DYNAMIC RESPONSE OF RUTTBA SAND USING EQUIVALENT LINEAR MODEL

Asma Y. al-Tae’e
Assistant Lecturer, Civil Eng. Dept.
College of Eng./ University of Baghdad

ABSTRACT
In this paper, the dynamic response of Ruttba sand with different relative densities is studied by using equivalent linear model. The equivalent linear model is adopted in which the soil properties are strain dependent but cycle independent. The shear modulus tends to decrease as the shear strain increases, while the damping ratio tends to increase as the shear strain increases. The computer program SHAKE, developed at the University of California, Berkley, is used for predicting and analyzing the problem. The study reached that the magnitude of the acceleration is increasing as the relative density is decreasing and the natural frequency of the structures that would be built on Al- Ruttba soil must not be equal to (4 c/sec.) to avoid resonance condition.

الخلاصة
في هذه المقالة، تم دراسة الاستجابة الديناميكية لتربة الرطبة الرملية بكتافات نسبية مختلفة باستخدام النموذج الخطي المكافئ. في النموذج الخطي المكافئ يعتمد كل من معدل القصف ونسبة الإخماد على مقدار الإنتقال الحاصل ولكنهما لا يعتمدان على طبيعة الحمل المسلط حيث إن معدل القصف يقل بزيادة انفعال القفص، بينما تزداد نسبة الإخماد بزيادة انفعال القفص. لقد استخدم برنامج SHAKE لتخمين وتحليل استجابة النموذج الخطي المكافئ.

في هذه الدراسة تم التوصل إلى إن مقدار التعجيل بزداد بلقصان الكثافة النسبية وان النشاطات التي تنبى على هذه الترب (تربة الرطبة) يجب ان لا يكون ترددها الطبيعي مساوٍ إلى (4 c/sec.) لتجنب حالة الرنين.

KEY WORDS
Earthquake, shake, spectra, acceleration and Ruttba sand.

INTRODUCTION
In recent years engineers and geologists have become increasingly aware of the need for evaluating the effect of soil condition on the ground surface accelerations, and thus the intensity of shaking, to which structures are subjected during earthquakes. For design purposes, the expected earthquake motions at a site are usually characterized by response spectra curves, such as those proposed by Housner (1970), Newmark et al (1973), Seed et al (1976). These spectra are presumably equivalent to an ensemble of earthquake time histories that can occur on a site, and therefore represent a more
generalized form of seismic design input than any single earthquake accelerogram (Singh and Ashtiany, 1980).

EQUIVALENT LINEAR MODEL.

Theory
The model considers the responses associated with vertical propagation of shear waves through the linear viscoelastic system shown in Fig. (1) [Schnable, Lysmer and Seed, 1972]. The system consists of N horizontal layers, which extend to infinity in the horizontal direction and have a half-space at the bottom layer. Each layer is homogenous and isotropic and is characterized by the thickness, h, mass density, \( \rho \), shear modulus, \( G \), and damping factor, \( \xi \).

Fig. (1) One-Dimensional System (Schnable, Lysmer, and Seed, 1972).

Program
The program SHAKE (developed at the University of California, Berkeley) computes the response in a horizontally layered soil rock system subjected to transient, vertical travelling shear waves. The method is based on Kani’s solution to the wave equation and the Fast Fourier Transform algorithm. The motion used as basis for the analysis can be applied to any layer in the system. Systems with elastic base and with variable damping in each layer can be analyzed. Equivalent linear soil properties are used with an iterative procedure to obtain soil properties compatible with the strains developed in each layer. A varied set of operations of interest in earthquake response analysis can be performed (Schnable, Lysmer, and Seed, 1972).
Propagation of Harmonic Shear Waves in a One-Dimensional System

Vertical propagation of shear waves through the system shown in Fig. (1) will cause only horizontal displacements:

\[ u = u(x, t) \]

where:

\[ t = \text{time} \]

which must satisfy the wave equation (Schnable, Lysmer, and Seed, 1972):

\[ \rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial x^2} + \eta \frac{\partial^3 u}{\partial x^2 \partial t} \]  \hspace{1cm} (2)

where:

\[ \eta = \text{viscosity}. \]

Harmonic displacements with frequency \( \omega \) can be written in the form:

\[ u(x, t) = U(x)e^{i\omega t} \]  \hspace{1cm} (3)

Substituting equation (3) into equation (2) results in an ordinary differential equation:

\[ (G + i\omega \eta) \frac{d^2 U}{dx^2} = \rho \omega^2 U \]  \hspace{1cm} (4)

which has the general solution:

\[ U(x) = Ee^{ikx} + Fe^{-ikx} \]  \hspace{1cm} (5)

in which:

\[ k^2 = \frac{\rho \omega^2}{G + i\omega \eta} = \frac{\rho \omega^2}{G^*} \]  \hspace{1cm} (6)

where:

\[ k = \text{complex wave number, and} \]

\[ G^* = \text{complex shear modulus}. \]

The critical damping ratio, \( \xi \), is related to the viscosity \( \eta \) by:

\[ \omega \eta = 2G\xi \]  \hspace{1cm} (7)

Experiments on many soil materials indicate that \( G \) and \( \xi \) are nearly constant over the frequency range, which is of main interest in the analysis. It is therefore convenient to express the complex shear modulus in terms of the critical damping ratio instead of the viscosity:

\[ G^* = G + i\omega \eta = G(1 + 2i \xi) \]  \hspace{1cm} (8)

where \( G^* \) can be assumed to be independent of frequency.

