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STANDARD METHOD OF RMSE CALCULATION FROM POLYNOMIAL RECTIFICATION

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ABSTRACT
Image quality and geometric accuracy of SPOT data are essential elements in cartographic applications. The evaluation of geometric accuracy of SPOT data is carried out by analysis and quantification of the errors in geometric correction. For this purpose, many programs are designed to correct the image geometrically (polynomial transformation) and to calculate the root mean square error (RMSE) by the standard method.
The standard method of calculating the RMSE is shown to be capable of providing accurate estimates of geometric error when a modest number of control points is available (between 10-15 points). This method also provides an indication of the effects of choosing different polynomial orders.

الخلاصة
أن نوعية بيانات القمر الصناعي (SPOT) ودقتها الهندسية عناصر أساسية في التطبيقات الكارتوكرافية. ولتقييم الدقة الهندسية لهذه البيانات تم تحليل وتحديد الأخطاء في التصحيح الهندسي بطريقة التحويل المتعددة الحدود ولهذا الغرض صممت عدّة برامج لتصحيح الصور هندسياً بطريقة التحويل المتعددة الحدود و حساب جذر متوسط مربع الخطأ (RMSE) بالطريقة القياسية (Standard Method).
أثبتت الطريقة القياسية لحساب جذر متوسط مربع الخطأ قدرتها على التخمين الدقيق للأخطاء الهندسية عندما يتوفر عدد معتدل من نقاط الضبط الأرضي (10-15 نقطة) مع اعطاء دلالة حول تأثير اختيار الدرجات المختلفة لطريقة التحويل المتعددة الحدود.

KEY WORDS
Polynomial transformation, Control points, Geometric accuracy, Geometric error, Image quality.

INTRODUCTION
A standard method of estimating errors in polynomial geometric transformation is presented. Polynomial rectification is widely applied in both remote sensing and geographical information system (GIS) applications. Polynomial rectification calculates a global mathematical transformation for converting image to map coordinates system and vice versa. The area selected for this research is in Mosul in the north of Iraq, **Fig.(1)**.

Fig. (1). Scene of study area from Mosul SPOT PAN image in 1992

POLYNOMIAL RECTIFICATION

Polynomial model is an approach to correct the image geometrically. It is sensor independent and based on statistical principles. In the correction process, numerous points are located both in the distorted image (column, row numbers such as a street intersection) and in the reference map or master image (ground coordinates). The original image is shifted, rotated, scaled and warped to fit the reference points. The polynomial equation can be solved after a sufficient number of points have been collected. In case of redundant information, a least square adjustment of the measurements is applied to determine the best fitting polynomial and its accuracy. The linear least squares function is used to express (Mather, 1987):

- x as a function of c and r (a coefficient matrix)
- y as a function of c and r (b coefficient matrix)
- c as a function of x and y (av coefficient matrix)
- r as a function of x and y (bv coefficient matrix)

Once the coefficients of each of these functions are known, it will be possible to transform from (x, y) to (c, r) coordinates or from (c, r) to (x, y). Procedures for estimating the coefficients (a_{ij} and b_{ij}) in the least squares functions relating the two coordinates systems can now be considered. The map easting (X) and northing (Y) estimate from image column (c) and row (r) coordinates for a set of control points. The set of control points (map easting and northing) are denoted by the vector X and Y respectively, while the powers and cross-products of the c and r values are considered to form the matrix P. The coefficients a_{ij} and b_{ij} will be the elements of vector a and b are respectively. The method of the least squares is used to find the vector of estimates X and Y according to the following model:

$$\left.\begin{aligned} X &= Pa \\ Y &= Pb \end{aligned}\right\} \qquad (1)$$

The standard formula for the evaluation of a and b is:

$$\left.\begin{aligned} a &= (P'P)^{-1} P'X \\ b &= (P'P)^{-1} P'Y \end{aligned}\right\} \qquad (2)$$

The first and the second order polynomial have the system shown in **Table (1)**. The map easting vector X and northing vector Y are to be estimated from the powers and cross-products of the image column and row (c and r) vectors, forming the matrix P. The values x, y, c and r are measured at N control points. For moderate distortions in a relatively small area of an image, a six-parameter affine transformation is sufficient to rectify the imagery to a geographic frame of reference such as a subimage used in this study from a large SPOT image.

