



ASSESSMENT OF EQUIVALENT GRAIN DIAMETER FOR SOIL SPECIFIC SURFACE DETERMINATION

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ABSTRACT

A procedure is presented to calculate an equivalent diameter for soil grains to be used to calculate the specific surface of the soil. The typical grain size distribution curve is expressed as a normal probability distribution cumulative curve and the frequency corresponding to the equivalent diameter is accordingly found. This frequency is adopted as the percent finer corresponding to the equivalent diameter. A relation is given for the calculation of the specific surface using the equivalent diameter. Grain size distribution curves of many soil samples are collected. A value for the specific surface of each soil is determined summing the surface area of subintervals in the distribution curve. The values of specific surface obtained from these gradation curves are compared to those calculated using the proposed values of the equivalent diameter for each soil. The results have shown good agreement for using the equivalent diameter that presented in this paper to determine the specific surface for soils instead of using the usual long procedure.

الخلاصة

تم تقديم منهج عملي لحساب القطر المكافئ لذرات التربة لكي يتم استخدامها في حساب المساحة السطحية النوعية للتربة. وتم التعبير عن منحني التوزيع الحبيبي للتربة كمنحني احتمالات طبيعي متجمع و كذلك تم ايجاد التكرار المقابل للقطر المكافئ تبعاً لذلك . وتم اعتماد هذا التكرار كمصحح مؤوي مقابل للقطر المكافئ. وقد اعطيت علاقة لحساب المساحة السطحية النوعية باستخدام القطر المكافئ . تم جمع بيانات لمنحني التوزيع الحبيبي للعديد من النماذج وتم تعيين قيمة المساحة السطحية النوعية لكل عينة بحساب المساحة السطحية للمسافات الثانوية في منحني التوزيع . اجريت مقارنة لقيم المساحة السطحية النوعية التي تم الحصول عليها من منحنيات التوزيع الحبيبي هذه بتلك المحسوبة باستخدام القيم المقترحة للقطر المكافئ لكل عينة. وقد ظهرت نتائج جيدة لاستخدام القطر المكافئ المقدم في هذا البحث لحساب المساحة السطحية النوعية عوضاً عن استخدام المنهج العملي الطويل المعتاد.

KEY WORDS

Effective diameter, equivalent diameter, specific surface, grain size distribution

INTRODUCTION

The equivalent or the "effective" diameter or size is a well-known parameter in soil classification and permeability determination. Hazen (1892), on the basis of his study of filter sands, found that the diameter of which 10% by weight of soil grains are finer may cause the same effects as the given soil. Denoted as D_{10} , Hazen called this diameter the "effective diameter". The main consideration was the effect of this diameter on flow characteristics and a famous formula known as "Hazen's formula" was presented to estimate the coefficient of permeability. Related to soil classification, the effective diameter is used with two other diameters D_{60} and D_{30} to describe the gradation behavior of the soil through the definition of the well-known coefficient of uniformity D_{60}/D_{10} and the coefficient of curvature $D_{30}^2/(D_{60} * D_{10})$ by which a granular soil may be classified.

Terzaghi and Peck (1948) suggested the use of D_{70}/D_{20} for the coefficient of uniformity and stated that the characteristics of fine-grained soils depend on the finest 20%. The term, D_{50}/D_{10} , is also suggested, by Kezdi (1980). Anyhow, the above definition of effective diameter is related to flow characteristics and classification of the soil and not exactly to the determination of the surface area of the particles.

Many soils undergo chemical reactions on the surface of their particles. These chemical reactions may be dissolution, adsorption, reaction of the grains with the surrounding chemicals, decay, etc. Most of these reactions depend on the surface area of the soil particles as one of the parameters controlling the rate of the process.

The goal of this study is to determine the value of the percent finer that corresponds to the equivalent diameter of soil grains that may be used directly to calculate, as accurately as possible, the surface area of the soil solids. The equivalent or the effective diameter would certainly vary according to the gradation of the soil grains, the wider the range of particle diameters included in the soil matrix the smaller would be the effective diameter.

DEFINITION OF THE EQUIVALENT DIAMETER AND THE SPECIFIC SURFACE

The usual method to calculate the specific surface of a soil is the summation of the surface area of several sub divisions of the soil grains according to corresponding intervals on the gradation curve.

