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TRANSIENT FLOW IN A RECTANGULAR OPEN CHANNEL

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ABSTACT

In this paper, the transient flow in a rectangular open channel has been studied for different conditions. The velocity and depth of water have been calculated at different sections in the channel. A computer program (Channel) was developed to solve the mathematical model using characteristic method .The initial conditions have been calculated using Runge-Kutta method.

الخلاصة

في هذا البحث، لقد تمت دراسة الجريان المضطرب في القنوات المفتوحة ذات المقطع المستطيل لمختلف الظروف. لقد تم أيضا في هذا البحث حساب سرعة وعمق الجريان لمقاطع مختلفة في القناة. برنامج الحاسبة (Channel) طور لحل النموذج الرياضي باستخدام أسلوب characteristic. الشروط الأولية لقد تم حسابها باستخدام أسلوب Runge-Kutta .

KEY WORDS

Open Channel, Rectangular, Transient Flow

INTRODUCTION

Unsteady uniform flow rarely occurs in open channel flow. Unsteady non-uniform flow is common but is difficult to analyze. Wave motion is an example of this type of flow, and its analysis is complex when friction is taken into account (Streeter & Wylie 1983).

If the depth and /or velocity vary at any point with time, the flow is termed unsteady flow. Examples of unsteady flow are: floods in rivers, tides in oceans and in estuaries surges in power canals, and storm runoff in sewers (Amein & Chu 1975).

Unsteady flow in compound channel is complicated by large difference in hydraulic properties (flow depth, resistance) and cross sectional geometry of the main channel, and the flood plains (Ayyoubzadeh & Zahiri 2004).

Two different numerical flux, namely, flux difference splitting and flux vector splitting schemes, are compared in numerical computation of one-dimensional open-channel transition free surface flows (Saint Wongsa 2000).

A one –dimensional theoretical approach is used for the treatment of transient open-channel flow to lead to the development of three techniques for analytically simulating both natural and artificially induced transient flows, in rivers and in estuaries, studied (Baitzer & Lai 1968) using of a high-speed digital computer.

The performance of the numerical solution has been studied (Patridge & Brebbia 1976).The influence of the different terms in the shallow water equations have been examined. Comparisons have been made between the exact solution and the three-node finite element solution. Methods for the numerical solution of the set of equations of unsteady flow in open channels may be appropriately classified as :(1) Direct methods; and (2)characteristic methods. In the direct methods, the finite difference representation is based directly on the primary equations. In the characteristic methods, the equations are first transformed into the characteristic form, which is then used to develop the finite difference representations (Chaudhry 1987).
The aim of this work is to obtain a mathematical model to find out the velocity and depth of flow variation along an open channel .A characteristics method is used to analyze and calculate these variables.

MATHMETICAL MODEL

The dynamic and continuity equations describing the one-dimensional transient flows are derived in this section (Chaudhry 1987).
The following assumptions are made in deriving these equations:-

1- The slope, θ, of the channel bottom is small so that sinθ≈tanθ≈θ and cosθ≈1.
2- The pressure distribution at a section in hydrostatic. This is true if the vertical acceleration is small, i.e., if the water-surface variation is gradual.
3- The transient-state friction losses may be computed using formulas for the steady-state friction losses.
4- The velocity distribution at a channel cross section is uniform.
5- The channel is straight and prismatic.

Let us consider the control volume shown in **Fig. (1)**.The x-axis lies along the bottom of the channel and is positive in the downstream direction. The depth of flow, y, is measured vertically from the channel bottom. Thus, the x and y-axes are not orthogonal. However, as the channel is assumed to have a small bottom slope, this discrepancy does not introduce significant errors.

Continuity Equation

If γ is the specific weight of the water, then referring to **Fig. (1-a)**:

The rate of mass inflow into the control volume= $\frac{\gamma}{g}AV$ (1)

The rate of mass outflow from the control volume= $\frac{\gamma}{g}\left(A+\frac{\partial A}{\partial x}\Delta x\right)\left(V+\frac{\partial V}{\partial x}\Delta x\right)$ (2)

By neglecting Second-order terms, then the net rate of mass inflow:-

$$=-\frac{\gamma}{g}V\frac{\partial A}{\partial x}\Delta x-\frac{\gamma}{g}A\frac{\partial V}{\partial x}\Delta x \quad (3)$$

The rate of increase of the mass of the control volume= $\frac{\gamma}{g}\frac{\partial A}{\partial t}\Delta x$ (4)

As the time rate of increase of the mass of the control, volume must equal the net rate of mass inflow into the control volume then by equation 3, 4 and rearrangement we get:-

$$\frac{\partial A}{\partial t}+V\frac{\partial A}{\partial x}+A\frac{\partial V}{\partial x}=0 \quad (5)$$

Since the channel is assumed prismatic, the flow area, A is a known function of depth, y. Therefore, the derivatives of A may be expressed in terms of y as follows:-

$$\frac{\partial A}{\partial x}=\frac{dA}{dy}\frac{\partial y}{\partial x}=B\frac{\partial y}{\partial x} \quad (6)$$

$$\frac{\partial A}{\partial t}=\frac{dA}{dy}\frac{\partial y}{\partial t}=B\frac{\partial y}{\partial t} \qquad (7)$$

Substituting Eq. (6) and Eq. (7) into Eq. (5), we get

$$\frac{\partial y}{\partial t}+\frac{A}{B}\frac{\partial V}{\partial x}+V\frac{\partial y}{\partial x}=0 \qquad (8)$$

Since discharge Q=AV, then

$$\frac{\partial Q}{\partial x}=V\frac{\partial A}{\partial x}+A\frac{\partial V}{\partial x} \qquad (9)$$

On the basis of Eq. (6) and Eq. (7), Eq. (9) becomes

$$\frac{\partial Q}{\partial x}=BV\frac{\partial y}{\partial x}+A\frac{\partial V}{\partial x} \qquad (10)$$

From Eq. (8) and Eq. (10) .The continuity equation for flow in open channel state that

$$\frac{\partial Q}{\partial x}+B\frac{\partial y}{\partial t}=0 \qquad (11)$$

Dynamic Equation

The following forces are acting on the water in the control volume shown in **Fig. (1-b)**.

$$F_1=F_2=\gamma A\bar{y} \qquad (12)$$

$$F_3=\gamma A\frac{\partial y}{\partial x}\Delta x \qquad (13)$$

$$F_4=\gamma AS_f\Delta x \qquad (14)$$

Note that the pressure force acting on the downstream face is divided into two components, F2 and F_3,and the term of higher order are not included in the expression for F_3.In **Fig.(1-b)** .F_1,F_2 and F_3 are forces due to pressure :F_4=force due to friction;F_5=x component of the weight of the water in the control volume; θ=angle between the channel bottom and horizontal axis(positive downstream);and S_f=slope of the energy grade line.

