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THE INFLUENCE OF TENSION STIFFNENING MODELS ON THE DYNAMIC ANALYSIS OF REINFORCED CONCRETE SLABS

Dr.K.S. Mahmood
Professor
Baghdad University

Dr.A.A. Abdul-Razzak
Assistant Professor
Mosul University

M.K. Kasim
Assistant Professor
Tikrit University

ABSTRACT

In the present work, the finite element method has been used to investigate the behavior of reinforced concrete slabs subjected to dynamic loads. Eight-node Serendipity degenerated elements have been employed. This element is based on isoperimetric principles with modifications, which relax excessive constraints. The modifications include reduced order integration to overcome the shear locking.

A layered approach is adopted to discretize the concrete through the thickness. Both an elastic-perfectly plastic and strain hardening plasticity approaches have been employed to model the compressive behavior of the concrete. A tensile strength criterion is used to initiation of crack and a smeared fixed crack approach is used to model the behavior of the cracked concrete. Five models are used to consider the effect of tension stiffening in the cracked concrete.

Implicit Newmark with corrector-predictor algorithm is employed for time integration of the equation of motion.

Several examples are analyzed using the proposed model. The numerical results showed good agreement with other sources.

ألخلاصة

استخدمت طريقة العناصر المحددة لدراسة التصرف اللاخطي الديناميكي للبلاطات الخرسانية المسلحة. استخدمت العناصر ثمانية العقد في هذا البحث. هذه العناصر تعتمد على مبدأ توحيد المتغيرات مع اعتماد مجموعة من التعديلات التي تخفف القيود الإضافية. هذه التعديلات تتضمن قواعد التكامل المخفض وذلك لتفادي حالة القفل بالقص

استخدم مبدأ الطبقات لتمثيل الكونكريت وحديد التسليح على امتداد ارتفاع مقطع البلاطة. و مثل سلوك الخرسانة في حالة الانضغاط كمادة مرنة- تامة اللدونة أو كمادة مرنة مع انفعالات لدنة متصلدة. تم استخدام سلوك مقاومة الخرسانة للشد للتنبؤ بحدوث تشقق واستخدم اسلوب الشق الثابت لتمثيل الخرسانة المتشققة مع تثبيت حدود مقاومة تصلد الشد للتنبؤ بحدوث الشق. أخذت في الاعتبار خمسة نماذج لتمثيل تأثير صلابة الشد في الخرسانة المتشققة.

تم أعتماد طريقة نيومارك الضمنية مع طريقة التنبؤ-التصحيح لحل معادلة الحركة التفاضلية.

تم حل عدة أمثلة أظهرت النتائج توافقا" جيدا" مع النتائج المستحصلة بالطرق الأخرى

KEY WWORDS

Dynamics, Finite elements, Nonlinear analysis, Reduced integration, Reinforced concrete slabs, Tension stiffening models.

NOTATION

B	Strain-nodal displacement matrix.
B_b	Bending strain- nodal displacement matrix.
B_s	Transverse shear strain- nodal displacement matrix.
D	Flexural (or Shear) Rigidities.
D_b	Flexural rigidities.
D_s	Shear rigidities.
d	Displacements.
$\dot{d}$	Velocities.
$\ddot{d}$	Accelerations
E_c	Initial modulus of elasticity of concrete.
E_s	Modulus of elasticity of steel.
E_s'	Second modulus of elasticity of steel (hardening coefficient).

f_c'	Uniaxial compressive strength of concrete.
f_t'	Uniaxial tensile strength of concrete.
Gc	Fracture energy of concrete.
K	Elastic stiffness matrix.
K^*	Effective stiffness matrix.
K_T	Tangential stiffness matrix.
M_x, M_y, M_{xy}	Generalized stress components(moments).
N	Shape function.
ρ	Mass density.
Q_x, Q_y	Generalized stress components (shear forces).
R.C.	Reinforced concrete.
β, γ	Newmark's integration parameters.
ε_x, ε_y	Strains in x and y-direction.
ε_b	Bending strain tensor.
ε_s	Transverse shear strain tensor.
ε_u	Crushing strain.
ν	Poisson's ratio.
σ_x, σ_y	Normal stress components.

INTRODUCTION

The finite element method was introduced for structural analysis many years ago. It has been recognized as a powerful and widely used approach for analysis of R.C. structures [1,5,10,11,20].

Farag and Leach [9] analyzed reinforced concrete structures under transient dynamic loading. The three dimensional isoparametric element with 20 nodes is used to simulate the concrete. The smeared approach is used to represent the reinforcement in the elements. A viscoplastic model is used to simulate the concrete in compression with two surfaces, the failure surface expressed as a function of the first and the second deviatoric stress invariant.

Shirai et al. [21] investigated and proposed a method to improve impact resistance of reinforced concrete plates against projectile impact, and the damage of double-layered reinforced concrete plates was examined experimentally and simulated analytically.

