SURFACE TEMPERATURE EFFECT ON THE THERMOHYDRODYNAMIC PERFORMANCE OF JOURNAL BEARING IN HEAVY DUTY MACHINERY

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ABSTRACT

Increasing high demands for concept design requires journal bearing to work under several operating condition. The purpose of this work is to study the effect of surface temperature on the performance of journal bearing for heavy duty machines. Steady state thermohydrodynamic model (THD) for journal bearings has been developed. The generalized Reynold's equation, energy equation in the oil film, and the heat transfer equation in the bush and shaft are solved simultaneously. It was found that the shaft temperature has a great effect on the performance of the bearing.

الخلاصة :

أن الاعتماد المتزايد على مبدأ التصميم يحتاج إلى مساند مقعديه تعمل تحت ظروف عمل متعددة. أن الغاية من هذا البحث هو دراسة تأثير درجة حرارة السطح على أداء المسند المقعدي المستخدم في المكائن ذات الأحمال العالية. تم اعتماد النموذج الثرموهيدروديناميكي المستقر لأغراض هذا البحث. تم حل معادلة رينولدز ،التي تأخذ بنظر الاعتبار تغير لزوجة الزيت مع درجة الحرارة، معادلة الطاقة لطبقة الزيت ، ومعادلة انتقال الحرارة خلال جداري البوشة الخارجية ومحور نقل الحركة آنيا للحصول على توزيع الضغط وتوزيع درجات الحرارة خلال طبقة الزيت. لوحظ أن لدرجة حرارة المحور تأثير كبير على أداء المسند المقعدي.

KEY WORDS: Hydrodynamic lubrication, Thermal effect, Partial journal bearing

INTRODUCTION:

Journal bearings are designed for heavy-duty machinery to work under several operating conditions. In the past two decades, a great deal of research had been conducted to investigate the performance of journal bearings under a variety of operating conditions, Oscar Pinkus and Sargit S. (1979), Suganami and Szeri (1979), Cowking(1981), Seireg and Dandage (1982). Many works had been performed to investigate the thermal effect in a finite journal bearing theoretically and experimentally, Ferron et.al.(1983), Mitsui et.al. (1983), Lund and Hansen (1984), Boncompain et.al. (1986), Costa et.al. (2000). Khonsari and Esfahanian (1988) extended the thermohydrodynamic theory to include the effect of solid particles carried by the oil in the hydrodynamically lubricated journal bearings. An experimental investigation of thermal effects in circular and elliptical plain bearings has been executed by M.T. Ma and C. M. Taylor (1996). The results obtained through this work shows that thermal effects are significant in both bearings .The effects of various geometric factors and operating conditions on the thermal performance of journal bearing had been studied by Pierre and Fillon (2000), and Nassab and Moayeri(2002). Another works had been carried out investigating the methods of solving the energy equation, Hatakenaka and Tanaka (2002) and Jang and Khonsari (2004). M Fillon and J. Bonyer (2004) made a thermohydrodynamic analysis to a worn plain journal bearing. They showed that the worn bearing presents not only some disadvantages but also advantages , such as lower temperature. U. Singh et.al. (2008) analyze the steady state thermo-hydrodynamic of cylindrical fluid film journal bearing with an axial groove. From the parametric study it was found that the temperature of the fluid film raises due to frictional heat therby viscosity and load capacity decreases.

A thermohydrodynamic performance of grooved oil journal bearing has been made by L. Roy (2009). It had been shown during this work that feeding of oil from the bottom is very less preferable since the load capacity is lesser and the temperature development is more.

The present work represents an attempt to study the effect of journal surface temperature, as an external heat source, affecting the performance of journal bearing.

MATHEMATICAL MODEL

Hydrodynamic-lubrication sub model

The modified Reynold's equation that considers the variation of oil viscosity with temperature is employed to describe the relation ship between the hydrodynamic pressure and the lubricant film thickness Khonsari and Esfahanian (1988):

$$\frac{\partial}{\partial \bar{x}}\left(\bar{F}\,\bar{h}^3\frac{\partial \bar{P}}{\partial \bar{x}}\right)+\left(\frac{R}{L}\right)^2\frac{\partial}{\partial \bar{z}}\left(\bar{F}\,\bar{h}^3\frac{\partial \bar{P}}{\partial \bar{z}}\right)=\frac{\partial}{\partial x}\left(\bar{G}\,\bar{h}\right) \qquad (1)$$

Where:

$$\bar{F}=\frac{\int_0^1\left(\int_0^1\frac{\bar{y}}{\bar{\mu}}d\bar{y}\int_0^{\bar{\bar{y}}}\frac{1}{\bar{\mu}}d\bar{y}-\int_0^1\frac{\bar{y}}{\bar{\mu}}d\bar{z}\int_0^1\frac{1}{\bar{\mu}}d\bar{y}\right)d\bar{y}}{\int_0^1\frac{1}{\bar{\mu}}d\bar{y}} \qquad (2)$$

$$\bar{G}=\frac{\int_0^1\left(\int_0^{\bar{\bar{y}}}\frac{1}{\bar{\mu}}d\bar{y}\right)d\bar{y}}{\int_0^1\frac{1}{\bar{\mu}}d\bar{y}} \qquad (3)$$

The oil film thickness can be evaluated as:

$$\bar{h}=\frac{h}{c}=1+\varepsilon\cos\bar{x} \qquad (4)$$

The viscosity in equation (1) follows the empirical viscosity- temperature relationship suggested by Ferron et.al:

$$\bar{\mu}=k_o-k_1\,\bar{t}+k_2\,\bar{t}^2 \qquad (5)$$

The solution adopted in this work considers the viscosity variation across the film thickness of the lubricant.