The value of S_f may be computed using any standard formula for the steady state losses ,such as Manning's formula .Since θ is assumed small ,$\sin\theta\approx\theta=S_o$ in which S_o=bottom slope. Hence ,

$$F_5=\gamma A\Delta xS_o \qquad (15)$$

Referring to **Fig. (2-b),** the resultant force acting on the water in the control volume in the positive X-direction is:-

$$F=F_1-F_2-F_3-F_4+F_5 \qquad (16)$$

Substituting expressions for F_1 to F_5 from Eq. (12) to Eq. (15), we obtain

$$F=-\gamma A\frac{\partial y}{\partial x}\Delta x+\gamma AS_o\Delta x-\gamma AS_f\Delta x \qquad (17)$$

Momentum entering the control volume$=\frac{\gamma}{g}AV^2$ (18)

Momentum leaving the control volume$=\frac{\gamma}{g}\left[AV^2+\frac{\partial}{\partial x}\left(AV^2\right)\Delta x\right]$ (19)

Therefore, net influx of momentum into the control volume$=-\frac{\gamma}{g}\frac{\partial}{\partial x}\left(AV^2\right)\Delta x$ (20)

The time rate of increase of momentum$=\frac{\partial}{\partial t}\left(\frac{\gamma}{g}AV\Delta x\right)$ (21)

According to the law of conservation of momentum, the time rate of increase of momentum is equal to the net rate of momentum influx plus the sum of the forces acting on the water in the control volume. Hence, from Eq. (17), Eq. (20) and Eq. (21) we get:-

$$\frac{\partial}{\partial t}(AV)+\frac{\partial}{\partial x}(AV^2)+gA\frac{\partial y}{\partial x}=gA(S_o-S_f) \quad (22)$$

Expansion of two terms on the left-hand side. division by A, and rearrangement of the terms yields:-

$$g\frac{\partial y}{\partial x}+V\frac{\partial V}{\partial x}+\frac{\partial V}{\partial t}+\frac{V}{A}\left(\frac{\partial A}{\partial t}+V\frac{\partial A}{\partial x}+A\frac{\partial V}{\partial x}\right)=g(S_o-S_f) \quad (23)$$

On the basis of the continuity equation, Eq. (5) , the sum of the terms within the brackets on the left-hand side of Eq. (23) is equal to zero .Hence, Eq. (23) becomes:-

$$g\frac{\partial y}{\partial x}+\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}=g(S_o-S_f) \quad . (24)$$

Equations 8 and 24 are referred to as *St. Venant* Equations (Chaudhry 1987).

METHOD OF CHARACTERSTIC

In the method of characteristics, the *St. Venant* equations are converted into characteristic equations, which are then solved by a finite-difference scheme.

The wave speed relative to the medium in which it is traveling called wave celerity, C which is different from the flow velocity V. The absolute wave velocity V_w is equal to the vectorial sum of the wave celerity and the flow velocity, i.e,

$$V_w=V\pm C \quad (25)$$

Where the positive sign for wave traveling in the downstream direction and the negative sign for a wave traveling upstream direction.

Therefore multiplying Eq. (8) by ± CB/A, adding it to Eq. (24), and rearranging the terms, we obtain the following so-called characteristic equations.

$$\left[\frac{\partial V}{\partial t}+(V+C)\frac{\partial V}{\partial x}\right]+\frac{g}{C}\left[\frac{\partial y}{\partial t}+(V+C)\frac{\partial y}{\partial x}\right]=g(S_o-S_f) \quad (26)$$

and

$$\left[\frac{\partial V}{\partial t}+(V-C)\frac{\partial V}{\partial x}\right]-\frac{g}{C}\left[\frac{\partial y}{\partial t}+(V-C)\frac{\partial y}{\partial x}\right]=g(S_o-S_f) \quad (27)$$

In which

$$C=\sqrt{gA/B}, \quad (28)$$

Equations 26 and 27 could be converted to ordinary differential equation by defining

$$\frac{dx}{dt}=V+C \quad (29)$$

for positive characteristic curve at x-t plane, and

$$\frac{dx}{dt}=V-C \quad (30)$$

for negative characteristic curve at x-t plane

Referring to **Fig. (2)**, these equations may be expressed in the finite-difference form as.

$$\frac{V_P-V_M}{\Delta t}+(V_M+C_M)\left(\frac{V_M-V_L}{\Delta x}\right)+\frac{g}{C_M}\left[\frac{y_P-y_M}{\Delta t}+(V_M+C_M)\frac{y_M-y_L}{\Delta x}\right]=g(S_o-S_{fM}). \quad (31)$$

$$\frac{V_p - V_M}{\Delta t} + (V_M - C_M)\left(\frac{V_R - V_M}{\Delta x}\right) - \frac{g}{C_M}\left[\frac{y_p - y_M}{\Delta t} + (V_M - C_M)\frac{y_R - y_M}{\Delta x}\right] = g(S_o - S_{fM}) \quad (32)$$

The subscript L, M, P, and R refer to the variables at various points in the x-t plane, the terms of Eq. (31)and Eq.(32) can be rearranged to yield the following equations:-