Sziveri et al. [22] analyzed reinforced concrete plates under transient dynamic loading. A layered triangular element was considered for determining the dynamic transient nonlinear response of reinforced concrete plates.

Manjuprasad et al. [18] analyzed reinforced concrete rectangular slabs and containment shell subjected to seismic load. A 20-noded three-dimensional, solid isoparametric finite element is used for spatial discretisation.

Agbossou and Mougin [3] used a layered approach to the non – linear static and dynamic analysis of rectangular reinforced concrete slabs. The proposed model considers the slab as a layered structure and leads to explicit relations, which account for the macroscopic linear and non-linear behavior of slabs on lines of simple supports.

Xu and Lu [24] analyzed reinforced concrete plates subjected to blast loading using three-dimensional nonlinear finite element. Pseudo-tensor concrete/geological model is employed to model the concrete, taking into account the strain rate effect. A strain rate multiplier is used to modify the dynamic yield strength of the concrete material.

BASIC THEORY

By Mindlin thick plate element, the variation of displacements and rotations are given by the expression as[12]:

$$[w,\ \theta_x,\ \theta_y\ ]^T = \sum_{i=1}^{n} N_i d_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The plate curvature-displacement and shear strain-displacement relations are then written as:

$$\varepsilon_b = \sum_{i=1}^{n} B_{bi} d_i \quad , \quad \varepsilon_s = \sum_{i=1}^{n} B_{si} d_i \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

The moment-curvature and shear force–shear strain relations can be written as:

$$[M_x,\ M_y,\ M_{xy}\ ]^T = D_b \varepsilon_b \quad , \quad [Q_x,\ Q_y\ ]^T = D_s \varepsilon_s \qquad \text{……………(3)}$$

Based on the energy minimization, the elastic stiffness and the mass matrices can be determined from the relations:

$$[K] = \int_v [B]^T [D][B] dv \qquad \text{………………………………………(4)}$$

$$[M] = \rho \int_v [N]^T [N] dv \qquad \text{………………………………………(5)}$$

REDUCED INTEGRATION

Based on the work of Dohestry et al. [8] to eliminate the parasitic shear on plane quadrilateral elements, the implementation of the reduced integration for the degenerated shell element was firstly introduced by Zeinkiewics et al. [25]. Then many papers about the reduced integration technique have been published[13,19].

Using the full integration rule a shear-locking problem will appear. Therefore reduced integration rule (2x2) for 8-node Serendibity element is applied in this study to overcome this problem.

MATERIAL MODELING

Based on the flow theory of plasticity, the nonlinear compressive behavior of concrete is modeled. Adopting Kupfer's results[16], the yield condition for the slab can be written in term of the stress components as[10]:

$$f(\sigma) = \left\{1.355\left[(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y) + 3(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)\right] + 0.355\sigma_o(\sigma_x + \sigma_y)\right\}^{0.5} = \sigma_o \qquad \text{…...(6)}$$

where (σ_o) is the equivalent effective stress taken as the compressive strength ($f_c^{'}$) which is obtained from uniaxial test.

The crushing of concrete is a strain control phenomenon. A simple way of incorporation in the model is to convert the yield criterion of stresses directly into the strains, and the crushing condition can be expressed in terms of the total strain components as:

$$1.355\ \left\{(\varepsilon_x^2+\varepsilon_y^2-\varepsilon_x\varepsilon_y)+0.75\ (\gamma_{xy}^2+\gamma_{xz}^2+\gamma_{yz}^2)\right\}+0.355\varepsilon_u(\varepsilon_x+\varepsilon_y)=\varepsilon_u^2 \quad ....(7)$$

The concrete is assumed to lose all its characteristics of strength and rigidity when (ε_u) reaches the specified ultimate strain.

Tension stiffening, illustrates that the cracked reinforced concrete as a result of bond mechanisms carries, between cracks, a certain amount of tensile stress normal to the cracked plane. The concrete between cracks adheres to the reinforcing bars and contributes to the overall stiffening of the structure. The strain softening or descending branch of the stress strain curve of concrete in tension, in one form or another, may be used to simulate this "tension stiffening" effect. In the present study five types of tension stiffening models are used.

(i) Model (1) linear model. In this model a linear gradual release of the concrete stress component normal to the cracked plane is assumed. as shows in Fig (1).

(ii) Model (2) tension stiffening (parabolic model). The relationship between stress and strain after cracking is given in Fig(2) as cited in reference [1,4,5] :

$$\sigma = f_t^{'}\ \left(\frac{\varepsilon_1-\varepsilon_m}{\varepsilon_t-\varepsilon_m}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

The maximum tensile strain (ε_m) can be evaluated from equation (9) as:

$$\varepsilon_m = \frac{3G_c}{h_c\ f_t^{'}}+\varepsilon_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

The typical values for (G_c) lie in the range $(200 f_t^{'2}/E_c)$ to $(400 f_t^{'2}/E_c)$. In the present study (G_c) is taken equal to 100 N/m.