Table (1) Solution of the least square estimation procedure

First order polynomial least squares

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix} \quad a = \begin{bmatrix} a_o \\ a_1 \\ a_2 \end{bmatrix} \quad P = \begin{bmatrix} 1 & c_1 & r_1 \\ 1 & c_2 & r_2 \\ 1 & c_3 & r_3 \\ . & . & . \\ . & . & . \\ . & . & . \\ 1 & c_n & r_n \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_n \end{bmatrix} \quad b = \begin{bmatrix} b_o \\ b_1 \\ b_2 \end{bmatrix}$$

Second order polynomial least squares

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ . \\ . \\ x_n \end{bmatrix} \quad a = \begin{bmatrix} a_o \\ a_1 \\ a_2 \\ . \\ . \\ . \\ a_5 \end{bmatrix} \quad P = \begin{bmatrix} 1 & c_1 & r_1 & c_1^2 & c_1 r_1 & r_1^2 \\ 1 & c_2 & r_2 & c_2^2 & c_2 r_2 & r_2^2 \\ 1 & c_3 & r_3 & c_3^2 & c_3 r_3 & r_3^2 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & c_n & r_n & c_n^2 & c_n r_n & r_n^2 \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ . \\ . \\ y_n \end{bmatrix} \quad b = \begin{bmatrix} b_o \\ b_1 \\ b_2 \\ . \\ . \\ . \\ b_5 \end{bmatrix}$$

GROUND CONTROL POINTS (GCPs)

It is generally accepted that the number, spatial distribution and location accuracy of GCPs influence the accuracy of the correction. Different numbers of control points are needed depending on the degree of the polynomial. The minimum number of points required to perform a transformation of order (d) equals (ERDAS, 1994):

$$\frac{((d+1)(d+2))}{2} \qquad (3)$$

For the best rectification results, more than the minimum number of GCPs is required.
The optimal distribution of GCPs is generally thought to be uniform over the entire image, evenly spread, as far as possible, over the image area (Borgeson et al, 1985) and (Manual of remote sensing,1983), **Fig.(2)**.

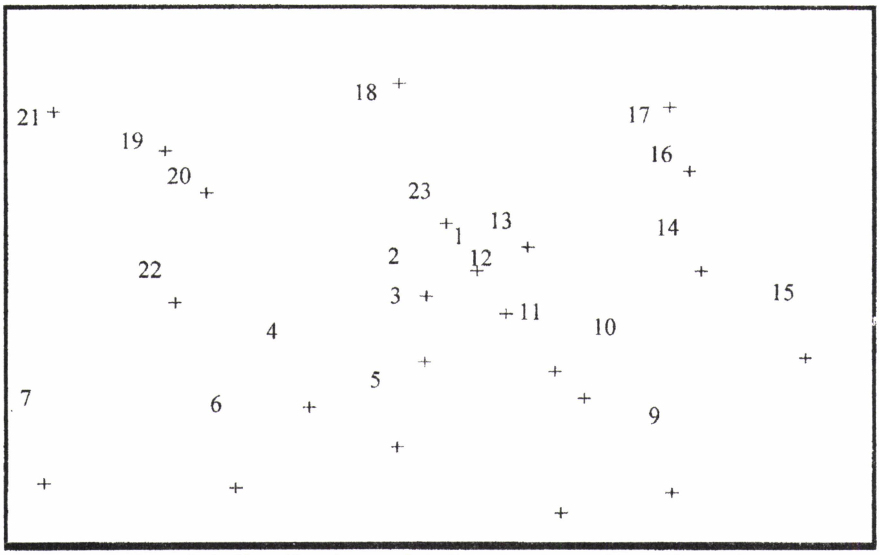

Fig. (2) Spatial distribution of GCPs in the selected study area

Image coordinates can be identified either manually by visual identification using the cursor on a display or sometimes by means of enlargement of pixels displayed on the CRT, making it possible to mark them within an error standard deviation (σ_{ij}) of 0.5 pixel.
Ground coordinates are obtained from the map at scale 1:50,000 by the computer-controlled tablet digitizing. The computer–controlled tablet digitizing converts digitizer coordinates from inches to meters on the ground. The corner locations of each map are measured to establish the actual scale and orientation of the map.

STANDARD METHOD

RMSE is often calculated separately for the x and y components of each ground control point in order to provide information about the relative error associated with that point. Total RMSE based on transformation residuals is an accuracy statistics for the entire image. Because the GCPs are not independent from the transformation coefficients, RMSE will underpredict the actual error found elsewhere in the transformed image when the degrees of freedom are small. As the number of GCPs increases, the transformation coefficients will converge towards an improved estimate and RMSE will asymptotically approach actual error. The RMSE estimated from transformation residuals increased consistently with the number of control points. This relationship is portrayed in **Fig. (3)** (McGwire , 1996).

