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# DIFFERENT APPROACHES TO ESTIMATE THE SUCTION HEAD AT THE WET FRONT IN THE GREEN AND AMPT WATER INFILTRATION EQUATION

**Mahdi I. Aoda** **Abbas H. Thiab** **Nameer T. Mahdi**
Dept. Soil and water Sciences, College Agric., Baghdad Univ.

## ABSTRACT

This study was conducted to estimate the suction head at the wetting front $\tau_w$ in the Green and Ampt infiltration equation using uniformly packed soil columns of three different – textured soils. Four different approaches were used for this estimation. Cumulative depths of infiltration I along with the visual wet front advance Z was recorded with time t. Soil water content profiles were also obtained by which the sharpness of the wet front was tested using an empirical model adopted from van Genuchten (1980) model for soil moisture-suction head relation.

The Green and Ampt equation did fit the infiltration data very well with significant correlation of its parameter $K_1$ with the physical measure of saturated hydraulic conductivity $K_s$.

Four different methods were used to estimate $\tau_w$: from fitting the Green and Ampt equation to infiltration data (I vs. t), numerical integration using hydraulic conductivity as afunction of suction head $K(\tau)$ in the basic definition equation of defining $\tau_w$, numerical integration using conductivity as afunction of volumetric water content $K(\theta)$, and from the soil sorptivity S evaluated from early time infiltration data using Philip one-term equation alons with Green and Ampt equation for horizontal infiltration.

No significant differences were found between these four methods in estimating $\tau_w$ at 0.01 level. Significant differences, however, were found at the same level between the values of $\tau_w$ for the three soils as it was expected.

## الخلاصة

نفذت هذه الدراسة لاختبار طرق مختلفة لتقدير الشد عند جبهة الابتلال $\tau_w$ في معادلة Green and Ampt لغيض الماء. قيس الغيض في اعمدة تربة متجانسة واستعملت ثلاث ترب ذات نسجات مختلفة. اثناء حركة الماء قيس كل من عمق ماء الغيض المتراكم I ومسافة تقدم جبهة الابتلال Z مع الزمن t. استخرجت كذلك مقدات محتوى رطوبة التربة ومنها تم تحديد حادية جبهة الابتلال باستخدام نموذج محور عن معادلة van Genuchten (1980).

طابقت معادلة Green and Ampt بيانات الغيض بشكل متميز، واظهرت النتائج وجود علاقة تطابق بين احد معاييرها $K_1$ مع الايصالية المائية المشبعة $K_s$ المقاسة مختبرياً.

قدرت قيم الشد باتباع اربع طرائق لقياس الشد عند جبهة الابتلال. كان اولها من مطابقة معادلة Green and Ampt مع بيانات الغيض التراكمي. ومن اجراء التكامل العددي وباستعمال الايصالية المائية غير

المشبعة كدالة للشد K(τ) في المعادلة الاساسية لتعريف الشد عند جبهة الابتلال. ومن اجراء التكامل العددي باستعمال الايصالية المائية كدالة للمحتوى الرطوبي الحجمي K(θ) . ومن قيم الامتصاصية المستخرجة من معادلة Philip ذات الحد الواحد لبيانات الغيض خلال الازمان المبكرة مع معادلة Green and Ampt للغيض الافقي.

اظهرت الدراسة عدم وجود فروقات معنوية بين قيم الشد عند جبهة الابتلال المستخرجة بالطرق الاربع المختلفة عند مستوى 0.01 . ولكن توجد فروقات معنوية عند نفس المستوى لقيمها للترب المختلفة وكما هو متوقع.

**KEY WORDS**
Green and Ampt, hydraulic conductivity, infiltration, moisture profiles, sorptivity, suction head.

## INTRODUCTION

### Green and Ampt water infiltration equation:

The Green and Ampt (1911) infiltration model was developed based on the Darcy's (1856) equation in the early twentith century. It has been the subject of considerable developments in soil physics and hydrology owing to its simplicity and satisfactory performance for a great variety of water infiltration problems. It has been applied to infiltration into homogeneous soils from constant rainfall (Mien and Larson, 1973; Swartzendruber, 1974) as well as from unsteady rainfall (James and Larson, 1976; Chu, 1978). It has also been extended to soils of non-uniform initial water content (Bouwer, 1969), to layered soils (Bouwer, 1976; Childs and Bybordi, 1969), and to crust-topped soils (Hillel and Gardner, 1970; Ahuja, 1983). These and other studies have established the utility of the Green and Ampt model under a number of circumstances. None of these studies, however, has given information about details of water content profiles during infiltration, but does offer estimates of the infiltration rate and cumulative infiltration functions of time.