If a grain size distribution curve such as the one shown in **Fig. (1)** is divided into n intervals, the specific surface of the soil, assuming spherical particles, is calculated according to the following:

The average surface area of particles in an interval i of this gradation curve is

$$S_{p_{iav}} = \pi D_{iav}^2 \quad (1)$$

Where D_{iav} is the average diameter in this interval, while the average volume of a particle in this interval is

$$V_{p_{iav}} = \frac{\pi}{6} D_{iav}^3 \quad (2)$$

Hence, the total surface area of particles of this interval will be: -

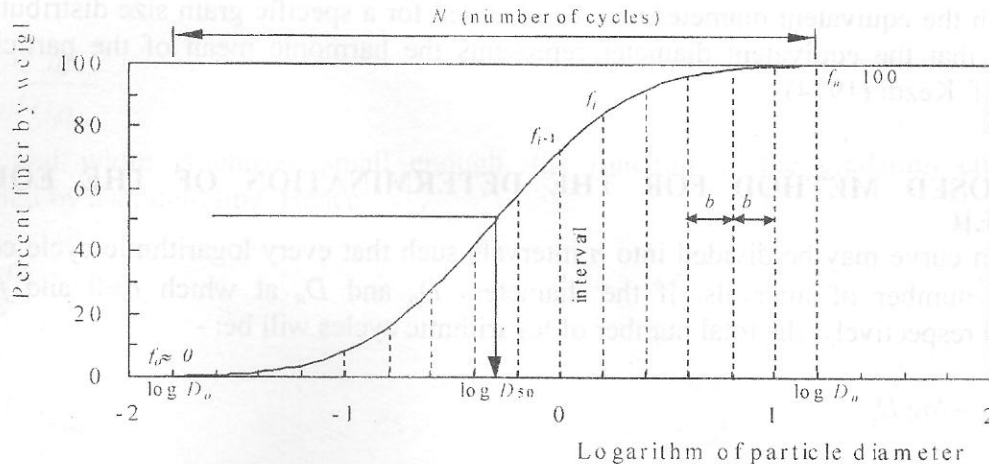


Fig. (1). A typical gradation curve of a soil divided into n equal intervals

$$S_i = \frac{(f_i - f_{i-1})W}{V_{piav}G\rho_w} S_{piav} = \frac{6(f_i - f_{i-1})W}{D_{iav}G\rho_w} \quad (3)$$

where W is the total weight of soil particles in grams, f_i and f_{i-1} are the cumulative percentages by weight of the particles finer in diameter than those at the beginning and the end of the interval i respectively, substituted in decimals, G is the average specific gravity of soil particles and ρ_w is the density of water = 1gm/cm^3 . To unify the units, diameters are substituted in centimeters and the surface areas are obtained in cm^2 . The specific surface of the soil, in cm^2 per 100gm of soil, may then be computed as: -

$$S_s = \left(\sum_{i=1}^n S_i \right) * \frac{100}{W} = \frac{6}{G\rho_w} \sum_{i=1}^n \frac{f_i - f_{i-1}}{D_{iav}} * 100 \quad (4)$$

The above formula does not take into consideration the effect of shape and roughness of particles. In fact, the exact surface area is not the same as that found by Eq. (1), because the soil particles are not spherical indeed. This effect is usually overcome empirically. Shape factors for surface area and volume of a soil particle are usually used to correct the calculations as much as possible, (see e.g. Marsal, 1973; Harr, 1977; Lee *et al.*, 1983).

In this paper, as the goal is to predict an effective diameter, only the ratio of the predicted to the calculated specific surface is needed. This cancels the need to use exact surface area values and only spherical particles will be assumed. An equivalent diameter is defined as the diameter that may substitute the whole soil grains for calculating the specific surface. The specific surface in cm^2 per 100gm of a soil may be calculated from the equivalent diameter as: -

$$S_s = \frac{100S_{s,e}}{G\rho_w V_{pe}} = \frac{600}{G\rho_w D_e} \quad (5)$$

where S_{pe} and V_{pe} are the surface area in cm^2 and the volume in cm^3 respectively, of a particle having a diameter equal to the equivalent diameter, D_e in cm.

Equating the specific surface from Eq. (5) and Eq. (4), the following is obtained: -

$$\frac{1}{D_e} = \sum_{i=1}^n \frac{(f_i - f_{i-1})}{D_{iav}} \quad (6)$$

From which the equivalent diameter may be obtained for a specific grain size distribution curve. It is obvious that the equivalent diameter represents the harmonic mean of the particle diameters available. cf. Kezdi (1974).

