ESTIMATION OF THE LIFE OF THICK CYLINDER SUBJECTED TO INTERNAL PRESSURE WITH MANUFACTURING CRACKS USING J-INTEGRAL METHOD

Dr. Adnan. N. Jamel
Baghdad University
College of Engineering
Mechanical Engineering
Department

Dr. Imad. A. Husain
Saddam University
College of Engineering
Mechanical Engineering Department

B.Sc. Akeel A. Abtan
Mechanical Engineering

ABSTRACT

Cracks may appear in structures due to manufacturing processes and some time are appeared in the structure product from casting, these strutures may be used but in the life less than the design life according to crack propagation on it.

In this research a thick cylinder has one crack or more is investigated to estimate the life under pulsation internal pressure. Finite element method with J-integral approach has been used to evaluate the numerical strain energy release rate (J) for the thick cylinder and the stress intensity factor (SIF).

J-integral method is most accurate method to evaluate the SIF for the elastic-plastic materiel by considering the local plastic zone near crack tip.

Software developed using the FEM; J-integral; SIF and Paris formula to estimate the life of the component is presented with many working examples.

الخلاصة

في اغلب الأحيان تظهر الشقوق في الهياكل المصنعة بواسطة عمليات إنتاجيه و قد تظهر الشقوق أو العيوب أيضا في الهياكل المصنعة بواسطة السباكة. هذه الهياكل قد تكون قابله للاستعمال و لكن بعمر استخدام اقل من العمر المصمم بسبب عمليه توسع الشقوق فيها.

في هذا البحث تم دراسة الاسطوانات السميكة التي تحتوي على شق أو عدة شقوق و المعرضة إلى ضغوط داخليه مكرره و ذلك لحساب عمر الاستخدام لها. تم استخدام نظرية العناصر المحددة و طريقة (J-integral) لحساب كل من معدل طاقة الانفعال (J) و معامل شدة الإجهاد (K) عددياً.

تم بناء برنامج باستخدام نظرية العناصر المحددة و طريقة التكامل المحوري (J-integral) و كل من صيغة (Paris) و صيغة (Forman) وذلك لحساب عمر الاستخدام للهياكل، وتم تجربته على عدة أمثلة مدروسة.

KEY WORDS

Fatigue, crack, fracture mechanics, J-integral, stress intensity factor, finite element method.

INTRODUCTION

Mechanical failures have caused many injuries and much financial loss. Fatigue has accounted for many of these mechanical failures; most of these are unexpected fractures.

Fatigue failure is characterized by three stages crack initiation, crack growth, and fast fracture. A fatigue crack usually starts at the surface perpendicular to the maximum tensile stress. Under the repeated action of the tensile stresses, the crack grows, weakening the section. As the section gradually weakens, the crack grows faster until sudden fracture occurs.

The word "fatigue" was first introduced in 1840s and 1850s to describe failures occurring due to repeated stresses. In Germany, during the 1850s and 1860s Wöhler performed many fatigue tests under repeated stresses. He showed from stress versus life (S-N) diagrams how fatigue life decreased with higher stress amplitudes and that below a certain stress amplitude, the test specimen did not fracture. Thus, Wöhler introduced the concept of (S-N) diagram and the life limit. He pointed out that for fatigue, the range of stresses is more important than the maximum stress.

In 1920, Griffith published the results of his theoretical calculations and experiments on brittle fracture using glass. His results gave the critical tensile stress $\sigma_c = \sqrt{2E\gamma/\pi a}$, in which the quantity, γ, referred to the specific surface energy of the material, E, to the Young's modulus of elasticity and, a, represented the crack length. By introducing line cracks of length 2a in a given material and recording the various loads at incipient fracture, Griffith had shown that the product $\sigma_c\sqrt{\pi a}$ remained essentially constant.

In 1957, [Irwin] contribute another major advance by showing that the energy approach is equivalent to the stress intensity approach, i.e. failure occur when the stress intensity factor reaches its maximum value denoted by Kc.