Aoda and Swartzendruber (1987) have given a complete theoretical analysis of the Green and Ampt infiltration equation. They reported that the equation of water infiltration into initially air-dried soil could be written as:

$$t = K_1^{-1}\left[ I - a \ln\left(1 + \frac{I}{a}\right)\right] \qquad (1)$$

Where I is the cumulative depth of water infiltrated into soil at time t; $K_1$ is the hydraulic conductivity of the wet region behind the wet front which corresponds to water content $\theta_1$; a is a constant which equal to $(\theta_1-\theta_0)(H+\tau_w)$, where $\theta_0$ is the initial soil water content, H is the constant pressure head of water at the soil surface, and $\tau_w$ is the suction head at the wet front which is defined as:

$$\tau_w = \int_0^{\tau_0} K_r(\tau)d\tau = K_1^{-1}\int_0^{\tau_0} K(\tau)d\tau \qquad (2)$$

Where $K_r(\tau)$ is the relative hydraulic conductivity as a function of the suction head τ which equals to $[K(\tau)/K_1]$, where $K(\tau)$ is the unsaturated hydraulic conductivity.

The practical difficulty in using Green and Ampt model is how to accurately determine the saturated conductivity ($K_s$ or $K_1$) and the suction head at the wetting front, (Zhang, *et al.* 1999).

Generally, $K_s$ can be measured by the constant pressure head method in laboratory (Klute, 1965), but $\tau_w$ can not be measured directly. Parameter $\tau_w$ can be predicted based on soil data such as particle size, organic matter content, and bulk density (Rawl and Brakensiek, 1983). However, the parameters which are needed for predicting $\tau_w$ may be difficult to obtain.

## The Suction head at the wetting front

Philip (1954, 1958) called $-\tau_w$ the capillary potential at the wetting front, while Gardner (1967) in effect reversed the sign by using exactly the same words to define $\tau_w$. Since then, authors practice has been fairly consistent relative to the wet-front condition, either by following Childs (1967) in identifing $+\tau_w$ as the constant suction head (Mein and Larson, 1973; Swartzendruber, 1974; Panikar and Nanjappa, 1977; Brakensiek, 1977; Aoda and Swartzendruber, 1987; Aoda, 1992), or by following Bouwer (1967) in identifying $-\tau_w$ as the constant pressure head (Whisler and Bouwer, 1970; Youngs, 1972; Neuman, 1976; and Aggelides and Youngs, 1978).

The meaning of $+\tau_w$ and $-\tau_w$, however, has probably been the most difficult aspect of the Green and Ampt's approach to interpret and elucidate. However, derivation of Green and Ampt equation along with the lines of Aoda and Swartzendruber (1987) in which $\tau_w$ becomes expressed as an integral, does provide a straightforward and physically meaningful interpretation.

Equation (2) shows clearly that $\tau_w$ is not simply equal to $\tau_0$ and hence the Philip (1958) contention on the physical unreality of $\tau_w$ is unnecessory. In particular, since $K(\tau)$ decreases as $\tau$ increases, $\tau_w$ is always less than $\tau_0$ -- generally much less for low $\theta_0$ (high $\tau_0$) and the most common shapes of $K(\tau)$. Equation (2) also shows how $\tau_w$ will change with initial suction head $\tau_0$, and hence, because $\tau_0=\tau(\theta_0)$, with initial moisture content $\theta_0$. For initially dry soils, $\theta_0$ will be low, $\tau_0$ will thus be high, and $\tau_w$ will assume its maximum value – the reasonable circumstonces for the maximum effect of capillarity. For initially wet soils, the reverse will be true.

In this paper new methods for determining $\tau_w$ were developed based on van Genuchten (1980) conductivity model [$K=K(\tau)$ and $K=K(\theta)$] along with the integral definition of $\tau_w$ [Eq.(2)]. Parameter $\tau_w$ was also determined using sorptivity value estimated by using early times data of downword infiltration and the derivation of Green and Ampt (1911) for horizontal infiltration.