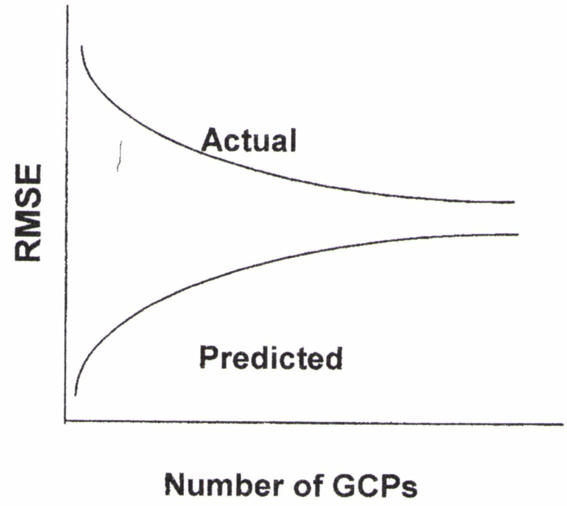

Fig.(3). Idealized curves of estimated and actual RMSE versus number of GCPs in the standard method

From the residuals, the following calculations are made to determine the total RMSE, the RMSEx and the RMSEy (ERDAS, 1994):

$$\left.\begin{aligned} RMSEx &= \left\{ \frac{1}{N} \sum_{i=1}^{N} \Delta x_i^2 \right\}^{1/2} \\ RMSEy &= \left\{ \frac{1}{N} \sum_{i=1}^{N} \Delta y_i^2 \right\}^{1/2} \end{aligned}\right\} \qquad (4)$$

$$RMSE = \left\{ RMSE_x^2 + RMSE_y^2 \right\}^{1/2} \qquad (5)$$

A normalized value representing each point's RMSE in relation to the total RMSE is (ERDAS, 1994):

$$E_i = \frac{RMSE_i}{RMSE} \qquad (6)$$

where E_i = error contribution of GCP_i

ERROR ANALYSIS

In this research, the polynomial rectification is used to rectify SPOT data to the UTM coordinates system. The three major sources of errors that limit the accuracy of fitting are:

- Location errors caused by the spatial resolution of SPOT data.
- Map errors attributable to the scale, quality, projection / coordinates system, and to the digitizing procedures.
- Errors caused by terrain relief.

In the study area scene, the 10 m spatial resolution of SPOT PAN data, land cover, and variation in terrain, make it difficult to define GCPs location better than ±0.5 pixel (±5 m).

The current accuracy of a map is about 0.2 mm and standard deviation of digitizing is 0.2 mm. So, the root mean square of the digitized data residuals is $0.2\sqrt{2}$ mm, which is 14m and 7 m respectively for 1:50,000 and 1:25,000 scale map.
With a relief of about 100 m or less (i.e. the study area), the resulting planimetric error should be less than ± 0.1 pixel. In this case, the GCPs used for the rectification are selected at mid-range elevations. The geometrical impact of this value can be neglected(Welch et al, 1985). The overall error budgets for the study area can be approximated as follows (Welch et al, 1985):
:

$$RMSE \approx \sqrt{(location\ error)^2 + (map\ error)^2 + (relief\ error)^2} \qquad (7)$$

$$\approx \sqrt{(5)^2 + (14)^2 + (1)^2} = \pm 15\ m \ (\pm 1.5 \text{ pixels}) \text{ for SPOT PAN data}$$

EVALUATING RMSE

After each computation of a transformation matrix and RMSE, there are four options:

1- Throw out the GCP with highest RMSE, assuming that this GCP is the least accurate. Another transformation matrix can then be computed from the remaining GCPs. A closer fit should be possible. However, if this is the only GCP in a particular region of the image, it may cause greater error to remove it.
2- Select only the most confidence points.
3- Tolerate a higher amount of RMSE.
4- Increase the order of transformation, creating more complex geometric alterations in the image.

A transformation matrix can then be computed that can accommodate the GCPs to less error.
The residual errors are used to detect gross errors in the acquisition of GCP locations. If the residual error is greater than the corresponding standard deviation error (σ_{ij}) by more than three times (for example, σ_{ij} = 0.5 pixel), that is, if

$$|r_{ij}| > 3\sigma_{ij} \qquad (8)$$

then GCP is considered as "suspect" and is examined to determine if an error is made in determining its location in the image or map (Ford and Zanelli , 1985)

APPLICATIONS AND RESULTS

The Standard method is accomplished by using first and second order polynomials. The least squares method fits the SPOT images to the reference map with the aid of GCPs. The procedure can be iterated until the RMSE is reduced to less than ±1 pixel. 23 GCPs are located on the map (UTM easting and northing of each point) by using the tablet digitizing. The same GCPs are then identified in the SPOT data according to their row and column coordinates, **Table (2)**. The GCPs are located throughout the region to be rectified and not congested into one small area. These GCPs are entered to the least squares regression procedure to identify the:

1- Coefficients of the coordinates transformation.
2- Individual and total residual error (r_{ij}) in X and Y.
3- Individual and total RMSE associated with the GCPs.
4- Relative error and scale error.

