THE EFFECT OF SELF –EQUILIBRATING STRESSES ON THE NATURAL FREQUENCIES OF ELASTICALLY RESTRAINED RECTANGULAR PLATE

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ABSTRAC

An investigation has been made into the effect of residual stresses on the vibration, characteristics of thin rectangular plates elastically restrained against rotation along three edges and free on the fourth edge .General frequency equation with and without including the effect of residual stresses has been obtained. Exact frequency expressions including the effect of residual stresses for the cases: S-S-S-F,S-S-C-F,C-C-S-F,C-S-S-F, C-S-C-F,C-C-C-F were also obtained. The effect of the position of welding along the width of the plate for all cases was also included. Actual plate models were tested and the results were compared with the theoretical predictions giving good agreement.

الخلاصة

تم في هذا البحث دراسة تأثير الاجهادات المتبقية على خواص الاهتزازات و الكلل للصفائح الرقيقة ذات حدود التثبيت المرنة من ثلاثة حواف والحافة الرابعة حرة . تم التوصل الى اشتقاق معادلة عامة لتأثير الاجهادات المتبقية على التردد الطبيعي و كذالك الحصول على معادلات للحالات التالية:-
S-S-S-F, S-S-C-F, C-C-S-F,C-S-S-F, C-S-C-F,C-C-C-F
تم بناء نموذج باستخدام طريقة العناصر المحددة لتدقيق نتائج تاثير الاجهادات المتبقية نتيجة خط لحام موازي لطول الصفيحة مع النتائج التي تم الحصول عليها من المعادلات. تم اختيار نماذج حقيقية تجريبيا لدراسة لتأثير الاجهادات المتبقية على التردد الطبيعي و مقارنتها مع النتائج النظرية حيث اظهرت تطابقا جيدا.

KEY WORDS

Vibration, Fatigue, Rectangular Plate

INTRODUCTION

fatigue life, Residual stresses are induced at each stage of the life cycle in most engineering components, from original material production to final disposal. Residual stresses are created by welding, forging, casting, rolling, machining, surface treatment and heat treatment. Residual stresses are important in distortion, corrosion resistance, dimensional stability and brittle fracture. Compressive stress increases both fatigue strength and resistance to stress-corrosion cracking, but causes a decrease in the buckling load. Tensile residual stresses may reduce the performance or cause either distortion or cracks. However, literature does not exhibit an analytical treatment of the subject and the present paper presents an attempt in this direction.

Jubb et al, (1975), indicated that the boundary condition of the plate element is critical factors to determining the effect of the introduction of residual stresses.

Porter Golf,(1976) , described an experimental and a theoretical study on the effect of self equilibrating stress system, induced by running a weld on the longitudinal centre-line on the torsion and flexural modes of a rectangular plate with all edges free.

Laura et al,(1978), determined the fundamental frequency of vibration of thin rectangular plates with edges possessing different rotational flexibility coefficients using a very simple polynomial expression which identically satisfies the boundary conditions. Laura et al, studied the transverse vibration of a rectangular plate elastically restrained against rotation along three edges and free on the fourth edge. The problem was solved by Ritz deflection function method which is a simple polynomial.

Wu, et al.(2001), presented a finite element(FE) simulation of the welding process yielding the welding-induced residual stresses in a butt-welded plate .

Bambach and Rasmussen, (2001), tested rectangular plate with a simply support to three sides, leaving the remaining (longitudinal) edge free. In many applications such plate elements exist as flange outstands in thin-walled sections whose fabrication process involves welding and possibly flame cutting. In order to accurately predict the behavior of un-stiffened elements in this condition, equivalent residual stress profiles must be induced in the plate specimens prior to testing.

Al-Ammri et al, (2001), presented a theoretical and experimental investigation to determine the residual stresses for various isotropic (thin/thick) plates combined with finite element calculation. The technique of the hole drilling was extended and used to calculate the residual stresses for thin orthotropic (composite) plates.

Al-Ammri ,(2004), studied the effect of residual stresses on the vibration, stress and fatigue characteristics of thin rectangular plate with different boundary conditions. Theoretical method based on the theory of bending of thin plates was used to obtain the governing differential equation. A general frequency equation including the effect of residual stresses for plate elastically restrained against rotation along three edges and free on the fourth edge, simply supported plates with edge possessing different rotational flexibility coefficients at all edges, and clamped ends were obtained.

THEORETICAL CONSIDERATIONS

The strain energy stored in a plate element **Fig (1)** is the sum of the work done by the bending moments, $M_x dy$ and $M_y dx$ and by the twisting moments $M_{xy} dy$ and $M_{xy} dx$, neglecting the work done by shearing forces and by any stretching of the middle plane of the plate.

The work done by the bending moments is 1/2 x moment x angle between the sides of the element after bending.

In the xz plane the angle is $-(\frac{\partial^2 w}{\partial x^2})dx$ and in the yz plane $-(\frac{\partial^2 w}{\partial y^2})dy$,

The negative sign occurs because a sagging downwards curvature (positive) has a decreasing slope as x increases

The energy stored due to bending (dU_b) is therefore given by:

$$dU_b = -\frac{1}{2}(M_x \frac{\partial^2 w}{\partial x^2} + M_y \frac{\partial^2 w}{\partial y^2})dxdy \qquad \text{eq. (1)}$$

The relative rotations of the element faces due to twist are $\frac{\partial^2 w}{\partial x \partial y}dx$ and

Since M_{xy}dy and M_{yx}dx are the twisting moments, and $M_{xy} = -M_{yx}$, the $\frac{\partial^2 w}{\partial x \partial y} dy$ same a mount of energy is stored by both couples. Then, total energy due to twisting (dU_t) is given by:

$$dU_t = M_{xy} \frac{\partial^2 w}{\partial x \partial y} dxdy \qquad \text{eq. (2)}$$

Substituting the expressions for the moments and adding (dU_b) and (dU_t) to produce the total energy stored in an element (dU) , we get :

$$dU = \frac{1}{2} D[(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2})^2 - 2(1-\mu)[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - (\frac{\partial^2 w}{\partial x \partial y})^2]]dxdy \qquad \text{eq (3)}$$

The function is composed of the following elements:

$\frac{D}{2}[(\frac{\partial^2 w}{\partial x^2})^2 + \mu(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2})] 1/2$ * x-direction bending moments X rotation:

$\frac{D}{2}[(\frac{\partial^2 w}{\partial y^2})^2 + \mu(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2})] 1/2$ * y-direction bending moments X rotation :

$\frac{D}{2}[(2-\mu)(\frac{\partial^2 w}{\partial x \partial y})^2] 1/2$ * twisting moments X rotation

The strain energy stored in a complete plate is obtained by integration of equation (3) over the surface.

$$U = \frac{1}{2} D \iint [(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2})^2 - 2(1-\mu)[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - (\frac{\partial^2 w}{\partial x \partial y})^2]]dxdy \qquad \text{eq. (4)}$$

If the maximum kinetic energy of the element is,

$$dT = \frac{1}{2} \rho h \omega_o^2 w^2 \partial x \partial y , \qquad \text{eq. (5)}$$

of the plate may be deduced from the energy balance: ω_o then the angular frequency

$$\int dU = \int dT \qquad \text{eq.(6)}$$

RESIDUAL STRESS DISTRIBUTION

It is assumed in the following analysis that any cross –section x = constant, the plate is under longitudinal compression distributed uniformly a cross the breadth of the plate, with the equilibrating tension concentrated on the line of welding at (y = r).

