PERFORMANCE IMPROVEMENTS OF ADAPTIVE FIR EQUALIZER USING MODIEFED VERSION OF VSSLMS ALGORITHM

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ABSTRACT:
In this paper possible improvements in the performance of adaptive Linear Equalizer (LE) and Decision Feed Back Equalizer (DFE) are reported. A modified Least Mean Square (LMS) algorithm incorporating a recursively adjusted adaptation step size based on rough estimate of the performance surface gradient square is proposed. The first proposed algorithm was called Adjusted Step Size LMS (ASSLMS) which used single adjusted step size for all weight coefficients. The second proposed algorithm was called Distributed Step Size LMS algorithm (DSSLMS). This algorithm (i.e. DSSLMS) will distribute the resultant variable step size in an exponential form among all weights of the adaptive filter such that each weight coefficient has its own step size. These proposed algorithms through computer simulation results shows favorable performance than traditional LMS algorithm and another Variable Step Size LMS (called VSSLMS) algorithm in terms of fast convergence time, less miss-adjustment in steady state, and good tracking ability.

KEYWORDS

Linear Adaptive Equalizer, DFE Equalizer, LMS Adaptive algorithm,

Variable Step Size LMS algorithm.

INTRODUCTION

Adaptive equalizer was widely used in digital communication systems in order to reduce or eliminate the channel distortions like additive noise or intersymbol interference ISI before demodulation at the receiver. The simple structure for adaptive equalizer was the Finite Impulse Response filter (FIR) which can be trained by the Least Mean Square adaptive algorithm (LMS). This LMS algorithm, which was first proposed by Widrow and Hoff in 1960, is the most widely used adaptive filtering algorithm (Farhang 1999) This widely applications of LMS algorithm are as a result of its simplicity. Furthermore, it does not require matrix inversion, nor does it require measurements of the pertinent correlation functions (Farhang 1999). But this algorithm suffers from slow convergence since the convergence time of LMS algorithm is inversely proportional to the step size(Widrow and S.Stearns 1985). However if large step size is selected, then fast convergence will be obtained but this selection results in deterioration of the steady state performance (i.e. increased the miss-adjustment (excess error). Also small value of the step size will cause slow convergence but will enhance or decrease the steady state error level(Widrow and S.Stearns 1985).

Numerous modifications of the LMS algorithm have been reported [3, 4, 5, 6, 7, 8, 9 and 10]. In these works, the optimization issue concerning the step size is discussed, and several methods of varying the step size to improve performance of the LMS algorithm especially in time varying environments are proposed. In such environments the step size must be adjusted automatically in order to obtain the following features :

- Adaptive filter must be able to track any change in the system, i.e. to reduce the lag factor which was a decreasing function of the step size.
- Reduce the trade off between the low level of miss-adjustment and fast convergence , i.e. both requirements , must be obtained .
- Reduce the sensitivity of the algorithm to variations in the level of non-stationary, i.e. Low level of miss-adjustment must be obtained when high level of non-stationary occurred.

Optimum step Size can be used but it is undesirable approach since in practical implementation; optimum step size is approximated by trial and error. In addition to the optimum step size cannot track any sudden change in the system environments because it has a fixed value. Some papers used active taps detection techniques with traditional LMS algorithm to enhance the performance of the system. But this technique suffers from overhead operations which is not suitable for hardware implementation. Therefore this paper tries to improve the performance of the LMS algorithm and to get rid of the previous main drawbacks. The proposed algorithms used recursively adjusted adaptation step size based on the performance surface gradient square. The first proposed algorithm was called ASSLMS. Then the obtained step size is distributed among all the weights of the adaptive filter. This proposed algorithm will be refereed to as Distributed Step Size LMS algorithm (DSSLMS) all over in this paper. These proposed algorithms then will be applied to two types of adaptive equalizers which are Linear Equalizer (LE) and Decision Feed Back Equalizers (DFE).

