Number 1 Volume 11 March 2005 Journal of Engineering

GLOBAL BUCKLING LOAD OF STEEL COLUMNS STRENGTHENED BY FIBER REINFORCED POLYMER

Abbas Abdel-Majid Allawi
Assistant Lecturer
Engineering College, Baghdad University

ABSTRACT

The need for strengthening structural members is well known and research is progressing in this field. In recent years the use of fiber reinforced polymers (FRP) for strengthening has shown to be a efficient method both regarding structural performance and economical aspects. However, most of the research in this field has been undertaken on concrete of old and damaged structures and for flexural and shear strengthening. So, this paper presents axially loaded steel columns strengthened for increased load capacity and improved stability. The topic is studied theoretically. The theory covers analytical method, and a numerical finite elements (FE) analysis. Different types of most common commercial FRP systems have been examined and used in this study.

الخلاصة

إن الحاجة إلى تقوية الأعضاء الإنشائية معروفة و البحوث مستمرة في التطور في هذا المجال. في السنوات الأخيرة تبين إن طريقة استعمال قماش البوليمر المسلح بالالياف لأغراض التقوية فعالة من خلال زيادة الكفاءة الإنشائية و من الناحية الاقتصادية. معظم البحوث في هذا المجال أجريت على الخرسانة المسلحة القديمة والمتضررة لتحسين و زيادة مقاومة القص و الانحناء . في هذا البحث تم دراسة استخدام هذه الألياف لتقوية الأعمدة الحديدية المحملة محورياً و تحسين إستقراريتها بصورة نظرية. الدراسة النظرية شملت الطرق التحليلية و الطرق العددية باستخدام طريقة العناصر المحددة. أنواع مختلفة من هذه الاقمشة البوليمرية الشائعة الاستعمال تجارياً جرى اختبار مدى فائدتها في هذا البحث.

KEY WORDS

Buckling load, FRP, stability, steel column, strengthening.

INTRODUCTION

Columns are the most important members in a structural system. Slender columns subjected to compression should be designed to carry the load requirement without failing by yielding or instability, i.e. buckling. However, for cases of changing, for example due to increased loads or change in use, it is required to improve the load capacity. In addition, structures may be affected by accidents. If the function of a structure becomes inadequate by one of the above reasons, it might be possible to keep it in service by repairing or strengthening. It should be determined whether it is more economical in strengthening the structure compared to replacement.

If a steel column that subjected to compression needs to be strengthened, many methods exist. For example, extra steel sections can be welded or bolted to the column, but sometimes this method is

too expensive. One method that has become established for strengthening existing concrete structures is to bond fiber composite materials to the surfaces (ACI 440:2002).
The most common materials to be used are fiber reinforced polymers (FRP) in combination with epoxy resin. In this study the possibility of using fiber composite for increasing global buckling load for steel pipe under pure compression has been investigated. The work includes a theoretical study consisting of both analytical and numerical finite element solutions.

THEORY

For the theories, Young's modulus (E) and moment of inertia (I) of the involved materials are needed. Young's modulus for the steel is well known. For the composite, the modulus is dependent on the fiber content. The moment of inertia is only dependent on geometry and for a pipe section with notation as in **Fig. (1)** it is calculated in accordance to Eq. (1).

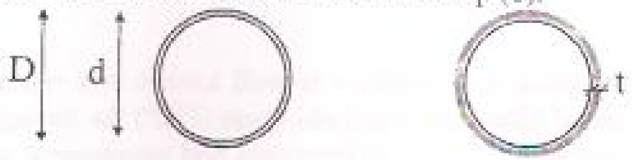


Fig. (1) Cross-section of nonstrengthened (left) and strengthened (right) member.

$$I = \frac{\pi(D^4 - d^4)}{64} \qquad (1)$$

For the strengthened members, the modulus will be varied over the cross-section and it is most convenient to calculate the product of the Young's modulus and the moment of inertia (EI). Here, the property $(EI)_{str}$ of the strengthened cross-section may be calculated as in Eq. (2). This is possible since the two materials have the same center of gravity and is based upon the assumption that plane cross-sections remain plane (Popov 1990).

$$(EI)_{str} = E_{frp}\frac{\pi((D+2t_{frp})^4 - D^4)}{64} + E_s\frac{\pi(D^4 - d^4)}{64} \qquad (2)$$

