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# MATERIAL NONLINEAR BEHAVIOR OF REINFORCED CONCRETE SHELL FOUNDATIONS

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## ABSTRACT

Shell foundations are generally found to be economic under conditions of heavy loads and weak soils as small amount of reinforcement is needed due to occurrence of compressive stresses in most parts of the foundation. Depending upon their size, conical shells can serve as footings for columns while inverted domes shells can serve as rafts for tanks supported on a circular row of columns. This paper describes 3-D finite element models, the eight nodes degenerated shell and the twenty nodes brick elements which are used herein. The models which may be adopted in the material nonlinear analysis of reinforced concrete shell foundations are described briefly in this paper. The present study results give good agreement compared with available experimental values about 5% in displacements.

## الخلاصة

تكون الأسس القشريةالمصنوعة من الخرسانة المسلحة بصورة عامة اقتصادية في حالة الأحمال الثقيلة المستندة على تربة ضعيفة لأكتفاء هذه الأسس على حديد تسليح قليل بسبب حصول اجهادات ضغط في معظم اجزاءها. اعتمادا على أحجام الأسس فان القشريات المخروطية يمكن استخدامها كأساس لعمود بينما القباب المقلوبة يمكن استخدامها كأساس لخزان مستند على سطر دائري من الأعمدة.

ان هذا البحث يتناول طريقة لتحليل الأسس القشرية الخرسانية المسلحة باستخدام طريقة العناصر المحددة في الأبعاد الثلاثيةحيث تم استخدام العنصر القشري المنحل ذو العقد الثماني لتمثيل الاساس والعنصر الطابوقي ذو العقد العشرون لتمثيل التربة وقد تضمنالتمثيل التصرف اللاخطي للقشريات الخرسانية المسلحة المستخدمة كأسس.

أظهرت نتائج هذه الطريقة كفاءة ودقة بالمقارنة مع النتائج العملية بحدود 5% فرق في الازاحات.

**KEYWORDS**
Finite elements, material nonlinearity, shell, foundations.

## INTRODUCTION

Shell foundations have been established as economic alternatives to plain (flat) shallow foundations in situation involving heavy superstructural loads to be transmitted to weaker soils. The use of shells in foundation, as in roofs, leads to considerable saving in materials. In the case of axisymmetric shells such as the cone and the inverted dome, this is achieved without much extra input of labor. A theoretical work using a material nonlinear finite element analysis is carried out to study the behavior of reinforced concrete shell foundations. In the case of reinforced concrete shells, material nonlinearity as a result of tension cracking, strain softening after cracking, the nonlinear response of concrete in compression, crushing of concrete and the yielding of reinforcement have been considered. A layered approach is used in the present finite element analysis. The concrete is divided into a number of layers through the thickness and the smeared reinforcement layer is included within these concrete layers at an appropriate position. The elastoplastic constitutive relations with a single yield surface are employed for modeling the concrete and the reinforcement. The soil-structure interaction between the shell elements and the supporting media are modeled by using a three-dimensional computational model as interface element. The eight and the twenty node isoparametric brick elements are used for the soil. Also, the eight and the twenty node thin layer elements are used to model the interface. The elastoplastic constitutive relations with a cap yield surface are employed for modeling the soil (Al-Azzawi 2000).

## FINITE ELEMENT MODELS

Three approaches to the finite element representation of general shell structures have traditionally been used:

1- The "faceted" form, with flat elements.
2- Via elements formulated on the basis of curved shell theory.
3- Degenerated isoparametric elements.