A PROPOSED METHOD FOR THE DETERMINATION OF THE EQUIVALENT DIAMETER

A gradation curve may be divided into n intervals such that every logarithmic cycle could include an integer number of intervals. If the diameters D_o and D_n at which $f_o \approx 0$ and $f_n = 100\%$ are determined respectively, the total number of logarithmic cycles will be: -

$$N = \log D_n - \log D_o \tag{7}$$

and each interval will have a width of: -

$$h = N / n \tag{8}$$

Although n is chosen as an integer, the number of logarithmic cycles N is governed by the gradation curve and may be a real number. As n is known, the "percent finer" values f_{i-1} and f_i for each interval are determined from the gradation curve.

After selection of an average diameter for each interval, Eq. (6) may be used to determine the effective (the equivalent) diameter required. The selection of the average diameter of the interval should be as accurate as possible to lessen the approximation error.

THE AVERAGE DIAMETER OF AN INTERVAL

A typical interval of the grain size distribution curve is shown in Fig. (2). The surface area of particles for a slice of a width dx in the interval will be: -

$$S(x) = \frac{df W}{\frac{4}{3} \pi \left(\frac{10^x}{2}\right)^3 G \rho_w} * 4 \pi \left(\frac{10^x}{2}\right)^2 \tag{9}$$

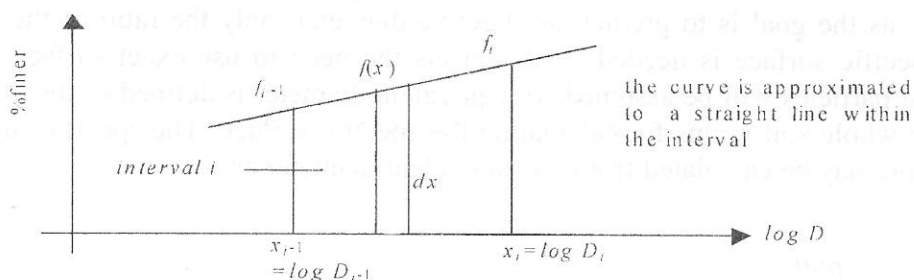


Fig. (2). A typical interval i of a gradation curve.

which may be reduced to: -

$$S(x) = \frac{6Wdf}{10^x G \rho_w} \tag{10}$$

Then, the surface area of particles in the whole interval is: -



$$S_i = \int_{x_{i-1}}^{x_i} \frac{6W \cdot df}{10^x G\rho_w} \tag{11}$$

If the interval width is chosen small enough, the function of the gradation curve may be approximated by a straight line. Hence:

$$\frac{df}{dx} = \frac{f_i - f_{i-1}}{x_i - x_{i-1}} \tag{12}$$

or

$$df = \frac{f_i - f_{i-1}}{x_i - x_{i-1}} dx \tag{13}$$

By substituting Eq. (13) in Eq. (11), the following is obtained: -

$$S_i = \frac{6W (f_i - f_{i-1})}{G\rho_w (x_i - x_{i-1})} \int_{x_{i-1}}^{x_i} \frac{dx}{10^x} \tag{14}$$

Substituting $\log D$ for x : -

$$S_i = \frac{6 (f_i - f_{i-1}) W}{\ln(10) (x_i - x_{i-1}) G\rho_w} \left[\frac{D_i - D_{i-1}}{D_{i-1} D_i} \right] \tag{15}$$

Comparing the latter equation with Eq.(3) and substituting b for $(x_i - x_{i-1})$, the average diameter for the interval i will be: -

$$D_{i,av} = b \ln(10) \left(\frac{D_{i-1} D_i}{D_i - D_{i-1}} \right) \tag{16}$$

As D_{i-1} may be substituted by $10^{-b} D_i$, Eq. 16 may be written in a simpler form: -

$$D_{i,av} = \frac{b \ln(10)}{10^b - 1} D_i \tag{17}$$

It is now possible to estimate the surface area of the particles of a small interval as: -

$$S_i = \frac{6 (f_i - f_{i-1}) W 10^b - 1}{G\rho_w D_i b \ln(10)} \tag{18}$$

and the specific surface may be calculated using its definition in Eq. 4 or

$$S_s = \frac{600 - 10^b - 1}{G\rho_w b \ln(10)} \sum_{i=1}^n \frac{f_i - f_{i-1}}{D_i} \tag{19}$$

with D_i in centimeters and S_s in cm^2 per 100gms. Again f and df in all of the above equations are in decimals.

THE EQUIVALENT DIAMETER OF THE SOIL

The aforementioned steps were devoted to determine an average diameter for a specific interval of a small width within the grain size distribution curve. Next, it is required to assess the value of the percentage finer corresponding to the diameter closest to the effective equivalent diameter, the latter being adopted to calculate the average specific surface of the soil as a whole. It is plausible to assume that the required value of the percentage finer is dependent on the number of logarithmic cycles defined in Eq. 7 and on the properties of the cumulative distribution of the grains.