Paris [Paris] in the early of 1960s showed that crack growth rate (da/dN) could be best described using the stress intensity factor range (ΔKI). On the other hand, Paris argued that the growth rate should be a function of the stress intensity factor on the ground that this factor defines the elastic stress field around the crack tip.

The application of principles of fracture mechanics to practical problems requires knowledge of crack size, the service stress, the appropriate properties of the materials, and the stress intensity factor. In practical problems, structural geometry and loading are often so complex that the available stress intensity factor solutions are inadequate.

Although FEM is a powerful technique in determining the values of displacement, strain and stress at any point in the domain of elasticity problems, but the results of which are rather distorted at the zones very closed to the crack tip due to the singularity of the solution there, where the values of stresses tend to infinity. So the subsequent determination of the stress intensity factor based on these result are not reliable enough, especially when precise and accurate stress intensity factors are required in important and expensive engineering applications.

So the need of an effective technique appears. It was the J-integral method, which represents the energy release rate or the rate of change of potential energy. It was proved that its value is equal to the rate of energy required for crack extension.

The path independence of the J-integral expression allows calculation along a contour remote from the crack tip. Such as a contour can be chosen to contain only elastic loads and displacements. Thus, an elastic plastic energy release rate can be obtained from an elastic calculation along a contour for which loads and displacement are known.

J-Integral were first be driven by Eshelby [Eshelby] in 1956. Cherepanor [Cherepanor] and Rice [Rice] were apply such an integral to crack problems, Rice used it in 1968 as a fracture criterion, and as a technique for calculating stress intensity factor, since under linear elastic fracture mechanics (LEFM) conditions, the J value may be equated to the strain energy release rate, G, which can be related by simple expression to the stress intensity factor, K.

In 1975, Knott [Knott] showed that the J-integral approach could be employed to characterize fracture in some ways, which may be open to discussion, but should be tested by experiment.

In 1993, Azodi and Bachmann [Azodi] investigated and analyzed the fatigue crack growth in pressure vessel under cyclic loading using FE analysis based on the calculation of the J-integral. Comparing the results to a standard solution, high accuracy was obtained.

AL-Edani [AL-Edani] in 1996 developed a method of implementing the rotational effects in the J-integral equation. This method was based on applying the integration by parts theorems to the equilibrium equation in terms of internal displacements and their derivatives.

THE BASIC EQUATIONS

Griffith Equation may be rearranged in the form [Marc]:

$$\frac{\pi\sigma^2 a}{\mathrm{E}} = 2\gamma_e \tag{1}$$

The left-hand side in eq (1) has been designated the energy release rate, G, and represents the elastic energy per unit crack surface area that is available for infinitesimal crack extension. The right hand side in eq (1) represent the surface energy increases that would occur owing to infinitesimal crack extension, and is designated the crack resistance, R.

It follows that G must be at least equal to R before unstable crack growth occurs. If R is constant, this means that G must exceed a critical value G_c.

Or in other words, fracture occurs when:

$$\frac{\pi\sigma^2 a}{\mathrm{E}} \geq \frac{\pi\sigma_c^2 a}{\mathrm{E}} = G_c = R \tag{2}$$

Where the critical value G_c can be obtained by measuring the stress σ_c required fracturing a plate with a crack of size 2a.

Owing to the practical difficulties of the energy approach Irwin made a major advance when he developed the stress intensity approach. First, from linear elastic theory, Irwin showed that the stress in the vicinity of a crack tip take the form:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + ...+ \tag{3}$$

Where:

r, θ are the polar coordinates of a point with respect to the crack tip.

K is a constant that gives the magnitude of the elastic stress field. It is called the stress intensity factor.

Irwin then demonstrated that if a crack is extended by amount da, the work done by the stress field ahead of the crack when moving through the displacement corresponding to a crack of length $(a+da)$ is formally equivalent to the change in strain energy Gda. Thus the achievement of a critical stress intensity factor, K_c, is exactly equivalent to the Griffith-Irwin energy balance approach which requires the achievement of a stored elastic strain energy equal to G_c.