Heat transfer sub model

The temperature in the lubricant can be determined from the following steady state energy equation:-

$$\lambda_1 \bar{u}\frac{\partial \bar{t}}{\partial \bar{x}} + \lambda_1\left(\frac{\bar{v}}{\bar{h}} - \bar{u}\frac{\bar{y}}{\bar{h}}\frac{\partial \bar{h}}{\partial \bar{x}}\right)\frac{\partial \bar{t}}{\partial \bar{y}} = \frac{1}{\bar{h}^2}\lambda_2\frac{\partial^2 \bar{t}}{\partial \bar{y}^2} + \frac{\bar{\mu}}{\bar{h}^2}\lambda_3\left[\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \left(\frac{\partial \bar{w}}{\partial \bar{y}}\right)^2\right] \tag{6}$$

Where:

$$\lambda_1 = \frac{\rho U C_o R}{k_{oil}} \tag{7}$$

$$\lambda_2 = (\frac{R}{c})^2 \tag{8}$$

And

$$\lambda_3 = (\frac{R}{c})^2 \frac{\mu_i U^2}{k_{oil} t_{in}} \tag{9}$$

The temperature distribution through the solid boundaries (journal and bearing) can be evaluated by solving the heat conduction equation. The steady state heat conduction equation can be written as:

$$\frac{\partial^2 \bar{t}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{t}}{\partial \bar{r}} + \frac{1}{\bar{r}^2}\frac{\partial^2 \bar{t}}{\partial \theta^2} = 0 \tag{10}$$

Bearing parameters:

The fluid film forces are calculated as follows:.

$$\bar{w}_{\bar{r}} = \int_0^1 \int_0^{2\pi} \bar{p} \cos \bar{x} d\bar{x} d\bar{z} \tag{11}$$

$$\bar{w}_{\bar{t}} = \int_0^1 \int_0^{2\pi} \bar{p} \sin \bar{x} d\bar{x} d\bar{z} \tag{12}$$

Where $\bar{w}_{\bar{r}}$ and $\bar{w}_{\bar{t}}$ are the components of dimensionless load in the direction of line of centers of the journal and the normal to it.

The attitude angle (ϕ) can be computed as:

$$\phi = \tan^{-1}(-\frac{\bar{w}_{\bar{t}}}{\bar{w}_{\bar{r}}}) \tag{13}$$

BOUNDARY CONDITIONS

In the case of the 120° partial arc journal bearing, as shown in fig.(1), the following boundary conditions are used to solve the Reynolds' equation:

1. At $\bar{x} = 2\pi/3 - \phi$ $\quad \bar{p} = 0.0$ (14)
2. At $\bar{x} = 4\pi/3 - \phi$ $\quad \bar{p} = 0.0$ (15)
3. At the cavitation zone $\quad \dfrac{\partial \bar{p}}{\partial \bar{x}} = 0.0 \quad \bar{p} = 0.0$ (16)

The temperatures distribution through the oil film can be determined by solving the energy equation subjected to the following boundary conditions :

The oil film temperature at the inlet is assumed to be constant:

At $\bar{x} = 2\pi/3 - \phi$ $\quad t_{mix} = t_{in}$ (17)

At the bearing oil film interface:

$$\left.\frac{\partial \bar{t}}{\partial \bar{r}}\right|_{\bar{r}=1} = -\frac{k_{oil}}{k_b}\frac{r_{bin}}{c}\frac{1}{\bar{\bar{h}}}\left.\frac{\partial \bar{t}}{\partial \bar{y}}\right|_{\bar{y}=0} \tag{18}$$

For the shaft oil film interface, assuming that the shaft temperature is independent on circumferential direction, then:

$$\left.\frac{\partial \bar{t}}{\partial \bar{r}}\right|_{\bar{\bar{r}}=1} = -\frac{1}{2\pi}\frac{k_o}{k_s}\frac{R}{c}\int_0^{2\pi}\frac{1}{\bar{\bar{h}}}\left.\frac{\partial \bar{t}}{\partial \bar{y}}\right|_{\bar{y}=1} d\bar{x} \tag{19}$$

The temperature distribution at outer surface of the bearing is:

$$\left.\frac{\partial \bar{t}}{\partial \bar{r}}\right|_{\bar{\bar{r}}=\bar{r}bout} = -\frac{h_{conv}}{k_b} r_{bin}\left(\bar{t}_{bo}-\bar{t}_a\right) \tag{20}$$

NUMERICAL SOLUTION

Governing equations of the problem have been discritized and solved simultaneously using iterative scheme with successive under relaxation. Reynold's equation is made discrete at the spaced grid points in the coordinates ($\bar{x}, \bar{y}, \bar{z}$) to make it suitable for the finite difference method in order to gate the pressure distribution ($\bar{P}$) through the oil film. The grid size of (360) in circumferential direction, (6) across the oil film thickness and (20) across the length of the bearing have been adopted.