For the first order transformation, a threshold of 1 pixel is not satisfied until 10 GCPs are deleted from the SPOT PAN data set analysis. The order in which the 10 GCPs are deleted and the total RMSE found after each deletion are summarized in, **Table (2)**. The 13 GCPs finally selected produce an acceptable RMSE <1 pixel which are shown in, **Table (3)**. The six coefficients derived from 13 suitable GCPs are found in the same table.

Finally, the RMSE is ± 9.7 m (± 0.977 pixel, 13 GCPs) for the SPOT PAN data set. These values are checked with the GIS software packages. Results of applying second order polynomial provide bad results for rectification of the SPOT data due to the moderate distortion in a relatively small area of an image (i.e. subimage used in this study from the large SPOT image). The analysis shows that first order transformation suggests better results than higher order polynomials.

The standard RMSE is a function of the number of GCPs ,see **Fig. (4)**. Five subsets are tested and the mean of these subsets is calculated in table 4. The subsets are chosen according to the spatial distribution of GCPs. As expected, the RMSE for the standard method based on transformation residual starts low and increases to a maximum of approximately 15 points for the first order polynomial. The RMSE based on transformation residuals is stabilized at approximately seven control points but it converges towards a more accurate estimate of actual error within 7–15 control points for the first order transformation.

Table (2) GCPs locations on both map and image with the total standard RMSE after each deletion

GCP No. i	Order of points deleted	Observed Map GCPs locations (UTM – m)		Observed Image GCPs locations (pixel)		Total RMSE after this point deleted (pixel)
		x	y	r	c	
1	kept	332424	4026319	240	166	–
2	10	332113	4026217	214	182	0.977
3	Kept	332025	4025815	213	224	–
4	Kept	331376	4025660	154	253	–
5	Kept	331750	4025306	199	278	–
6	8	330887	4025278	116	304	1.146
7	7	329956	4025515	18	301	1.278
8	Kept	332460	4024730	283	320	–
9	Kept	333063	4024729	340	307	–
10	Kept	332759	4025402	295	247	–
11	Kept	332648	4025613	280	230	–
12	4	332521	4025974	255	193	1.665
13	5	332739	4026400	266	151	1.544
14	Kept	333545	4026053	355	166	–
15	9	333925	4025385	408	221	1.012
16	6	333640	4026633	349	103	1.394
17	2	333645	4026979	339	62	2.358
18	Kept	332331	4027549	200	47	–
19	Kept	331023	4027392	80	90	–
20	1	331210	4026995	101	117	3.039
21	Kept	330540	4027763	23	65	–
22	Kept	330845	4026462	85	186	–
23	3	332324	4026696	224	136	1.869
Total RMSE with all 23 GCPs used =						3.582

Table (3) Results of applying the standard method to 13 selected GCPs

GCP No. i	Observed Map GCPs locations (UTM – m)		Observed Image GCPs locations (pixel)		Estimated Image GCPs locations (pixel)		Residuals errors (pixel)		$RMSE_i$ of GCP_i (pixel)	GCPi error contrib-ution
	x	y	c	r	c`	r`	Δx	Δy		
1	332424	4026319	240.000	166.000	240.062	165.173	0.062	-0.827	0.830	0.849
3	332025	4025315	213.000	224.000	213.859	223.252	0.859	-0.748	1.139	1.165
4	331376	4025660	154.000	253.000	154.040	252.633	1.040	-0.367	1.103	1.128
5	331750	4025306	199.000	278.000	199.737	279.107	0.737	1.107	1.330	1.361
8	332460	4024730	283.000	320.000	282.252	319.965	-0.748	-0.035	0.748	0.766
9	333063	4024729	340.000	307.000	340.435	306.865	0.435	-0.135	0.455	0.466
10	332759	4025402	295.000	247.000	294.717	247.624	-0.283	0.624	0.685	0.701
11	332648	4025613	280.000	230.000	278.870	229.395	-1.130	-0.605	1.282	1.312
14	333545	4026053	355.000	166.000	354.664	166.681	-0.336	0.681	0.759	0.777
18	332331	4027549	200.000	47.000	201.121	46.778	1.121	-0.222	1.143	1.170
19	331023	4027392	80.000	90.000	78.792	90.777	-1.208	0.777	1.436	1.469
21	330540	4027760	23.000	65.000	23.166	65.023	0.166	0.023	0.168	0.172
22	330845	4026462	85.000	186.000	84.285	185.731	-0.715	-0.269	0.765	0.782