1- Negative characteristic equation **Fig.(2-a)**:-

$$V_p = C_n + C_a y_p \quad (33)$$

In which

$$C_n = V_M + \frac{\Delta t}{\Delta x}(V_M - C_M)(V_M - V_R) - \frac{g}{C_M}\left[y_M - \frac{\Delta t}{\Delta x}(V_M - C_M)(y_R - y_M)\right] + g(S_o - S_{fM})\Delta t \quad (34)$$

$$C_a = \frac{g}{C_M} \text{ and } C_M = \sqrt{\frac{gC_M}{B_M}} \quad (35)$$

2- Positive characteristic equation **Fig.(2-b)**:-

$$V_p = C_p - C_a y_p \quad (36)$$

In which

$$C_p = V_M - \frac{\Delta t}{\Delta x}(V_M + C_M)(V_M - V_L) + \frac{g}{C_M}\left[y_M - \frac{\Delta t}{\Delta x}(V_M + C_M)(y_M - y_L)\right] + g(S_o + S_{fM})\Delta t \quad (37)$$

In Eq. (33) through Eq. (37), conditions at M are those at the boundary at the beginning of the time step. Since the values of all variables are known at L, M, and R, the value of constants C_n and C_p can therefore be completed.

In Eq. (33) or Eq. (36), there are two unknowns (i.e. y_p and V_p).

BOUNDARY CONDITIONS

As discussed above, Eq. (33) and Eq. (36) are used to determine the conditions at the interior sections. At the boundaries, however, special boundary conditions are developed by solving the positive or negative characteristic equations, or both, simultaneously with the conditions imposed by the boundary.

Constant-Head Reservoir at Upstream End

If the head losses and the velocity head at the entrance are negligible then y_p=y_{res};V_p May be computed from Eq. (33) as shown in **Fig. (3-a)**.

Dead End at Downstream End

When the gate at downstream of the channel suddenly closed V_p=0; y_p may be computed from Eq. (36) as shown in **Fig. (3-b)**.

INTIAL CONDITIONS

The initial conditions are determined by solving the ordinary differential equation describing the gradually varied flow in open channels.

The steady-state gradually varied flow in open channels is described by the following equation.

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{Q^2 B}{gA^3}} \quad (38)$$

Where S_f is calculated from Manning formula which states that:-

$$S_f = \frac{Q|Q|}{K^2} \tag{39}$$

In which the conveyance factor, K, in the SI units may be written as:-

$$K = \frac{1}{n} A R^{2/3} \tag{40}$$

Equation (38) is a first-order differential equation. The flow depth along the channel may be computed by integrating this equation. For this purpose, the fourth-order Runge-Kutta method may be used as follows:-

Let the depth of flow, y_i at $x=x_i$, be known (**Fig. (4)**); then the depth of flow at $x=x_{i+1}$ is

$$y_{i+1} = y_i + \frac{1}{6}(a_1 + 2a_2 + 2a_3 + a_4) \tag{41}$$

$$a_1 = \Delta x f(x_i, y_i) \tag{42 a}$$

$$a_2 = \Delta x f\left(x_i + \frac{1}{2}\Delta x, y_i + \frac{1}{2}a_1\right) \tag{42 b}$$

$$a_3 = \Delta x f\left(x_i + \frac{1}{2}\Delta x, y_i + \frac{1}{2}a_2\right) \tag{42 c}$$

$$a_4 = \Delta x f(x_i + \Delta x, y_i + a_3) \tag{42 d}$$

$$f(x,y) = \frac{dy}{dx} \tag{42 e}$$

By starting from the known depth at a control section, and by repeating application of Eq. (41), the flow profile along the whole channel is computed in steps of length, Δx .

STABILITY CONDITIONS

Using the technique presented by Courant et al (Chaudhry 1987), it has been shown that the diffusive scheme is stable if

$$\Delta t = \frac{\Delta x}{|V| \pm C} \tag{43}$$

This is called the Courant-Friedrichs-Lewy condition or simply the Courant condition.

It is necessary that the Courant Stability condition (Eq. (43)) is satisfied at each time step.

COMPUTER PROGRAM

By using (Q-basic) language a computer, program which is named (Program Channel) is developed to solve numerical one-dimensional linear partial differential equations, unsteady state using characteristic method .The flow chart of the computer program is shown in **Fig.(5).**

CASE STUDY

To determine transient conditions in an open channel. Fig.(6) illustrated the dimensions of the channels which is as follows:

RESULTS AND DISCUSSIONS

In this section, the numerical results are obtained for several case of steady to validate the described algorithm. The case study assumed that a rectangular channel cross-section area with different width and Manning's Coefficient. The treatments of the boundary conditions are based on the theory of Characteristics.

The variation of the flow velocity and depth of water at different sections in the channel with the time have been studied.

Figs. (7&8) show the variation of the channel depth and velocity with the time at different sections in the channel from the dam when the gate at downstream suddenly closed respectively.
It is found that the maximum depth of water in the channel at the gate reaches after a certain time (120 sec) while the velocity is zero at the gate, this means that the potential energy will be accumulated the pressure head. While for the other sections such as (200m,500m) from the dam the maximum depth happened when the velocity is zero after a certain time (110 sec) and the velocity of the flow repeating itself as sine wave with different amplitude due to wave celerity.
Figs. (9&10) show that the Maximum depths and velocities at different sections will increase due to the change of channel widths, this is due to increase the flow rate and the energy consumed.
Figs. (11&12) show that the maximum depths and velocities at different sections will be decreased due to the increase the friction (Manning's coefficient) between the flow and bed of the channel also the maximum depth and velocity at all sections reached after a certain time (200 sec).

CONCLUSIONS

In working with actual field problems, the worse case have been studied for design an open channel, which depends on the width , Manning's coefficient, the slope, and the time of closing the gate .
Increasing the width of the channel will increase the maximum pressure and velocity while increasing the Manning's coefficient (friction) the maximum and velocity will decrease at downstream.