(iii) Model (3) is used in the present study, and is given by Collins and Vecchio [23] and by Collins and Mitichell [7], see Figure(3)

$$\sigma = \frac{\alpha_1\alpha_2\ f_t^{'}}{1+\sqrt{500\varepsilon_1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

where

α_1= factor accounting for bond characteristics of reinforcement equal (1.0) for deformed reinforcing bars and (0.7) for plain bars, wires, or bonded strands and (0) for unbounded reinforcement.

α_2= factor accounting for sustained or repeated loading equal to (1.0) for short-term monotonic loading and (0.7) for sustained and or repeated loading.

It must be noted that the stress in the reinforcement at a crack location cannot exceed the yield stress of reinforcement.

(iv) Model (4) This model is developed by Abrishami [2] tack the splitting effect into account. After cracking the maximum tension is limited to the yield force of the reinforcement. To account for the detrimental effects of the influence of splitting cracks on the tension stiffening, the factors $(\alpha_1, \alpha_2, \alpha_3)$ [2] are including, see Fig (4).

$$\sigma = \frac{\alpha_1 \alpha_2 \alpha_3 \; f_t'}{1+\sqrt{500\varepsilon_1}} \quad \text{...(11)}$$

where

α_1 and α_2 are defined as in equation (10)

α_3= 1.0 for $c/d_b > 2.5$

α_3= $0.8c/d_b - 1$ for $1.25 \le c/d_b \le 2.5$

α_3 $=0$ for $c/d_b < 1.25$

where (c) is the concrete cover to the centroid of steel and (d_b) is the diameter of the bar, (f_t') is the tensile strength of concrete and (ε_1) is the strain in concrete in direction (1). Both models (3 & 4) have a limited value of $\varepsilon_m = \varepsilon_y$

(v) Model (5) is a linear model. This model is developed by Johanson [5] and adopted in the present analysis where by assuming unloading and reloading of cracked concrete, the behavior is linear which is shown in Fig. (5)

CRACKED SHEAR MODULUS

In the present study, the cracked shear modulus is assumed to be a function of the current tensile strain. In this approach a value of $(G^{'})$ linearly decreasing with the current tensile strain is adopted by Cedolin and Deipoli [6] and used by many investigators [1,5,10] .

For concrete cracked in direction 1.

$$G_{12}^{'} = 0.25G(1-\varepsilon_1 / 0.004) \quad \text{for} \quad \varepsilon_1 < 0.004$$
$$G_{12}^{'} = 0 \quad \text{for} \quad \varepsilon_1 > 0.004$$
$$G_{13}^{'} = G_{12}^{'}$$
$$G_{23}^{'} = \frac{5}{6}G \quad \text{……………………………….(12)}$$

where (G) is the uncracked shear modulus and (ε_1) is the tensile strain in direction (1). For concrete cracked in both directions:

$$G_{13}^{'} = 0.25G(1-\varepsilon_1 / 0.004) \quad \text{for} \quad \varepsilon_1 < 0.004$$
$$G_{13}^{'} = 0 \quad \text{for} \quad \varepsilon_1 > 0.004$$
$$G_{23}^{'} = 0.25G(1-\varepsilon_2 / 0.004) \quad \text{for} \quad \varepsilon_2 < 0.004 \quad \text{……..(13)}$$
$$G_{23} = 0 \quad \text{for} \quad \varepsilon_2 > 0.004$$
$$G_{12}^{'} = 0.5G_{23}^{'} \quad \text{for} \quad G_{23}^{'} < G_{13}^{'}$$

NEWMARK METHOD

The Newmark method as cited in reference[11], and adopted in this work, is an extension of the linear acceleration method. The dynamic equilibrium equation is linearized and written at time t_{n+1} as:

$$M\ddot{d}_{n+1} + C\dot{d}_{n+1} + Kd_{n+1} = f_{n+1} \quad \text{……………………………………..(14)}$$

and $$[C] = c\int_v [N]^T [N] dv \quad \text{………………………………………(15)}$$

where c is a damping coefficient (per unit volume).

The following assumptions on the variation of displacements and velocities are made within a typical time step:

$$d_{n+1} = d_n + \Delta t \dot{d}_n + \frac{\Delta t^2}{2}[(1-2\beta)\ddot{d}_n + 2\ \beta\ \ddot{d}_{n+1}] \quad \text{.....................(16)}$$

$$\dot{d}_{n+1} = \dot{d}_n + \Delta t[(1-\gamma)\ddot{d}_n + \gamma\ \ddot{d}_{n+1}] \quad \text{.....................(17)}$$

The Newmark family of direct integration includes, as particular cases, many well known integration schemes.