The coefficients of the least squares regression procedure are

$a_o = 330471.494$	$b_o = 4028442.366$	$av_o = 66283.620$	$bv_o = 401660.509$
$a_1 = 9.812$	$b_1 = -2.194$	$av_1 = 0.096$	$bv_1 = -0.021$
$a_2 = -2.441$	$b_2 = -9.666$	$av_2 = -0.024$	$bv_2 = -0.097$

Total RMSEx = 0.777 pixel
Total RMSEy = 0.593 pixel
AE = Total RMSE = 0.977 pixel = 9.77 m
RE = 0.410
SE = 9.721×10^{-4}

Table (4) GCPs subsets for testing the RMSE in the standard method

No. of points	Set 1		Set 2		Set 3		Set 4		Set 5		Mean of RMSE
	Points	RMSE (pixel)	Points	RMSE (pixel)	Points	RMSE (pixel)	Points	RMSE (pixel)	Points	RMSE (pixel)	
4	1,4,14, 18	0.437	3,4,14, 22	0.315	1,5,10, 11	0.523	2,5,18, 22	1.017	1,2,11, 14	0.594	0.577
5	+22	0.816	+5	0.679	+4	0.562	+14	0.946	+5	0.956	0.791
6	+10	0.764	+2	0.742	+2	0.793	+9	0.990	+22	0.880	0.833
7	+9	0.768	+19	0.834	+14	0.880	+8	0.953	+18	1.009	0.888
8	+19	0.910	+10	0.827	+3	0.900	+4	0.998	+8	0.958	0.918
9	+8	0.895	+18	1.081	+8	0.938	+3	1.011	+10	0.918	0.966
10	+5	0.955	+9	1.036	+22	0.958	+19	1.064	+9	0.936	0.989
11	+21	0.915	+21	1.000	+9	0.936	+21	1.023	+21	0.898	0.954
12	+3	0.938	+8	0.950	+19	0.949	+1	1.012	+19	0.940	0.968
13	+11	0.977	+1	0.991	+21	0.943	+11	1.033	+4	0.991	0.974
14	+2	1.012	+11	1.012	+18	1.012	+10	1.012	+3	1.012	1.012

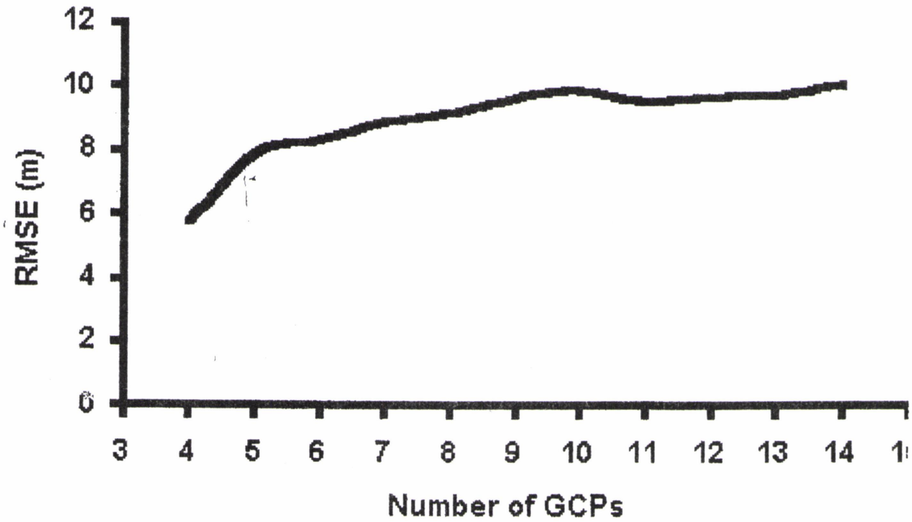

Fig. (4) Standard RMSE as a function of the number of GCPs

CONCLUSIONS

1- The standard RMSE is found to provide good estimates of geometric error when the number of control points is between 10-15 points.

2- The RMSE is affected mainly by the number of GCPs, the distribution of GCPs and the RMSE for each GCPs.

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NOMENCLATURE

AE	Absolute Error
c	Column
GCPs	Ground Control Points
GIS	Geographical Information System
PAN	Panchromatic
r	row
RE	Relative Error
RMSE	Root Mean Square Error
SE	Scale Error
SPOT	Système Pour l' Observation de La Terre
UTM	Universal Transverse Mercator
XS	Multispectral