By defining the Young's modulus for the FRP as a ratio of Young's modulus of steel as:

$$E_{frp} = \alpha E_s \qquad (3)$$

Then eq. (2) may be written as:

$$(EI)_{str} = \frac{E_s\pi}{64}\left[\alpha(D+2t_{frp})^4 + D^4(1-\alpha) - d^4\right] \qquad (4)$$

From Eq. (4), the moment of inertia I^*_{str} for the strengthened cross-section corresponding to a cross-section with Young's modulus for steel can be identified as:

$$I^*_{str} = \frac{\pi}{64}\left[\alpha(D+2t_{frp})^4 + D^4(1-\alpha) - d^4\right] \qquad (5)$$

A stiffness improvement constant (c) is defined as the stiffness of the strengthened cross-section in relation to the stiffness of the non-strengthened cross-section.

$$c = \frac{(EI)_{str}}{E_s I_s} = \frac{I^*_{str}}{I_s} = \frac{\alpha(D + 2t_{frp})^4 + D^4(1-\alpha) - d^4}{(D^4 - d^4)} \quad (6)$$

ANALYTICAL ANALYSIS

The critical stability load for a member with uniform stiffness over its whole length can easily be analytically described by the Euler formula, Eq. (7). The stiffness is uniform for a member without strengthening and for a member with uniform strengthening over the whole length. For a hinged member with length (L), the critical load (P_{cr}) can be described by Euler formula (Popov, 1990).

$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad (7)$$

However, an examination of the bending-moment diagram for a buckled column indicates that a uniform cross-section along the length is not the most economical form for strengthening for increasing stability. The greater part of the strengthening material should be applied in the mid-portion of the column. The buckling mode of a column with higher moment of inertia in the mid-portion, shown in **Fig. (2-a)**, is presented in **Fig. (2-b)**. By comparing **Fig. (2-b)** and **Fig. (2-c)**, it is found that the column can be studied as in **Fig. (2-d)**.