In the present work an unconditionally stable time stepping scheme is adopted with $\gamma = 0.5$ and $\beta = 0.25$

Huang[12] and Hughes et al[14] have developed a predictor-corrector form of the Newmark method which is most suitable for nonlinear transient analysis.

The Newmark formulas can be written in terms of predictor and corrector values as

$$d_{n+1} = d_{n+1}^p + \Delta t^2 \beta\ \ddot{d}_{n+1} \quad \text{.....................(18)}$$

$$\dot{d}_{n+1} = \dot{d}_{n+1}^p + \Delta t\ \gamma\ \ddot{d}_{n+1} \quad \text{.....................(19)}$$

with predictor values given as

$$d^p{}_{n+1} = d_n + \Delta t \dot{d}_n + \frac{\Delta t^2}{2}(1-2\beta)\ddot{d}_n \quad \text{.....................(20)}$$

$$\dot{d}^p{}_{n+1} = \dot{d}_n + \Delta t(1-\gamma)\ddot{d}_n \quad \text{.....................(21)}$$

The terms d_{n+1} , $\dot{d}_{n+1}$ are corrector values and $d^p{}_{n+1}$, $\dot{d}^p{}_{n+1}$ are the predictor values. The corrector values for the acceleration values can be obtained from equations (18) and (19) as:

$$\ddot{d}_{n+1} = (d_{n+1} - d_{n+1}^p)/(\beta \Delta t^2) \quad \text{.....................(22)}$$

Substituting equations(18), (19) and (22) into equation(14) an effective static problem is formed in terms of unknown Δd where:

$$K^* \Delta d = \psi \quad \text{.....................(23)}$$

and where the effective stiffness matrix is

$$K^* = M/(\beta\ \Delta t^2) + \gamma\ C_T/(\beta\ \Delta t) + K_T \quad \text{.....................(24)}$$

and the residual forces are

$$\psi = f_{n+1} - M\ddot{d}^{p}_{n+1} - C_T\dot{d}^{p}_{n+1} - p(d^{p}_{n+1}) \qquad \text{……………………………(25)}$$

where $p(d) = Kd$ …………………………..…………..(26)

When solving nonlinear problems, the linearization makes it necessary to perform iterative correction to Δd to achieve equilibrium at time $t+\Delta t$. A Newton-Raphson type scheme is used in this work.

NUMERICAL EXAMPLES

Example(1): Clamped rectangular R.C. slab subjected to a jet force

The clamped reinforced concrete slab shown in Fig.(6) is subjected to a jet force at the center. The percentage of reinforcement placed near the upper and lower surfaces in each direction is 1.5% .

From symmetry only one quarter of the slab is considered. The finite element mesh is shown in Fig.(7). Nine elements with six concrete layers and four steel layers are used in the thickness direction.

The selected time step is approximately 1/25 of the elastic fundamental period. Thus the time step is nearly 0.001 second. The material properties of the concrete and steel are given in Table(1).

The dynamic response for different tension stiffening models (for cracking strains 0.00015 and 0.0002) are shown in Figs.(8-9).(assume deformed bar of diameter 25mm is used). Numerical results are in good agreement with reference[5,18].

Example(2): Clamped circular R.C. slab

The clamped reinforced concrete slab shown in Fig.(10) is subjected to a uniformly distributed load of intensity 0.14 N/mm^2. The slab has a radius of 10m and a thickness of 1m. The load is applied with a rise time equal to half of the elastic fundamental period (T=0.06 second). The materials properties are given in Table(2).

The percentage of reinforcement placed near the upper and lower surfaces in the radial and tangential directions is 1% .

From symmetry only one quarter of the slab is considered. The finite element mesh is shown in Fig.(11). Eight elements with six concrete layers and four steel layers are used in the thickness direction.

The selected time step is approximately 1/100 of the elastic fundamental period this is nearly 0.0005 second.

The dynamic response for different tension stiffening models (for cracking strains 0,00015 and 0.0002) are shown in Figs.(12-13). Numerical results are in good agreement with reference[5,11,17].

CONCLUSIONS

A finite element technique has been used successfully for the nonlinear dynamic analysis of reinforced concrete slabs. No locking was observed in the results due to adopting the 8-node Serendipity element with reduced integration.

An good agreement is found between the present results and other source results throughout the entire structural response. This demonstrates the effectiveness of the proposed element and the solution procedure.

Model(2) gives lower deflection and less amplitude compared to other models. The difference in the central displacement for different tension stiffening models is due to difference post-cracking energy from the different models.