Among all of the shell elements, the Ahmad type "degenerated " isoparametric shell element (Ahmad et al. 1970) based on an independent rotational and translational displacement interpolation has become popular in recent years. In this element, the Mindlin-type theory for thick plates and shells is employed. The "normal" to the middle surface of the three-dimensional element is constrained to remain straight after deformation in order to overcome the numerical difficulty associated with the large stiffness ratio in the through-thickness direction. Also, this element neglects the strain energy associated with stresses perpendicular to the local $x' - y'$ surface and constrains the normal stress component to zero to simplify the constitutive equations. By adopting the isoparametric geometric description, the element can be used to represent thin and thick shell components with arbitrary shapes, circumventing the complexities of classical shell theory and differential geometry **Fig. (1)**.

The general finite element procedure to fully three-dimensional problems of stress analysis is rarely used. In many problems the various two dimensional approximations give an adequate and more economical model. It can be pointed out that in most practical analyses, the use of isoparameteric elements satisfying the constant strain states, including all the rigid body modes and maintaining $(C^o)$ continuity is preferred.

The three dimensional computational model for soil is adopted in the present study. The eight and the twenty node hexahedral isoparametric elements shown in **Fig. (2)** are used.

The thin layer interface element developed by Desai et al. in 1984 is used in the present study. The eight and the twenty node isoparametric brick elements with small thickness are used to model the shell-soil interface **Fig. (3)**. The formulation of the thin layer element is essentially the same as for the solid soil element, but its constitutive relationship is different.

## CONSTITUTIVE MODELS

### Material Modeling of Concrete

In the used model, linear elasticity is used for the recoverable part of the strain, and a strain hardening plasticity approach is employed for the irrecoverable part of the deformation (Cervera et al. 1987). A hardening-plasticity model requires the description of a yield condition, a hardening rule, a flow rule and a crushing condition.

**Yield Criterion**

The yield criterion for concrete under triaxial stress state is generally assumed to be dependent on the three stress invariants. Nevertheless, a yield function depending only on $I_1$ and $J_2$ the first stress and the second deviatoric stress invariants can be adequate for most practical situations. The yield criterion adopted in this study is based on the modified Drucker-Prager yield function; it can be written in the form:

$$F(I_1, J_2) = [\alpha_o \cdot I_1 + 3\beta_o \cdot J_2]^{0.5} = \sigma_o \quad (1)$$

where $F$ is the yield surface, $\alpha_o$ and $\beta_o$ are two material parameters and $\sigma_o$ is the equivalent effective stress.

The parameters $\alpha_o$ and $\beta_o$ can be obtained from biaxial experimental results. If Kupfer's results **Fig. (4)** are to be fitted for the principal stress ratios $\sigma_1 / \sigma_2 = \infty$ (uniaxial compression test) and $\sigma_1 / \sigma_2 = 1.0$ (equal biaxial compression test) the values obtained are ($\alpha_o = 0.355 \cdot \sigma_o$ and $\beta_o = 1.355$) (Chen 1982).

It is more realistic to use a hardening model in which $\sigma_o$ is a function of a hardening parameter. A value $\sigma_o = \alpha_1 \cdot f_c^{'}$ defines the initial yield surface for concrete, which limit the elastic behavior. When this surface is reached, inelastic deformation begins and a hardening rule monitors the expansion of the yield surface under further loading. In this way, a whole family of loading surfaces is defined as shown in **Fig. (5)**. Unloading inside the current loading surface occurs elastically. Under loading, the loading surface expands until either the failure surface $\sigma_o = f_c^{'}$ or, the crushing surface in the strain space is reached **Fig. (5)**.

The yield surface equation can be rewritten as:

$$F(\underline{\sigma}) = [2c\sigma_o \cdot I_1 + 3\beta_o \cdot J_2]^{0.5} = \sigma_o \quad (2)$$

with $c = \alpha_o / 2\sigma_o$. This can be solved for $\sigma_o$, giving

$$F(\underline{\sigma}) = cI_1 + [c^2 \cdot I_1^2 + 3\beta_o \cdot J_2]^{0.5} = \sigma_o \quad (3)$$

**Hardening Rule**

The hardening rule defines the motion of the loading surfaces during plastic deformation. A relationship between the accumulated plastic strain and the effective stress $\sigma_o$ is assumed. In this way, the concepts of effective plastic strain and effective stress allow for extrapolation of results from uniaxial test to the multiaxial situation.