The component of maximum strain energy of the element due to the mid-plane forces is:

$$\partial U_r = \frac{1}{2} N_x (\frac{\partial w}{\partial x})^2 \partial x \partial y \qquad \text{eq (7a)}$$

The strain energy due to the forces may therefore be written as:

$$U_r = \frac{1}{2} N_x [\int_0^a \int_0^b (\frac{\partial w}{\partial x})^2 dxdy - b\int_0^a (\frac{\partial w}{\partial x})^2 |_{y=r} dx] \qquad \text{eq(7b)}$$

In this case N_x is a negative constant, [P. Golf, 1976].
In the case of the mechanical system shown in **Fig (2)**, equation (6), can be written as:

$$\int_0^a \int_0^b \frac{D}{2}\{[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}]^2-2(1-\mu)[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-(\frac{\partial^2 w}{\partial x\partial y})^2]\}dxdy+$$

$$\frac{1}{2}N_x[\int_0^a \int_0^b (\frac{\partial w}{\partial x})^2 dxdy - b\int_0^a (\frac{\partial w}{\partial x})^2 dx]=\frac{1}{2}\rho h\omega_o^2\int_0^a\int_0^b w^2 \partial x\partial y \quad \text{eq .(8)}$$

In general the form of w is not known. However, if an assumed form, normally chosen to satisfy the boundary conditions, is substituted in equation (8), an approximate (too high) value of the frequency is obtained.
The governing boundary conditions are given by:

$$w(0,y,t)=w(x,0,t)=w(a,y,t)=0 \quad \text{eq .(9a)}$$

$$\frac{\partial^2 w}{\partial y^2}(x,y,t)+\mu\frac{\partial^2 w}{\partial x^2}(x,y,t)=0 \quad (y=b) \quad \text{eq.(9b)}$$

$$\frac{\partial^3 w}{\partial y^3}(x,y,t)+(2-\mu)\frac{\partial^3 w}{\partial x^2\partial y}(x,y,t)=0 \quad (y=b) \quad \text{eq.(9c)}$$

$$\frac{\partial w}{\partial x}=\Phi_1 D(\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial y^2}) \quad (x=0) \quad \text{eq.(9d)}$$

$$\frac{\partial w}{\partial x}=-\Phi_2 D(\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial y^2}) \quad (x=a) \quad \text{eq. (9e)}$$

$$\frac{\partial w}{\partial y}=\Phi_3 D(\frac{\partial^2 w}{\partial y^2}+\mu\frac{\partial^2 w}{\partial x^2}) \quad (y=0) \quad \text{eq.(9f)}$$

It is quite difficult to construct coordinate functions which satisfy identically conditions (9b) and (9c). It is convenient to replace them by:

$$\frac{\partial^2 w}{\partial y^2}(x,y,t)=0\,(y=b) \quad \text{eq. (9b)}$$

$$\frac{\partial^3 w}{\partial x^3}(x,y,t)=0\,(y=b) \quad \text{eq. (9c)}$$

The maximum strain and kinetic energies are given by:

$$U_{max}=\frac{D}{2}\int_0^a\int_0^b\{[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}]^2-2(1-\mu)[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-(\frac{\partial^2 w}{\partial x\partial y})^2]\}dxdy+$$

$$\frac{D}{2}\{[\int_0^b \frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial x}\Big|_{x=0}dy-\int_0^b \frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial x}\Big|_{x=a}dy+\int_0^a \frac{\partial^2 w}{\partial y^2}\frac{\partial w}{\partial y}\Big|_{y=0}dx\}+ \quad \text{eq. (10)}$$

$$\frac{1}{2}N_x[\int_0^a\int_0^b(\frac{\partial w}{\partial x})^2 dxdy - b\int_0^a(\frac{\partial w}{\partial x})^2\Big|_{y=r}dx]$$

$$T_{max} = \frac{\rho h \omega_o^2}{2} \int_0^a \int_0^b w^2 \, dx dy \qquad \text{eq. (11)}$$

When the plate executes normal modes of vibration one takes:

$$w(x,y,t) = w(x,y) \cos \omega_0 t \qquad \text{eq. (12a)}$$

In case of fundamental mode of vibration (1978):

$$w(x,y) \cong w_o(x,y) = A_{00}(\alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \alpha_4 x^4)(\beta_1 y + \beta_2 y^2 + \beta_3 y^3 + \beta_4 y^4) \qquad \text{eq. (12b)}$$

Substituting expression (12b) in equations (9a), (9b), (9c) and (9d) through (9f) one obtains:

$$\alpha_1 = \frac{2a}{k_1}\alpha_2 \; , \; \beta_1 = \frac{2b}{k_3}\beta_2$$

$$\alpha_2 = \alpha_2 \; , \quad \beta_2 = \beta_2$$

$$\alpha_3 = -\frac{2}{3}\alpha_2 \frac{3k_2 + 5k_1 + k_1 k_2 + 12}{k_1(6+k_2)} \quad , \quad \beta_3 = -\frac{2}{3b}\beta_2$$

$$\alpha_4 = \frac{\alpha_2}{a^2}\frac{4k_1 + 4k_2 + k_1 k_2 + 12}{k_1(6+k_2)} \quad , \quad \beta_4 = \frac{1}{6b^2}\beta_2$$

Where

$$k_i = \frac{a}{\Phi_i D} \text{ for } i = 1,2 \quad \text{and} \quad k_3 = \frac{b}{\Phi_3 D}$$

Use will now be made of the "energy approach".
Replacing $w(x,y)$ in equations (10) and (10) one obtains the following frequency equation.

$$A + \frac{4}{k_1}R_1 + \frac{4}{k_3}R_6\left(\frac{a}{b}\right)^2 - S_2 R_1\left(\frac{a}{b}\right)^2 + \frac{S_1}{D} = \frac{\rho h \omega_o^2 a^4}{D} R_1 R_6 \qquad \text{eq.(13)}$$

Where

$$A = (2 - 2\mu)R_7 R_8 + 2\mu R_3 R_4 + R_5 R_6 + R_1 R_2$$

$$R_1 = \left(\frac{b}{a}\right)^2 \left[\frac{4}{3}\frac{1}{k_3^2} + \frac{26}{45}\frac{1}{k_3} + \frac{26}{405}\right]$$

$$p_1 = \frac{3k_2 + 5k_1 + k_1 k_2 + 12}{k_1(6+k_2)} \quad , \quad p_2 = \frac{4k_1 + 4k_2 + k_1 k_2 + 12}{k_1(6+k_2)}$$

$$n = \frac{r}{b} \; , m = \rho h$$

$$R_2 = 4 - 24p_1 + 16p_2 + 48p_1^2 - 72p_1 p_2 + \frac{144}{5}p_2^2$$

$$R_3 = \frac{1}{3}\frac{1}{k_3} + \frac{1}{21}$$

$$R_4 = \frac{2}{k_1} - \frac{8p_1}{k_1} + \frac{6p_2}{k_1} + \frac{2}{3} - 4p_1 + \frac{14}{5}p_2 + \frac{24}{5}p_1^2 - 6p_1p_2 + \frac{12}{7}p_2^2$$

$$R_5 = \frac{4}{5}(\frac{a}{b})^2$$

$$R_6 = \frac{4}{3}\frac{1}{k_1^2} + \frac{1}{k_1} + \frac{1}{5}(1 - \frac{8p_1}{k_1}) + \frac{1}{3}(\frac{2p_2}{k_1} - 2p_1) + \frac{1}{7}(4p_1^2 + 2p_2) - \frac{1}{2}p_1p_2 + \frac{1}{9}p_2^2$$