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Table(1) Material properties for simply supported R.C. beam of example(1)

E_c MPa	ν	f_c' MPa	ε_u	cracking strain	ρ N.sec^2/mm^4	E_s MPa	f_y MPa
28000	0.2	35.0	0.0035	0.00015 & 0.0002	0.245E-8	200000	460

Table(2) Material properties for clamped R.C. circular slab of example(2)

E_c MPa	ν	f_c' MPa	ε_u	cracking strain	ρ N.sec^2/mm^4	E_s MPa	f_y MPa
280000	0.2	35.0	0.0035	0.00015 & 0.0002	0.245E-8	210000	460

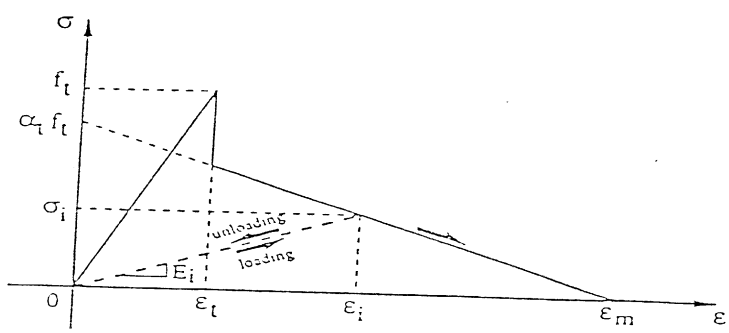
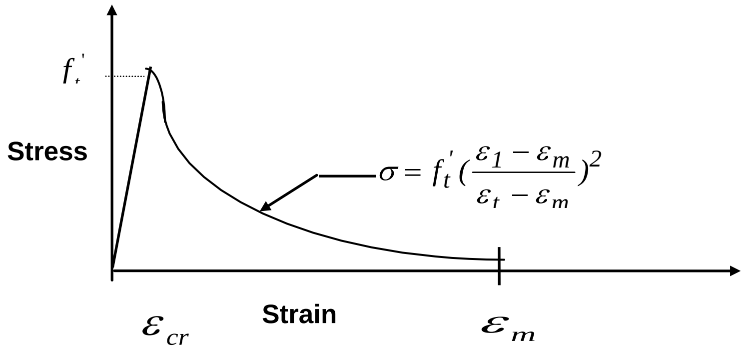
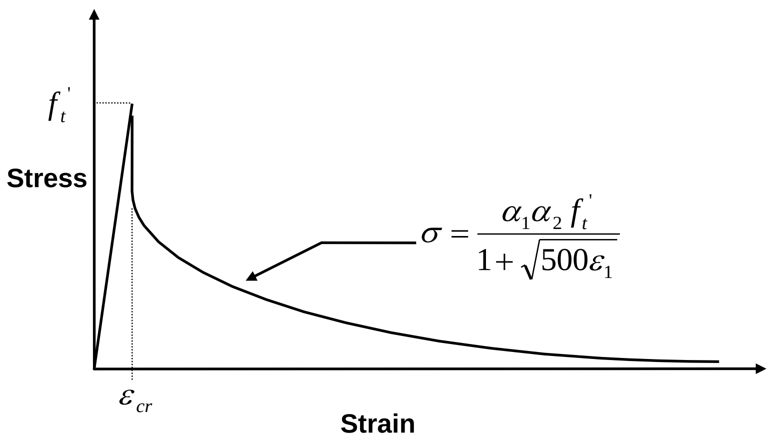


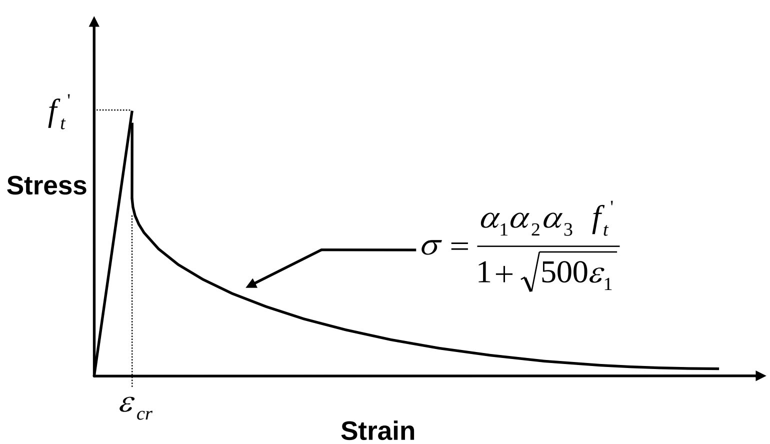
Fig. (1) Loading and unloading behavior of cracked concrete illustrating tension stiffening behavior[14].



Figure(2) Tension softening (parabolic model), model 2.