LINEAR AND DECISION FEED BACK EQUALIZER WITH LMS ALGORITHM

Figure (1) shows the classical model of the LE .As shown in this figure there are two modes of operations, namely, the training mode and decision-directed mode (Thamer el al 1997).

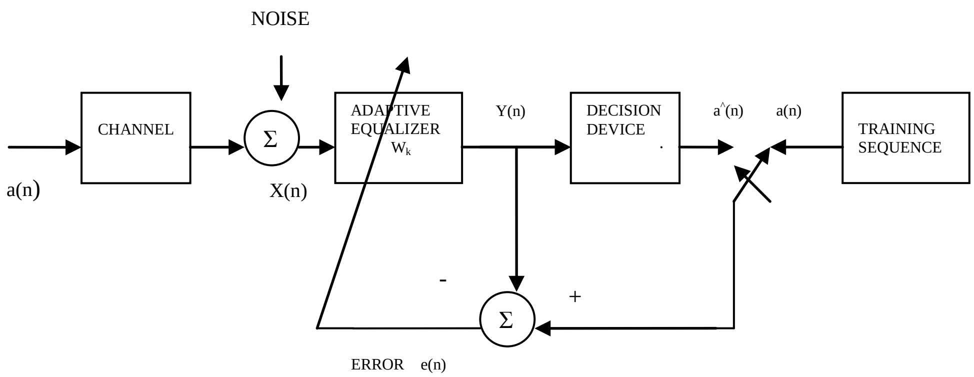


Fig (1) Classical Model of LE

During the training mode, the transmitter generates a data symbol sequence known to the receiver. The receiver therefore, substitutes this known training signal in place of the decision device output. Once an agreed time has elapsed, the decision device output is substituted and the actual data transmission begins. When the training process is completed, the adaptive equalizer is switched to its second mode of operation: the decision-directed mode. In this mode of operation, the error signal is defined by (Thamer el al 1997).

$$e(n) = a^{\wedge}(n) - y(n) \qquad \text{………………} \ (1)$$

Where y (n) is the equalizer output and $a^{\wedge}(n)$ is the final correct estimate of the transmitted symbol $a(n)$. In figure (2), there is a general structure of the adaptive decision feedback equalizer. This figure shows that DFE equalizer consists of two sections, a feed forward section and a feedback section connected together (Thamer el al 1997)

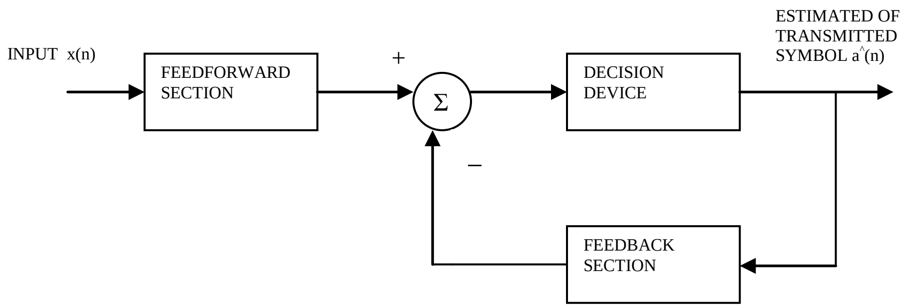


Fig (2) Block Diagram of DFE

The feedback section consists of a tap-delay line filter (such as FIR type) whose taps are spaced at the reciprocal of the signaling rate. The data sequence to be equalized is applied to this section. The feedback section consists of another tapped-delay-line filter whose taps are also spaced at the reciprocal of the signaling rate. The input applied to the feedback section consists of the decision made on previously detected symbols of the input sequence. The function of the feedback section is to subtract out that portion of the ISI produced by previously detected symbols from the estimates of the future samples (Thamer el al 1997). As in the case of the linear adaptive equalizer, the coefficients of the feed forward filter and the feedback filter in a decision-feedback equalizer may be adjusted recursively. Also in the case of a linear equalizer, a training sequence was used to adjust the coefficients of the DFE initially. Upon convergence to optimum coefficients, the system is switched to a decision-directed mode where the decisions at the output of the detector are used in forming the error signal and fed to the feedback filter. This is the adaptive mode of the DFE.