For tensile loading, the relation between G_c and K_c are [Marc]:

$$G_c = \frac{K_c^2}{\mathrm{E}} \quad \text{plane stress}$$

$$G_c = \frac{K_c^2}{\mathrm{E}}\left(1-\upsilon^2\right) \quad \text{plane strain} \tag{4}$$

Or in general, it was proved:

$$G_i = \frac{1+\lambda}{8\mu} K_I^2 \tag{5}$$

λ is either equal to $(3-\upsilon)/(1+\upsilon)$ for plane stress, or equal to $(3-4\upsilon)$ for plan strain

Fatigue crack Growth curve, da/dN – ΔK

Linear elastic fracture mechanics concepts are most useful to correlate fatigue crack growth behavior. The form of this correlation for constant amplitude loading is usually a log-log plot of fatigue crack growth rate, da/dN, in m/cycle, versus the opening mode stress intensity factor range ΔK, or (ΔKI) in Mpa √m where ΔK_I is defined as

$$\left.\begin{aligned} \Delta K_I &= \Delta K = K_{\max} - K_{\min} \\ &= Y\sigma_{\max}\sqrt{\pi a} - Y\sigma_{\min}\sqrt{\pi a} \end{aligned}\right\} \qquad (6)$$

If σmin is taken as zero;

$$\Delta K = K_{max}$$

Sigmoid shaped of da/dN – ΔK curve

A typical complete log-log plot of da/ dN versus ΔK is shown in **Fig (1)** This curve has a sigmoid shape that can be divided into three major regions Region I indicates a threshold value ΔKth, below which there is no observable crack growth. Below ΔKth, fatigue cracks behave as non-propagating cracks Region II shows a linear relation ship between log da/dN and log ΔK, which corresponds to a large number of formulas, but first suggest by Paris [Paris]

$$\frac{da}{dN} = A(\Delta K)^m \qquad (7)$$

Here, m is the slope of curve and A is the coefficient found by extending the straight line to ΔK = 1 Mpa √m. In region III the crack growth rates are very high and little fatigue crack growth life is involved. Region III may have the least importance in most fatigue situations. The fatigue crack growth behavior shown in **Fig (1)** is essentially the same for different specimens taken from a give materials, because ΔK is the principal controlling factor in fatigue crack growth. Knowing the stress intensity factor expression, KI for a given component and loading, the fatigue crack growth life of the component can be obtained by integrating eq(7), for example, between limits of initial crack size and final crack size. The greatest usage of **Fig (1)** data has been in-fail-safe design of aircraft and nuclear energy system.

The effect of mean load on fatigue crack initiation and propagation behavior can studied by using the stress ratio parameter, R, where R is equal to

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} = \frac{K_{\min}}{K_{\max}}$$

Available experimental data shows no systematic change in fatigue crack growth rate with changes in R value from 0 to 0.82 [Newman]. The data also show that this change in R value has negligible effects on the rate of crack growth. The best equation that indicates R effect is Forman equation [Marc].

$$\frac{da}{dN} = \frac{A(\Delta K)^m}{\left(1 - R - \dfrac{\Delta K}{Kc}\right)} \qquad (8)$$

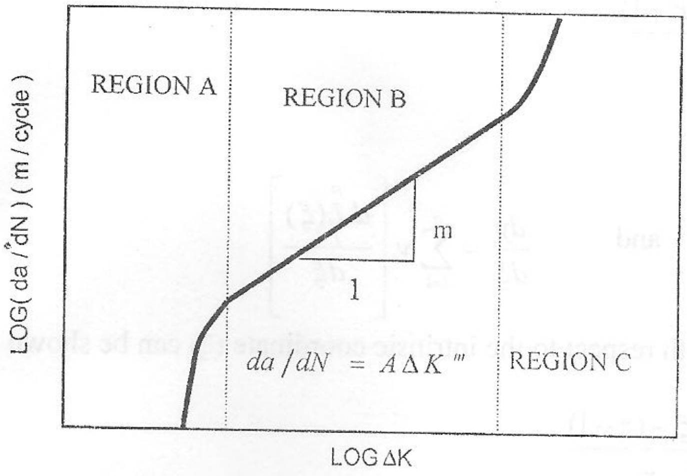

Fig (1) Schematic sigmoid behavior of fatigue crack growth rate

J-INTEGRAL METHOD

From [Rice] it can obtain, the expression for J-integral as:

$$J = \int_{\Gamma_o} [\frac{1}{2}\underline{\sigma}'\,\underline{\varepsilon} dy - \underline{T}'\frac{\partial \underline{u}}{\partial x} ds]$$