The purpose of the work reported here is to estimate the suction head at the wetting front $\tau_w$ by using three different approaches and evaluate the accuracy of these approaches with the results obtained by the fitting of the Green and Ampt equation (using least square estimates procedure) with the experimental data of I versus t for three different textured soils.

## MATERIALS AND METHODS:

Plexiglas columns of 100 cm long and 3.17 cm inside diameter were uniformly packed with three different-textured soils (their properties are listed in **Table (1)**. Averaged values of bulk density and coefficient of variation (% CV) is also listed in the table.

Table (1). Particale size analysis and soil texture for the three soils studied along with the results of column packing.

| Texture | Sand | Silt | Clay | Wet front depth (cm) | Average bulk density (Mg/m$^3$) | Standard deviation (Mg/m$^3$) | Coefficient of variation %C.V |
|---|---|---|---|---|---|---|---|
| | (g/Kg) | | | | | | |
| Sand | 910 | 55 | 35 | 40 | 1.630 | 0.016 | 0.960 |
| | | | | 80 | 1.635 | 0.020 | 1.238 |
| Loamy sand | 738 | 195 | 67 | 40 | 1.579 | 0.017 | 1.087 |
| | | | | 80 | 1.584 | 0.020 | 1.240 |
| Clay loam | 257 | 352 | 391 | 40 | 1.316 | 0.014 | 1.097 |
| | | | | 80 | 1.322 | 0.016 | 1.223 |

The water applicator used was the same of that designed by Al-Duri (2002). It was designed to minimize the resistance to flow and disturbance of soil surface during application of an instantaneous constant head of free water to soil surface. The applicator is connected to a Mariotte-type water supply reservoir, which was adjusted to produce the desired value of fixed head H (1.0 cm of water).

The infiltration experiment started by moving the water applicator down against the top end of the soil column to initiate flow. Water reservoir burette readings were recorded with time, as was the depth of the visual wet front. These readings were continued until the waterfront reached a depth of 40 cm for some columns and 80 cm depth for some others. When the wet front reached each fixed position (40 cm or 80 cm), the water stopcock was shut off and the soil column was sectioned into pieces of 2-cm length to determine the bulk density ($\rho_b$) and the volumetric water content ($\theta$) with depth (Z). Shorter columns of the same corresponding infiltration columns were packed to measure the satiated (less than saturated) hydraulic conductivity (Satiation was first defined by Miller and Bresler, 1977) using the constant head procedure for laboratory soil columns (Klute, 1965).

Capillary rise experiments were performed by applying water from the bottom end of other packed soil columns. Water was applied with a constant pressure head of 1 cm of water to the inlet bottom end of the soil column. Cumulative depth of water infiltrated and the upward rise of visual wet front were recorded with time. Each experiment was ended when the advance of the wet front stopped and no water continued to enter the soil. The soil column then sectioned to determine $\rho_b$ and $\theta$ along the height of the soil column. The suction head $\tau$ at each point in the wet region above the inlet bottom end was calculated by substracting H (1cm) from each height Z ($\tau = Z - H$).

The unsaturated hydraulic conductivity [$K(\tau)$] as a function of suction head ($\tau$) was calculated using van Genuchten (1980) closed – form equation:

$$K_r(\tau) = \frac{\left\{1-(\alpha\tau)^{n-1}\left[1+(\alpha\tau)^{-m}\right]\right\}^2}{\left[1+(\alpha\tau)^n\right]^{\frac{m}{2}}} \qquad (3)$$

Where $\alpha$, n, and m are fitted parameters obtained by fitting the following van Genuchten (1980) equation to data of $\theta$ vs. $\tau$ :

$$\theta = \theta_0 + (\theta_1 - \theta_0)\left[1+(\alpha\tau)^n\right]^{-m} \qquad (4)$$

Where

$$m = 1 - \frac{1}{n}$$

The suction head at the wet front $\tau_w$ was estimated using the following techniques:

1- From the fitting of the Green and Ampt infiltration equation [Eq (1)] to infiltration data (t vs. I) using least – square fitting analysis (SAS, 1990). Value of $\tau_w$ can be estimated using the value of estimated parameter a [Since $a = (H+\tau_w)(\theta_1 - \theta_0)$].
2- Using the basic definition of $\tau_w$ [Eq (2)] by substituting Eq (3) in Eq (2) to obtain:

$$\tau_w = \int_0^{\tau_0} \frac{\left\{1-(\alpha\tau)^{n-1}\left[1+(\alpha\tau)^{n}\right]^{-m}\right\}^2}{\left[1+(\alpha\tau)^n\right]^{\frac{m}{2}}}\, d\tau \qquad (5)$$

A numerical integration was used to estimate $\tau_w$ from the $\theta$ - $\tau$ data.