The LMS or Gradient algorithm is used to implement adaptive equalization. It is a stochastic gradient optimization algorithm based on a traditional optimization technique called the Method of Steepest Descent. Let us define the input vector at the equalizer input as $X_n=[x(n), x(n-1),\ldots\ldots,x(n-N-1)]^T$, and the vector of the weight coefficients as $W(n)=[w_0(n)\ w_1(n)\ w_2(n)\ldots w_{N-1}(n)]^T$ of the adaptive filter at an instant n, N is the order of the adaptive FIR filter. Moreover, the signal samples at the equalizer input are of the form:

$$x(n)=\sum_j hn(j)a(n-j)+noise(n) \quad \ldots\ldots\ldots. (2)$$

Where a (n) denotes the n-th data transmitted sample, noise (n) is the additive noise with the variance σ^2, and hn (j) is the channel impulse response. The equalizer output at the n-th iteration instant is:

$$y(n)=W^T(n)X(n) \quad \ldots\ldots\ldots\ldots. (3)$$

The output y(n) is used in estimating the transmitted data symbol a(n-D) , with D denoting to the delay. The equalizer output error at the n-th iteration instant in training mode is:

$$e(n) = y(n) - a(n - D) \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

The weighting coefficients in the LMS algorithm are obtained from the following expression:

$$W(n+1) = W(n) + \mu e(n) X(n) \quad \ldots\ \ldots. \quad (5)$$

Where μ represents the algorithm fixed step size. The output Mean Square Error (MSE) is(Zens 1989.)

$$\varepsilon(n) = E(e^2(n)) = W^T(n) R W(n) + E(a^2(n)) - 2W^T(n) E(X(n) a(n_0 - D)) \qquad (6)$$

With R=E(X (k) X^T (k)). Where E (.) is expected operation and R is defined as the square matrix. The average output MSE after n-th iteration can be expressed as:

$$\varepsilon_{avr} = \varepsilon_{MIN} + E\left(V^T(n) R V(n)\right) \qquad (7)$$

Where ε_{MIN} is the MSE minimum (Zens1989) ,for optimal weighting coefficients vector $W^*(n)$, and V (k) = W (n)-W^*(n) is the weighting coefficients error vector. In the steady state, the MSE above ε_{MIN} is known as the excess MSE.
As shown in (Widrow and S.Stearns1985), the excess MSE (also called miss-adjustment) for LMS algorithm is given by:

$$\varepsilon_{exess} = \frac{1}{2} \mu \sigma_n^2 tr(R) \qquad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

It may be observed that from (Zens1989) that the excess MSE is due to the gradient noise and is proportional to the step size. The step size must be selected to balance the coefficient goals of the fast convergence (large step size) and small steady state error, i.e. small excess MSE (small step size) (Widrow and S.Stearns1985) .

PROPOSED VARIABLE STEP SIZE LMS ALGORITHMS

ASSLMS Algorithm

The proposed algorithm in this paper is called Adjusted Step Size LMS (ASSLMS) algorithm which used variable step size that will be adjusted according to the square of the gradient of the performance surface (i.e. $e_n X_n)^2$. Using the gradient of the performance surface as a guide to adjust the step size was first developed in(Kang and Johnstone 1992) [9]. Their formula for adjusting the step size was(Simon haykin 1983):-

$$\mu_k = \mu_{\max} .(1 - \exp(-\alpha \| e_k X_k \|) \quad \ldots\ldots\ldots.. \ (9)$$

Where α is a constant called damping factor and $\| \|$ is regular norm vector. This algorithm has practical hardware implementation since equation (9) has exponential factor and require an approximate formula. Another formula to adjust the step size was developed in (Simon haykin 1983). The algorithm used in (Simon haykin 1983) was called Variable Step Size LMS algorithm (VSSLMS) and their formula is:-

$$\mu_n = \alpha\mu_n + \delta e_n^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \ldots (10)$$