The boundary Γ_o can be divided into a number (ne) of the sub boundaries using piecewise discretization concept and the J-integral can be evaluated as:

$$J = \sum_{e=1}^{ne} \int_{\Gamma_e} (\frac{1}{2}\underline{\sigma}'\,\underline{\varepsilon} dy - \underline{T}'\frac{\partial \underline{u}}{\partial x} ds)$$

Or: $$J = \sum_{e=1}^{ne} J_e$$

Where, Je is the J-integral ever e[th] elements and can be written as:

$$J_e = \int_{\Gamma_e} (\frac{1}{2}\underline{\sigma}'\,\underline{\varepsilon} dy - \underline{T}'\frac{\partial \underline{u}}{\partial x} ds)$$

To evaluate Je, a boundary element is required to be defining over Γ_e. An n- nodded isoparametric boundary element is represented by n nodes, which lie on the same boundary within the x-y plane. The element may be transformed into a straight line of unit length in the intrinsic space and the parametric equations of the transformation of the element can, therefor, be expressed as Follows:

$$X(\xi) = \sum_{i=1}^{n} x_i \overset{n}{\underset{i}{L}}(\xi)$$

$$Y(\xi) = \sum_{i=1}^{n} y_i \overset{n}{\underset{i}{L}}(\xi)$$

Using Lagrange interpolation function, it can be shown that:

$$\overset{n}{\underset{i}{L}}(\xi) = \prod_{\substack{r=1 \\ r\neq i}}^{n} \frac{(n-1)\xi - (r-1)}{i-r}$$

Hence it can proved that:

$$\frac{dx}{d\xi} = \sum_{i=1}^{n} x_i \left[\frac{d\overset{n}{\underset{i}{L}}(\xi)}{d\xi} \right] \quad \text{and} \quad \frac{dy}{d\xi} = \sum_{i=1}^{n} y_i \left[\frac{d\overset{n}{\underset{i}{L}}(\xi)}{d\xi} \right]$$

Where, the derivative of L(ξ) with respect to the intrinsic coordinate (ξ) can be shown as:

$$\frac{d\overset{n}{\underset{i}{L}}(\xi)}{d\xi} = \sum_{\substack{s=1 \\ s\neq i}}^{n} \frac{n-1}{i-s} \prod_{\substack{r=1 \\ r\neq i \\ r\neq s}}^{n} \frac{(n-1)\xi - (r-1)}{i-r}$$

The directional cosines of the outward normal to the surface Γcan define as follows:

$$l_x = \frac{dy}{d\xi} / \frac{ds}{d\xi} \qquad l_y = -\frac{dx}{d\xi} / \frac{ds}{d\xi}$$

Evaluation of J_e not possible unless the valves of $\underline{\sigma}$, $\underline{\varepsilon}$ and $\underline{u}$ are given at the boundary nodes of the e^{th} element. Let the e^{th} element be n- nodded element as described before.
Using isoparametric equations, it can be proved that:

$$\underline{\sigma}(\xi) = \sum_{i=1}^{n} \underline{\sigma}_i \overset{n}{\underset{i}{L}}(\xi) \quad \text{and} \quad \underline{\varepsilon}(\xi) = \sum_{i=1}^{n} \underline{\varepsilon}_i \overset{n}{\underset{i}{L}}(\xi)$$