3- Since $K_r(\tau) = K_r[\tau(\theta)] = K_r(\theta)$, $K(\theta) = K_1(\theta/\theta_1)^N$, and $K_r(\theta) = [K(\theta)/K_1] = [K_1(\theta/\theta_1)^N/K_1] = (\theta/\theta_1)^N$, these relations can be used in the basic equation of defining $\tau_w$ [Eq (2)] to obtain:

$$\tau_w = \int_0^{\tau_0} \left(\frac{\theta}{\theta_1}\right)^N d\tau \qquad (6)$$

Where N is a fitted parameter evaluated from fitting $K(\theta) = (\theta/\theta_1)^N$. Numerrical integration can be performed to obtain $\tau_w$ using the $\theta$ - $\tau$ data.

4- The Green and Ampt equation for horizontal infiltration can be written as:

$$I = t^{\frac{1}{2}}\sqrt{2K_1 a} \qquad (7)$$

The fitting of $I=St^{1/2}$ with the early-time downward infiltration data (I vs. t) [(Collis-George (1980)] was performed to estimate the sorptivity which, in this case, can be written as:

$$S = \sqrt{2K_1(\tau_w + H)(\theta_1 - \theta_0)} \qquad (8)$$

The suction head $\tau_w$ can simply be estimated from Eq (8) by using the fitted value of S. Similar approach for this method was also used by Koorevaar, *et al* (1983).

An empirical model adopted from van Genuchten (1980) equation for $\theta$ - $\tau$ relation was used to test the sharpness of the wetting front data ($\theta$ vs. Z). This model can be written as:

$$\theta = b + \frac{f}{\left[1 + (CZ)^{\upsilon}\right]^{\omega}} \qquad (9)$$

Where b, c, f, $\upsilon$ and $\omega$ are fitted parameters.

## RESULTS AND DISCUSSION

The model presented by Green and Ampt (1911) is based on the assumption of sharp wetting front, a constant hydraulic conductivity in the wetted zone, and a constant suction head at the wet front.

Fitting of the wet front ($\theta$ vs. Z) to equation (9) was very good with very small values of residual mean squares of $\theta$ (RMS$\theta$) which ranged from $2.4 \times 10^{-4}$ to $7.4 \times 10^{-4}$ and high values of the coefficient of determintion ($R^2$) which ranged from $0.926^{**}$ to $0.971^{**}$ for all soils and for both depth of wet front (40 and 80 cm). A typical example of the sharpness of the wetting front is shown in Figure 1 for loamy sand soil and for 40 and 80-cm depths of the wet front. From the results of the fitting and the shapes of the $\theta$ - Z profiles, one may conclude that the wet fronts were sharp enough to satisfy the Green and Ampt assumption of piston – like soil moisture profile.