In the present work, the relationship is the conventional Madrid parabola (Cervera et al. 1987):

$$\sigma_o = E_o \cdot \varepsilon - \frac{1}{2}\frac{E_o}{\varepsilon_o}\varepsilon^2 \quad (4)$$

where $\sigma_o$ is the effective stress, $E_o$ is the initial Young's modulus, $\varepsilon$ is the current total strain, and $\varepsilon_o$ is the total strain at peak stress $f_c^{'}$, where : $\varepsilon_o = 2f_c^{'} / E_o$.

Substituting the elastic strain $\varepsilon^e = \sigma_o / E_o$ in eq.(4), the desired relation is obtained:

$$\sigma_o = -E_o \cdot \varepsilon^p + (2E_o^2 \cdot \varepsilon_o \cdot \varepsilon^p)^{0.5} \qquad \alpha_1 \cdot f_c' \le \sigma_o \le f_c' \tag{5}$$

where $\varepsilon^p$ is the plastic component of strain. From Eq.(5) the hardening parameter can be obtained:

$$H' = \frac{d\sigma_o}{d\varepsilon^p} = E_o[(\frac{\varepsilon_o}{2\varepsilon^p})^{0.5} - 1] \tag{6}$$

**Flow Rule**

In plasticity model a flow rule is defined so that the increments of plastic strain can be evaluated from given stress state. The associative flow rule will be employed in the present study. This means that the plastic deformation rate vector will be assumed to be normal to the yield surface. The plastic strain increment is defined as:

$$d\underline{\varepsilon}^p = d\lambda \frac{\partial F(\sigma)}{\partial \underline{\sigma}} = d\lambda \cdot \underline{a} \tag{7}$$

where $d\lambda$ is a factor determining the size of the plastic strain increment and $\partial F / \partial \underline{\sigma}$ is a vector normal to the current loading surface.

The flow vector $\underline{a}$ can be computed as:

$$\underline{a}^T = \left[ \frac{\partial F}{\partial \sigma_x}, \frac{\partial F}{\partial \sigma_y}, \frac{\partial F}{\partial \tau_{xy}}, \frac{\partial F}{\partial \tau_{xz}}, \frac{\partial F}{\partial \tau_{yz}} \right]$$

The plastic multiplier can be found to be:

$$d\lambda = \frac{1}{H' + \underline{a}^T [D]_e \cdot \underline{a}} \underline{a}^T [D]_e d\underline{\varepsilon} \tag{8}$$

where $H'$ is the hardening parameter, $[D]_e$ is the elastic constitutive matrix, and $d\underline{\varepsilon}$ is the total strain increment vector.

The complete elastoplastic incremental stress-strain relationship is given by:

$$d\underline{\sigma} = [D]_{ep} \cdot d\underline{\varepsilon} \tag{9}$$

with the elastoplastic material constitutive matrix defined by:

$$[D]_{ep} = [D]_e - \frac{[D]_e \cdot \underline{a}^T \underline{a} \cdot [D]_e}{H' + \underline{a}^T [D]_e \cdot \underline{a}} \tag{10}$$

**Crushing Criterion**

The crushing type of fracture is a strain-controlled phenomenon. A failure surface in the strain space must be defined so that this kind of fracture is taken into account. A simple way of doing so, despite the lack of experimental data on concrete ultimate deformation capacity under multiaxial loading, is to assume a crushing surface in the strain space whose size is related to a maximum equivalent strain extrapolated from uniaxial tests.

The following failure surface may be used:

$$3J_2^{'} = \varepsilon_{cu}^2 \tag{11}$$

where $J_2^{'}$ is the second deviatoric strain invariant and $\varepsilon_{cu}$ is an ultimate total strain value obtained from uniaxial tests. When concrete reaches the crushing surface it is assumed to release all stresses and lose its stiffness.