The energy equation (6) and the heat conduction equation (10) with the equation of state (5) have been solved simultaneously with the Reynold's equation.

Temperature and pressure distribution in the oil film and solid parts at the mid-plan of the 120 arc partial journal bearing are obtained using the following solution procedure:

1. An initial value of the attitude angle (ϕ) is assumed.
2. Temperature of oil film, bearing bush and the shaft grid points are assumed.
3. The dimensionless value of the oil viscosity for all the points are computed by using equation (5)
4. .The oil film thickness $(\bar{h})$ calculated using equation (4).
5. Initial values of oil pressure for all the grid points are assumed to have zero values except at the inlet zone.

Then iterative scheme with successive under relaxation method is employed to solve the Reynold's equation with the boundary conditions (14),(15) and (16) to obtain the dimensionless pressures of the oil film .The negative values of pressure set to zero during the computation of the of the pressure field. These iterations are stopped when the following convergence criterion for the pressure is obtained.

$$\in \bar{p} = \frac{\sum\sum \left| \bar{p}^{\,n}_{i,k} - \bar{p}^{\,n-1}_{i,k} \right|}{\sum\sum \left| \bar{p}^{\,n}_{i,k} \right|} < 10^{-4} \tag{21}$$

6. Equations (11 and 12) have been solved and used to get a new value of the attitude angle using equation (13), which compared with the old one. The solution procedure is repeated using the new value of the attitude angle until the difference between the angles of the last two steps are reached less than one degree.

7. Dimensionless values of the fluid velocities ($\bar{u}, \bar{v}, \bar{w}$) are then calculated.

8. The energy equation (6) and the heat transfer equation at the solids (10) with the boundary conditions (17),(18), and (19) are solved simultaneously to get the temperature filed for the whole region.

9. The new oil-film temperature was used to compute a new viscosity field which is subsequently used in Reynolds' equation and simultaneous solutions for the equations are obtained iteratively until the convergence criterion of the temperatures for all points on the boundary between the oil film and the bush (inner bush face) for two successive iteration steps is less than (10^{-6}).

$$\in \bar{t} = \frac{\sum\sum \left| \bar{t}^{\,n}_{i,j} - \bar{t}^{\,n-1}_{i,j} \right|}{\sum\sum \left| \bar{t}^{\,n}_{i,j} \right|} < 10^{-6} \tag{22}$$

RESULTS AND DISCUSSION

A suitable computer program was prepared and written in (FORTRAN – 90) language , to solve the governing equations of the problem.

Numerical results of a steady state performance of a partial journal bearing with hollow shaft, in condition pertinent to that used in supporting the two necks of cement ball mills was studied. The main data related to this bearing are presented in table (1).

The effect of the temperature of the hollow shaft which receives the heat from the ball mill body and the hot fluid passing through on the performance of the journal bearing coupled with the other operating parameters were studied as follows:

The effect of the temperature of the hollow shaft (t_{si}) on the behavior of the bearing can be shown in figures (2,3). For the same applied load and journal rotational speed, when the shaft temperature(t_{si}) increases the maximum oil film pressure increases as a result of increasing the oil film temperature which causes an increase in eccentricity ratio and hence, reduced minimum oil film thickness (hmin) as shown in figure (2). More clear insight onto the effect of the inner temperature of the hollow shaft on the temperature distribution in the bearing bush can be gained by examining figure (3). It can be shown that as the shaft inner temperature was changed from $50^{\circ}C$ to $60^{\circ}C$ the maximum bearing bush temperature was increased by $7.1^{\circ}C$. This is due to the increase of the temperature difference across the bearing bush.

The effect of the inlet oil temperature on the temperature distribution in bearing bush metal can be shown in figures (4,5). It can be shown from fig.(4) that when the inlet oil temperature increased by $10^{\circ}C$ (from 35to $45^{\circ}C$) the maximum bearing bush temperature increased only by $4.9^{\circ}C$ due to reduction in viscous dissipation in oil film with the decrease in the oil viscosity. Fig.(5) shows the combined effect of the inlet oil temperature and the inner temperature of the

hollow shaft on temperature distribution in the bearing bush metal . It can be shown from the figure that the increase of the inlet oil temperature causes a remarked effect on the temperature distribution in the first half of the bearing bush metal. At the inlet zone, the temperature of the bearing bush was effected by the inlet oil temperature. At the second half of the bearing, it is clear that a small difference in the temperature distribution has been noticed. The effect of bearing surrounding temperature on the temperature distribution can be shown in figures (6,7). As shown in fig. (6) when the bearing surrounding temperature increases by $10^{\circ}C$ (from 35 to $45^{\circ}C$) the maximum bearing bush temperature increased by $2.6^{\circ}C$. This can be attributed to the effect of the surrounding temperature on the heat lost from the bush surface. When the inner surface temperature of the hollow shaft reaches $60^{\circ}C$ with the same surrounding temperature the maximum bearing bush temperature at the mid-plane increased by $2.4^{\circ}C$ as shown in fig. (7).
The effect of lubricant type on the behavior of the partial journal bearing at the same applied load and journal rotational speed can be shown in figures (8,9). Two types of lubricant were used in this case (No.1,and No2). Fig.(8) shows the influence of lubricant type on circumferential temperature distribution in bearing bush metal. Maximum bush metal temperature in this case was increased by $2^{\circ}C$. In other case as the effect of inner temperature of the hollow shaft is taken into consideration a small increase in maximum bush temperature is noticed and there is shifting in the temperature distribution curve. The computer program used in this work has been tested by comparing the results obtained for the full journal bearing with that published by Ferron etal.