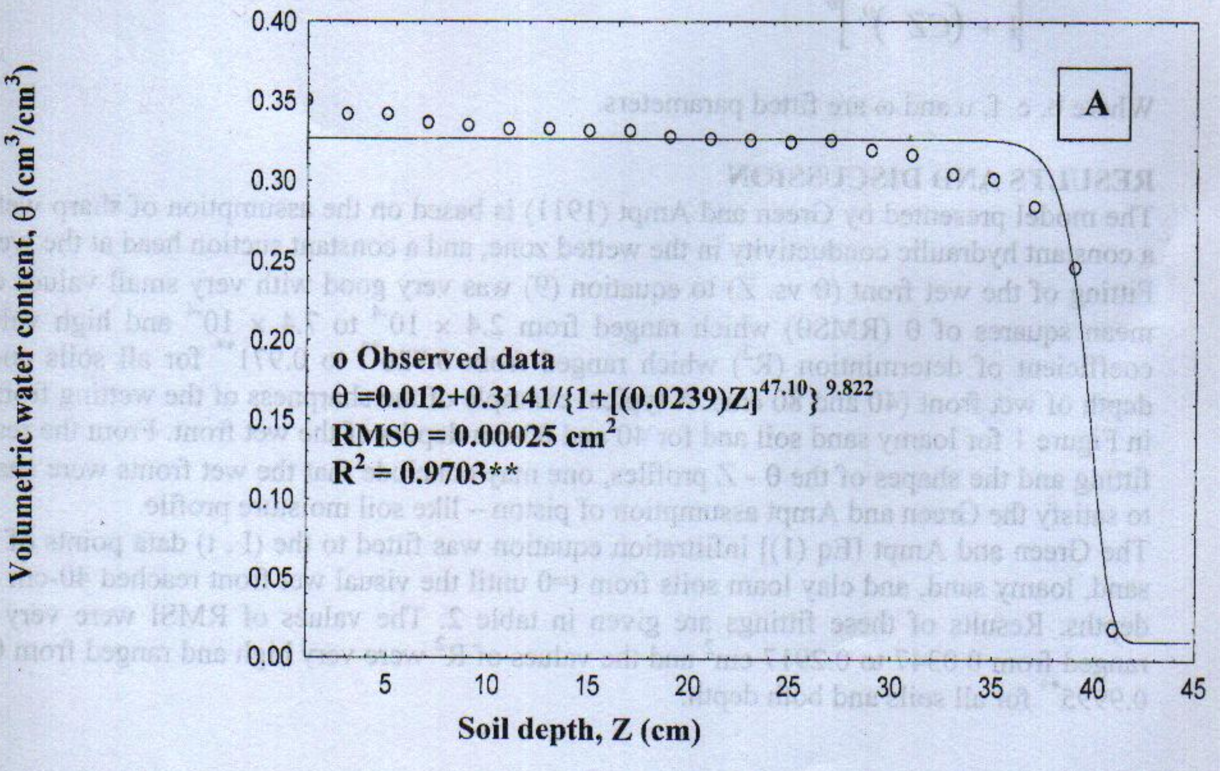
The Green and Ampt [Eq (1)] infiltration equation was fitted to the (I , t) data points of air – dried sand, loamy sand, and clay loam soils from t=0 until the visual wet front reached 40-cm and 80-cm depths. Results of these fittings are given in table 2. The values of RMSI were very small and ranged from 0.0347 to 0.2917 cm$^2$ and the values of $R^2$ were very high and ranged from $0.9999^{**}$ to $0.9995^{**}$ for all soils and both depth.

Table (2). Results of fitting the Green and Ampt infiltration equation for the three soils with two wet front depths.

| Texture | Wet front depth (cm) | Fitted parameters | | RMSt ($min^2$) | $R^2$ | RMSI* ($cm^2$) | Measured $K_s$ (cm/min) |
|---|---|---|---|---|---|---|---|
| | | $K_1$ (cm/min) | a (cm) | | | | |
| Sand | 40 | 0.1016 | 3.873 | 0.312 | 0.9999** | 0.0374 | 0.1152 |
| | 80 | 0.1124 | 4.608 | 4.370 | 0.9998** | 0.0470 | |
| Loamy sand | 40 | 0.0097 | 24.891 | 2.332 | 0.9998** | 0.0472 | 0.0107 |
| | 80 | 0.0085 | 24.245 | 9.880 | 0.9995** | 0.2917 | |
| Clay loam | 40 | 0.0029 | 38.917 | 24.040 | 0.9997** | 0.0816 | 0.0032 |
| | 80 | 0.0021 | 41.896 | 109.80 | 0.9995** | 0.2638 | |

** Significant at 0.01 level.

$$* \text{ Residual mean square of I} = \frac{\text{Total Sum of squares residual}}{\text{Number of data points} - 2}$$