**Crack Modeling**

The main feature of plain concrete material behavior is its low tensile strength, which results in tensile cracking at very low stress compared with the failure stress in compression. In the finite element method, two main mathematical models are used for crack representations; discrete crack model and smeared crack model.

The smeared approach is used for most structural engineering applications, since it offers:

1- Unchanging of topology of the mesh throughout the analysis, and only the stress–strain relationship needs to be updated when cracking occurs.
2- Complete generality in any possible crack direction.
3- Computational efficiency

A smeared crack model is adopted in this study. To be fully described such a model requires the following items:

**Cracking criterion**

In the present work, concrete in tension is modeled as a linear elastic strain softening material and the maximum tensile stress criterion will be employed to distinguish elastic behavior from tensile fracture. For a previously uncracked sampling point, when a principal stress exceeds a limiting value, a crack is formed in plane orthogonal to this stress. Thereafter, the behavior of the concrete is no longer isotropic, it becomes orthotropic, and the local material axes coincide with the principal stress directions. It should be noted that the direction of the crack is assumed to remain fixed "fixed crack approach". A maximum of two sets of cracks are allowed to form at each sampling point. For simplicity, the crack directions are assumed to be orthogonal.

The limiting value required to define the onset of cracking is established as follows:

1- In the tension-tension zone,

$$\sigma_i = f_t^{'} \qquad i = 1,2 \tag{12}$$

2- In the tension-compression zone,

$$\sigma_i = f_t^{'}\left(1 + \frac{\sigma_{i+1}}{f_c^{'}}\right) \qquad \sigma_{i+1} \le 0 \tag{13}$$

This expression incorporates the fact that compression in one direction favors microcracking in the orthogonal directions, thus reducing tensile capacity.

**Strain softening rule**

The early studies on numerical analysis of reinforced concrete structures assumed concrete to be elastic-brittle material in tension. When cracking occurred, the stress normal to the crack direction was immediately released and dropped to zero. It was soon discovered that this procedure leads to great convergence difficulties, and more importantly, to results that depend strongly on the size of the finite elements used in the analysis. Concrete can take tensile stresses between cracks, this effect is known as tension stiffening. It can be incorporated into the computational model by considering concrete as an elastic strain softening material in tension, and has been extensively used in computational analysis of reinforced concrete structures.

In this study, an exponential function **Fig. (6)** is used to simulate the strain softening effect, so that:

$$\sigma = f_t'[\exp(-(\varepsilon - \varepsilon_o')/\alpha)] \tag{14}$$

where $f_t'$ is the tensile strength of concrete, $\varepsilon_o'$ is the strain at cracking, $\alpha$ is the softening parameter, and $\varepsilon$ is the nominal tensile strain in the cracked zone.

The softening parameter, $\alpha$ can be determined from:

$$\alpha = (G_f - \frac{1}{2} f_t' \varepsilon_o' l_c) / f_t' l_c \quad > 0 \tag{15}$$

where $G_f$ is the fracture energy for concrete (0.05-0.20 N/mm), $l_c$ is the characteristic length.

In the present work the characteristic length is computed for each sampling point as:

$$l_c = (dV)^{1/3} \tag{16}$$

where dV is the volume of concrete represented by the sampling point.

**Shear transfer across crack**

Experimental studies show that a considerable amount of shear stress can be transferred across the rough surfaces of cracked concrete. In plain concrete the main shear transfer mechanism is the aggregate interlock and the main variables involved are the aggregate size and grading. In reinforced concrete, dowel action plays a significant role, the main variables being the reinforcement ratio, the size of bars and the angle between the crack and bars. A simplified approach is generally employed to take into account the shear transfer capacity of cracked concrete. The process consists of assigning to the shear modulus corresponding to the crack plane a reduced value, $G_c$, defined as:

$$G_c = \beta G_o \tag{17}$$

where $G_o$ is the shear modulus of uncracked concrete, $\beta$ is a reducing factor in the range of zero to one.