The cumulative distribution will be assumed following the well-known cumulative normal probability distribution. This enables the calculation of the cumulative function corresponding to a certain diameter and vice versa. The most significant range of the probability distribution will be assumed as from $\mu-3\sigma$ to $\mu+3\sigma$, μ and σ being the mean and the standard deviation of the distribution, giving a confidence level of 99.73%. This is called the "3 σ rule" advised by Duncan (2000) for reliability problems in geotechnical engineering. Thus, the N cycles will correspond to 6σ of the by weight distribution of the logarithms of diameters of particles. Then, it is possible to determine the standard deviation of the distribution as: -

$$\sigma = N \cdot b \quad (20)$$

The standard random variable of the standard normal probability distribution will be

$$z = \frac{\log D - \mu}{\sigma} \quad (21)$$

where: μ is the mean

Substituting Eq.(21) in Eq.(5) and Eq. (19) and equating the latter equations as in Eq. (6), the following is obtained: -

$$10^{z_e \sigma + \mu} = \frac{10^b - 1}{b \ln(10)} \sum_{i=1}^n \frac{f_i - f_{i-1}}{10^{z_i \sigma + \mu}} \quad (22)$$

The mean μ is the logarithm of D_{50} which may be easily determined. Nevertheless, its value is cancelled out from both sides if the equation is multiplied by (10^{μ}) . The variables z_e and z_i represent the standard normal variables that correspond to the effective diameter in question and the diameter at the end of the interval i in the gradation curve.

If a standard cumulative normal distribution curve is divided into n intervals within the range $z=-3$ to $z=+3$, z_e may be easily found as; -

$$z_e = \frac{1}{\sigma} \log \left\{ \frac{b \ln(10)}{10^b - 1} \left(\sum_{i=1}^n \frac{f_i - f_{i-1}}{10^{z_i \sigma}} \right)^{-1} \right\} \quad (23)$$

For a specified number of intervals, n , and a known value of the number of cycles N , the values of the standard deviation σ and the interval width b are determined. For each interval i , the value of z_i is determined from the inverse of the cumulative standard normal distribution function as the random variable corresponding to f_i , the cumulative frequency.

After z_e is found, the corresponding cumulative frequency f_e may be found from the cumulative standard normal distribution function as being the percentage finer that corresponds to the required effective diameter D_e . The specific surface of the soil may be determined through Eq. 5 using a single diameter obtained from the gradation curve.

To put a single equation for simple assessment of f_e , several trials to solve Eq. 23 are performed using different number of intervals and logarithmic cycles. The results of these trials are given in Table 1. Of course, the range of N for natural soils is from about one cycle for uniform soils to about five or six cycles for widely sorted soils (very well graded). The results have shown that 200 intervals would be enough to assess properly accurate values for f_e .

Table (1). The percentage finer f_e corresponding to the equivalent diameter

n	N					
	1	2	3	4	5	6
10	42.446	34.868	28.118	22.346	17.615	13.870
20	42.774	35.480	28.937	23.281	18.584	14.810
40	42.856	35.634	29.144	23.520	18.834	15.056
100	42.879	35.677	29.202	23.587	18.905	15.125
200	42.883	35.683	29.211	23.597	18.915	15.135

The values for the effective percentage of finer particles for the case of 200 intervals are plotted against the number of logarithmic cycles of the gradation curve in Fig. (3).

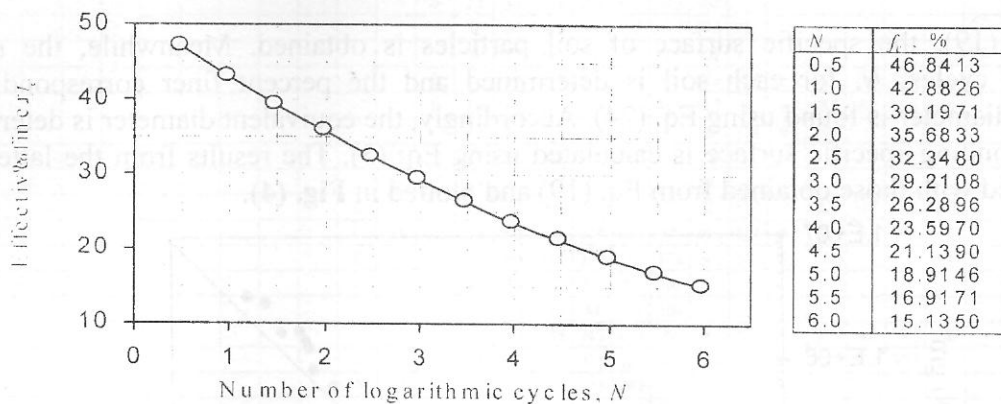


Fig. (3). The percent finer corresponding to the equivalent diameter for the case of 200 intervals for different N values