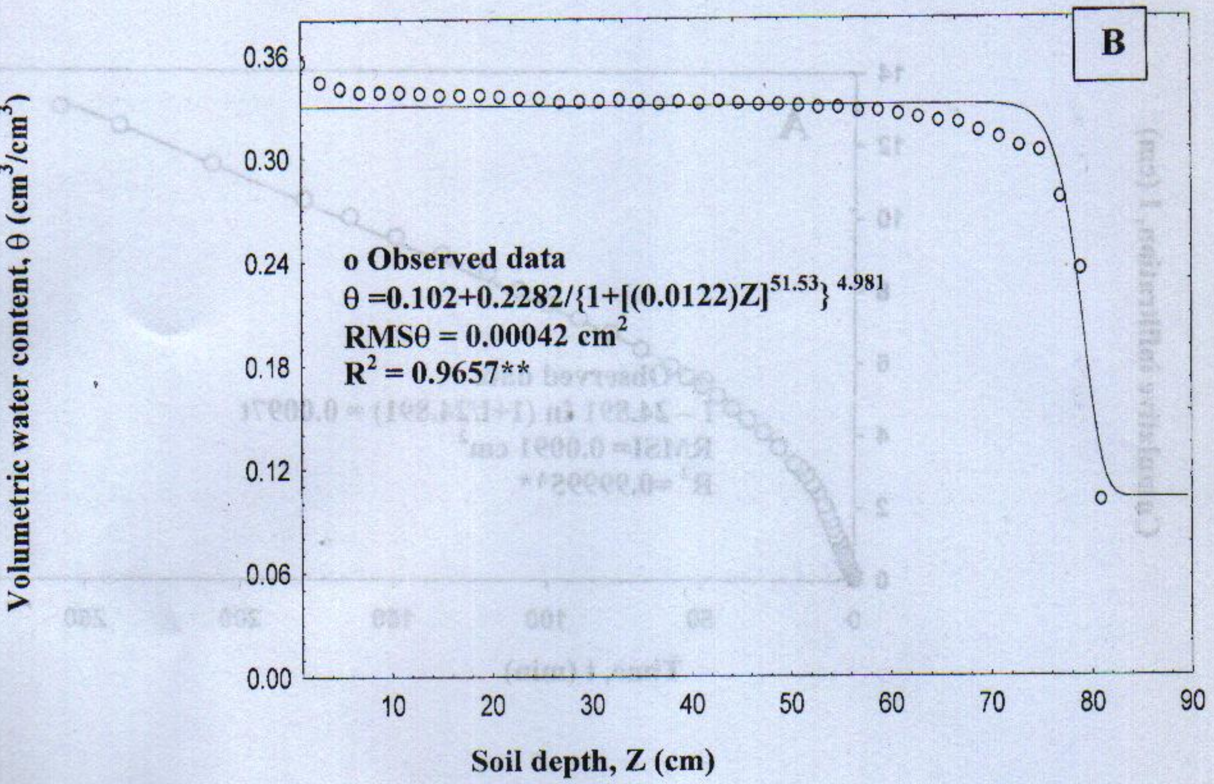


Fig. (1) Soil moisture profile of loamy sand soil for the wet front depth 40 cm (A) and 80 cm (B).

Experimental points are plotted in **Fig. (2)**, for the loamy sand soil and for both depths of wet front (as a typical example), along with their theoretical curves of the infiltration equation using the fitted values of the parameters shown on the figure. In each case the experimental points fall closely upon their fitted curves.