(1983) and Nassab etal. (2002) as shown in figures (10-13). It is clear that there is a good agreement between the obtaind and the published results

CONCLUSIONS

- The following concluding remark can be drawn:
- Maximum oil film and bearing bush temperatures increases with the increase of the hollow shaft temperature.
- 2.The maximum oil film pressure increases and the minimum oil film thickness decreases with the increase of the hollow shaft temperature.
- 3.Due to the effect of the shaft temperature on the bearing, a small change in the maximum bearing bush temperature remarked with the change of supplied oil temperature, bearing surrounding temperature, and lubricant viscosity.

Table 1: Partial bearing characteristics used in test case.

Parameter	Symbol	Unit	Value
Journal radius	R	m	0.9
External bearing radius	R_{bout}	m	0.972
Bearing length	L	m	0.9
Radial clearance	c	m	0.0016
Lubrication viscosity at 40 ^{o}C (No.2)	μ	Pa . s	0.241
	k_o		3.6714
Viscosity coefficients	k_1		3.5826
	k_2		0.9112
Lubrication density at 40 ^{o}C	ρ	kg/m^3	860
Lubrication specific heat	C_o	J/kg. ^{o}C	2000
Lubrication thermal conductivity	k_{oil}	W/m. ^{o}C	0.13
Bush thermal conductivity	k_b	W/m. ^{o}C	50
Shaft thermal conductivity	k_s	W/m. ^{o}C	50
Convection heat transfer coefficient	h_{conv}	W/m^2. ^{o}C	80
Inlet oil temperature	t_{in}	^{o}C	40
Ambient temperature	ta	^{o}C	40
Inlet lubricant pressure	P_s	Pa	0.0
Bearing arc angle	--	deg	120
Lubrication viscosity at 40 ^{o}C (No.1)	μ	Pa . s	0.12
Fig.(1) Partial journal bearing with hollow shaft journal			