Figure(3) Tension stiffening for model (3)



Figure(4) Splitting crack effect on tension stiffening model (4)

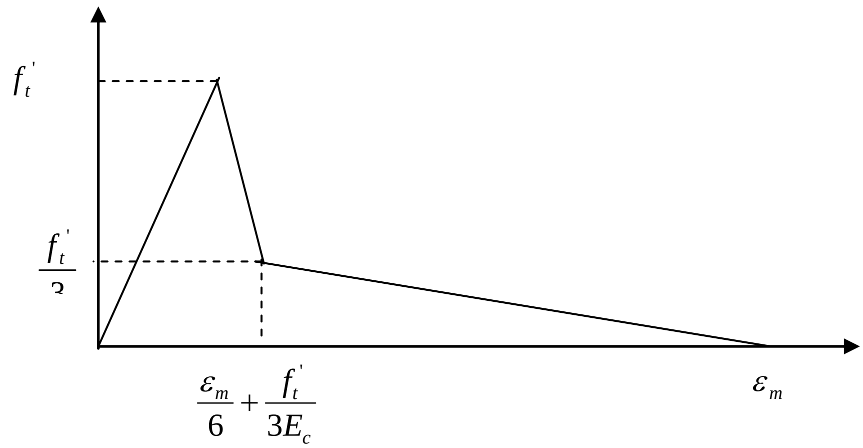
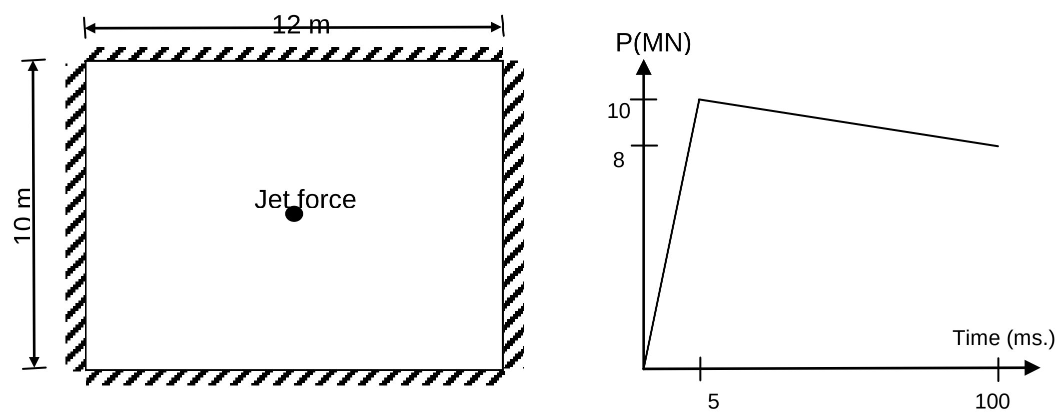
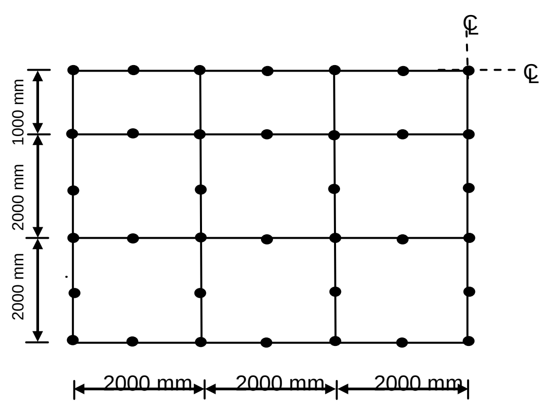


Figure (5) Tension stiffening for model (5).



Figure(6) Loading and geometry of clamped reinforced concrete slab.



Figure(7) Finite element mesh for example (1).

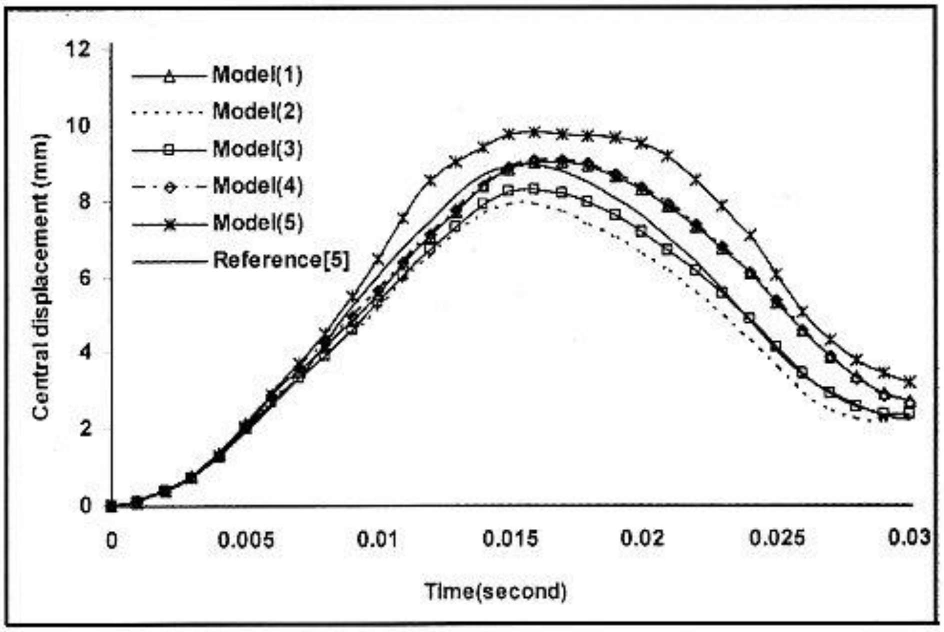


Fig.(8) Nonlinear dynamic response of example(1) (cracking strain 0.00015) for different tension stiffening models.