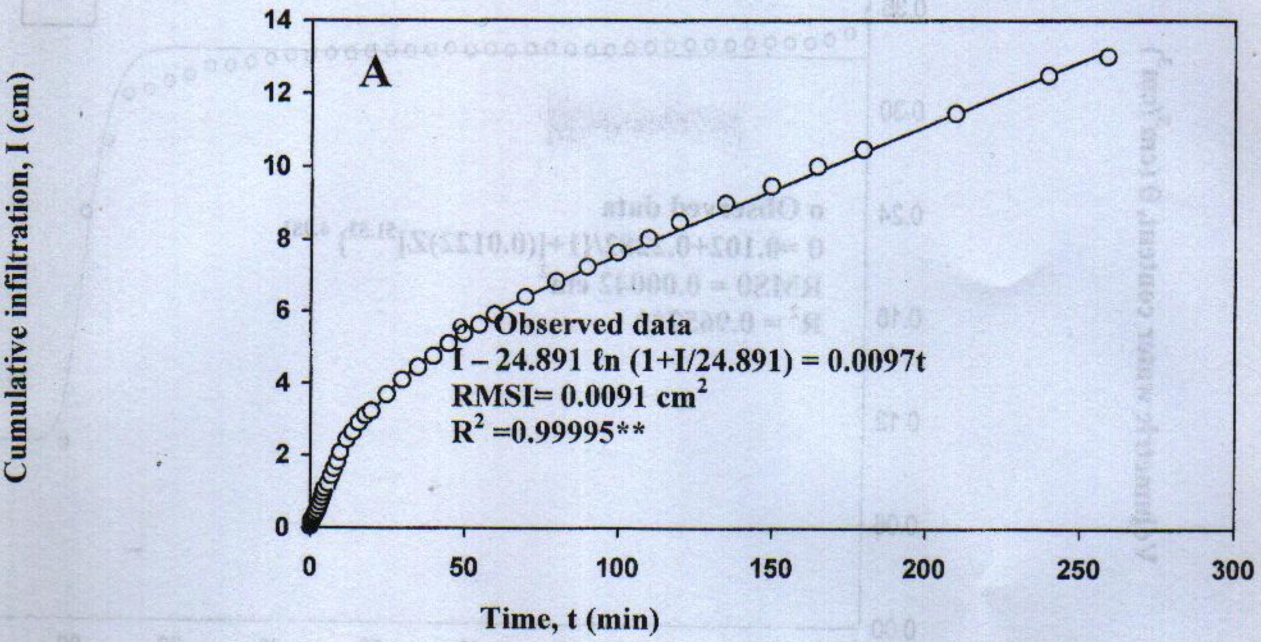
Another important aspect of the fitting is the meaning that can be attached to the fitted parameters. The parameter $K_1$ should be equal to the measured saturated hydraulic conductivity $K_s$. from **Table (2),** it can be noticed that the values of $K_1$ are close to the values of $K_s$ with the fitted $K_1$ values being lower than $K_s$ values. The smaller values of the fitted $K_1$ in comparison with $K_s$ values is probably related to the lack of reaching complete saturation during infiltration due to air entrapment (Miller and Gresler, 1977).

The Green and Ampt parameter a in table 2 was used along with the fillable porosity $(\theta_1 - \theta_0)$, $[(\theta_1 - \theta_0)$ is the slope of the line fitted between I and the depth of wet front Z) and H (1 cm $H_2O$) to calculate the suction head at the wet front $\tau_w$, $\{\tau_w = [a/(\theta_1 - \theta_0)] - H\}$. Values of $\tau_w$ are listed in `A wide variation was found in the values of $\tau_w$ among the soils. These values are used as references for the values obtained from the other methods of estimating $\tau_w$.

Table (3). Values of the suction head at the wet front $\tau_w$ (cm) estimated by four different methods for the three soils.

| Texture | Green and Ampt fitting<br>Wet front depths 40 cm | Green and Ampt fitting<br>Wet front depths 80 cm | Numerical integration<br>Function of θ | Numerical integration<br>Function of τ | Sorptivity |
|---|---|---|---|---|---|
| Sand | 11.01 | 13.90 | 13.11 | 13.32 | 12.35 |
| Loamy sand | 77.11 | 75.30 | 68.74 | 68.77 | 55.55 |
| Clay loam | 98.83 | 108.09 | 79.69 | 70.09 | 85.10 |

$LSD_{0.05}$ = 13.24 cm.