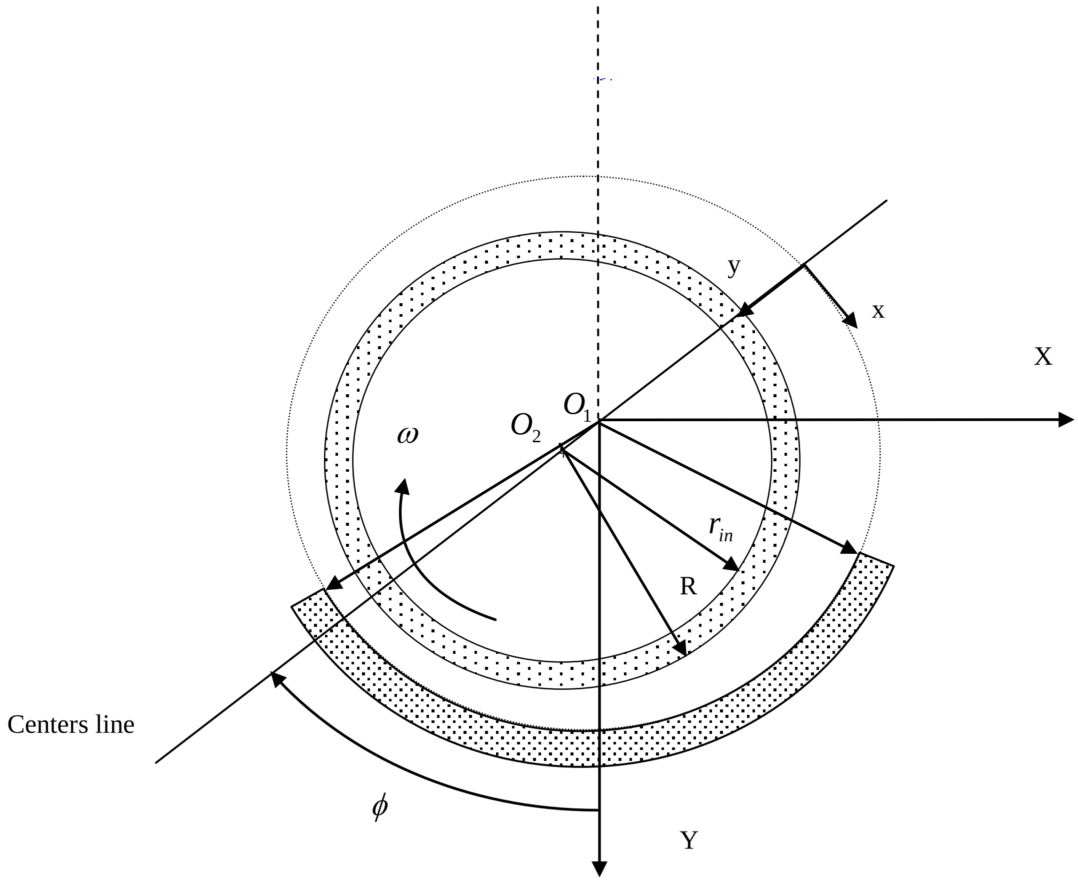

Fig.(1) Partial journal bearing with hollow shaft journal

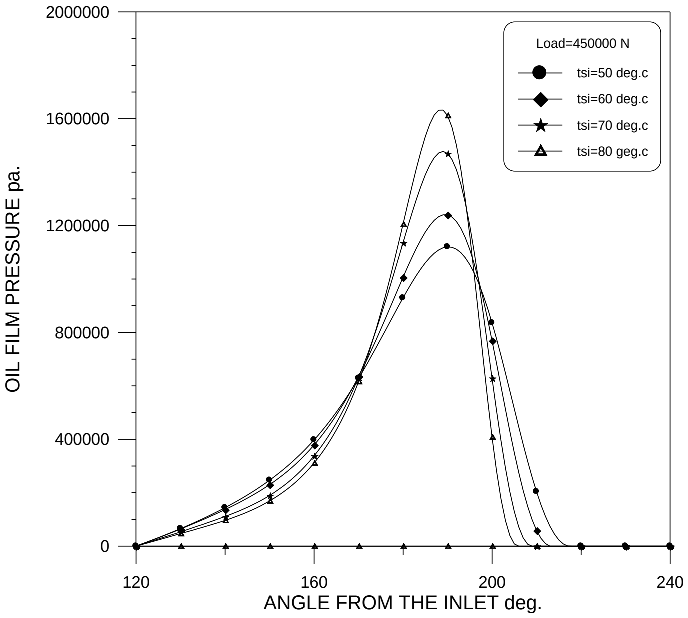

Fig.(2**)** Effect of inner temperature of the hallow shaft on the oil pressure distribution

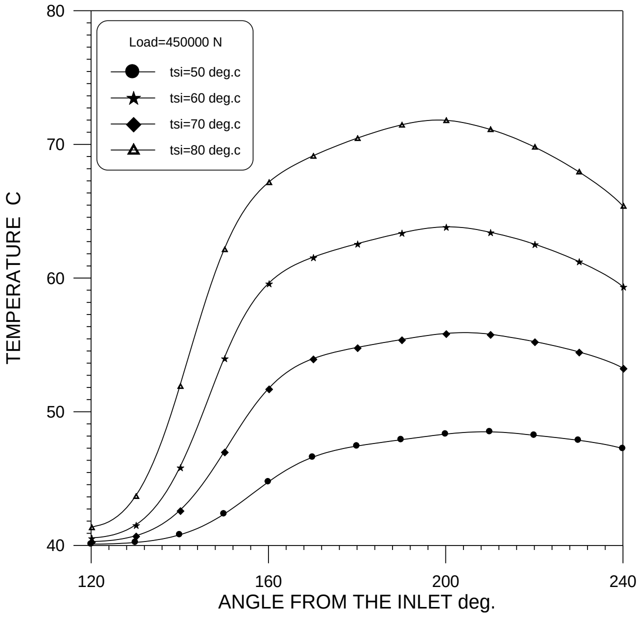

Fig.(3) Effect of inner temperature of the hallow shaft on the temperature distribution of bearing bush

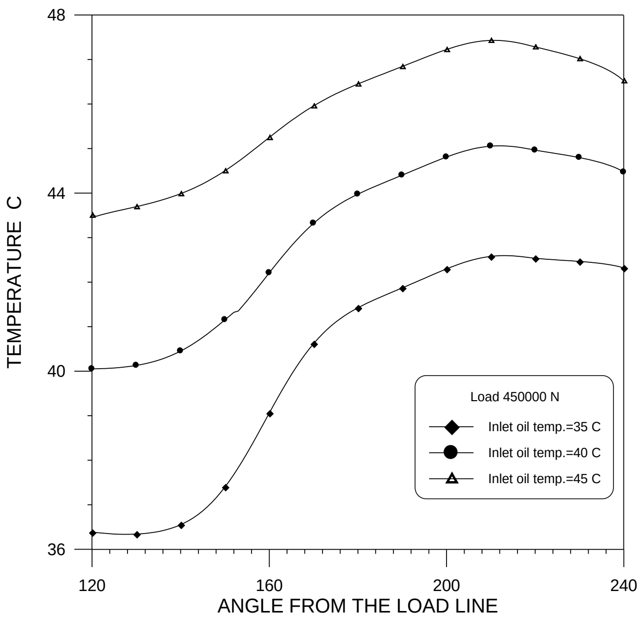

Fig.(4) Effect of inlet oil temperature on the temperature distribution of the bearing bush with out effect of inner temperature of the hallow shaft