Where $0\langle\alpha\langle 1$ and $\delta\rangle 0$, then:-

$\mu_{n+1} = \mu_{\max}$ if $\mu_{n+1}\rangle\mu_{\max}$, or $\mu_{n+1} = \mu_{\min}$ if $\mu_{n+1}\langle\mu_{\min}$, …….. (11)

Otherwise $\mu_{n+1} = \mu_{n+1}$

Where $\mu_{\min}$ is chosen to provide minimum level of miss-adjustment at steady state, and $\mu_{\max}$ ensures the stability of this algorithm (Simon haykin 1983). So this paper proposes to use the square of the gradient estimation $(e_n X_n)^2$ into equation (10) instead of using the square of the error. Then equation (10) will be modified to be as follow(Hulya 2004):-

$$\mu_n = \alpha\mu_n + \delta(e_n.X_n)^2 \qquad \ldots\ldots\ldots (12)$$

This proposed algorithm (ASSLMS) algorithm regard as modified version of the VSSLMS algorithm [12]. Involving the term (X_n) which represents the input signal in the updating step size formula in addition to error factor is favorite choice in order to speed up the estimation and adaptation process.

DSSLMS ALGORITHM

By theory for a stationary channel, the length of the window which tracks the channel is the length of the number of samples. However, channels that have a time- varying nature require a window which must adjust to the recent channel. There are several fundamental considerations that must be understood in the implementation of the LMS algorithm for time- varying channels. First of all, the LMS algorithm uses the most recent channel estimation error into equation (5). This, in turn, means it is severely affected by time-varying channels. Secondly, the most of the impulse response of the channel has decreasing values from higher values to smaller values. Also if all weight coefficients adjusted in equal form will lead to slower the convergence rate of the algorithm. Therefore this paper proposed to adjust the weight coefficients in non uniform manner such that the amount of adjusting the weight coefficients will be in decreasing manner. The only parameter that can perform this function was the variable step size (see equation (5)). This idea can be implemented by applying a sliding window to the obtained step size in equation (12) (Hulya 2004). Such that equation (12) can be distributed among all weight coefficients. There are several types of windows that can be used to do this idea. These can be rectangular, triangular, exponential, and such. In this paper, an exponential sliding window is considered due to its superiority over the other types of windows for the most types of channels. Then the step size that obtained from (12) will be distributed among the weights of the adaptive filter. Such that each weight coefficient has independent step size $\mu_{i(n)}$ where i=1,2,….,N (order of the FIR filter), and n is the iteration instants. The distribution of the step size μ_{n+1} will be according to the following:

$$\mu_{i(n+1)} = \xi^{i-1}\mu_{(n+1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

Where ζ is constant, determine the curvature of exponential distribution of μ_{n+1} among the weights of the adaptive filter. The length of this window is the same as the number of the weight coefficients (i.e. N). Such that each weight will be adjusted independently according to the following:

$$w_{i(n+1)} = w_{i(n)} + \mu_{i(n)} e_n y_{i(n)} \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

This proposed algorithm is called Distributed Step Size LMS (DSSLMS) algorithm. This algorithm adapts the filter weights having larger values by amounts more than the weights having smaller values at each iteration. Therefore the weights of coefficients for this proposed algorithm will convergence faster and estimate the inverse impulse response of the channel very quickly.

Simulation Results

<u>LE using Channel A</u>

In this case LE was simulated using different algorithms. The channel used here is called channel A which has a spectral null in the middle frequency region. The impulse response of the channel A is [0.2 -0.15 1.0 0.21 0.03]] . The order of FIR adaptive filter for all simulation was 11 taps and signal to noise ratio was 26 dB, the additive noise was Gaussian noise with mean zero, and variance $\sigma^2 = 0.001$. The training samples were 1000 samples then the adaptive process is switched to decision mode. Figure (3) shows the learning curve for this case with different algorithms. The optimum step size for LMS algorithm was chosen by trial and error to be 0.06.