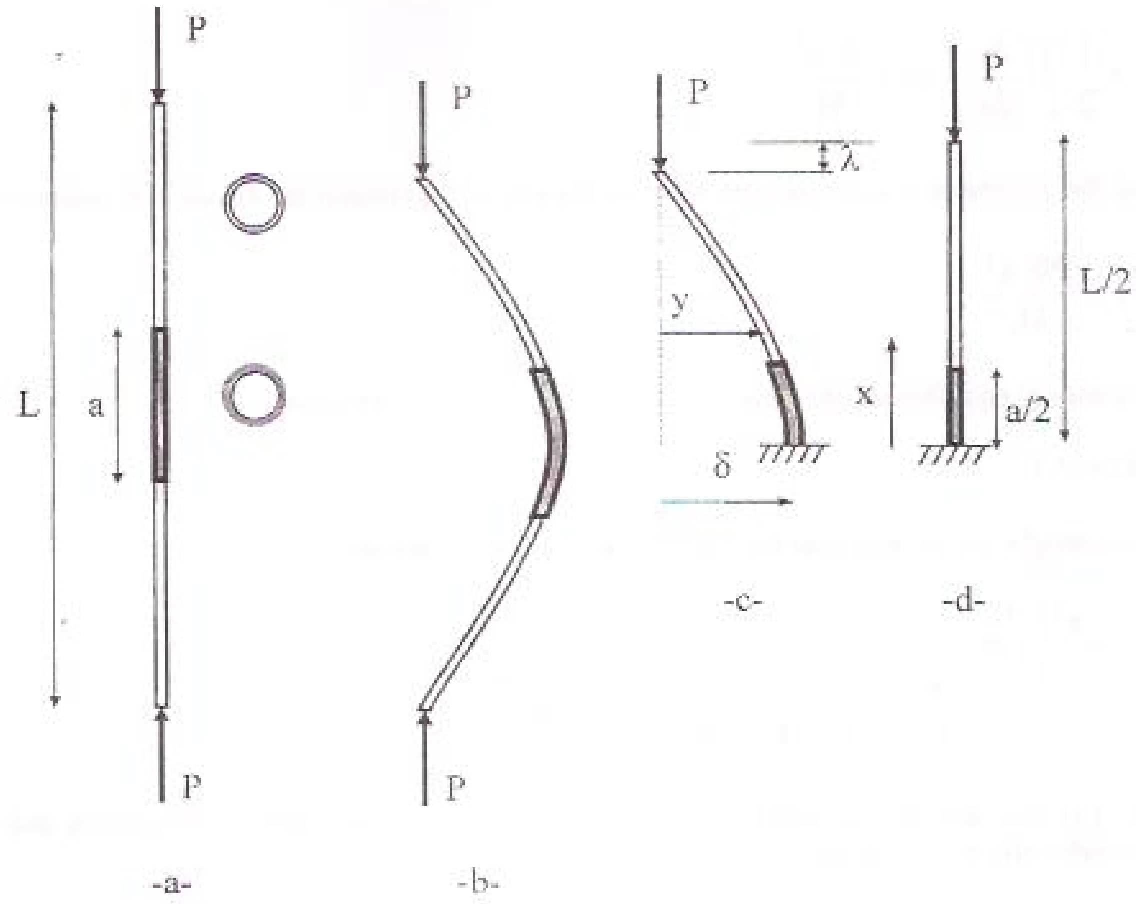


Fig. (2) Axially loaded column partially strengthened.

By using the energy approach, the critical load for a column, as in **Fig. (2-d)** may be analytically determined. The increase in internal work (or strain energy) to deform the column as in **Fig. (2-c)** may be expressed as:

$$\Delta U = \int_0^{L/2} \frac{M^2}{2EI} dx \tag{8}$$

where, $M = Py$. The deformation of the buckled column is approximated by:

$$y = \delta \cos\left(\frac{\pi x}{L}\right) \tag{9}$$

This gives $y = \delta$ at $x = 0$ (bottom end) and $y = 0$ at $x = L/2$ (top end).
By combining Eq. (8) and Eq. (9) and with notations as in **Fig. (2-d)**, the internal work becomes:

$$\Delta U = \frac{P^2 \delta^2}{2E_s I^*_{str}} \left[\int_0^{a/2} \cos^2\left(\frac{\pi x}{L}\right) dx + \frac{I^*_{str}}{I_s} \int_{a/2}^{L/2} \cos^2\left(\frac{\pi x}{L}\right) dx \right] \tag{10}$$

The vertical displacement (λ) is described by (Timoshenko and Gere, 1961) as:

$$\lambda = \frac{1}{2} \int_0^{L/2} \left(\frac{dy}{dx}\right)^2 dx = \frac{\delta^2 \pi^2}{8L} \tag{11}$$

and the increment in external work to move the end of the column the distance λ becomes:

$$\Delta T = \frac{P \delta^2 \pi^2}{8L} \tag{12}$$

For energy equilibrium, the internal and external work must be equal.

$$\Delta U = \Delta T \tag{13}$$

Accordingly, the critical load for the member may be written as:

$$P_{cr} = \frac{\pi^2 E_s I^*_{str}}{L^2} \cdot \frac{1}{\frac{a}{L} + \frac{L-a}{L}\frac{I^*_{str}}{I_s} - \frac{1}{\pi}\left(\frac{I^*_{str}}{I_s} - 1\right) \sin\frac{\pi x}{L}} \tag{14}$$

Eq. (14) only describes a member with two cross-sections where one is mid-portion and the other symmetrically placed on both sides.

FINITE ELEMENTS ANALYSIS

To solve the problem for a member subjected to axial load with non-uniform cross-section or non symmetrical strengthening, a numerical approach is suggested. The relation between forces an deformations is the stiffness matrix [k], as shown in Eq. (15) with notations as shown in Fig. (3), where Ri denotes forces and moments. Displacements and rotations are denoted by r_i.

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{Bmatrix} = [k] \begin{Bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{Bmatrix} \qquad (15)$$

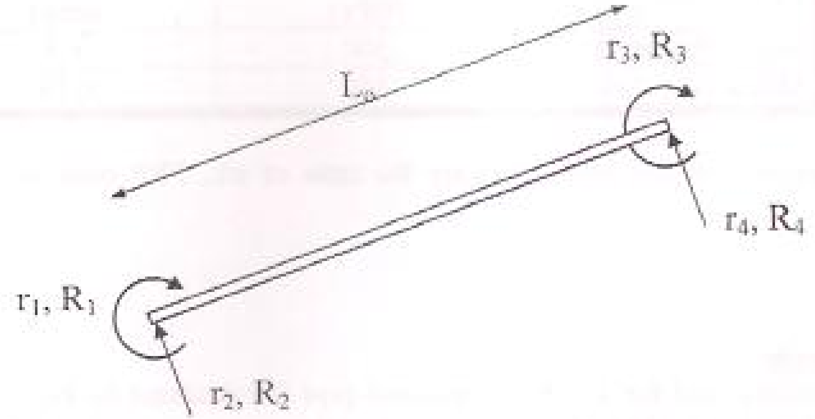


Fig. (3) Notations for member subjected to end bending loads

The symmetric stiffness matrix for the member subjected to axial load (P) and length (L_o) can be described as Eq. (16) (Gardner, 1968).