$$R_7 = \frac{4}{k_3^2} + \frac{2}{k_3} + \frac{2}{7}$$

$$R_8 = \frac{4}{k_1^2} + \frac{4}{k_1} - \frac{8p_1}{k_1} + \frac{4p_2}{k_1} + \frac{4}{3} - 6p_1 + \frac{36}{5}p_1^2 - 8p_1p_2 + \frac{16}{5}p_2 + \frac{16}{5}p_2^2$$

$$S_2 = (\frac{b}{a})^2[\frac{4}{k_1} + 4 - 36p_1 + 32p_2 - \frac{24p_1}{k_1} + 72p_1^2 - 120p_1p_2 + \frac{24p_2}{k_1} + 48p_2^2]$$

$$S_1 = N_x a^2(\frac{b}{a})^2 R_8[(\frac{a}{b})^2 R_1 - \frac{4n^2}{k_3^2} - \frac{1}{k_3}(4n^3 - \frac{8}{3}n^4 + \frac{2}{3}n^5) - (n^4 - \frac{4}{3}n^5 + \frac{7}{9}n^6 - \frac{2}{9}n^7 + \frac{1}{36}n^8)]$$

$$\therefore\ \omega_s^2 = \omega_{s'}^2 + N_x a^2(\frac{b}{a})^2 \frac{R_8}{R_1R_6}[(\frac{a}{b})^2 R_1 - \frac{4n^2}{k_1^2} - \frac{1}{k_3}(4n^3 - \frac{8}{3}n^4 + \frac{2}{3}n^5) - (n^4 - \frac{4}{3}n^5 + \frac{7}{9}n^6 - \frac{2}{9}n^7 + \frac{1}{36}n^8)]$$

eq.(14)

For

$r = 0$ $\quad S_1 = N_x a^2 R_1 R_8$

$r = b/2$ $\quad S_1 = N_x b^2 R_8(\frac{1}{3k_3^2} + \frac{161}{720}\frac{1}{k_1} + \frac{20}{609})$

$r = b$ $\quad S_1 = -N_x b^2 R_8(\frac{8}{3k_3^2} + \frac{64}{45}\frac{1}{k_1} + \frac{301}{1620})$

$$\therefore\ \omega_s^2 = \omega_a^2 + \frac{N_x}{ma^2}(\frac{R_8}{R_6}) \qquad \text{eq. (15)}$$

For a square plate, b = a , then the frequency equation , becomes:

$$A + \frac{4}{k_1}R_1 + \frac{4}{k_3}R_6 - S_2R_1 + \frac{S_1}{D} = \frac{\rho h \omega_n^2 a^4}{D}R_1R_6 \qquad \text{eq. (16)}$$

$$R_1 = [\frac{4}{3}\frac{1}{k_3^2} + \frac{26}{45}\frac{1}{k_1} + \frac{26}{405}]$$

$$S_2 = [\frac{4}{k_1} + 4 - 36p_1 + 32p_2 - \frac{24p_1}{k_1} + 72p_1^2 - 120p_1p_2 + \frac{24p_2}{k_1} + 48p_2^2]$$

$$S_1 = N_x a^2 R_8 [R_1 - \frac{4n^2}{k_3^2} - \frac{1}{k_3}(4n^3 - \frac{8}{3}n^4 + \frac{2}{3}n^5) -$$

$$(n^4 - \frac{4}{3}n^5 + \frac{7}{9}n^6 - \frac{2}{9}n^7 + \frac{1}{36}n^8)]$$

$$\therefore\ \omega_{nx}^2 = \omega_{of}^2 + N_x a^2 \frac{R_8}{R_1 R_6}[R_1 - \frac{4n^2}{k_3^2} - \frac{1}{k_3}(4n^3 - \frac{8}{3}n^4 + \frac{2}{3}n^5) -$$

$$(n^4 - \frac{4}{3}n^5 + \frac{7}{9}n^6 - \frac{2}{9}n^7 + \frac{1}{36}n^8)]$$

eq. (17)

For

$r = 0$ $\qquad S_1 = N_x a^2 R_1 R_8$

$r = b/2$ $\qquad S_1 = N_x a^2 R_8 (\frac{1}{3k_3^2} + \frac{161}{720}\frac{1}{k_3} + \frac{20}{609})$

$r = b$ $\qquad S_1 = -N_x a^2 R_8 (\frac{8}{3k_3^2} + \frac{64}{45}\frac{1}{k_3} + \frac{301}{1620})$

$$\therefore\ \omega_x^2 = \omega_{of}^2 + \frac{N_x}{ma^2}(\frac{R_8}{R_6})$$

eq. (18)

If the value of rotational stiffness is set to zero and infinity (as numerical values 10^{-2} and 10^4 respectively , (1985), the simply supported and clamped boundary conditions can be obtained respectively. By setting the rotation stiffness in the range of zero to infinity in the edges, different boundary conditions could be obtained .General closed-form solutions are given for vibration of a rectangular plate with various elementary boundary conditions on each of the four edges.

Simply Supported -Simply Supported-Simply Supported-Free (S-S-S-F) Plate

$$\omega_n^2 \cong (\frac{97.55}{a^4} + \frac{41.45}{a^2b^2})\frac{D}{m} + \frac{N_x}{ma^2}(9.870 - 153.58n^4 + 204.818n^5 - 119.483n^6 +$$
$$34.138n^7 - 4.267n^8)$$

eq. (19)

Now, if $N_x = 0$ then

$$\omega_{of}^2 \cong (\frac{97.55}{a^4} + \frac{41.45}{a^2b^2})\frac{D}{m}$$ eq. (20)

If r=0, then

$$\omega_n^2 \cong (\frac{97.55}{a^4} + \frac{41.45}{a^2b^2})\frac{D}{m} + \frac{N_x}{ma^2}(9.870)$$ eq. (21a)

$$\omega_{ox}^2 = \omega_{of}^2 + \frac{N_x}{ma^2}(9.870)$$ eq. (21b)

If r = b/2, then

$$\omega_o^2 \cong \left(\frac{97.55}{a^4}+\frac{41.45}{a^2b^2}\right)\frac{D}{m}+\frac{N_x}{ma^2}(2.467)$$ eq. (22a)

$$\omega_{ox}^2 \cong \omega_{of}^2+\frac{N_x}{ma^2}(2.467)$$ eq. (22b)

If r=b, then

$$\omega_o^2 \cong \left(\frac{97.55}{a^4}+\frac{41.45}{a^2b^2}\right)\frac{D}{m}-\frac{N_x}{ma^2}(19.742)$$ eq. (23a)

$$\omega_{ox}^2 \cong \omega_{of}^2+\frac{N_x}{ma^2}(19.742)$$ eq. (23b)

Simply Supported-Simply Supported-Clamped –Free(S-S-C-F) Plate

$$\omega_o^2 \cong \left(\frac{97.55}{a^4}+\frac{57.0097}{a^2b^2}+\frac{12.456}{b^4}\right)\frac{D}{m}+\frac{N_x}{ma^2}(9.870-153.58n^4+204.818n^5-119.483n^6+34.138n^7-4.267n^8)$$

eq. (24)

Now, If N_x=0 then

$$\omega_{of}^2 \cong \left(\frac{97.55}{a^4}+\frac{57.0097}{a^2b^2}+\frac{12.456}{b^4}\right)\frac{D}{m}$$ eq. (25)