In the present study, the following value of $\beta$ is used:

$$\beta = 1 - \left(\frac{\varepsilon_t}{0.005}\right)^{k_1} \tag{18}$$

where $\varepsilon_t$ is the fictitious tensile strain normal to the crack plane, $k_1$ is a parameter in the range of 0.3-1.0 (Cervenka 1985).

**Reinforcement Modeling**

In the present model, the steel reinforcement is simulated into equivalent steel layers with uniaxial properties in the bar direction. In contrast to concrete, the mechanical properties of steel reinforcement are well known. The uniaxial stress-strain curve for steel is idealized in this study as a bilinear curve, representing elastoplastic behavior with strain hardening. The curve is assumed to be identical in tension and compression, **Fig. (7)**. Unloading occurs elastically.

**Soil Modeling**

The critical state model can characterize the behavior of work hardening soils. This model is an elastoplastic constitutive law relating strain to effective stress. The law is incremental so that small changes of strains are related to corresponding stress changes. This model was originally developed under Roscoe's leadership at Cambridge University (Atkinson and Bransby 1978).

**Yield Function**

The projection of the yield surface on the meridional plane comprises two regions, as shown in **Fig. (8)**. They are on the form of an elliptical cap in the sub-critical region and a straight line, the critical state line, in the super-critical region. The meridional plane has an abscissa defined by the first stress invariant, and an ordinate defined by the square root of the second stress deviatoric invariant.

The equation of the yield function in the sub-critical region is:

$$F = \left(1 + \frac{\eta^2}{M^2}\right)(P + P_r) - (P_o + P_r) \tag{19}$$

where P is the mean stress or the first stress invariant $I_1/3$, of the state point, which is equal to:

$$P = \frac{I_1}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3} \tag{20}$$

$P_o$ is the initial mean stress, **M** is the slope of the critical state line in the (P, q) plane and $\eta$ is the slope of the stress path, which is equal to:

$$\eta = \frac{q}{P + P_r} \tag{21}$$

where $P_r$ is the deviation of the apex of the ellipse from center as shown in **Fig. (8)**, $q = \sqrt{3J_2}$ is the deviator stress.

**Hardening Rule**

The plastic or irrecoverable volumetric strain of soil associated with the movement of the elastic wall is identified as the hardening parameter **h**. The void ratio change $\Delta e$, is made of an elastic component:

$$\Delta e^{e} = -\kappa \log(\frac{\sigma_c}{\sigma_{co}}) \tag{22}$$

and plastic component, **Fig. (9)** :

$$\Delta e^{p} = -(\lambda - \kappa) \log(\frac{\sigma_c}{\sigma_{co}}) \tag{23}$$

The hardening parameter is equal to:

$$h = \varepsilon_v^p = \frac{\Delta e^p}{1+e_o} \tag{24}$$

where $e_o$ is the initial void ratio and $\varepsilon_v^p$ is the volumetric plastic strain (Al-Azzawi 2000).

**Flow Rule**

The associated flow rule is adopted in the present study. The flow vector can be determined by using chain rule:

$$\frac{\partial F}{\partial \underline{\sigma}} = \frac{\partial F}{\partial P}\cdot\frac{\partial P}{\partial \underline{\sigma}} + \frac{\partial F}{\partial q}\cdot\frac{\partial q}{\partial \underline{\sigma}} \tag{25}$$

The elastoplastic constitutive matrix is given by :

$$[D]_{ep} = [D]_e - \frac{[D]_e\left(\frac{\partial F}{\partial \underline{\sigma}}\right)^T [D]_e\left(\frac{\partial F}{\partial \underline{\sigma}}\right)}{\left(\frac{\partial F}{\partial \underline{\sigma}}\right)^T [D]_e\left(\frac{\partial F}{\partial \underline{\sigma}}\right) - \frac{1}{\frac{\partial \varepsilon_v}{\partial P_o} - \frac{1}{K_B}}\frac{\partial F}{\partial \underline{\sigma}_n}} \tag{26}$$

where $K_B$ is the bulk modulus.