Also, the displacement component along the boundary element can be expressed like above. From the equations of $\underline{\sigma}$ (ξ), σ_x, σ_y, τ_{xy} are known at any (ξ) hence, the tractions in x and y directions respectively can be evaluated as:

$$Tx(\xi) = l_x(\xi)\sigma_\chi(\xi) + l_y(\xi)\tau_{xy}(\xi)$$

$$Ty(\xi) = l_x(\xi)\tau_{\chi y}(\xi) + l_y(\xi)\sigma_y(\xi)$$

Finally the form of the J_e, can be shown as:

$$J_e = \frac{1}{2}\int_0^1 \underline{\sigma}'\underline{\varepsilon}\left(\frac{dy}{d\xi}\right)d\xi - \int_0^1 \left(T_x\frac{\partial u}{\partial x} + T_y\frac{\partial v}{\partial x}\right)|\underline{J}|d\xi$$

The above equation can be evaluated numerically using modified Gaussian quadrature, then: -

$$J_e = \sum_{q=1}^{n_Q} \int_0^1 \Delta J_e(\xi_q)$$

Where:

$$\Delta J_e = C_q\left[\frac{1}{2}\underline{\sigma}'\underline{\varepsilon} l_x - \left(T_x\frac{\partial u}{\partial x} + T_y\frac{\partial v}{\partial x}\right)|J|\right]_{(\xi=\xi_q)}$$

Cq : is the Gaussian's weight factor.
ξq : is the Gaussian's argument factor.

CASE STUDIES

To demonstrate the proficiency of two-dimensional J-integral program for LEFM, which are developed in this work for analysis of cracked structure, validation cases with known analytical solution and other case study have been performed.

Pressurized cylinder with outer crack

The geometry and dimensions for this case are shown in **Fig (2)**. Because of the symmetry around the x and y-axes, only one quarter of the thick cylinder domain has been analyzed. The mesh of the structure for this analysis is indicated in **Fig (3)**; it consists of (100) isoparametric 8-node element and (341) nodes.

Fig (4) shows the SIF for different crack length to thickness ratios, it is clear that the accuracy of J-integral program is better than ANSYS results, whenever (a/w) high. The analytical solution for this case is taken from [Rooke].

When the load is consider to be pulsating load then the fatigue crack growth rate can be calculated from Paris equation where ΔK=K. The results are shown in **Fig (5)** for alloy steel 4340 and for two different loads (P=10 Mpa and P=20 Mpa). It is clear that the two curves have the same trend and slop, from this it can be say that for different loads or crack length the slop of the curve is remain constant at certain K and it will function of material only.