If r=0, then

$$\omega_{of}^2 \cong \left(\frac{97.55}{a^4}+\frac{57.0097}{a^2b^2}+\frac{12.456}{b^4}\right)\frac{D}{m}+\frac{N_x}{ma^2}(9.870)$$ eq. (26a)

$$\omega_{ox}^2 \cong \omega_{of}^2+\frac{N_x}{ma^2}(9.87)$$ eq.(26b)

If r = b/2 ,then

$$\omega_{of}^2 \cong \left(\frac{97.55}{a^4}+\frac{57.0097}{a^2b^2}+\frac{12.456}{b^4}\right)\frac{D}{m}+\frac{N_x}{ma^2}(5.0482)$$ eq. (27a)

$$\omega_{ox}^2 \cong \omega_{of}^2+\frac{N_x}{ma^2}(5.0482)$$ eq. (27b)

If r=b ,then

$$\omega_{of}^2 \cong \left(\frac{97.55}{a^4}+\frac{57.0097}{a^2b^2}+\frac{12.456}{b^4}\right)\frac{D}{m}+\frac{N_x}{ma^2}(-28.565)$$ eq. (28a)

$$\omega_{ox}^2 \cong \omega_{of}^2+\frac{N_x}{ma^2}(-28.565)$$ eq. (28b)

Clamped-Clamped-Simply Supported-Free(C-C-S-F) Plate

$$\omega_o^2 \cong \left(\frac{593.59}{a^4}+\frac{50.38}{a^2b^2}\right)\frac{D}{m}+\frac{N_x}{ma^2}(11.995-35.98n^2)$$ eq. (29)

Now, if N_x=0, then

eq

$$\omega_{of}^2 \cong \left(\frac{593.59}{a^4}+\frac{50.38}{a^2b^2}\right)\frac{D}{m}$$ (30)

If r=0, then"

$$\omega_o^2 \cong \left(\frac{593.59}{a^4} + \frac{50.38}{a^2b^2}\right)\frac{D}{m} + \frac{N_x}{ma^2}(11.995) \qquad \text{eq. (31a)}$$

$$\omega_{os}^2 \cong \omega_{of}^2 + \frac{N_x}{ma^2}(11.995) \qquad \text{eq. (31b)}$$

If r = b/2 ,then

$$\omega_o^2 \cong \left(\frac{593.59}{a^4} + \frac{50.38}{a^2b^2}\right)\frac{D}{m} + \frac{N_x}{ma^2}(2.998) \qquad \text{eq. (32a)}$$

$$\omega_{os}^2 = \omega_{of}^2 + \frac{N_x}{ma^2}(2.998) \qquad \text{eq. (32b)}$$

If r=b, then

$$\omega_o^2 \cong \left(\frac{593.59}{a^4} + \frac{50.38}{a^2b^2}\right)\frac{D}{m} + \frac{N_x}{ma^2}(-23.99) \qquad \text{eq. (33a)}$$

$$\omega_{os}^2 = \omega_{of}^2 + \frac{N_x}{ma^2}(-23.99) \qquad \text{eq. (33b)}$$

Clamped-Simply Supported -Simply Supported –Free (C-S-S-F) Plate

$$\omega_o^2 \cong \left(\frac{238.64}{a^4} + \frac{47.73}{a^2b^2}\right)\frac{D}{m} + \frac{N_x}{ma^2}(11.995 - 35.98n^2) \qquad \text{eq. (34)}$$

Now, if N_x=0 then

$$\omega_{of}^2 \cong \left(\frac{238.64}{a^4} + \frac{47.73}{a^2b^2}\right)\frac{D}{m} \qquad \text{eq. (35)}$$

If r=0

$$\omega_o^2 \cong \left(\frac{238.64}{a^4} + \frac{47.737}{a^2b^2}\right)\frac{D}{m} + \frac{N_x}{ma^2}(11.995) \qquad \text{eq. (36a)}$$

$$\omega_{os}^2 = \omega_{of}^2 + \frac{N_x}{ma^2}(11.995) \qquad \text{eq. (36b)}$$

If r = b/2, then

$$\omega_o^2 \cong \left(\frac{238.64}{a^4} + \frac{47.737}{a^2b^2}\right)\frac{D}{m} + \frac{N_x}{ma^2}(2.8416) \qquad \text{eq. (37a)}$$

$$\omega_{os}^2 = \omega_{of}^2 + \frac{N_x}{ma^2}(2.8416) \qquad \text{eq. (37b)}$$

If r=b ,then

$$\omega_o^2 \cong \left(\frac{238.64}{a^4} + \frac{47.737}{a^2b^2}\right)\frac{D}{m} + \frac{N_x}{ma^2}(-22.732) \qquad \text{eq. (38a)}$$

$$\omega_{os}^2 = \omega_{of}^2 + \frac{N_x}{ma^2}(-22.732) \qquad \text{eq. (38b)}$$

Clamped- Simply Supported –Clamped-Free(C-S-C-F) Plate

$$\omega_o^2 \cong \left(\frac{238.64}{a^4} + \frac{65.748}{a^2b^2} + \frac{12.456}{b^4}\right)\frac{D}{m} + \frac{N_x}{ma^2}(11.366 - 0.0707n^3$$
$$-176.84n^4 + 235.84n^5 - 137.58n^6 + 39.3089n^7 - 4.9136n^8)$$

eq. (39)

Now, if $N_x=0$, then

$$\omega_{of}^2 \cong (\frac{238.64}{a^4}+\frac{65.748}{a^2b^2}+\frac{12.456}{b^4})\frac{D}{m} \quad \text{eq. (40)}$$

If r=0, then

$$\omega_o^2 \cong (\frac{238.64}{a^4}+\frac{65.748}{a^2b^2}+\frac{12.456}{b^4})\frac{D}{m}+\frac{N_x}{ma^2}(11.366) \quad \text{eq. (41a)}$$

$$\omega_{os}^2 \cong \omega_{of}^2+\frac{N_x}{ma^2}(11.366) \quad \text{eq. (41b)}$$

If r = b/2, then

$$\omega_o^2 \cong (\frac{238.64}{a^4}+\frac{65.748}{a^2b^2}+\frac{12.456}{b^4})\frac{D}{m}+\frac{N_x}{ma^2}(5.81286) \quad \text{eq. (42a)}$$

$$\omega_{os}^2 \cong \omega_{of}^2+\frac{N_x}{ma^2}(5.812) \quad \text{eq. (42b)}$$

If r=b, then

$$\omega_o^2 \cong (\frac{238.64}{a^4}+\frac{65.748}{a^2b^2}+\frac{12.456}{b^4})\frac{D}{m}+\frac{N_x}{ma^2}(-32.891) \quad \text{eq. (43a)}$$

$$\omega_{os}^2 \cong \omega_{of}^2+\frac{N_x}{ma^2}(-32.891) \quad \text{eq. (43b)}$$

Clamped- Clamped–Clamped-Free(C-C-C-F) Plate

$$\omega_o^2 \cong (\frac{503.96}{a^4}+\frac{69.424}{a^2b^2}+\frac{12.456}{b^4})\frac{D}{m}+\frac{N_x}{ma^2}(11.999-186.894n^4+249.197n^5-145.366n^6+41.533n^7-5.19163n^8) \quad \text{eq. (44)}$$

Now, if $N_x=0$, then

$$\omega_{of}^2 \cong (\frac{503.96}{a^4}+\frac{69.424}{a^2b^2}+\frac{12.456}{b^4})\frac{D}{m} \quad \text{eq. (45)}$$