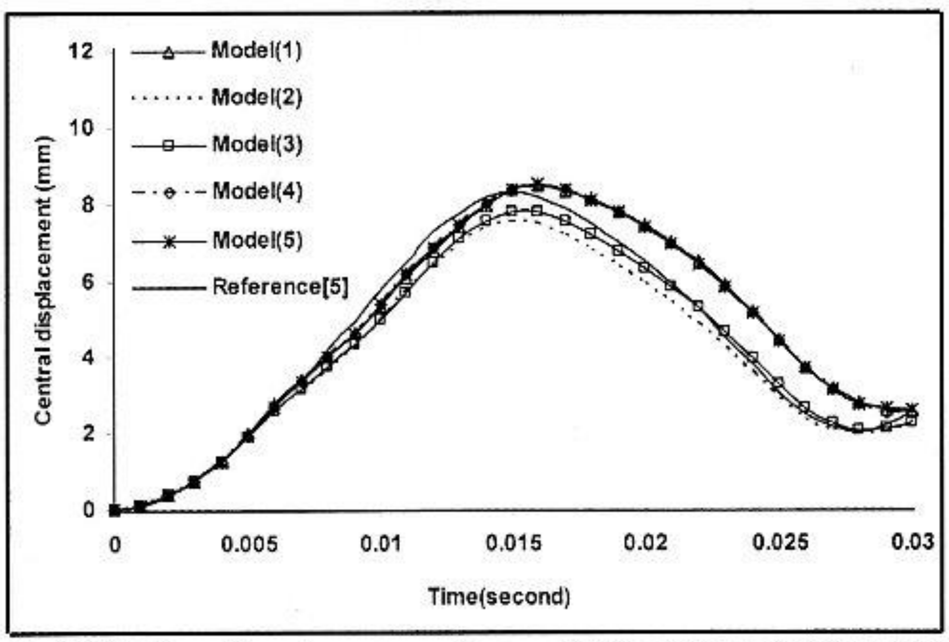


Fig.(9) Nonlinear dynamic response of example(1) (cracking strain 0.0002) for different tension stiffening models.

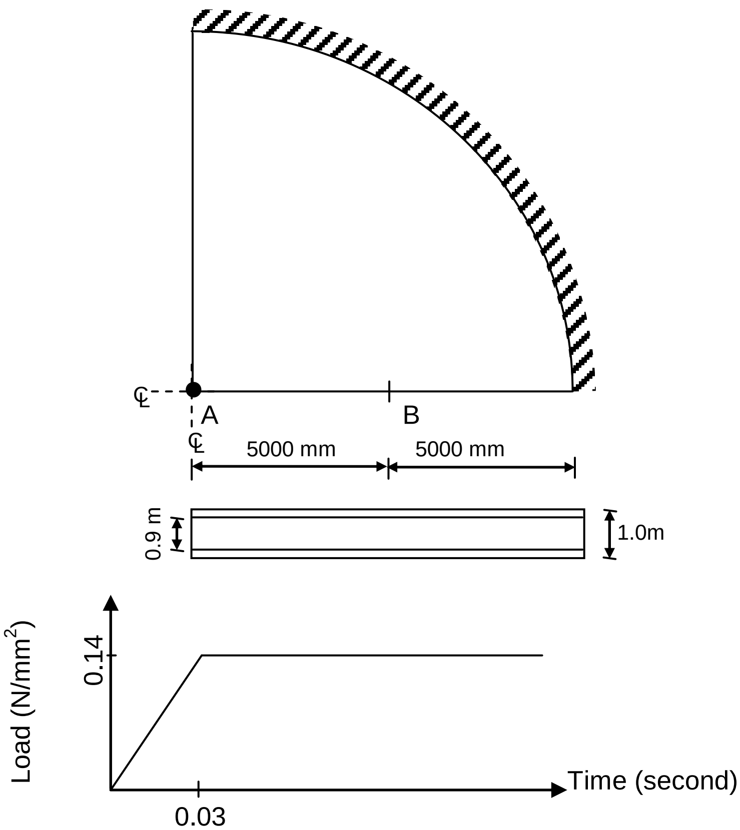


Figure (10) Geometry and Load-time history for example (2)