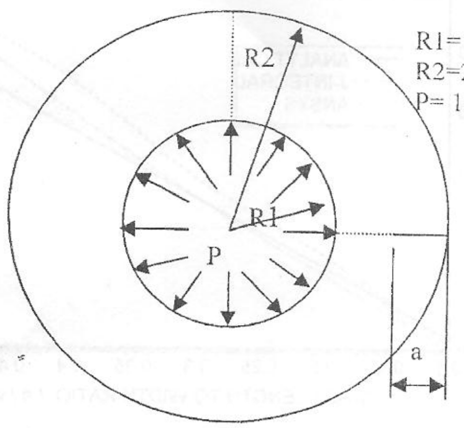

Fig (2) Pressurized cylinder with external crack

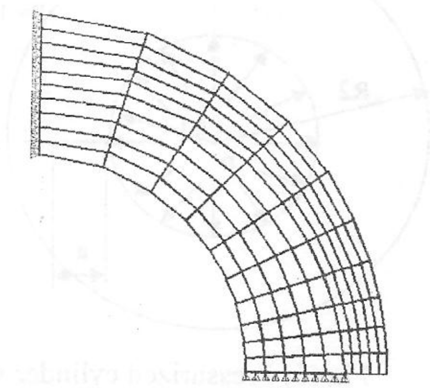

Fig (3) Finite element mesh for thick cylinder with external crack

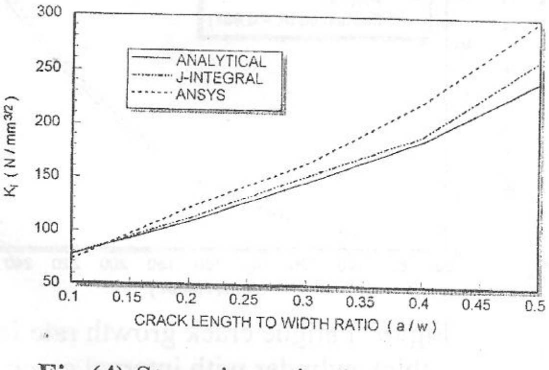

Fig (4) Stress intensity factor for thick cylinder with external crack

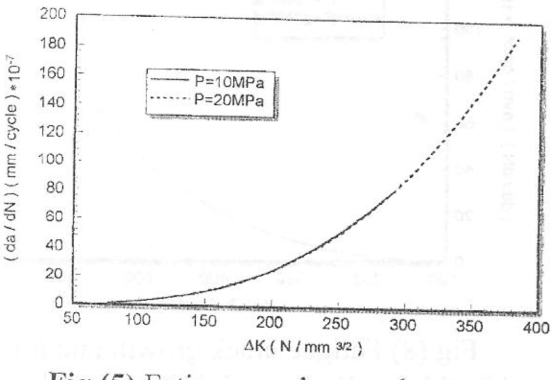

Fig (5) Fatigue crack growth rate for thick cylinder with external crack

Pressurized cylinder with internal crack

Fig (6) shows the geometry with all the information required to carry out a finite element analysis. **Fig (7)** shows the SIF for different crack length to thickness ratios. It is clear that the J-integral results have a good agreement with analytical results and the maximum error equals to 8 percent, it can be assumed that the error is high but it is better than the ANSYS results. The analytical solution of this case is taken from [Rooke].

Fatigue crack growth rate for different ΔK is shown in the **Fig (8),** two different curves are calculated from applying two pulsating loads (P=10 Mpa and P=20 Mpa). The two curves have the same configuration (i.e. they have the same slop at any point) this means that the effect of increasing load makes the fatigue crack growth rate increase but in the same slop, as explaining before.

To study the effect of mean stress on fatigue crack growth rate by indicated the stress ratio parameter, R, cyclic load is considered. **Fig (9)** shows the fatigue crack growth rate for different ΔK. It is calculated from Paris equation where the effect of mean stress is neglected and from Forman equation where the effect of mean stress is consider by indicated the parameter R. In this case R equals to (0.667) which it is the ratio between (Pmin =10 Mpa) and (Pmax =15 Mpa). It is clear that the effect of mean stress causes the increase in fatigue crack growth rate but the different between Paris equation and Forman equation results is not so much. The percentage error between cases that consider R or ignored R is not more than 4% and it can be neglected.