If r = 0, then

$$\omega_o^2 = (\frac{503.96}{a^4}+\frac{69.424}{a^2b^2}+\frac{12.456}{b^4})\frac{D}{m}+\frac{N_x}{ma^2}(11.999) \quad \text{eq. (46a)}$$

$$\omega_{os}^2 \cong \omega_{of}^2+\frac{N_x}{ma^2}(11.999) \quad \text{eq. (46b)}$$

If r = b/2, then

$$\omega_o^2 = (\frac{503.96}{a^4}+\frac{69.424}{a^2b^2}+\frac{12.456}{b^4})\frac{D}{m}+\frac{N_x}{ma^2}(6.138) \quad \text{eq. (47a)}$$

$$\omega_{os}^2 \cong \omega_{of}^2+\frac{N_x}{ma^2}(6.138) \quad \text{eq. (47b)}$$

If r=b, then

$$\omega_o^2 = (\frac{503.96}{a^4}+\frac{69.424}{a^2b^2}+\frac{12.456}{b^4})\frac{D}{m}+\frac{N_x}{ma^2}(-34.73) \quad \text{eq. (48a)}$$

$$\omega_{os}^2 \cong \omega_{of}^2+\frac{N_x}{ma^2}(-34.73) \quad \text{eq. (49b)}$$

VALIDITY OF THE PRESENT SOLUTION

Since the frequency equations derived in this investigation are new and general, their validity had to be checked. The checking method will be based on determining the characteristics for cases with known analytical solutions such as those with classical boundary conditions having no residual stresses. As another check, the results are compared with those obtained by the FEM. An advantage of using FEM simulation is that a complete picture of the residual stress fields is obtained. Programmatic residual stress estimates and the study of their effects were performed experimentally as well as computationally.

The circular frequency of the plate elastically restrained against rotation along three edges and free on the fourth edge, horizontally line heated at any position (r), Fig (1), is given in(rectangular plate Eq.(13) ,square plate Eq(16)) .Its relation with that of plate without residual stresses, the circular frequency is given in (rectangular plate Eq.(14) ,square plate Eq.(17)).

If the line heated is at positions (r=0, b/2, b) the circular frequency is given in (rectangular plate Eq(15) ,square plate Eq(18)).

If N_x is set to zero in Eq.(13), same frequency equation given in Ref [1978], will be obtained.

If the dimensionless rotational stiffness K_1, K_2 and K_3, are taken from 10^{-2} to 10^4, all the boundary conditions between the extremes of S-S-S-F and C-C-C-F will be obtained. The dimensionless frequency parameter of the first mode (λ^2_{11}) of stress free rectangular plate for five possible combination of the three elementary boundary conditions on the four edges of the plate are given in **Table (1).** This table shows a close agreement between the predicted results of this work and the numerical results presented by R.D.Blevins,1979.

NUMERICAL VERIFICATION

In this section a finite element model is developed to predict the effects of residual stresses due to one or more weld runs parallel to the plate edges. These effects include the determination of:-

1- Equivalent Stress, Shear Stresses, Equivalent Strain and Total Deformation together with their variation over the area of the plate.
2- Vibration characteristics including natural frequencies and mode shapes.
3- Fatigue characteristics including Life, Damage, Safety Factor, Biaxiality indication and Fatigue Sensitivity.

The tendon force concept as suggested by Porter Golf,(1976), is used to induce an equivalent stress pattern due to the welds .A model of 3.175 mm thick structure steel plate measuring 571.5 mm by 285.75 mm with movement of welding parallel to the x-axis was chosen to check the accuracy of the derived equations and to see whether welding residual stresses induced do have a significant effect on the stress, strain, vibration and fatigue characteristics. The line of welding is chosen parallel to the x-axis at (r=0,r=b/2 & r=b) as an area, distributed force.

Table (2), shows the results for the first natural frequency at different welding positions, calculated by the derived equations in this study and the calculated results using (FEM). These results indicated that a significant decrease on the natural frequencies was noticed when welding was at the free end (r=b) .The first three mode shapes, stresses, strains, deformation and fatigue characteristics due to the effect of welding are shown in Fig (2) through Fig (5).

MEASUREMENT OF NATURAL FREQUENCIES

Alloy steel plates (600 x 300 x 5 mm) were tested to find the first three natural frequencies before and after stress inducement. The heat treatment process was performed on the entire experimental models to get stress-free state. The models were put into an oven at 650 °C .Treatment time was 1 hr for each inch thickness and the models left to cool inside the oven after that time.

The residual stresses were induced by welding with the following characteristics:

Constant welding speed (100Amp, 220V, 250 mm/min).

Measurement of natural frequencies was performed before and after welding to find the shift in this frequency due to induced residual stress in welded plate. This shift is used to estimate the level of residual stresses. The instruments used in natural frequency and strain measurements, **Fig (6)**, are:-

1- Electromagnetic shaker type B&K 4810.
2- Sine generator type B&K 1023.
3- Accelerometer type B&K 4344.
4- Conditioning amplifier type B&K 4344.
5- Oscilloscope.

Fig (6) shows a block diagram for the instruments used in testing in the present investigation. The tested plate was excited by a shaker which was powered by a signal generator producing a sinusoidal wave signal in the frequency range from 0 to 20 kHz. The system response was picked up by a transducer whose output was fed through a conditioning amplifier to a double beam cathode ray oscilloscope. The theoretical and experimental results are shown in **Table (3).**

CONCLUSIONS

In this study the general energy method was employed for analyzing the vibration characteristics of thin rectangular plate with and without including the effect of residual stresses .Comparisons were made with published and finite element results. Sufficient results have been obtained to suggest that there is a general pattern of the behavior.

From the observations of this study of the following conclusions can be drawn.

1- The original contributions are the frequency equations with and without including the effect of residual stresses for a thin plate elastically restrained against rotation along three edges and free on the fourth edge, S-S-S-F , S-S-C-F , C-C-S-F , C-S-S-F , C-S-C-F , C-S-C-F .Also a general equation relating the free vibration frequencies with that including the effect of residual stresses for all the cases listed before are given.
2- Residual stresses have a very significant influence on the natural frequencies and fatigue characteristics for the free boundary conditions compared with clamped boundaries. Thus, the boundary condition is an important factor that influences the vibration, stress and fatigue properties.
3- The position of welding has a significant effect on the vibration and fatigue characteristics. This effect increases when welding is at free edges and this effect decreases when welding is at clamped edge.
4- Tensile residual stresses will increase the natural frequency while compression residual stresses will decrease the natural frequency.
5- The increase of residual stresses and loading ratio will increase the damage .The safety factor decreases as the tendon force and loading ratio increases. The residual stresses have a significant effect on the fatigue properties. Zero based loading ratios have less effect than the fully reversed on the fatigue characteristics.
6- The use of rotational flexibility boundaries enable different boundary conditions to be treated.
7- The methods presented in this paper provide exact techniques for determining the effect of residual stresses on the natural frequencies which satisfy many boundary conditions
8- The experimental results are within acceptable agreement with theoretical results.