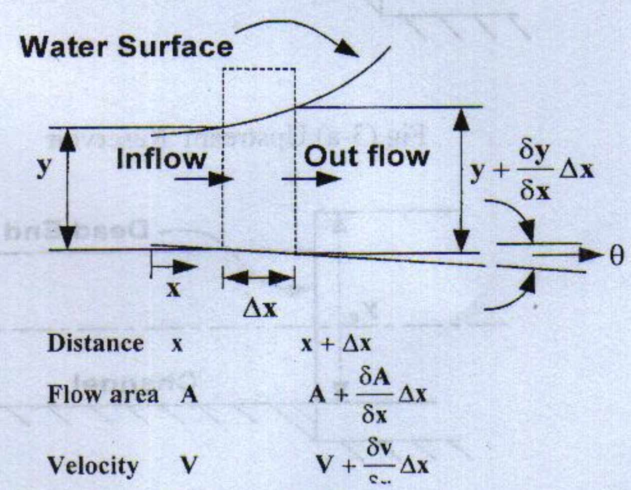


Fig. (1-a) Free Body Diagram for Continuity Equation

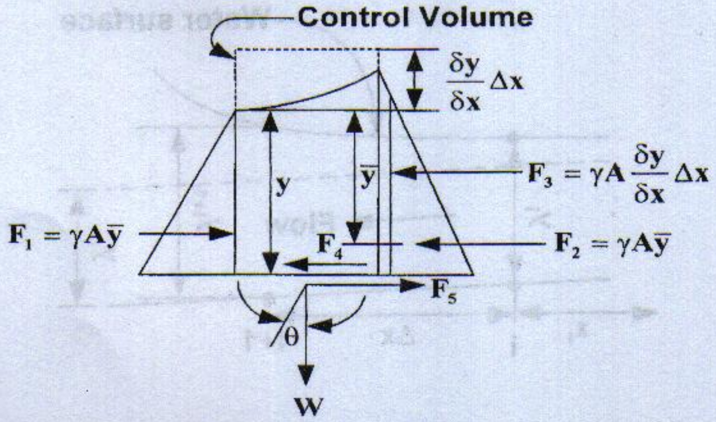


Fig. (1-b) Free Body Diagram for Dynamic Equation

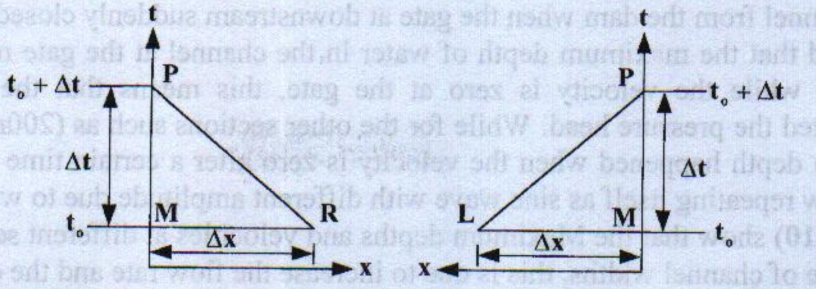


Fig.(2) Notation for Positive and Negative Characteristic

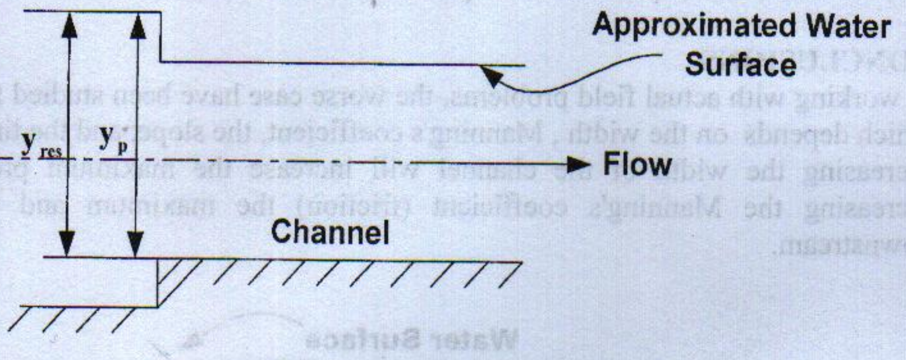


Fig.(3-a) Upstream Reservoir

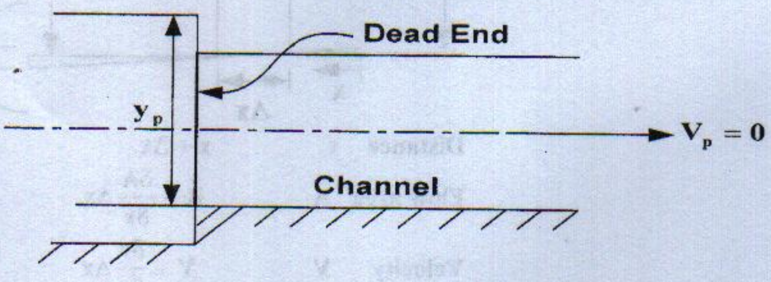


Fig. (3-b) Dead End at Downstream Reservoir

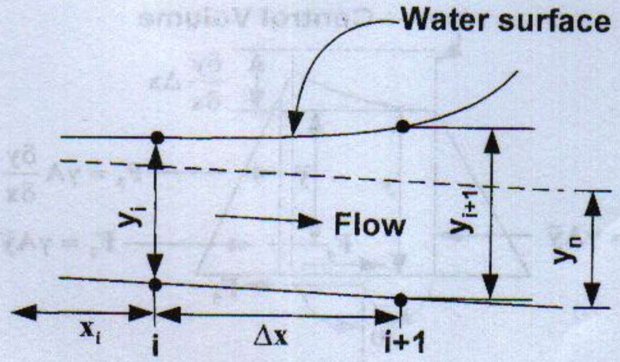