$$[k] = \frac{EI}{L_o} \underbrace{\begin{bmatrix} 4 & & & \\ \frac{-6}{L_o} & \frac{12}{L_o^2} & & \\ 2 & \frac{-16}{L_o} & 4 & \\ \frac{6}{L_o} & \frac{-12}{L_o^2} & \frac{6}{L_o} & \frac{12}{L_o^2} \end{bmatrix}}_{\text{Stiffness matrix}} - P \underbrace{\begin{bmatrix} \frac{2L_o}{15} & & & \\ \frac{-1}{10} & \frac{6}{5L_o} & & \\ \frac{-L_o}{30} & \frac{-1}{10} & \frac{2L_o}{15} & \\ \frac{1}{10} & \frac{-6}{5L_o} & \frac{1}{10} & \frac{6}{5L_o} \end{bmatrix}}_{\text{Geometric matrix}} \qquad (16)$$

For the calculations, ten elements have been used. Instability occurs when the load increases to such a level that singularity occurs for the stiffness matrix, i. e. buckling of the column occurs when the determinant of the stiffness matrix is no longer positive.

CASE STUDY

To investigate if a steel pipe can be strengthened for improved stability with FRP bonding, a case study is taken for a steel pipe with two simply supported ends served as a reference in this paper. The following data are assumed:

Pipe outer diameter(D): 150 mm
Pipe inner diameter (d): 142 mm.
Young's modulus (E_s): 210 GPa
Yield stress (f_y): 245 MPa.

Pipe length (L): 6000 mm.
There are numerous FRP systems commercially available from various manufactures. The properties of the FRP materials offered also vary from one manufacturers to another. The system Sika (2000) and MBrace FRP (2001) were used in this study with the following properties listed in **Table (1).**

Table (1) Material data.

FRP system	Young's modulus (GPa)	Thickness (mm)
Sika carbodur H	300	1.4
MBrace CF 530	640	0.19

Parameters suggested in this case study are the ratio of a/L, FRP material type, and number of layers.

RESULTS

Analytical Results

The critical buckling load for a non-strengthened pipe is calculated by Eq. (7) to be 282 kN. The yield load for the pipe is 449 kN and should be taken as an upper limit for strengthening. The critical buckling load for different strengthened ratios, layers and materials are calculated by Eq. (14) and presented in **Table (2).**

Table (2) Critical buckling load (kN) from analytical model.

	Sika carbodur H			MBrace CF 530		
a/L	One layer	Two layers	Three layers	One layer	Two layers	Three layers
0.1	291	298	301	286	287	291
0.2	303	316	323	290	296	301
0.3	316	336	345	294	304	312
0.4	330	359	379	298	312	324
0.5	343	385	419	302	320	336
0.6	359	416	454*	307	329	350
0.7	376	451*	511*	312	339	364
0.8	395	493*	579*	316	349	380
0.9	417	545*	667*	321	360	397
1.0	440	608*	787*	327	371	416

Note: (*) refers to the case which exceeds yielding load.

It is more convenient to use the term strengthening effect ratio for the buckling load for a strengthened member in comparison to a control member. This strengthening effect ratio is presented in **Table (3).**

Table (3) Strengthening effect ratio from analytical model.

a/L	Sika carbodur H			MBrace CF 530		
	One layer	Two layers	Three layers	One layer	Two layers	Three layers
0.1	1.04	1.06	1.07	1.01	1.02	1.03
0.2	1.07	1.12	1.15	1.03	1.05	1.07
0.3	1.12	1.19	1.22	1.04	1.08	1.11
0.4	1.17	1.27	1.34	1.06	1.11	1.15
0.5	1.22	1.37	1.49	1.07	1.13	1.19
0.6	1.27	1.48	1.69*	1.09	1.17	1.24
0.7	1.33	1.60*	1.81*	1.11	1.20	1.29
0.8	1.40	1.75*	2.05*	1.12	1.24	1.36
0.9	1.46	1.93*	2.37*	1.14	1.28	1.41
1.0	1.56	2.16*	2.79*	1.16	1.32	1.48

Note: (*) refers to the case which exceeds yielding load.