The plotted values in the figure are tabulated to the right of the plot in the same figure. A regression analysis has been performed and a best-fit curve is found to be the following: -

$$f_e = 0.43N^2 - 8.6N + 51.12 \tag{24}$$

with f_e in percent. The coefficient of correlation is found to be 0.999994, which is very high indeed. The equation is limited to the range $N \in [0.5, 6]$ where the trend of the relation with f_e is completely different for N values lower than 0.5, while for $N > 6$, the fitting equation should be changed (particle diameters in natural soils yield no values out of this range practically).

It is obvious now that for a certain soil, the equivalent diameter can be easily determined from the grain size distribution curve of that soil. It will be the diameter corresponding to the effective percentage finer determined from the table or from the graph given in Fig. (3) or using Eq.(24) directly. As the equivalent diameter is determined, the specific surface of the soil can be calculated using Eq. (5).

Here, it should be noticed that a proper choice for the value of the number of cycles, N , is very important as it may affect the results significantly. The grain size analysis performed in the laboratory should yield a curve as long as possible and at least the 10% passing diameter should be included. With such a curve, the value for N will be better determined. Anyhow, a proper value for N may be approximated through comparing the relative difference of logarithms of any two

diameters with that of the corresponding standard random variables in the cumulative standard normal distribution. That may be put in the following form: -

$$N = \frac{\log D_q - \log D_p}{z_q - z_p} \quad (25)$$

where p and q refer to two percentages of finer particles and z being the standard normal distribution random variable. For example, N may be approximated using the coefficient of uniformity as $3.986 \log(D_{60}/D_{10})$.

The results will be most accurate when the median of the range of logarithms of diameters is close to $\log D_{50}$ and when the curvature of the gradation curve is close to the curvature of that of the normal distribution curve.

COMPARISON WITH TEST RESULTS

Grain size distribution curves for 154 different soil samples have been analyzed. The particle diameter values are taken in centimeters and plotted on the usual logarithmic axis against which the percentage finer is plotted. The curves are subdivided into intervals each of width $b=0.2$. The tails of the curves in the direction of small diameters are extended to determine an approximate value for the D_0 to start calculation.

Table (2) represent these samples; the shaded cells are the extended to approximate values.

Using Eq. (19), the specific surface of soil particles is obtained. Meanwhile, the number of logarithmic cycles, N , for each soil is determined and the percent finer corresponding to the equivalent diameter is found using Eq. (24). Accordingly, the equivalent diameter is determined and the corresponding specific surface is calculated using Eq. (5). The results from the latter equation are compared with those obtained from Eq. (19) and plotted in Fig. (4).

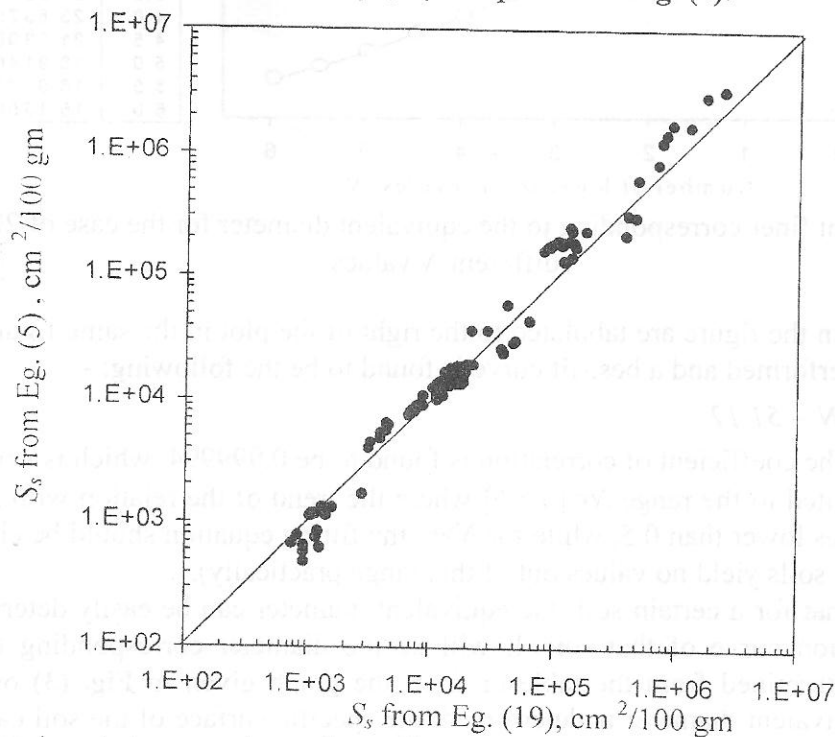


Fig. (4). Comparison between values of specific surface obtained from Eq. (19) and Eq. (5) for the analyzed soils