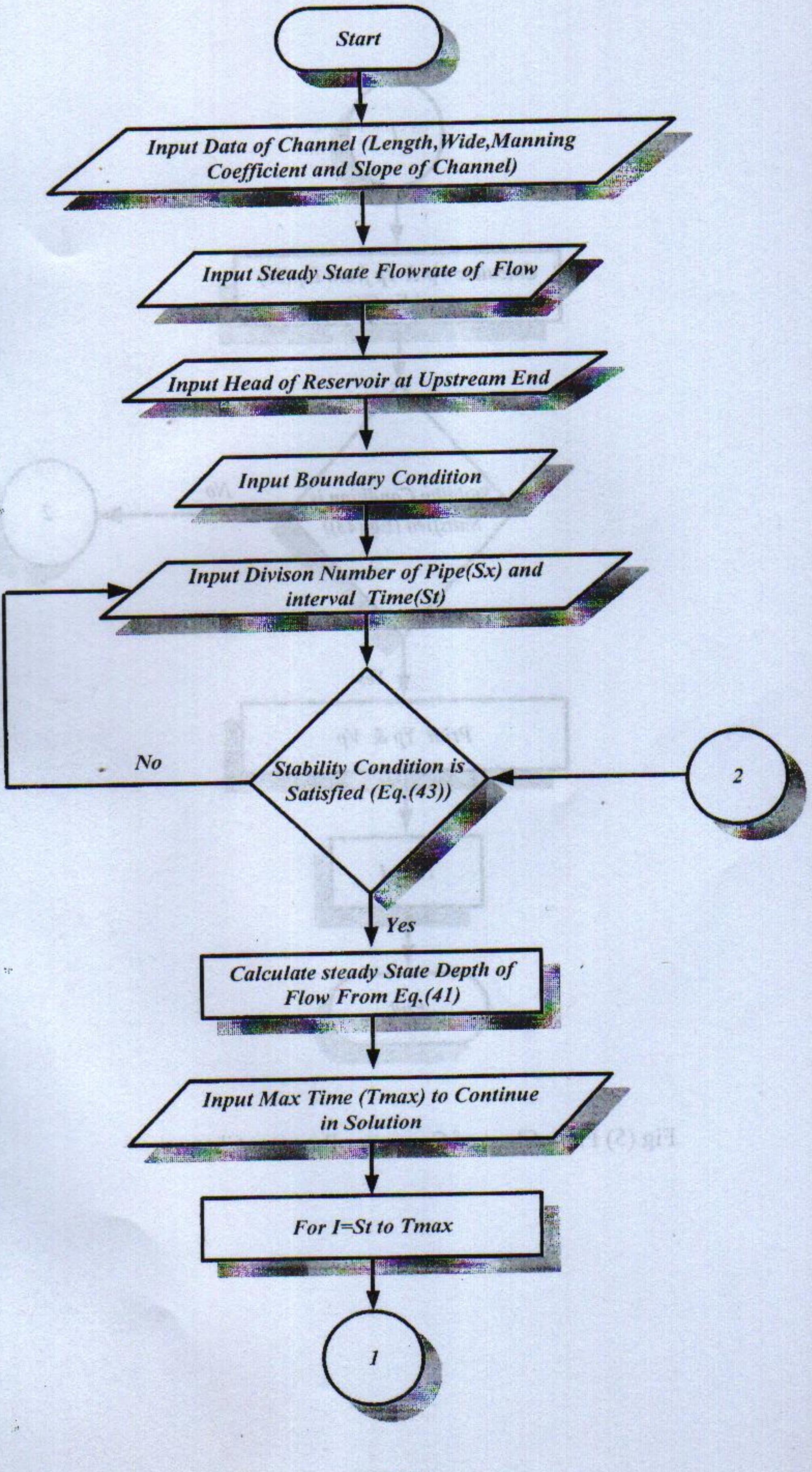


Fig. (4) Notation for Computing Steady-State Conditions


Start
Input Data of Channel (Length,Wide,Manning Coefficient and Slope of Channel)
Input Steady State Flowrate of Flow
Input Head of Reservoir at Upstream End
Input Boundary Condition
Input Divison Number of Pipe(Sx) and interval Time(St)
Stability Condition is Satisfied (Eq.(43))
No
Yes
2
Calculate steady State Depth of Flow From Eq.(41)
Input Max Time (Tmax) to Continue in Solution
For I=St to Tmax
1