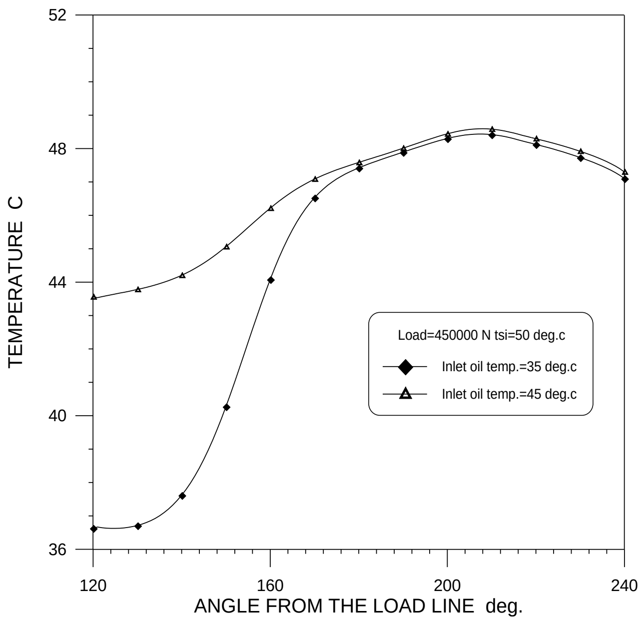

Fig.(5) Effect of inlet oil temperature on the temperature distribution of the bearing bush (tsi=50 ^{o}c)

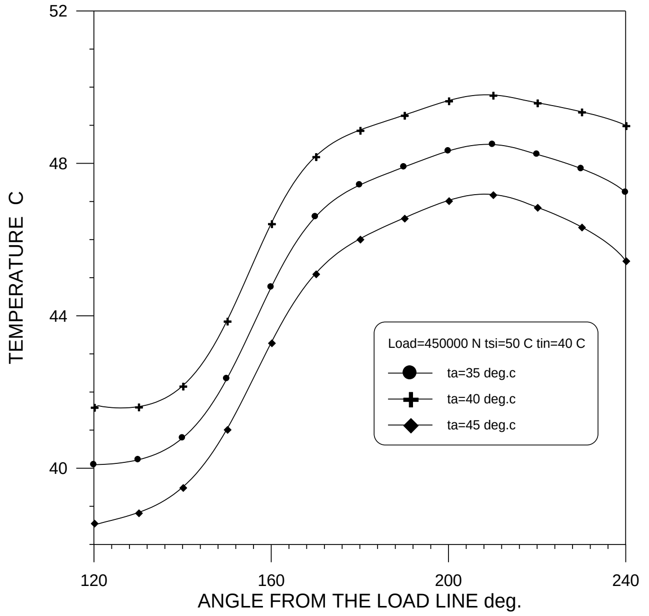

Fig.(6) Effect of ambient temperature on the temperature distribution of partial bearing bush (tsi=50 ^{o}c)

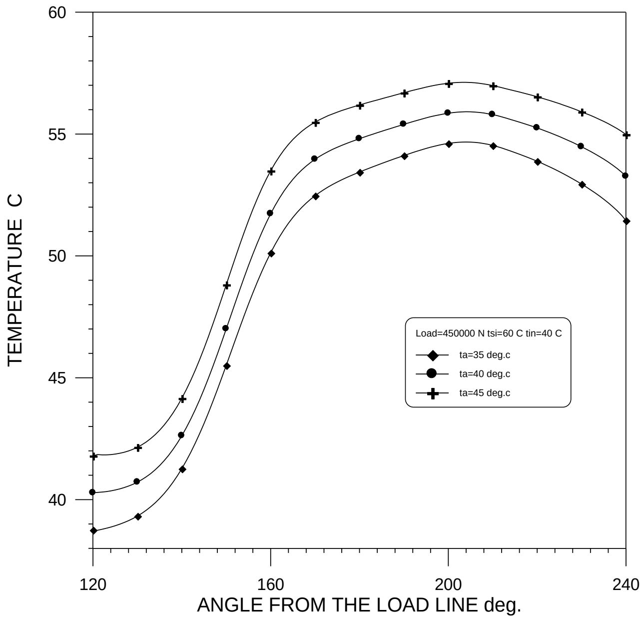

Fig.(7) Effect of ambient temperature on the temperature distribution of partial bearing bush (tsi=60 ^{o}c)

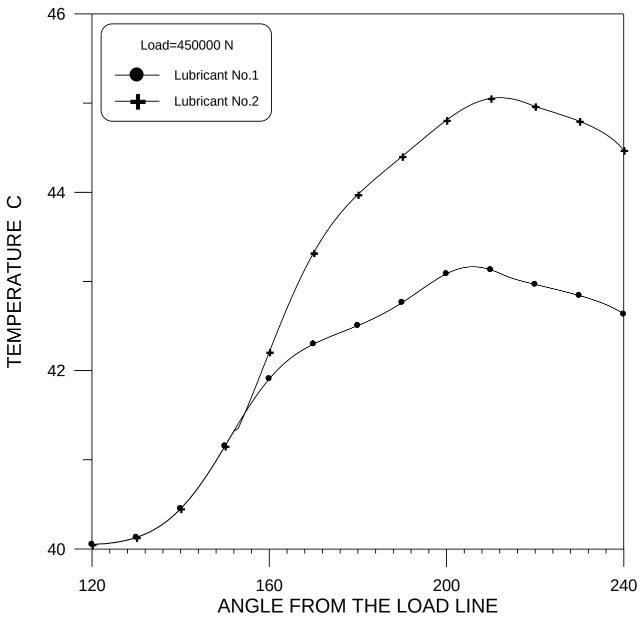

Fig.(8): Effect of lubricant viscosity on the temperature distribution of the bearing bush with out effect of inner temperature of the hallow shaft