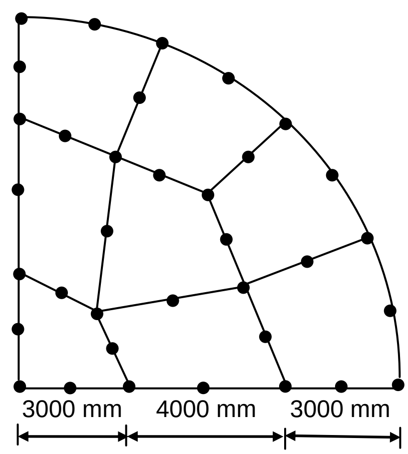


Figure (11) Finite element mesh for example(2).

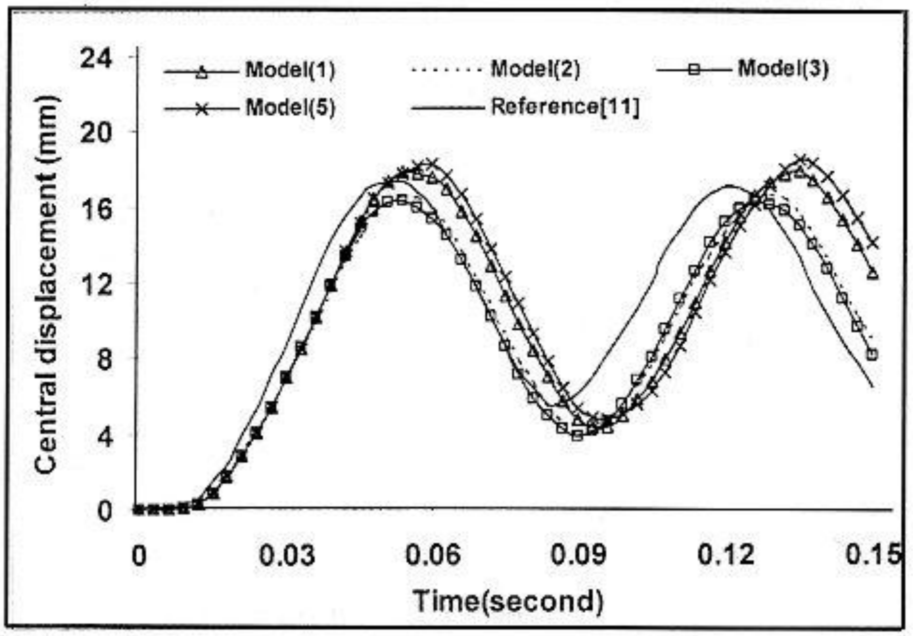


Fig.(12) Nonlinear dynamic response of example(2) (cracking strain 0.00015) for different tension stiffening models

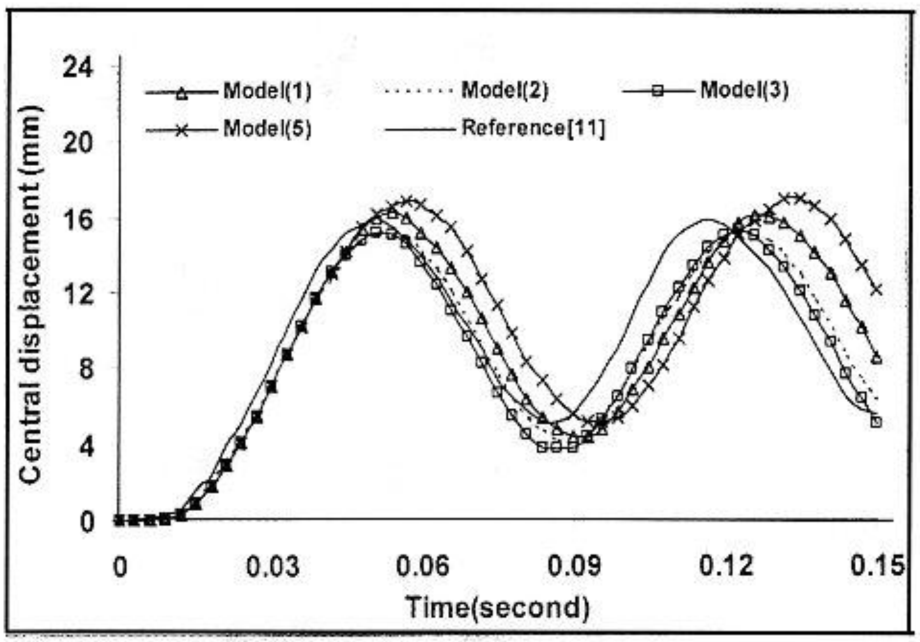


Fig.(13) Nonlinear dynamic response of example(2) (cracking strain 0.0002) for different tension stiffening models