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NOMENCLATURE

a, b	plate side lengths (mm)
$D=E h^3 /12(1-\mu^2)$	flexural rigidity(N.mm)
E	Young's Modulus of elasticity(N/mm^2)
F	tendon force (N)
.h	plate thickness (mm)
$m=\rho h$	Density per unit length (kg/mm^2)
$M_{x,y}$	bending moment on a plate element (N.mm)
M_{xy} , M_{xy}	twisting moments (N.mm)
$.n=r/b$	Ratio of welding position along the y-axis to the width of the plate
N_x	mid-plane forces per unit length(N/mm)
r	Welding position along the y-axis (mm)
t	time(sec)
T	maximum kinetic energy of the element
U	strain energy stored in a complete plate
U_b	energy stored due to bending

U_t	total energy due to twisting
U_r	strain energy due to the forces
w	deflection of the plate (mm)
x,y,z	co-ordinate axes (mm)
μ	Poisson's ratio
$\varphi 1, \varphi 2, \varphi 3$	rotational stiffnesses along the edges of the plate (N/rad)
ρ	mass density (kg/mm^3)
ω_o	angular frequency (rad/sec)
ω_{of}, ω_{os}	angular frequency without and with including residual stresses (rad/sec)
ω_{oc}, ω_{oe}	angular frequency (welding along center and edge of the plate) (rad/sec)
$\lambda^2_{ij} = \omega_0^2 a^2 (\frac{\rho h}{D})^{\frac{1}{2}}$	Dimensionless frequency parameter

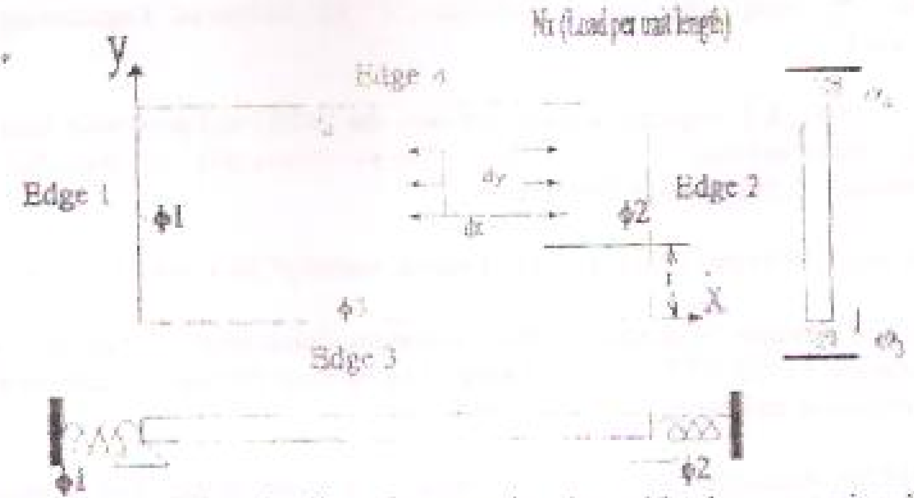

Fig.(1) Geometry and plane loading of rectangular plate with edge possessing different rotational flexibility coefficient

Welding at r=0

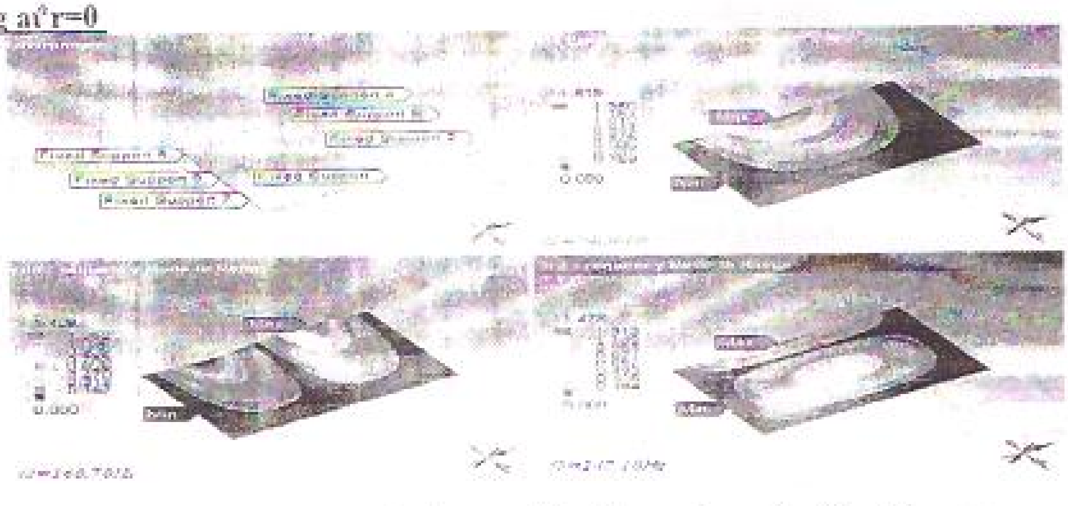

Fig .(2) First Three mode shapes of the Plate without Residual Stresses

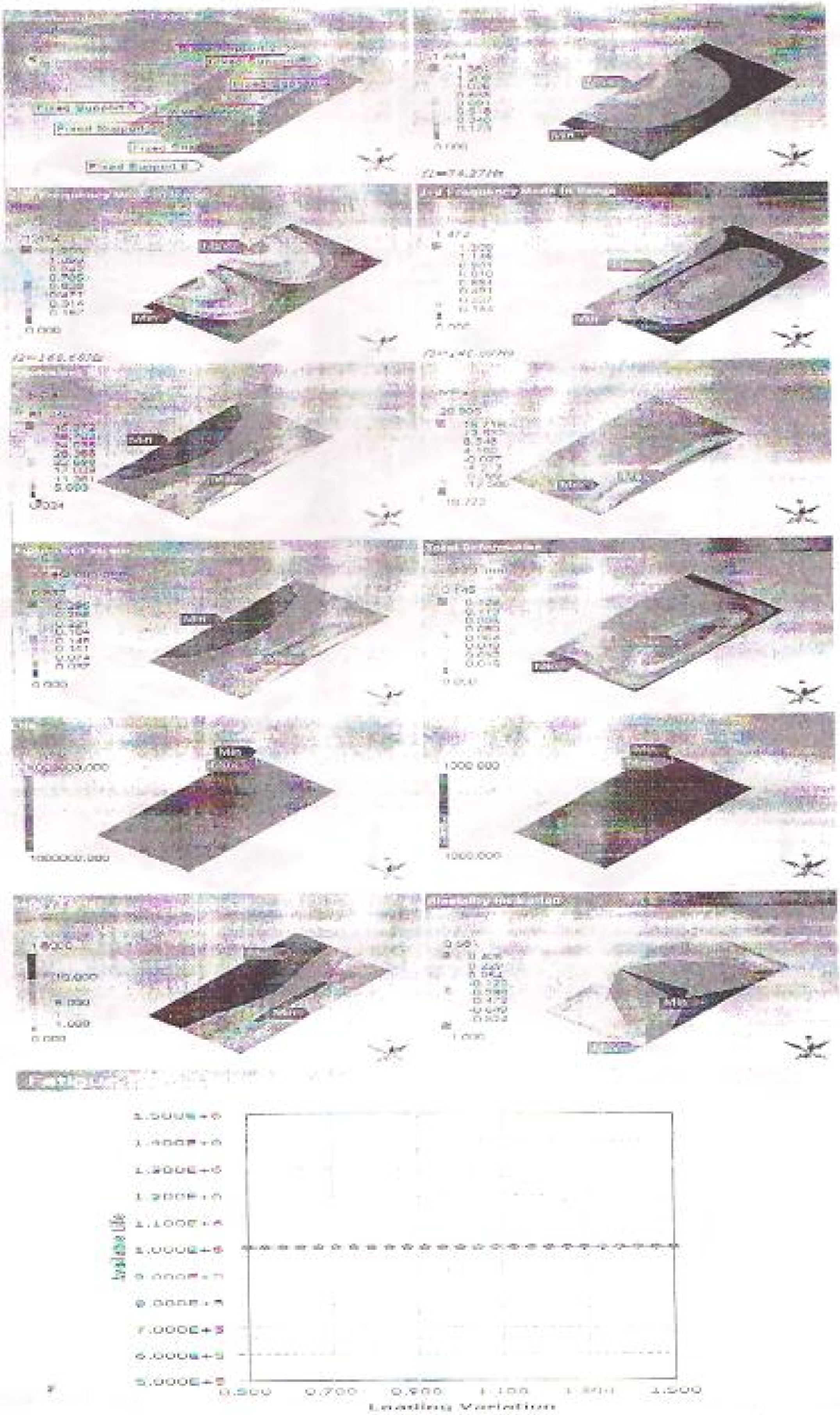

Fig (3), Vibration, Stress and Fatigue Characteristics (Welding at r=0)