Table (2). Grain size distribution of the samples

D ₉₀ (mm)	D ₈₅ (mm)	D ₇₅ (mm)	D ₆₀ (mm)	D ₄₅ (mm)	D ₃₀ (mm)	D ₂₅ (mm)	D ₂₀ (mm)	D ₁₅ (mm)	D ₁₀ (mm)	D _{7.5} (mm)	D ₆ (mm)	D _{4.75} (mm)	D _{4.25} (mm)	D _{3.75} (mm)	D _{3.0} (mm)	D _{2.5} (mm)	D _{2.0} (mm)	D _{1.5} (mm)	D _{1.18} (mm)	D _{0.85} (mm)	D _{0.6} (mm)	D _{0.425} (mm)	D _{0.3} (mm)	D _{0.25} (mm)	D _{0.15} (mm)	D _{0.075} (mm)	D _{0.075} (mm)	F%		
																												0	100	
1																													0	5.2
2																													0	0.8
3																													0	5.2
4																													0	1.7
5																													0	1.7
6																													0	3
7																													0	4.3
8																													0	3
9																													0	4.3
10																													0	2.5
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62																													0	1.2

Table (2). Continued

Table with multiple columns and rows of numerical data, likely representing soil properties. The table contains rows numbered 65 to 131 and columns with values ranging from 0 to 100.



Table (2). Continued

Table with 20 columns and 20 rows (132-151) containing numerical data for soil analysis.

The comparison has proved good agreement and the coefficient of correlation between the results of the two equations is found to be 0.98303. The values obtained for the ratio of the specific surface calculated using the equivalent diameter to the specific surface calculated through summing the interval surface area of particles (S_{pe}/S_{PT}) varied from 0.44 to 1.79 with an average of 1.003, which is very close to 1.0. The standard deviation of the ratio distribution is found to be 0.28. The effect of gradation of soil have been studied by redrawing the relation between specific surface calculated using Eq. (5) and that obtained from Eq. (19) for uniformly samples after separate the samples to uniformly samples, well graded and gap graded in Fig. (5, 6) and (7) respectively.

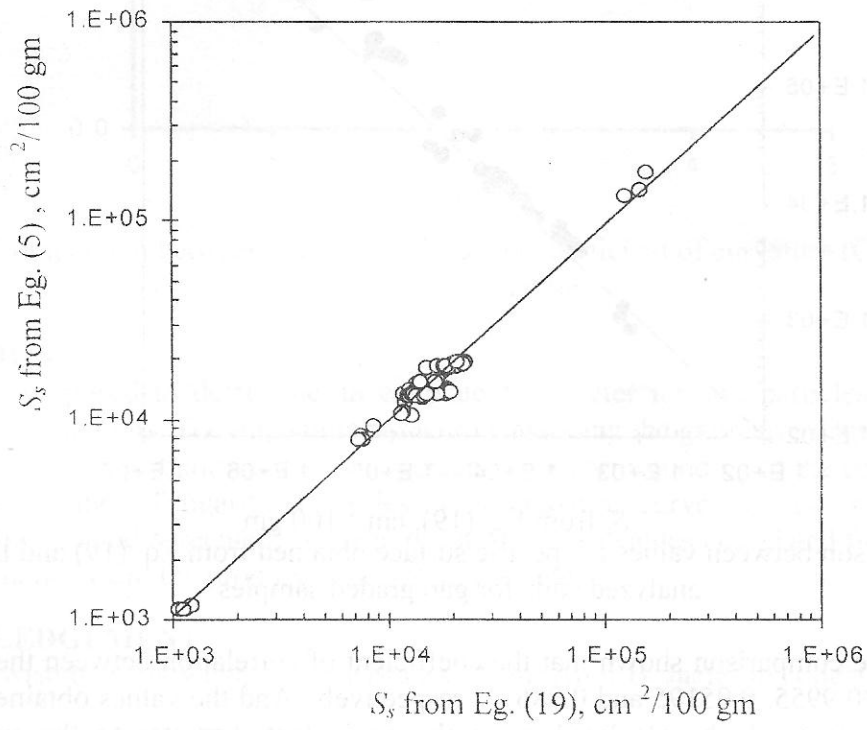


Fig. (5). Comparison between values of specific surface obtained from Eq. (19) and Eq. (5) for the analyzed soils for uniformly graded samples

