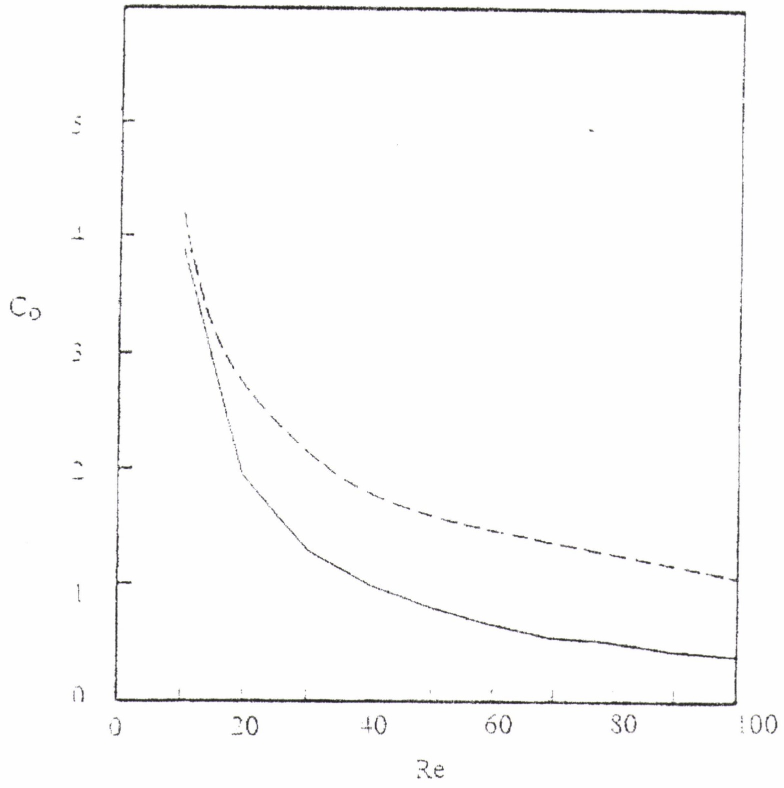

Figure (6) . Drag coefficient against Reynolds number for oblate spheroid of 0.9 aspect ratio.

______ , Eq. (2-19) for bubble and Eq. (2-20) for drop.
-------- , Numerical results of Masliyah and Epstein (1970).

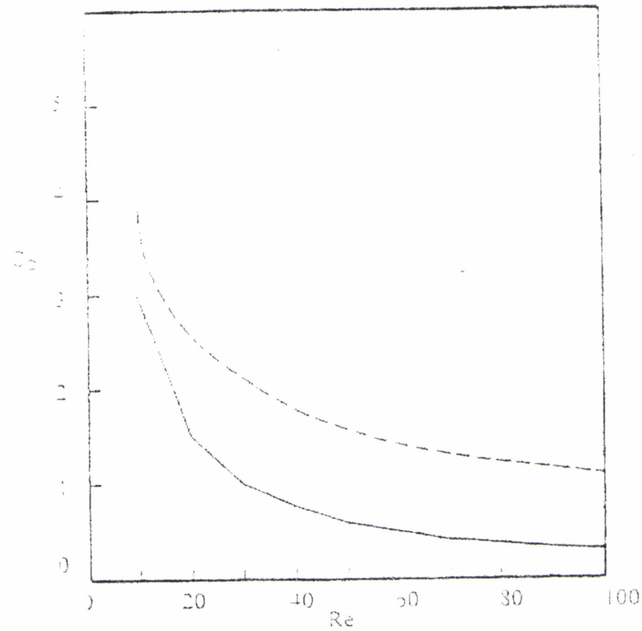

Figure (7). Drag coefficient against Reynolds number for oblate spheroid of 0.5 aspect ratio.