Welding at r=b/2

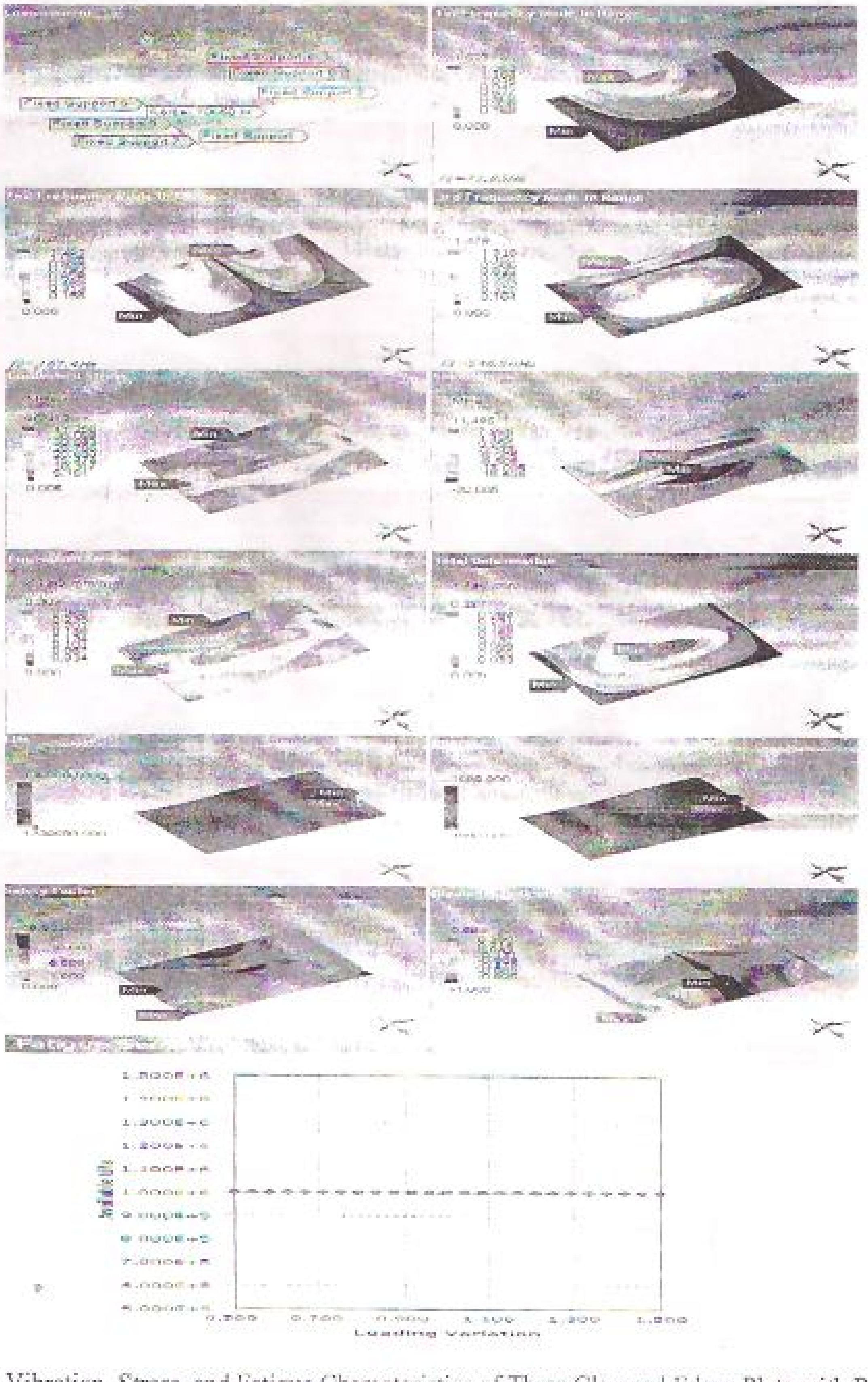

Fig (4), Vibration, Stress, and Fatigue Characteristics of Three Clamped Edges Plate with Residual Stresses (Welding at r=b/2)