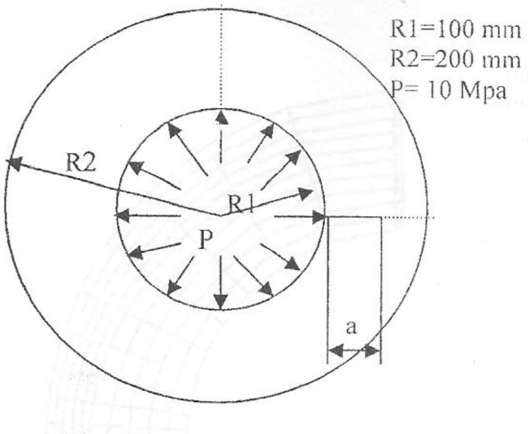

Fig (6) Pressurized cylinder with internal crack

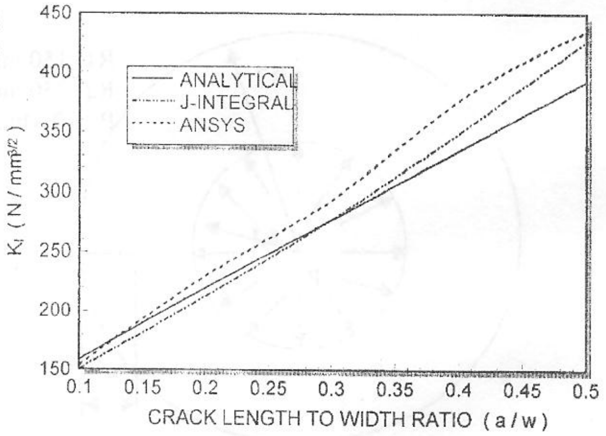

Fig (7) Stress intensity factor for thick cylinder with internal crack

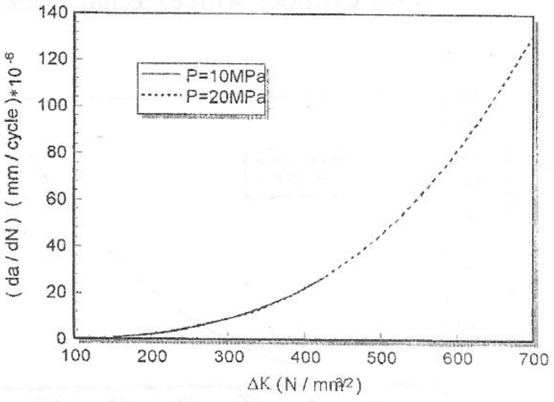

Fig (8) Fatigue crack growth rate for thick cylinder with internal crack

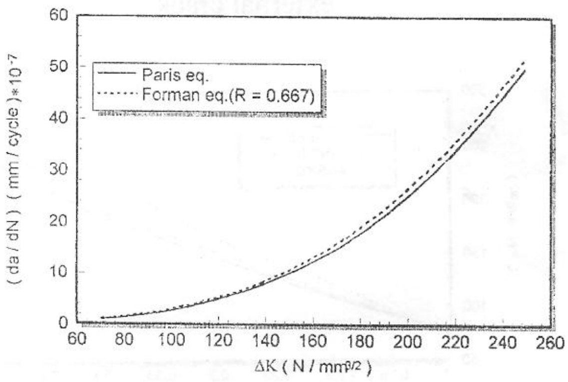

Fig (9) Fatigue crack growth rate for thick cylinder with internal crack under cyclic loading

Pressurized cylinder with internal and external cracks

The geometry and dimensions of this case are shown in **Fig (10)**, also it is shown all the information required to carry out the finite element analysis. Cin, Cout and Ctotal are the contours, which are employed in the SIF evaluation using the J-integral program.

SIF for different crack length to thickness ratios are shown in **Fig (11)**. It is clear that the SIF calculated using Cin contour is larger than SIF calculated using Cout contour, this may be came from load effect, in another words, the load effects on internal crack more than external crack.

Also, it is clear from the figure that there is large different between superposition SIF and total SIF (SIF calculated using Ctotal contour). The correct SIF is the one that calculated by superposition because the SIF calculated from Ctotal has a magnitude less than Cin, which is not the real case. Also, it is known that the selected contour must be continuous in all zone except in the crack surface may be discontinuous whilst in or case two discontinuous regions are appear and that is unaccepted in mathematical evaluations. Fatigue crack growth rate for different ΔK is shown in **Fig (12)** using superposition method.