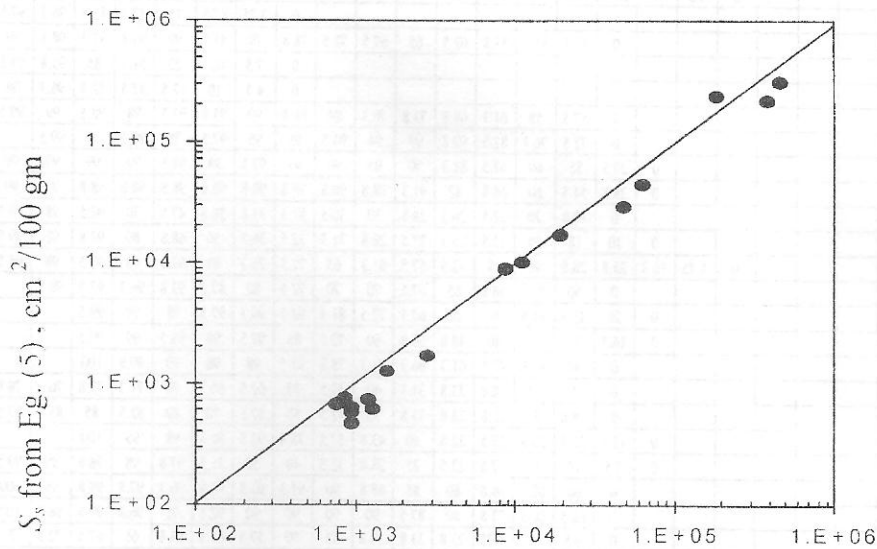


Fig. (6). Comparison between values of specific surface obtained from Eq. (19) and Eq. (5) for the analyzed soils for well-graded samples

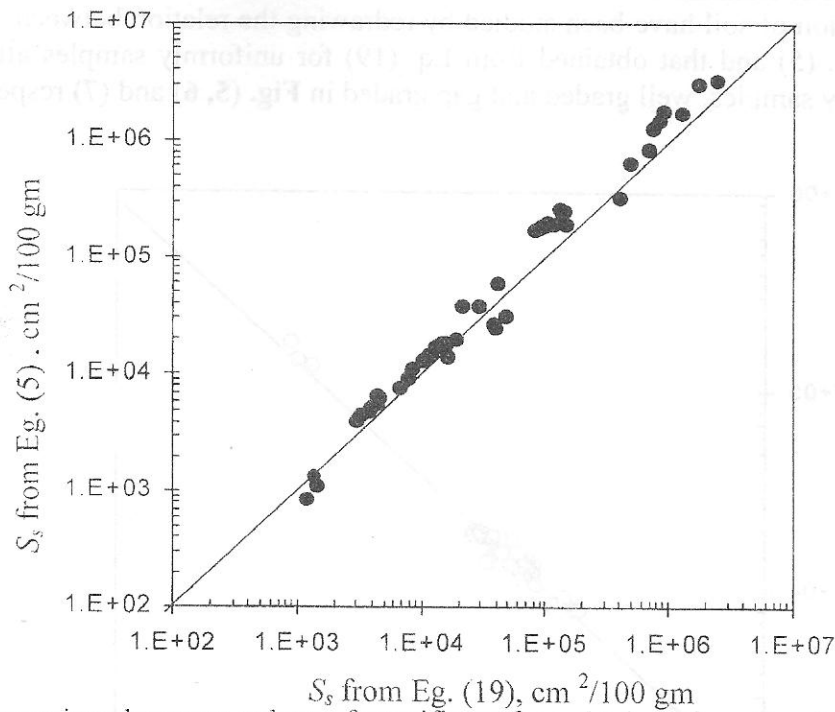


Fig. (7). Comparison between values of specific surface obtained from Eq. (19) and Eq. (5) for the analyzed soils for gap graded samples

In these figures the comparison shown that the coefficient of correlation between the results of the two equations by (0.9955, 0.95195 and 0.98613) respectively. And the values obtained for the ratio of the specific surface which calculated using the equivalent diameter to the specific surface calculated through summing the interval surface area of particles (S_{pe}/S_{PT}), varied from (0.68 to



1.13) with an average of 0.923 for uniformly samples, and between (0.43 to 1.20) with an average of 0.155 for well graded where its value lay between (0.57 to 1.79) with an average of 0.155 for gap graded

This results show that Uniformly graded soils yielded better results than for well or gap graded soils.