______ , Eq. (2-19) for bubble and Eq. (2-20) for drop.
-------- , Numerical results of Masliyah and Epstein (1970).

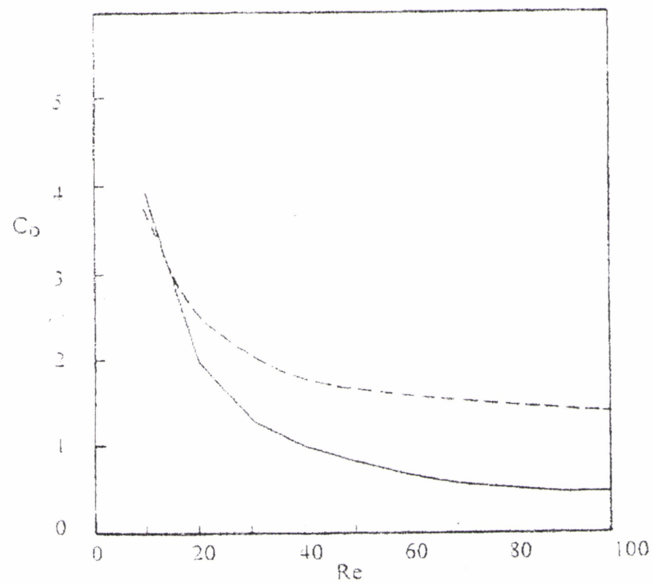

Figure (8). Drag coefficient against Reynolds number for oblate spheroid of 0.2 aspect ratio.
______ , Eq. (2-19) for bubble and Eq. (2-20) for drop.
-------- , Numerical results of Masliyah and Epstein (1970).

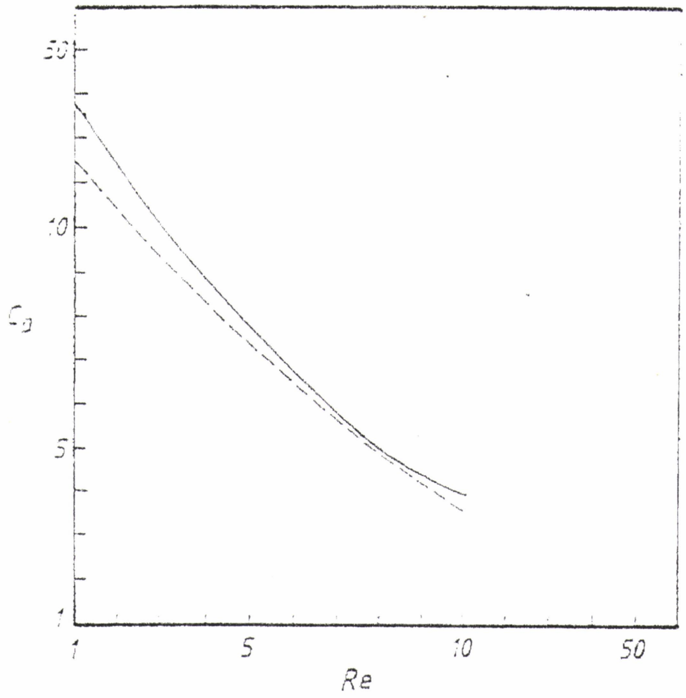

Figure (9) .Drag coefficient against Reynolds number for oblate spheroid of 0.9 aspect ratio.

______ . Eq. (2-19) for bubble and Eq. (2-20) for drop.

-------- . Numerical results of Masliyah and Epstein (1970).