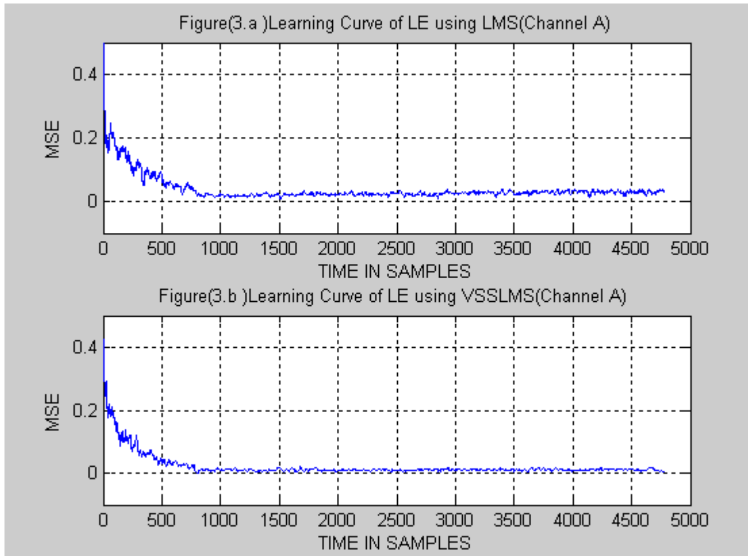


Fig (3)

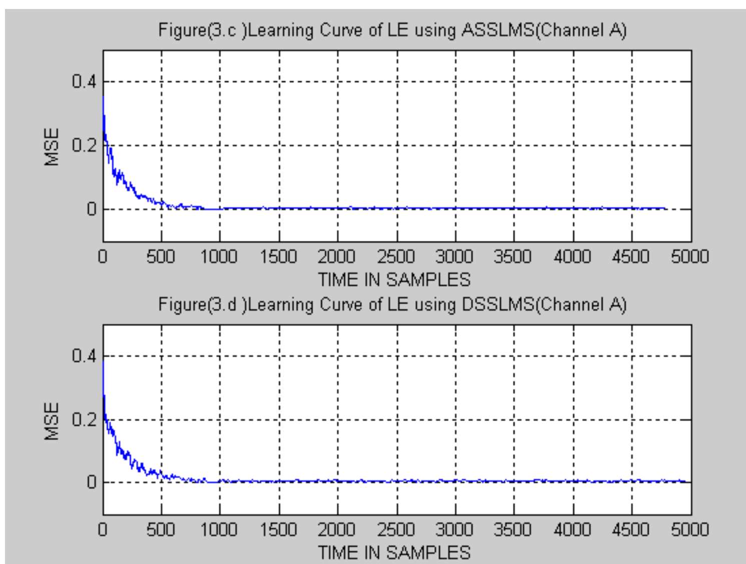


Fig (3) Learning Curves for LE using Channel A

The optimum values of μ_{max} and μ_{min} was chosen to be 0.06 and 0.0001 respectively for VSSLMS, ASSLMS and DSSLMS algorithms. The values of α and δ was chosen to be 0.97 and 0.001 respectively for all algorithms. The ζ for DSSLMS algorithm was equal to 0.05. As shown in figures (3.c and 3.d) the proposed algorithms have fast convergence time than LMS and VSSLMS algorithms (figures (3.a and (3.b)). Also these proposed algorithms have less miss-adjustment (excess error) in the steady state region than LMS and VSSLMS algorithms.

DFE USING CHANNEL A

In this case DFE is used with the same channel as used previously in LE. The order of feed forward section was 5 and for feedback section was 3. The optimum step size for LMS algorithm was chosen by trial and error to be 0.008. The optimum values of $\mu_{\max}$ and $\mu_{\min}$ was chosen to be 0.008 and 0.0000001 respectively for VSSLMS, ASSLMS and DSSLMS algorithms. The values of α and δ was chosen to be 0.97 and 1e-7 respectively for all other algorithms. The ζ for DSSLMS algorithm was equal to 0.05. Figure (4) shows the learning curves for this case.