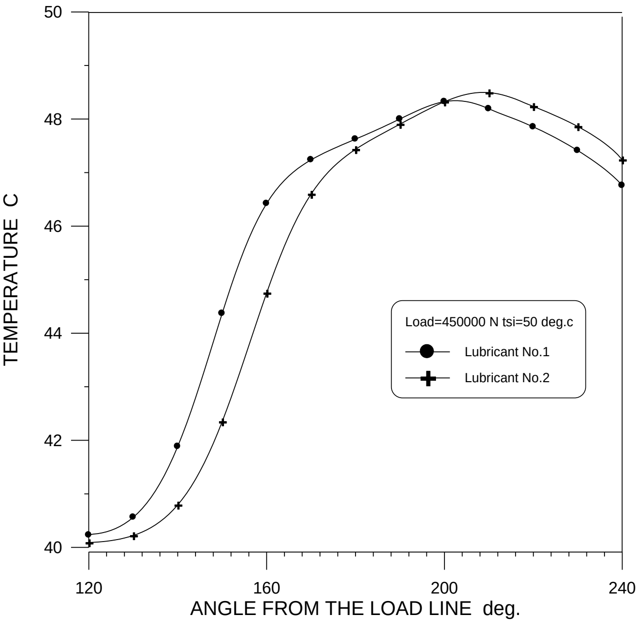

Fig.(9): Effect of lubricant viscosity on the temperature distribution of bearing bush (tsi=50 ^{o}c)