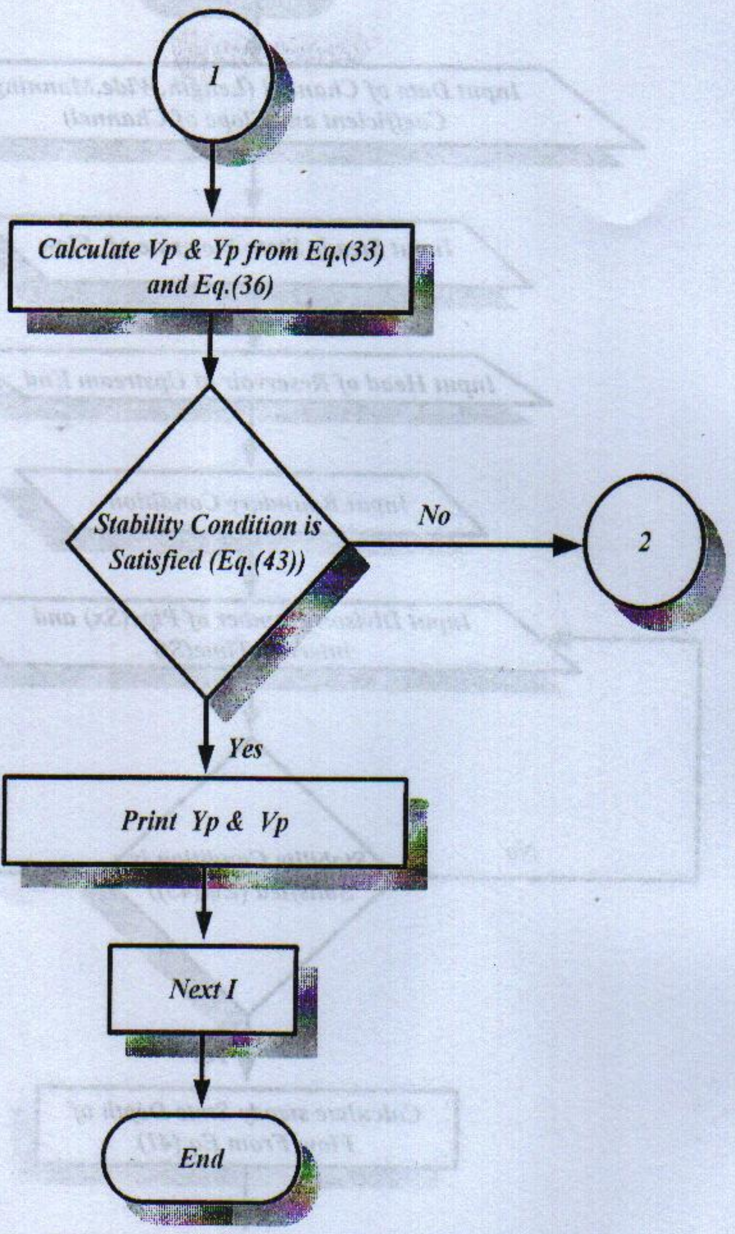


Fig.(5) Flow Chart of Computer Program Channel

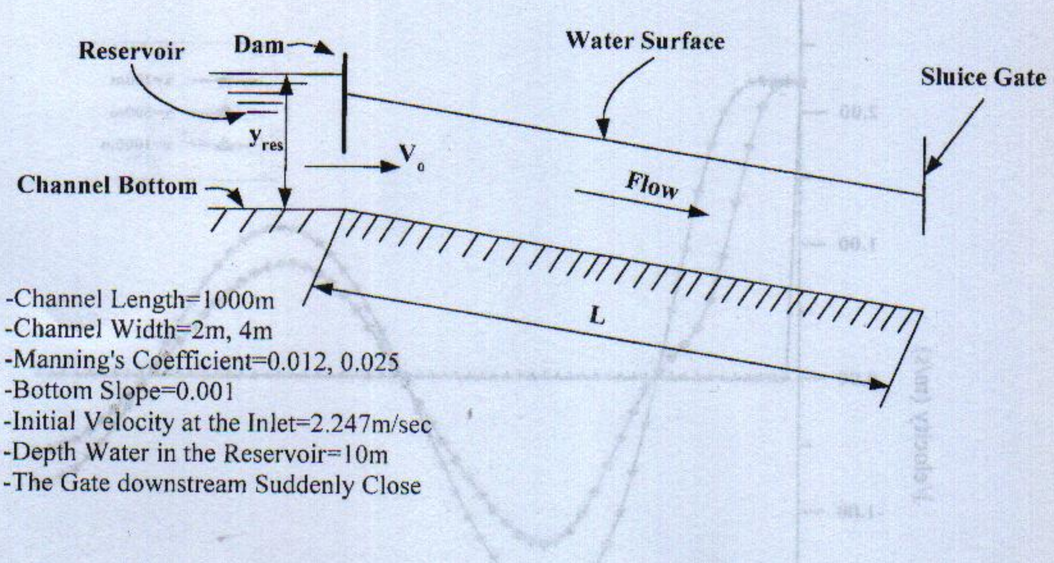


Fig. (6) An Open Channel with a Dam Upstream & Sluice Gate Downstream

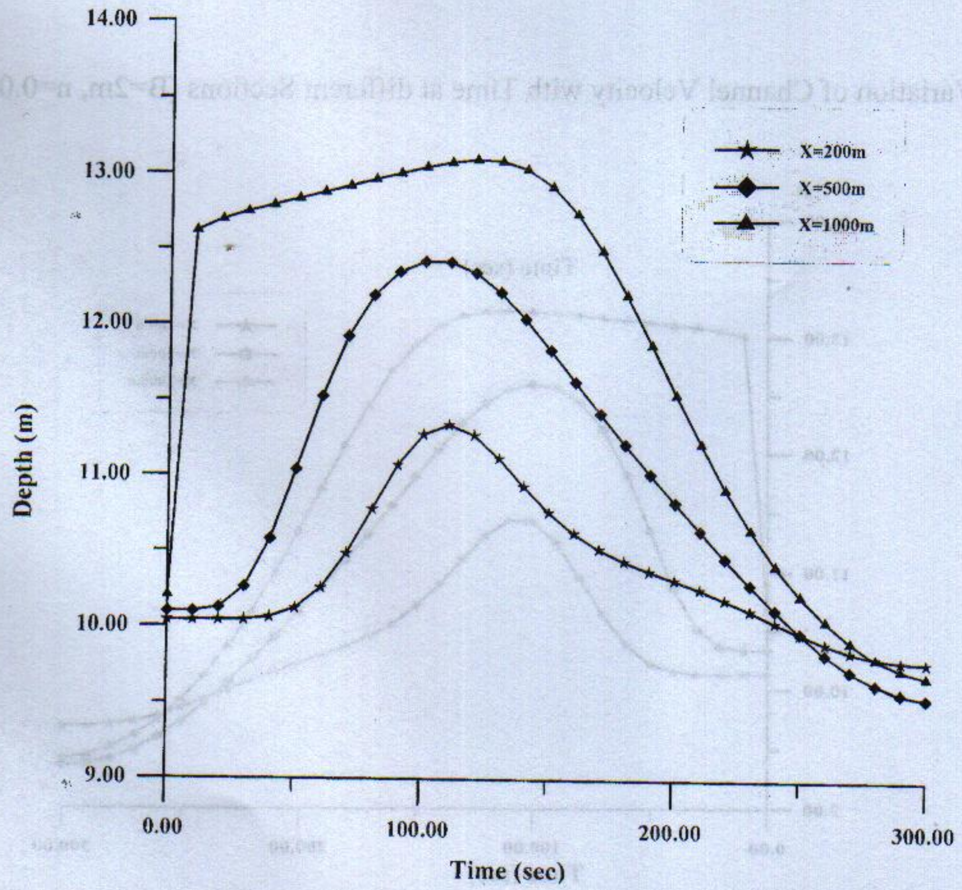


Fig.(7) The Variation of Channel Depth with Time at different Sections (B=2m, n=0.012)