Fig. (4) shows the strengthening effect on the critical buckling load of a steel pipe for different strengthening ratios, layers and materials of FRP.

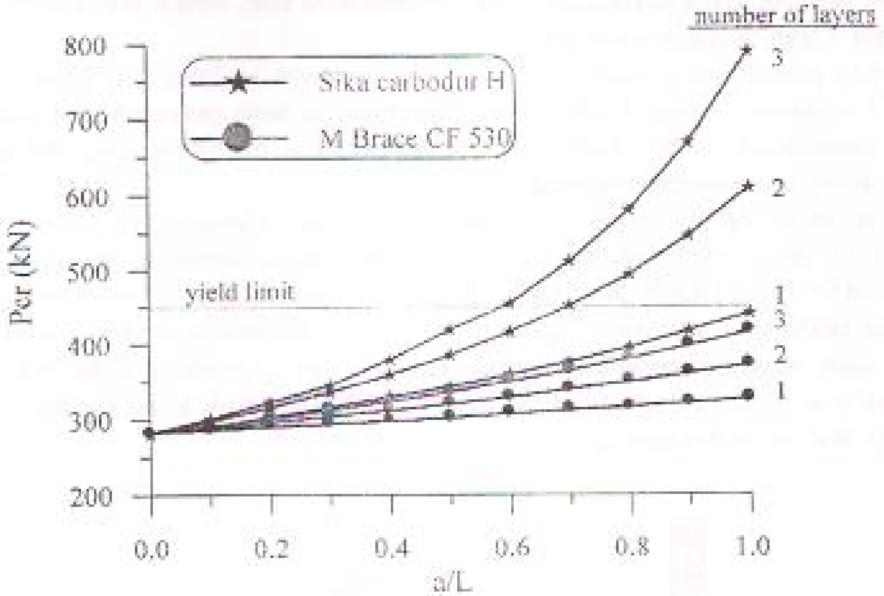


Fig. (4) The strengthening effect on the critical buckling load of a steel pipe for different strengthening ratios, layers and materials of FRP.

Tables (2) and (3) and Fig. (4) can be used to identify the strengthening length needed to achieve a desired strengthening effect.

FINITE ELEMENTS MODEL

The finite elements model gives a critical load of 282 kN for the control member without any strengthening. The calculated buckling loads are normalized with the theoretical buckling load for a non-strengthened member and presented in **Table (4).**

Table (4) Strengthening effect ratio from FE-model.

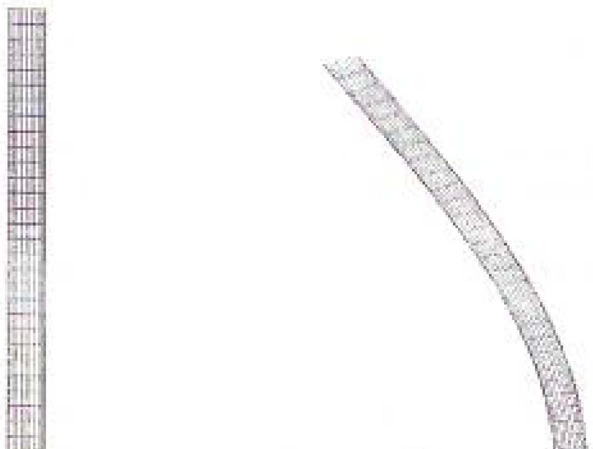
	Sika carbodur H			MBrace CF 530		
a/L	One layer	Two layers	Three layers	One layer	Two layers	Three layers
0.1	1.04	1.06	1.07	1.01	1.02	1.03
0.2	1.07	1.12	1.14	1.03	1.05	1.07
0.3	1.12	1.18	1.20	1.04	1.07	1.10
0.4	1.17	1.26	1.32	1.06	1.10	1.14
0.5	1.22	1.36	1.45	1.07	1.12	1.17
0.6	1.26	1.46	1.58*	1.09	1.16	1.22
0.7	1.33	1.58*	1.77*	1.10	1.08	1.27
0.8	1.39	1.73*	2.03*	1.11	1.23	1.34
0.9	1.47	1.92*	2.35*	1.14	1.27	1.40
1.0	1.56	2.16*	2.79*	1.16	1.32	1.48

Note: (*) refers to the case which exceeds yielding load

It was shown that the analytical model and the finite elements model give the same results for the non-strengthened member. The analytical model gives values 1-3% higher compared to finite elements model when using more than one layer. The reason for this difference is the divergence between the real buckling deformation and the assumed one, which was approximated as mentioned in Eq. (9) for a non-uniform member.