**APPLICATIONS**

The conical and inverted dome shell footings, which were previously tested by Rasheed 1998, are used in the present analysis.

**Conical Shell Footing**

The conical footing geometry and loading used in the present analysis is given in **Fig. (10)**. A mesh of seven eight-node shell elements to model the shell, seven twenty-node interface elements and 26 twenty-node soil elements are used. The material properties given in **Table (1)** are used.

The reinforced concrete conical shell and soil are modeled using finite elements. **Fig. (11)** shows the load-deflection curves at center for the reinforced concrete shell. The result shows good agreement with the experimental works of Rasheed 1998. Cracks first appeared at 35% of failure load, in a region at bottom layer of the shell, and soon spread upwards along the generators. As the applied load increases the value of circumferential tension increases and cracks extend progressively towards the center (or apex) of the shell. The crack patterns of the shell's bottom layer through increasing load levels are drawn in **Fig. (12)**. Crushed Gauss points are initiated at a load of 98% of the ultimate load 22900 N. When the Gauss points are considered crushed; zero stresses and stiffness are assigned to them.

**Fig. (13)** shows the effect of changing the rise to base ratio ($f / r_2$) on the ultimate strength loads for the reinforced concrete conical footing. The ultimate strength from both experimental and finite element results increases as ($f / r_2$) ratio increases. This is due to the fact that as ($f / r_2$) ratio increases, the hoop tensile stresses decreases.

**Inverted Spherical Dome Shell Footing**

The inverted dome footing geometry and loading used in the present analysis are given in **Fig. (14)**. A mesh of five eight-node shell elements to model the shell and the ring beam, five twenty-node interface elements and 18 twenty-node soil elements are used. The same material properties for concrete, interface and soil given in **Table (1)** are used except that $E_{so} = 68$ N / mm $^2$ and the failure slope M = 1.64.

**Fig. (15)** shows the load-deflection curves at center for the reinforced concrete inverted dome shell. The result shows good agreements with experimental work of Rasheed 1998. Cracks first appeared at 51% of failure load, in a region at bottom layer near the ring beam in the circumferential direction. Spreading cracks are not only in the hoop direction but also through the thickness of the shell resulting in neutral axis shifting upwards with increasing load. The crack patterns of the shell's bottom layer at failure load are drawn in **Fig. (16)**.

**Fig. (17)** shows the effect of changing the rise to base ratio ($f / r_2$) on the ultimate strength loads for the reinforced concrete inverted dome footing. The ultimate strength from both experimental and finite element results increases as ($f / r_2$) ratio increases.

**CONCLUSIONS**

The experimental, finite element results and also the parametric study in respect of the rise of the conical shell have shown that in the initial range of low rises the ultimate strength gains rapidly with the rise then followed by a slower increase. Thus, it is not advantageous to adopt unduly higher rises, as the gain in ultimate strength is not commensurate with the material used and constructional difficulty from that rising. The field practice of using rise-to-base radius ratios in the range of 0.5 to 1.0, as recommended in the relevant Indian National Code, can be considered as a desirable compromise between ultimate strength on the one hand and facility of construction on the other.

The parametric study in respect of the rise of the inverted spherical dome raft have shown that the ultimate strength gains rapidly with the rise in the initial range then followed by a slower increase.

The rise corresponding to a semi-vertical angle of $45^\circ$ is a desirable step in terms of both design and construction. The predominant membrane compression in both the meridional and hoop directions in the case of spherical sectors makes it an extremely efficient shell form for use in foundations.

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