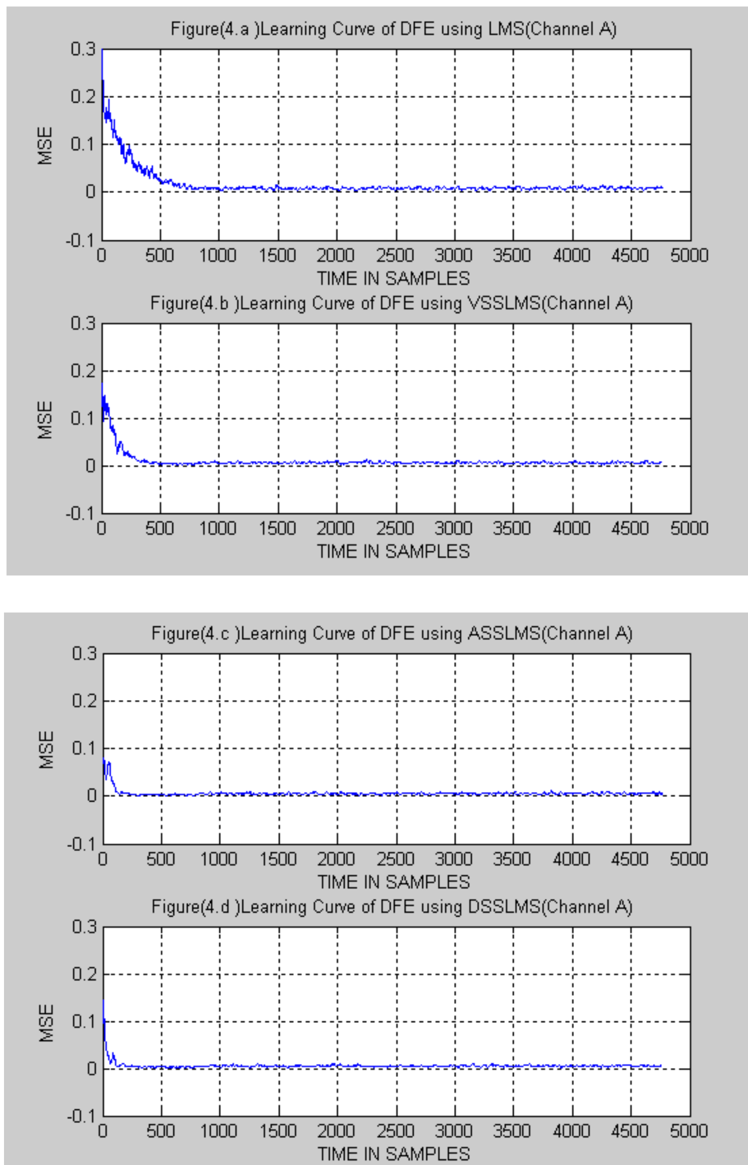


Fig (4) Learning Curves for DFE using Channel A

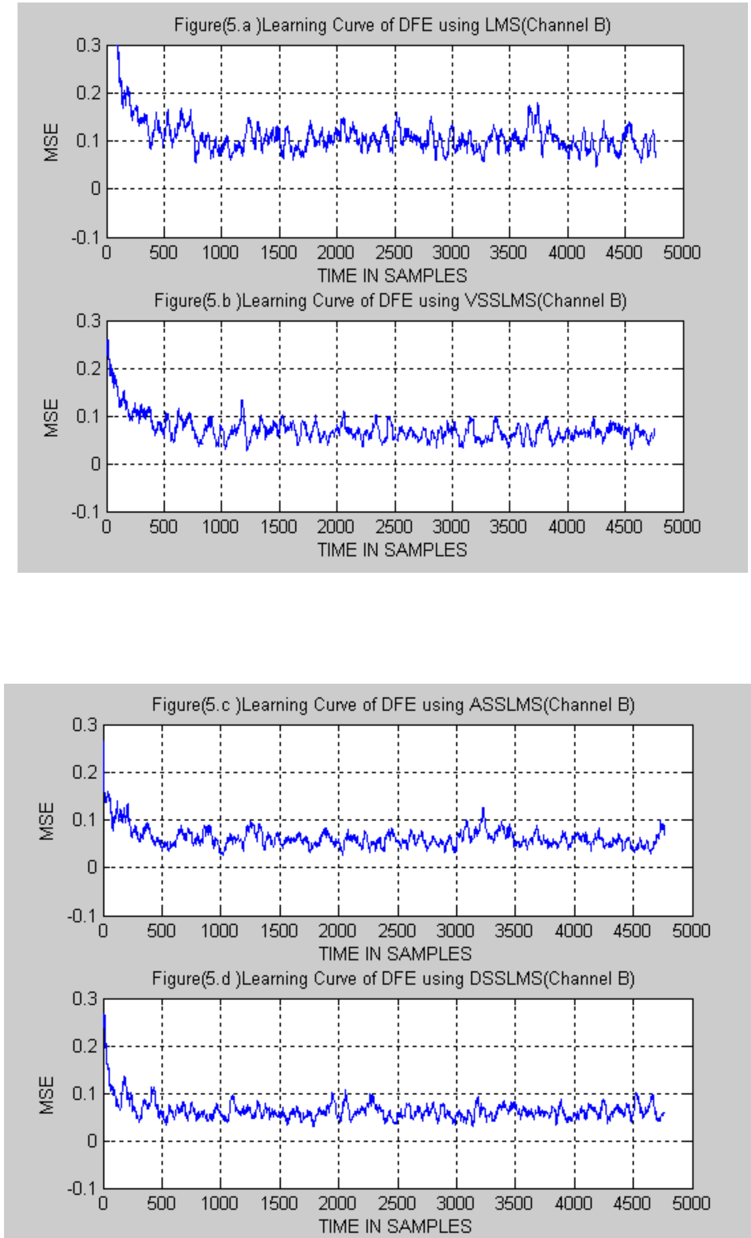
As shown in figure (4) the performance of adaptive equalizer was enhanced using different algorithms with the same channel compared with that obtained using LE type. Also here the performance of the proposed algorithms (figures (4.c and 4.d)) was also better than LMS and VSSLMS algorithms (figures (4.a and 4.b)) in terms of fast convergence and less miss-adjustment in the steady state region.

DFE USING CHANNEL B

In this case another channel response is used which has the following impulse response [1 0 1.2 0 0 -0.3]. This channel is called channel B which represent an example of non-minimal phase channel and it is characterized by a multipath that is stronger than the channel A. Figure (5) shows the learning curve of different algorithms using DFE type. The optimum step size for LMS algorithm was chosen by trial and error to be 0.001. The values of μ_{max} and μ_{min} was chosen to be 0.001 and 0.0000001 respectively for VSSLMS, ASSLMS and DSSLMS algorithms. The values of α and δ was chosen to be 0.97 and 1e-7 respectively for all algorithms. The ζ for DSSLMS algorithm was equal to 0.005. As shown in the figure (5) the good performance of the proposed algorithms (figures (5.c and 5.d) compared of the other algorithms (figures (5.a and 5.b)).

CONCLUSIONS

In this paper possible improvements of the adaptive LE and DFE are proposed by using two modified versions of VSSLMS algorithm. The first one was called ASSLMS algorithm and the second was called DSSLMS algorithm. These proposed algorithms used the square of the gradient estimation $(e_n X_n)^2$ instead of error square alone. Involving the term (X_n) which represents the input received signal in the updating step size formula is favorite choice in order to speed up the estimation and adaptation process. Also using individual step size (DSSLMS algorithm) for each weight coefficients was good idea to speed up and reduce the miss-adjustment in the steady state region. Through simulation results that used two different channels one can see the good performance of the proposed algorithms compared with traditional LMS and VSSLMS algorithm in terms of fast convergence and less miss-adjustment in the steady state region.



Fig(5) Learning Curves for DFE using Channel B

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