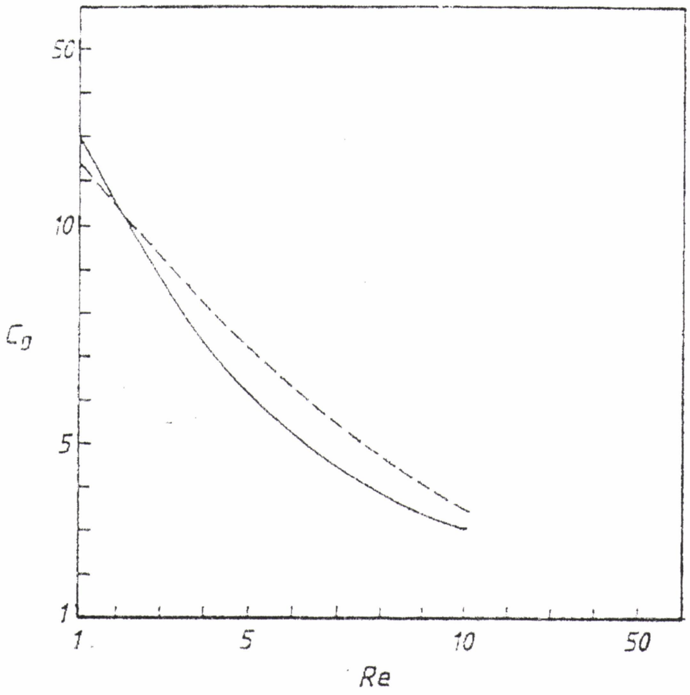

Figure (10) . Drag coefficient against Reynolds number for oblate spheroid of 0.5 aspect ratio.

______ . Eq. (2-19) for bubble and Eq. (2-20) for drop.

-------- . Numerical results of Masliyah and Epstein (1970).

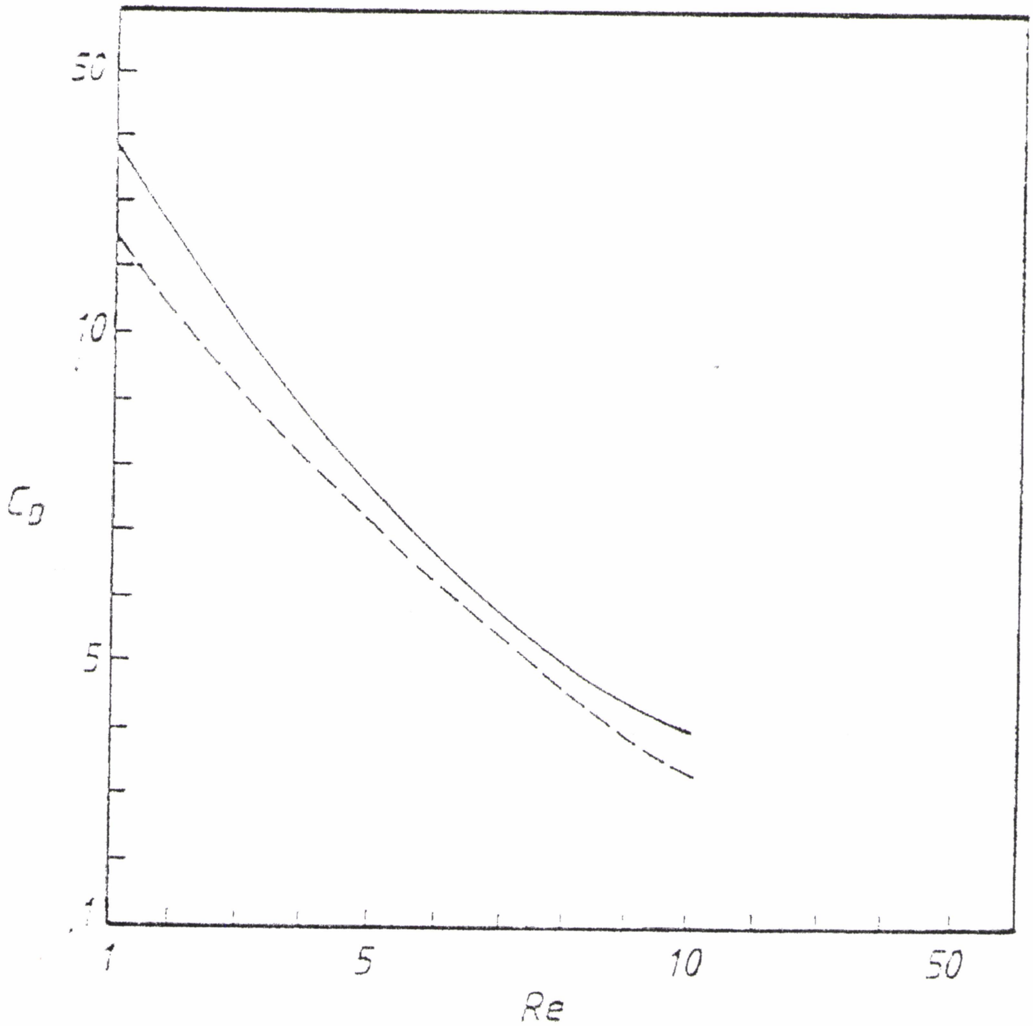

Figure (11) . Drag coefficient against Reynolds number for oblate spheroid of 0.2 aspect ratio.