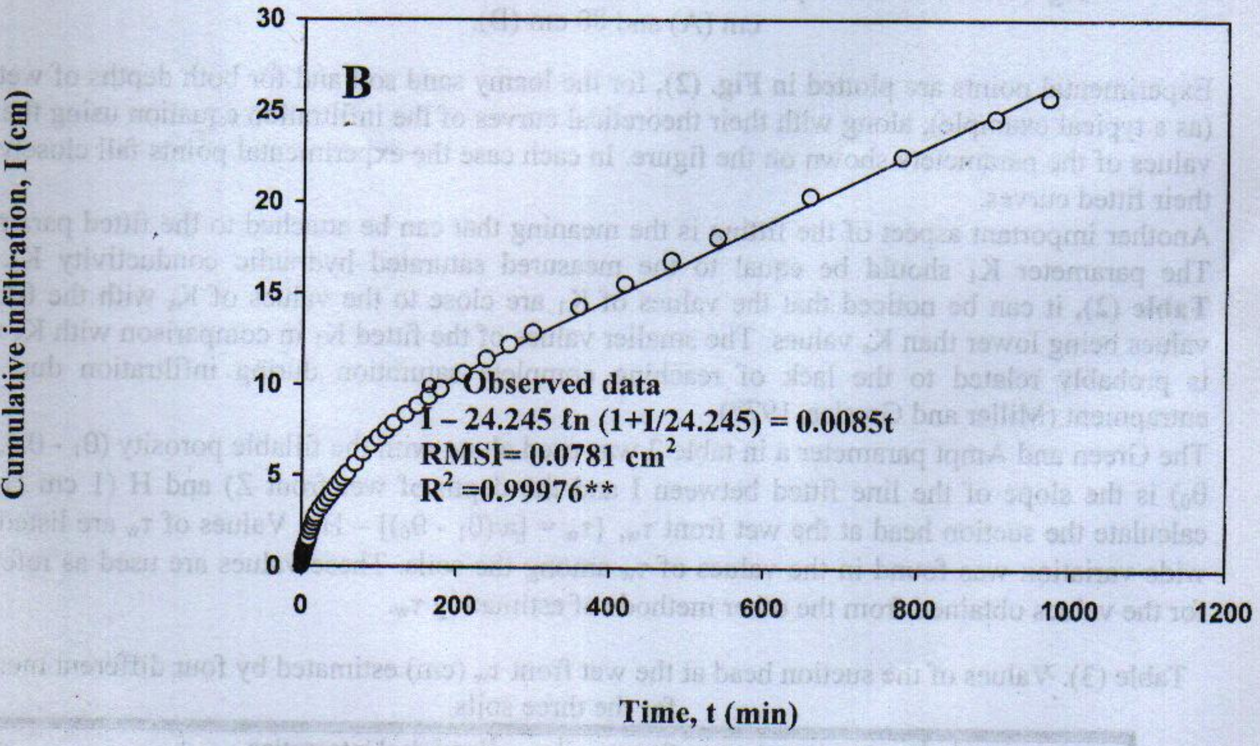


Fig. (2) Fitting of the Green and Ampt equation to infiltration data of loamy sand soil for wet front depth 40 cm (A) and 80 cm (B).

**Table (3)** shows the values of $\tau_w$ estimated by the four methods. As it is indicated in the table, no significant differences between the values of $\tau_w$ estimated by the four methods for each soil. However, significant difference was found between values of $\tau_w$ for the three soils for each method of estimation. Table 4 shows the fitted parameters used in each method for estimating $\tau_w$.
Comparison between the four methods in estimating $\tau_w$ shows that the sorptivity and the integration methods were close to each other in the values of $\tau_w$. Fitting the Green and Ampt equation seemed to result in a higher values of $\tau_w$ **Table (3).** This is probably related to the nature of the fitting using statistica program (least – square estimation fit) in which the convergence occures whenever the error is at its lowest level regardless of the values of the parameters in the model.
Finally, it can be concluded that using any of the four methods studied is adequately accurate for estimating the suction head at the wet front $\tau_w$ for a variety of practical instances.

Table (4) Fitted parameters used in each method for estimating $\tau_w$.

| Texture | Green and Ampt fitting<br>Wet – front depth | | | | Sorptivity method* | | |
|---|---|---|---|---|---|---|---|
| | 40 cm | | 80 cm | | | | |
| | a | $\theta_1$ - $\theta_0$ | a | $\theta_1$ - $\theta_0$ | S | $K_1$ | $\theta_1$ - $\theta_0$ |
| Sand | 3.873 | 0.322 | 4.608 | 0.310 | 0.993 | 0.1152 | 0.321 |
| Loamy sand | 24.891 | 0.318 | 24.245 | 0.318 | 0.661 | 0.0107 | 0.360 |
| Clay loam | 38.917 | 0.390 | 41.896 | 0.381 | 0.504 | 0.0032 | 0.464 |

| Texture | Numerical integration method | | | | | |
|---|---|---|---|---|---|---|
| | K is function of θ | K is function of τ | | | | |
| | N | $\theta_0$ | $\theta_1$ - $\theta_0$ | α | n | m |
| Sand | 3.508 | 0.070 | 0.268 | 0.012 | 2.362 | 0.577 |
| Loamy sand | 2.013 | 0.019 | 0.244 | 0.013 | 14.298 | 0.930 |
| Clay loam | 0.120 | 0.120 | 0.342 | 0.005 | 2.754 | 0.637 |

* Pressure head at the inlet (H) = 1 cm of water.

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