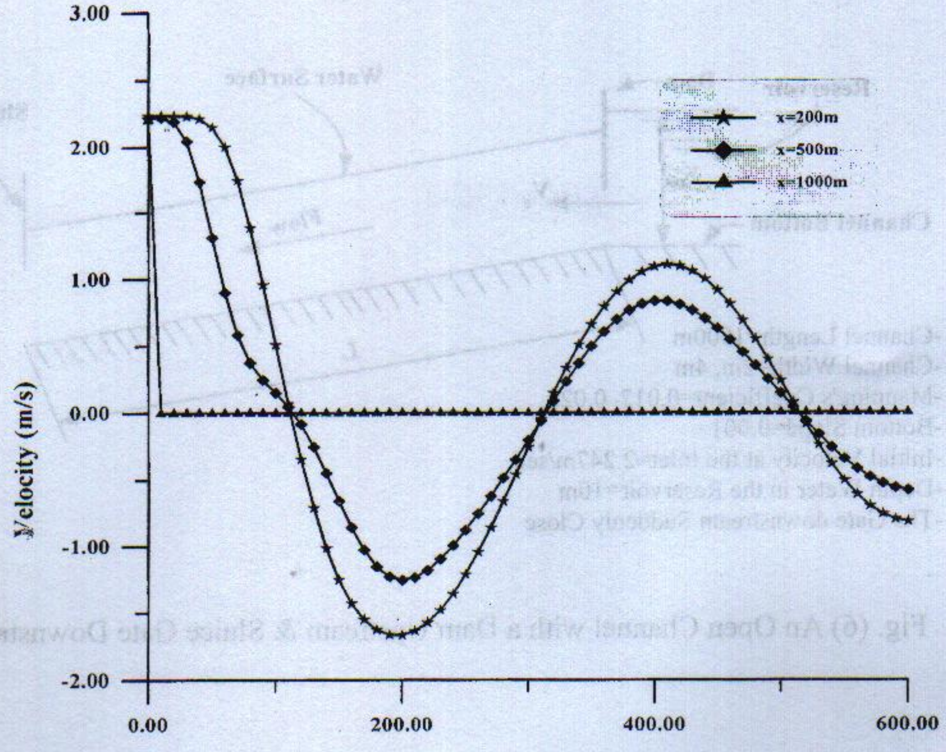


Fig. (8) The Variation of Channel Velocity with Time at different Sections (B=2m, n=0.012)

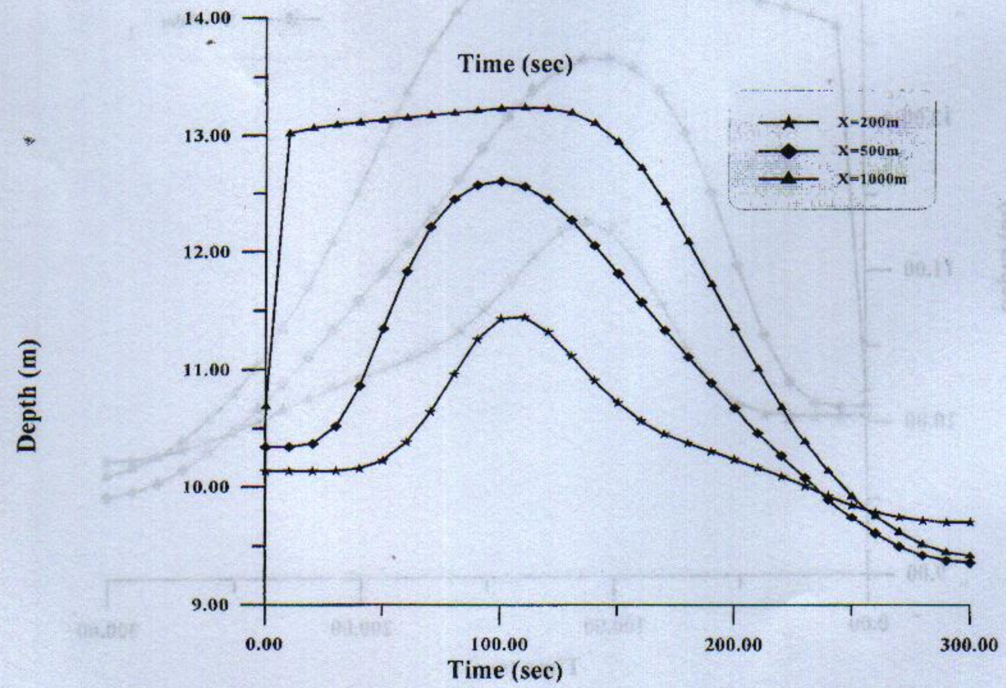


Fig. (9) The Variation of Channel Depth with Time at different Sections (B=4m, n=0.012)

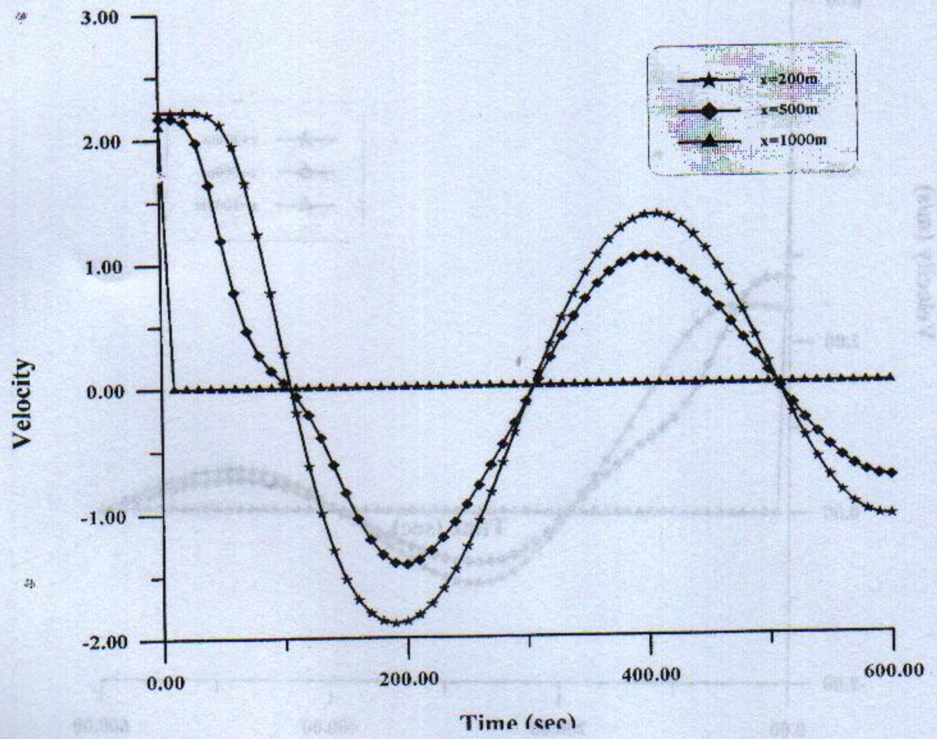


Fig. (10) The Variation of Channel Velocity with Time at different Sections (B=4m, n=0.012)