______ , Eq. (2-19) for bubble and Eq. (2-20) for drop.
-------- . Numerical results of Masliyah and Epstein (1970).

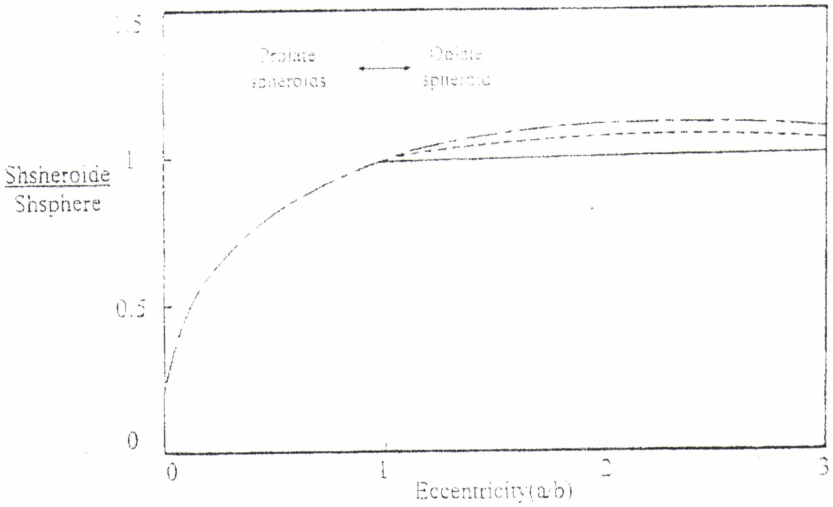

Figure (12). The influence of accentricity on mass transfer around solid spheroids.

_______ Eq. (-23) for various values of Weber number.
_._._._._ Theoretical results of Lochiel and Calderbank (1964).
-----------, Experimental results of Skelland and Cornish (1963).

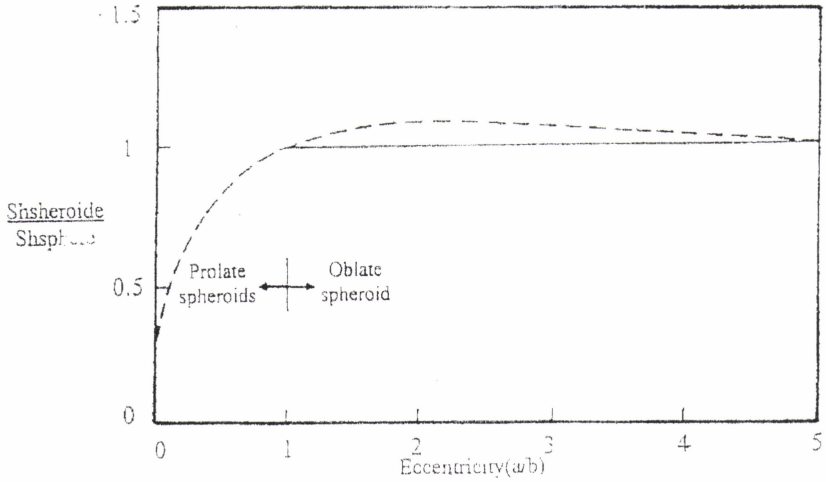

Figure (13). The influence of eccentricity on mass transfer around spheroid moving in a potential flow regime.

________, Eq. (-23) for various values of Weber number.
-----------, Theoretical results of Lochiel and Calderbank. (1964).