To check this procedure, a 3-dimensional finite element analysis has been done using the MSN-NASTRAN program version 4. Due to the symmetry in both geometry and loading, only half of the column is considered in the finite element idealization by introducing the appropriate boundary conditions along the column mid-length.

The steel is idealized by using 720 four-noded plate elements as shown in **Fig. (5).** FRP is represented by plate elements too, connected with steel elements by rigid elements. The rigid element in MSN-NASTRAN program is different than the other types of elements. It connects one independent node to a variable number up to 19 of dependent nodes. One independent and one dependent node must be specified at least, but all other dependent nodes are optional. In addition to the nodes, one or more degrees of freedom must be specified to be rigidly connected between the independent and all dependent nodes.



(a) (b)

Fig. (5) 3-dimensional finite element representation of the column
(a) before buckling (b) at buckling.

The load is applied in previously defined twenty increments up to the buckling load. The 3-dimensional finite elements model results approach the analytical results very closely. A stiffer behavior of the FE-model was observed when multi layers of FRP were used. **Table (5)** shows a comparison in the obtained critical buckling load from analytical and 3-D F. E. model for a column strengthened with MBrace FRP.

Table (5) Comparison in critical buckling load (kN) between analytical and 3-D F. E. models.

	MBrace CF 530					
	One layer		Two layers		Three layers	
a/L	Analytic.	3-D F.E.	Analytic.	3-D F.E.	Analytic.	3-D F.E.
0.1	286	285	287	287	291	292
0.2	290	289	296	297	301	302
0.3	294	294	304	305	312	313
0.4	298	298	312	314	324	327
0.5	302	301	320	322	336	339
0.6	307	306	329	331	350	354
0.7	312	312	339	342	364	369
0.8	316	317	349	352	380	386
0.9	321	320	360	364	397	404
1.0	327	325	371	376	416	424

DISCUSSION AND CONCLUSIONS

The analytical model and the finite element model show that fiber composites can be used for strengthening of steel columns. Derived models may be used to calculate the strengthening effects. If practical tests are undertaken it might be possible to asses the efficiency of the derived models used to calculate the strengthening effect.

REFERENCES

ACI Committee 440 (2002), Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening of Concrete Structures, American Concrete Institute, Michigan, USA.

Gardner W. Hicks (1967), Finite Elastic Buckling Analysis." ASCE,Vol.93, No. ST6,Dec., pp.71-86.

Master Builders Technology (2001), MBrace Material Properties, (http://www.masterbuilders.com).

Popov, E.P. (1990), Introduction to Mechanics of Solids, 2nd. Edition, Prentice-Hall, Inc., Englewood Cliffs, N. J., USA.

Sika Data Book Construction (2000), Building with a Safe System, (email :sikareg@cyberia.net.lb).

Timoshenko, S. and Gere (1961) Theory of Elastic stability, 2nd. Edition. McGraw-Hill, New York, USA.

NOTATIONS

c :	Stiffness improvement constant.
D :	Outer diameter of the pipe.
d :	Inner diameter of the pipe.
E :	Young's modulus.
E_s, E_{frp} :	Young's modulus for steel and FRP.
I :	Moment of inertia.
I_s :	Moment of inertia for steel pipe.
I_{str} :	Moment of inertia for the strengthened pipe.
[k] :	Stiffness matrix.
L :	Length of column.
M :	Moment.
P :	Applied axial load.
P_{cr} :	Critical load.
R_i :	Forces and moments at ends of beam element.
r_i :	Displacements and rotations at end of beam element.
t_{frp} :	FRP thickness.
y :	Deformation (deflection) of the buckled column.
α :	Modular ratio = E_{frp}/E_s.
ΔT :	External work increment.
ΔU :	Internal work increment.
δ :	Maximum deformation for the buckled column.
λ :	Vertical deformation.