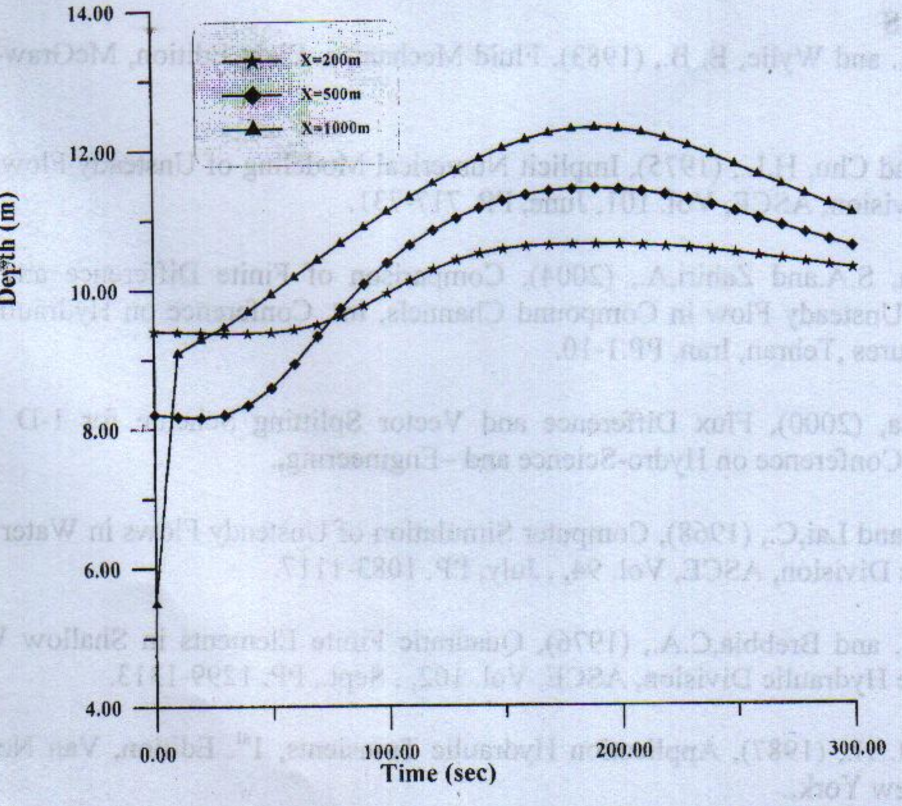


Fig. (11) The Variation of Channel Depth with Time at different Sections (B=2m, n=0.025)

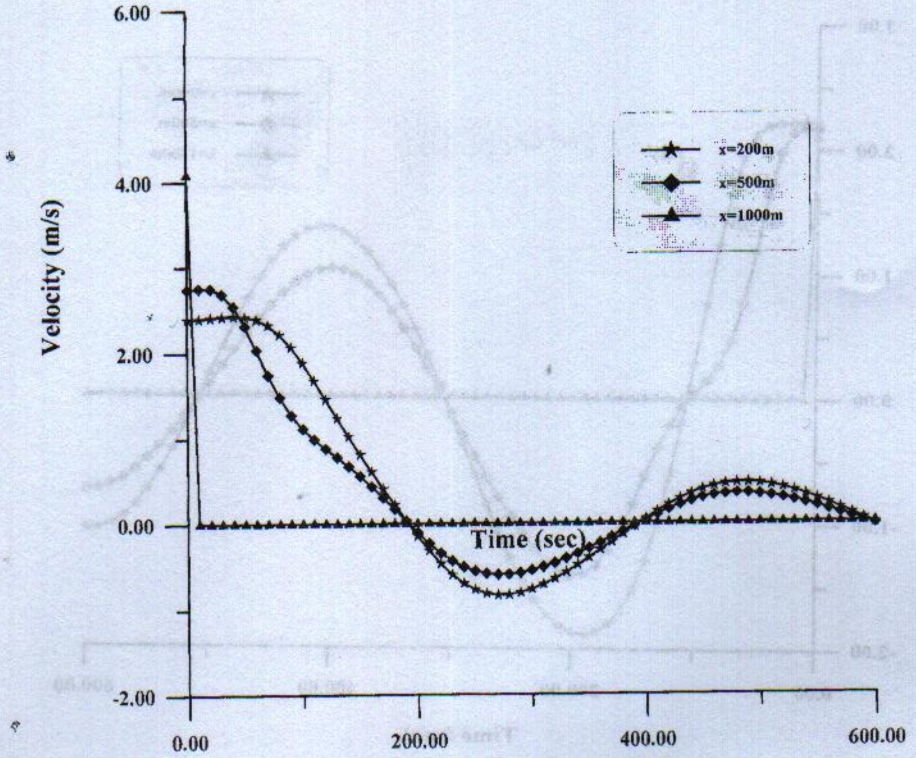


Fig. (12) The Variation of Channel Velocity with Time at different Sections (B=2m, n=0.025)

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NOTATION

The following symbols are used in this paper as shown in **Table (1):**

Table (1) Notation & Symbols

Symbol	Description	Unit
A	Cross-Section Area of Channel	m^2
B	Channel Width	m
C	Wave Celerity	m/sec
g	Acceleration	m/sec^2
L	Channel Length	m
n	Manning's Coefficient	___
Q	Flow rate	m^3/sec
R	Hydraulic Radius	m
S_f	Slope of The Grade Line	___
S_o	Bottom Slope	___
t	time	sec
V	Flow Velocity	m/sec
V_w	Wave Velocity	m/sec
y	Depth	m
y_{res}	Head of Reservoir	m
γ	Specific Weight	Kn/m^3
θ	Angle	___
Δx	Length of Element	m