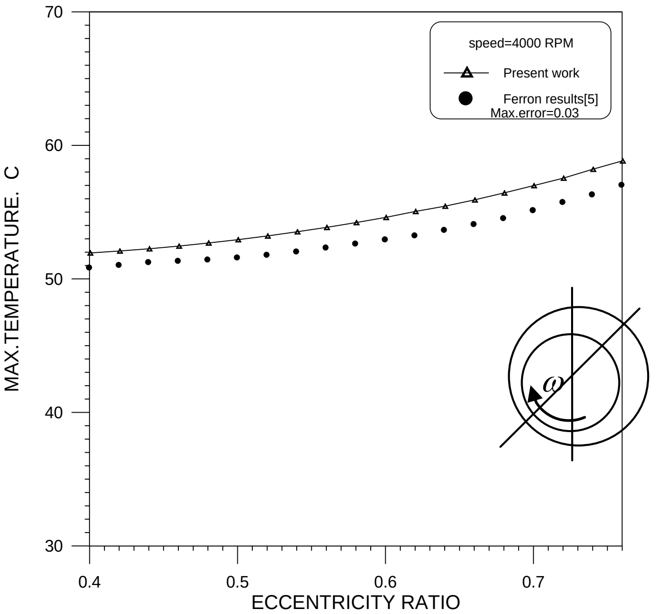

Fig..(10) The maximum oil film temperature versus eccentricity ratio

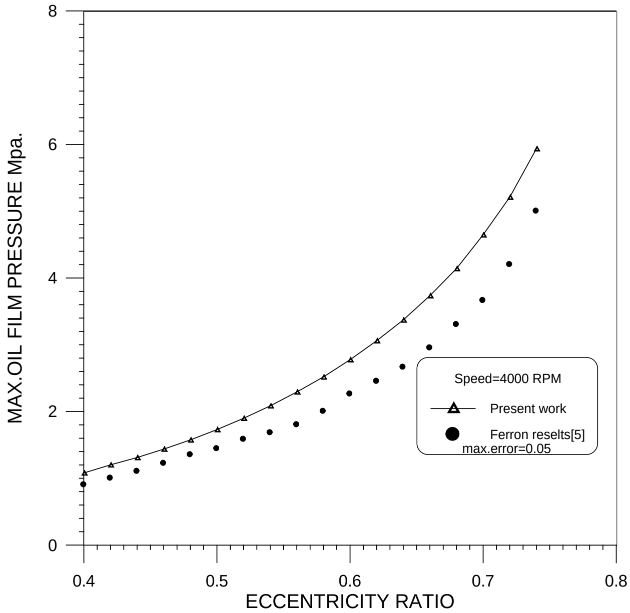

Fig.(11) Max. values of oil pressure versus eccentricity ratio

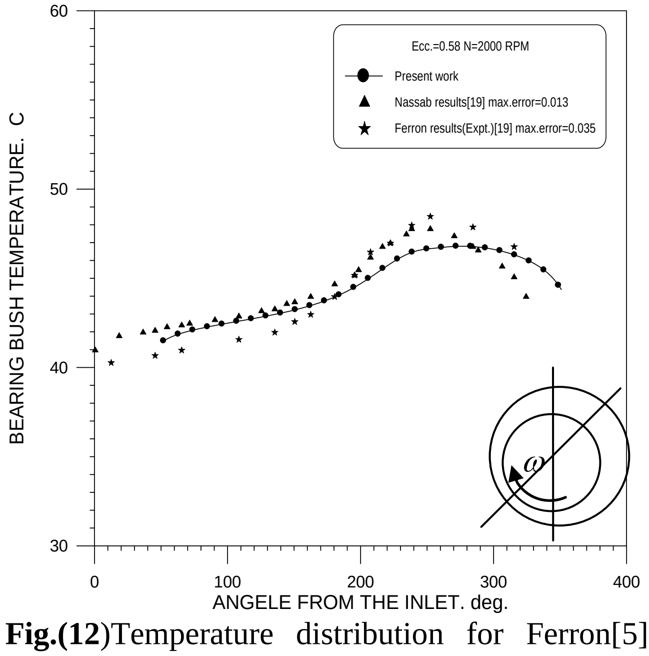

Fig.(12) Temperature distribution for Ferron[5] bearing bush

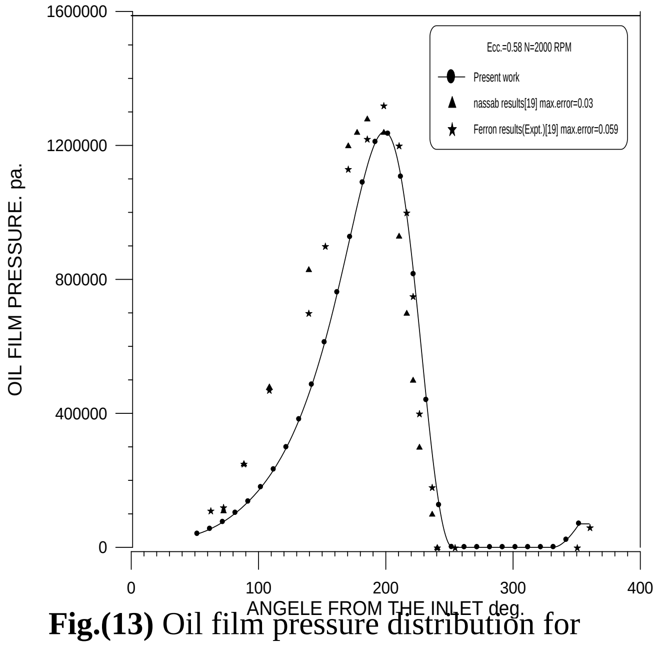

Fig.(13) Oil film pressure distribution for Ferron bearing

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NOMENCLATURE

SYMBOL	DESCRIPTION AND UNITS
c	Radial clearance m
C_o	Specific heat of lubricant J/kg. $^\circ C$
D	Diameter of journal m
e	Journal eccentricity m
h	Oil film thickness m
h_{conv}	Convective heat transfer coefficient W/m^2.^{o}C
h_{max}	Maximum oil film thickness m
h_{min}	Minimum oil film thickness m
$\bar{h}$	Dimensionless oil film thickness (h/c)
i,j,k	Finite difference mesh indices in circumferential, radial, and axial directions respectively
k_b	Thermal conductivity of the bush W/m^2.^{o}C
k_{oil}	Thermal conductivity of lubricant W/m^2.^{o}C
k_s	Thermal conductivity of the shaft W/m^2.^{o}C
k_o, k_1, k_2	Oil viscosity coefficient
N	Journal speed r.p.m
L	Bearing length m
P	Oil pressure N/m^2

p_{atm}	Atmospheric pressure N/m^2
p_s	Oil supply pressure N/m^2
$\bar{p}$	Dimensionless oil pressure $=(p/\eta_{in})(\frac{R}{U})(\frac{c}{R})^2$
$\bar{p}_s$	Dimensionless oil supply Pressure $=(p_s/\eta_{in})(\frac{R}{U})(\frac{c}{R})^2$
Q_{rec}	Recirculation oil flow rate m^3/s
Q_l	Axial leakage oil flow rate m^3/s
Q_{in}	Supply oil flow rate m^3/s
R	Journal radius m
r_b	Bush radius m
r_{bin}	Bush inner radius m
r_{bout}	Bush outer radius m
t	Oil temperature ^{o}C
ta	Ambient temperature ^{o}C
t_b	Bush temperature ^{o}C
t_{bo}	Bush outer surface temperature ^{o}C
t_{in}	Inlet oil temperature ^{o}C
t_r	Recirculating oil temperature ^{o}C
t_s	Shaft temperature ^{o}C
t_{si}	Inner temperature of the hallow shaft ^{o}C
$\bar{t}$	Dimensionless temperature = t / t_{in}
U	Shaft speed m/s
u	Fluid velocity component in x direction m/s
$\bar{u}$	Dimensionless velocity = u/U
v	Fluid velocity component in y direction m/s
$\bar{v}$	Dimensionless velocity = v/U(R/c)
w	Fluid velocity component in z direction m/s
$\bar{w}$	Dimensionless velocity = w/U
W	Bearing load capacity N
x,y,z	Cartesian coordinate system
r,θ	Cylindrical coordinate
$\bar{x},\bar{y},\bar{z}$	Dimensionless coordinate system

Greek symbols

Symbol	Description and Units	
ε	Eccentricity ratio	-
$\in \bar{p}, \in \bar{t}$	Errors ratios	-
μ	Lubrication viscosity	pa . s
μ_{in}	Inlet lubrication viscosity	pa . s
$\bar{\mu}$	Dimensionless viscosity = $\frac{\mu}{\mu_{in}}$	
ρ	Density of oil	kg/m^3
τ	Shear stress N/m^2	
ϕ	Bearing attitude angle rad	rad
ω	Journal rotational speed rad/s	rad/s

Superscript

$^-$ Dimensionless quantity