Fig. (8) show the relation between the ratio of the specific surface calculated using the equivalent diameter to the specific surface calculated through summing the interval surface area of particles (S_{pe}/S_{PT}) and the coefficient of curvature (C_C which equal to $(D_{30}^2/(D_{60} * D_{10}))$).

It is found that the ratio (S_{pe}/S_{PT}) decreases with the increase in the coefficient of curvature (C_C). Ratios closer to 1.0 are found for values of the coefficient of curvature in the range 1.0 to 3.0.

Considering the soil type and gradation, it is found that the results are less accurate in clayey soils than in sandy soils.

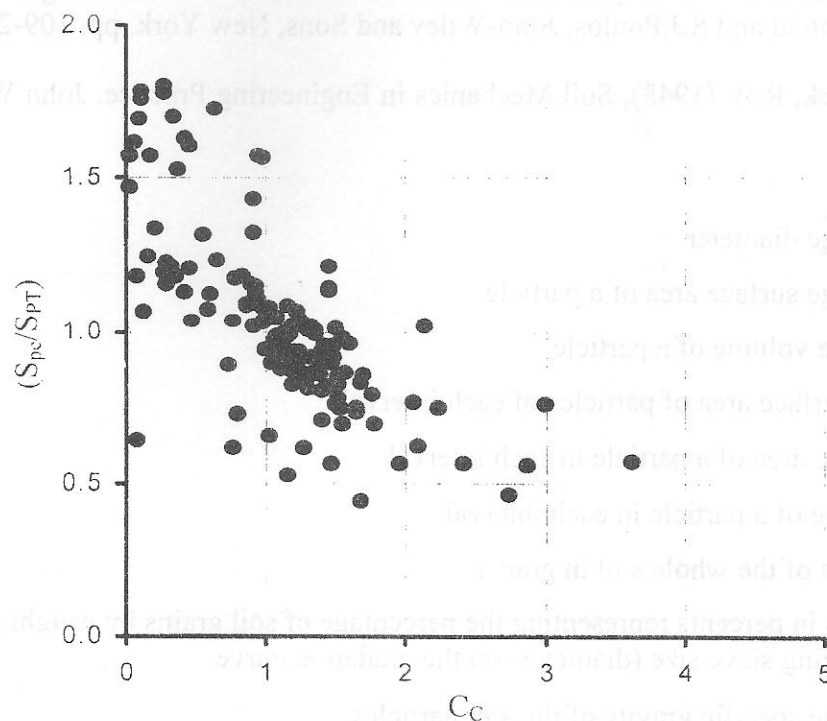


Fig. (8). Comparison between (S_{pe}/S_{PT}) values and coefficient of curvature (C_C) for uniformly samples

CONCLUSIONS

A method is proposed to determine an equivalent diameter for soil particles for the purpose of surface area calculations. The method is based on simulating the grain size distribution curve by the cumulative normal distribution curve. The percent finer corresponding to the equivalent diameter is related to the number of logarithmic cycles in the gradation curve. Analysis of 154 soil gradation curves has shown good agreement between the surface area values cumulated from gradation curves and those obtained from the proposed equivalent diameters.

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NOTATIONS

- D_{av} The average diameter
- S_{piaV} The average surface area of a particle
- V_{piaV} the average volume of a particle
- S_i the total surface area of particles of each interval
- S_{pi} The surface area of a particle in each interval
- V_{pi} The volume of a particle in each interval
- W_t The weight of the whole soil in grams.
- f The values in percents representing the percentage of soil grains by weight passing the corresponding sieve size (diameter) on the gradation curve.
- G The average specific gravity of the soil particles.
- ρ_w The density of water = 1 gm/cm³
- S_s The specific surface of the soil
- S_{pe} The specific surface of the particle having the equivalent diameter
- V_{pe} The volume of the particle having the equivalent diameter
- D_e The equivalent diameter
- S_{pt} The specific surface calculated through summing the interval surface area of particles
- N Number of logarithmic cycles
- n Number of classes
- b The width of each interval
- t_c The % finer on the gradation curve that yields a diameter which is the equivalent diameter



required

z The standard normal distribution variable

σ The standard deviation

μ The mean

π The constant ratio

C_c The coefficient of curvature