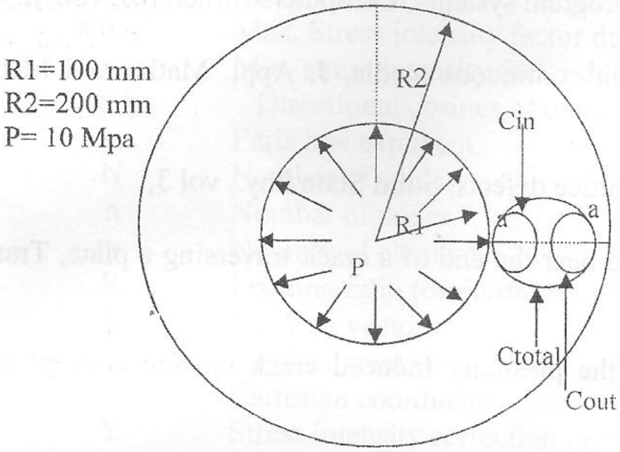

Fig (10) Pressurized cylinder with internal and external cracks

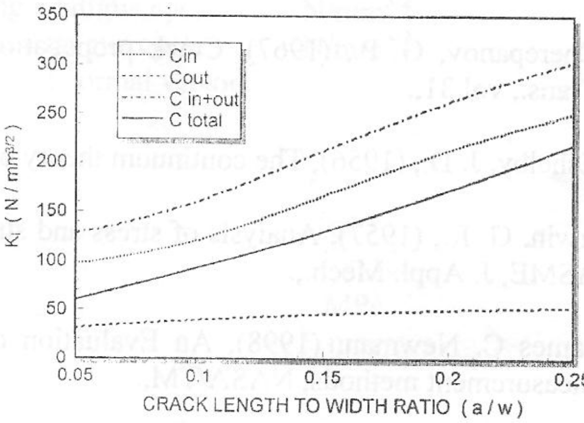

Fig (11) Stress intensity factor for case 3

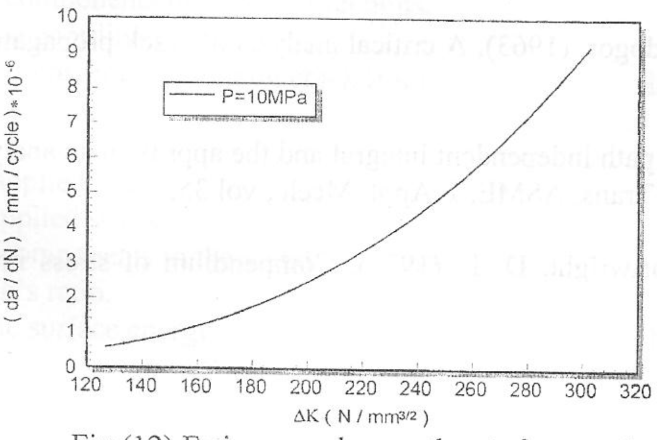

Fig (12) Fatigue crack growth rate for case 3

CONCLUSIONS

The most important conclusions that can be drawn from this work are as follows:

1- Integrity of the developed programs to handle the problems of cracked structures was proved.

2- The fracture parameters, J-integral and ΔKI could be utilized to predict the crack growth behavior of a material such as 4340 alloy steel in a complex test environment. Also the different between Paris and Forman results at cyclic load is less than 5% for cyclic stress ratios (R= 0-0.8). This means that the effect of mean stress in these ratios can be neglected.
3- To determine stress intensity factor for structures have two cracks or more never take total path for J-integral because it is mathematically illogical, cases like this must be determined using superposition.

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NOMENCLATURE

Symbol	Description	Units
A	Paris constant.	
da/dN	Fatigue crack growth rate.	mm/cycle
E	Modulus of elasticity.	GPa
G	Energy release rate.	N/mm
Gc	Critical energy release rate.	N/mm
J	J-integral value.	N/mm
Jc	Critical J-integral value.	N/mm
Kc	Critical stress intensity factor.	$N/mm^{3/2}$
KI	Mode I stress intensity factor.	$N/mm^{3/2}$
Kmax	Max. Stress intensity factor during a fatigue cycle.	$N/mm^{3/2}$
Kmin	Min. stress intensity factor during a fatigue cycle.	$N/mm^{3/2}$
lx, ly	Directional cosines of outward unit normal vector.	
m	Paris law exponent.	
N	Number of cycles.	
n	Number of nodes.	
ne	Number of elements.	
R	Loading ratio (σmin/σmax).	
T	Traction vector.	MPa
$\underline{u}$	Displacement vector.	mm
x, y	Cartesian coordinates.	
Y	Stress intensity correction factor.	
Γ	Boundary of the domain.	
ΔK	Stress intensity factor range.	$N/mm^{3/2}$
Δσ	Applied stresses range.	MPa
$\underline{\varepsilon}$	Strain vector.	
εx,εy	Strain components in x and y directions.	
ξ, η	Intrinsic coordinates.	
θ	Angular position of point on crack front.	Degree
μ	Shear modulus.	MPa
$\underline{\sigma}$	Stress vector.	MPa
σmax	Max. applied stress.	MPa
σmin	Min. applied stress.	MPa
σx, σy	Stress components in the x and y directions.	MPa
υ	Poisson's ratio.	
γe	Specific surface energy.	N/mm