Welding at r=b

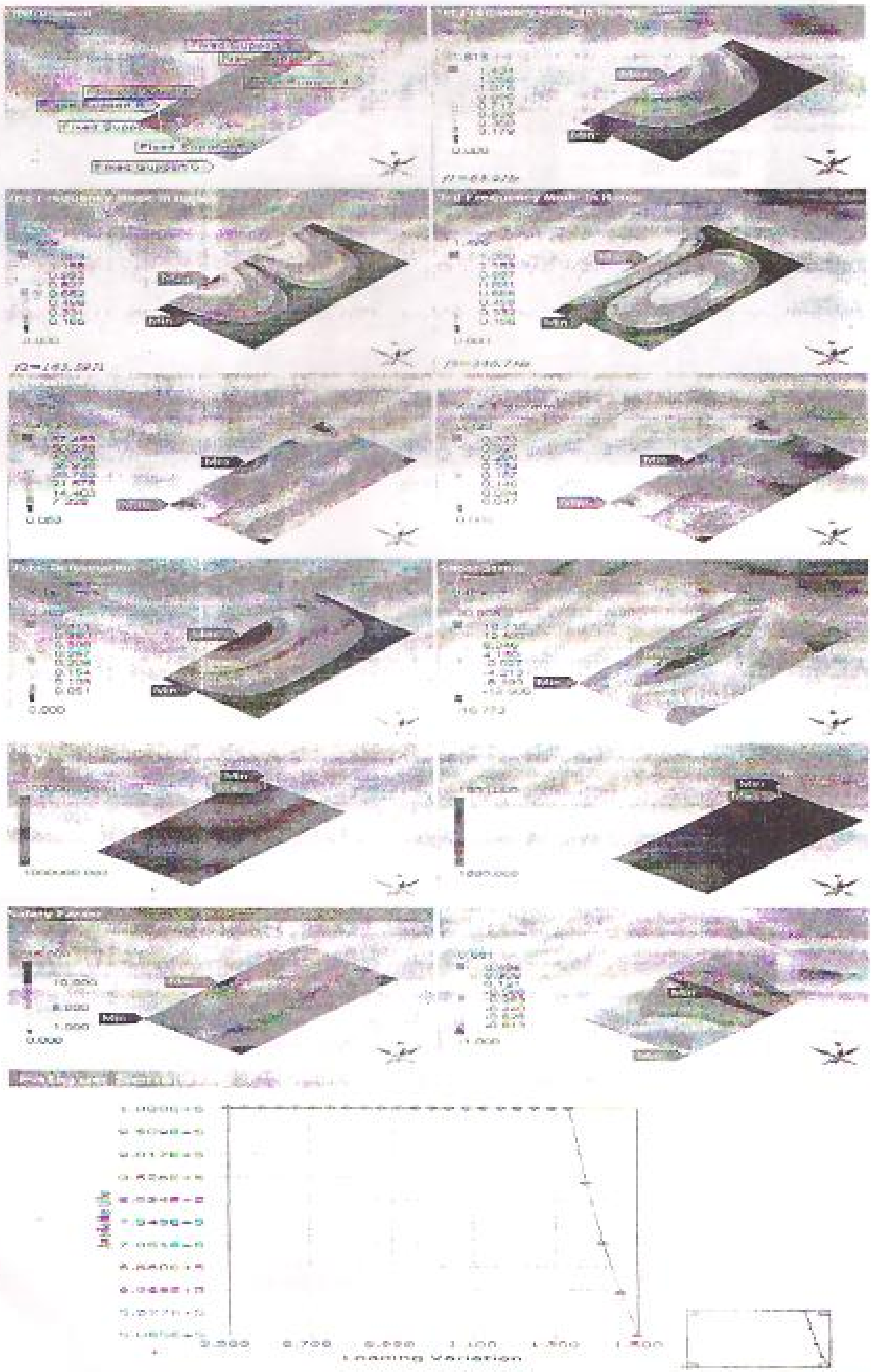

Fig (5), Vibration, Stress and Fatigue Characteristics of Three Clamped Edges Plate with Residual Stresses (Welding at r=b)

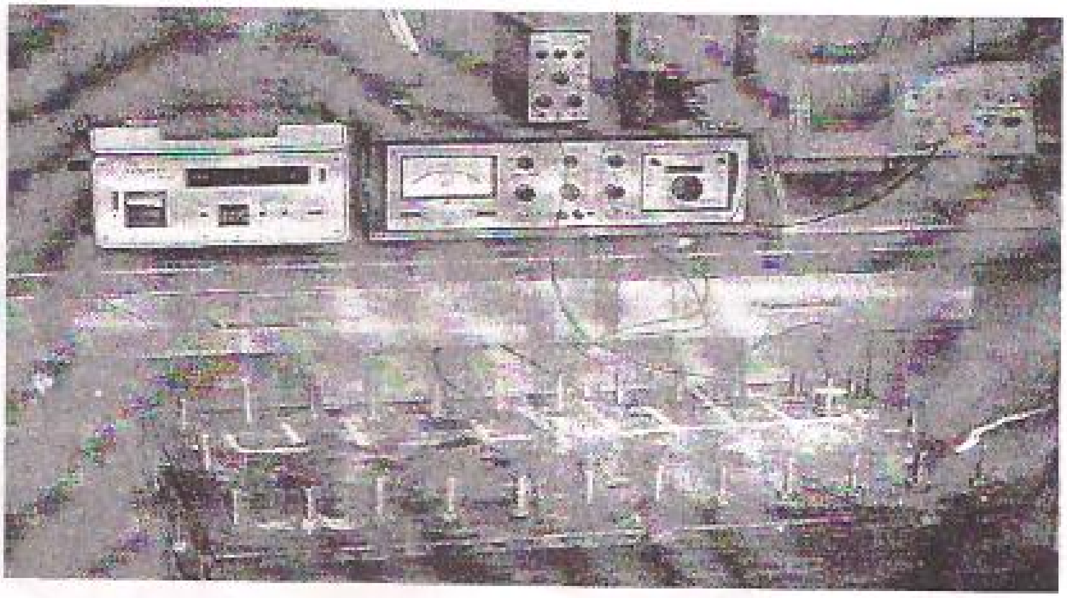

Fig.(6) Instrumentation used in natural frequency and strain measurements

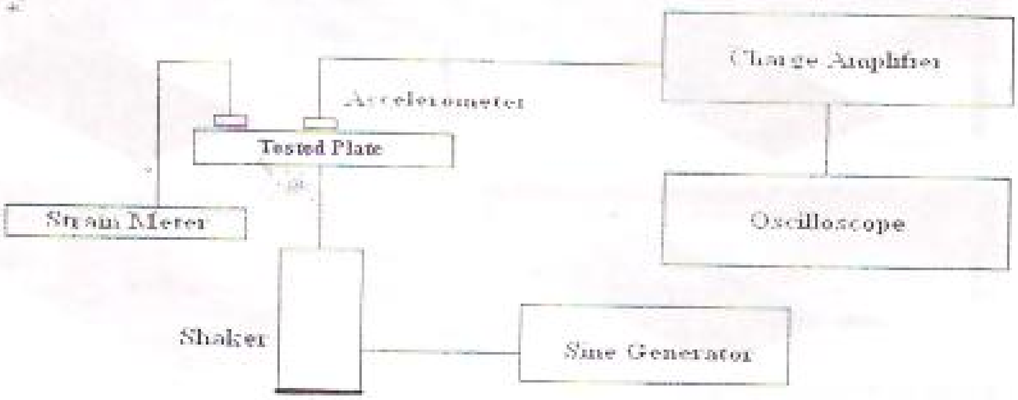

Fig.(7) block diagram for the testing elements

Table (1) dimensionless frequency parameter (λ^2_{11}) of a rectangular plate

$$\lambda^2_{11} = \varpi_n^{\ 2} a^2 (\frac{\rho h}{D})^{\frac{1}{2}}$$

No.	Description	Equation No.	K_1	K_2	K_3	$\frac{a}{b}$ = 0.4	$\frac{2}{3}$	1	1.5	2.5
1	S-S-S-F	(4-44)	0	0	0	10.707 [10.13]	10.76 [10.67]	11.79 [11.68]	13.81 [13.71]	18.88 [18.80]
2	S-S-C-F	(4-49)	0	0	∞	10.34 [10.19]	11.19 [10.98]	12.92 [12.69]	17.00 [16.82]	30.67 [30.63]
3	C-C-S-F	(4-54)	∞	∞	0	22.61 [22.54]	22.93 [22.86]	23.53 [23.46]	24.83 [24.78]	28.60 [28.56]
4	C-S-S-F	(4-59)	∞	0	0	15.69 [15.65]	16.12 [16.07]	16.92 [16.87]	18.60 [18.54]	23.17 [23.07]
5	C-S-C-F	(4-64)	∞	0	∞	15.79 [15.70]	16.44 [16.29]	17.80 [17.62]	21.20 [21.04]	33.70 [33.58]
6	C-C-C-F	(4-69)	∞	∞	∞	22.69 [22.58]	23.17 [23.02]	24.19 [24.02]	26.88 [26.73]	37.73 [37.66]

S -Simply supported, C-Clamped, F-Free

Table (2) Analytical and FEM results at three welding positions

Case	Present study (Nx=123.6 N/mm)	FEM Results (F=70650N)
No welding	74.0214	74.36
r=0	74.05	74.27
r=b/2	74.04	72.85
r=b	73.93	68.9

Table (3) Stress free and bead weld frequency

Mode No.	Free Stress Conditions Frequency (Hz)			Stress Condition Bead on plate Welding* Frequency (Hz)		
	Exact.	FEM	Exp.	Exact.	*FEM*	*Exp.*
1	105.75	107 [102.1]*	92	*105.85*	*93.58*	*100*
2	-----	243.1 [230.2]	222	----	*231.53*	*220*
3	-----	358.57 [337.6]	314	----	*357.09*	*399*

*Numerical Results,