Equations (3) and (5) give the solution to the wave equation for a harmonic motion of frequency \( \omega \):

\[ u(x, t) = Ee^{ik(x+\xi t)} + Fe^{-i(kx - \xi t)} \]  \hspace{1cm} (9)

where the first term represents the incident wave travelling in the negative \( x \)-direction (upwards) and the second term represents the reflected wave travelling in the positive \( x \)-direction (downwards).

Equation (9) is valid for each of the layers in the Fig. (1). Introducing a local coordinate system \( x \) for each layer, the displacements at the top and bottom of layer \( m \) are:

\[ u_m(x = 0) = (E_m + F_m)e^{iat} \]  \hspace{1cm} (10)

\[ u_m(x = h_m) = (E_m e^{ik_m h_m} + F_m e^{-ik_m h_m})e^{iat} \]  \hspace{1cm} (11)

The shear stress on a horizontal plane is:
\[ \tau(x, t) = G \frac{\partial u}{\partial x} + \eta \frac{\partial^2 u}{\partial x \partial y} = G \frac{\partial u}{\partial x} \]  
(12)

or by equation (9):

\[ \tau(x, t) = i \kappa G' (E e^{ikx} - F e^{-ikx})e^{iwt} \]  
(13)

\[ \gamma = \frac{\partial u}{\partial x} = i \kappa_y (E e^{i(kx+\omega t)} - F e^{-i(kx-\omega t)}) \]  
(14)

where:

\( \gamma \) = the shear strain.

**DESCRIPTION OF THE SOIL**

The soil in this study is a uniform silica sand from Al- Rutba region, in west of Iraq. The sand was washed and sieved on sieve No. 40 (0.42 mm), and retained on sieve No. 50 (0.297 mm). Thus uniform sand is obtained so that no segregation takes place during sample preparation. The properties of the soil can be seen in Tables (1) and (2) (Baho, 1989).

Hardin (1965) produced equations (15) and (16) based on the results reported by Hardin and Richart (1963) for the dynamic shear modulus, \( G \), of dry sands measured by steady-state vibration. These were:

For rounded grains:

\[
G = \frac{(32.17 - 14.8 \varepsilon)^2}{(1 + \varepsilon)} \sigma_0^{0.5} \quad \text{for } \sigma_0 \geq 9760 \text{ kg/m}^2
\]  
(15)

\[
G = \frac{(22.52 - 10.6 \varepsilon)^2}{(1 + \varepsilon)} \sigma_0^{0.6} \quad \text{for } \sigma_0 < 9760 \text{ kg/m}^2
\]  
(16)

where:

\( G \) = Dynamic shear modulus,

\( \sigma_0 \) = Confining pressure, and

\( \varepsilon \) = Void ratio.

Table (1) The index properties of Al- Rutba soil (after Baho, 1989).

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_s )</td>
<td>2.65</td>
</tr>
<tr>
<td>( C_u )</td>
<td>1.23</td>
</tr>
<tr>
<td>( C_c )</td>
<td>0.93</td>
</tr>
<tr>
<td>Effective size (D_{10})</td>
<td>0.38 mm</td>
</tr>
<tr>
<td>( \gamma e_{\text{min}} )</td>
<td>1.376 g/m³</td>
</tr>
<tr>
<td>( \gamma e_{\text{max}} )</td>
<td>1.675 g/m³</td>
</tr>
<tr>
<td>( e_{\text{min}} )</td>
<td>0.58</td>
</tr>
<tr>
<td>( e_{\text{max}} )</td>
<td>0.93</td>
</tr>
<tr>
<td>Damping factor (( \xi ))</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Table (2) The material properties of Al- Rutba sand (after Bahia, 1989).

<table>
<thead>
<tr>
<th>Dr %</th>
<th>E</th>
<th>( \gamma ) (kN/m(^3))</th>
<th>( G ) (kN/m(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \sigma_0 = 30 )</td>
<td>( \sigma_0 = 60 )</td>
</tr>
<tr>
<td>25</td>
<td>0.84</td>
<td>14.4</td>
<td>771.3</td>
</tr>
<tr>
<td>50</td>
<td>0.76</td>
<td>15.1</td>
<td>915</td>
</tr>
<tr>
<td>75</td>
<td>0.67</td>
<td>15.9</td>
<td>1065.5</td>
</tr>
</tbody>
</table>

\( \sigma_0 \): Confining pressure (kN/m\(^2\)),  
\( G \): Shear modulus calculated from equations (15) and (16).

**SELECTION OF SOIL SYSTEM AND INPUT MOTION:**

The soil system shown in Fig. (2) is selected for analysis. The factors for sand are estimated from relative densities.

Al- Hindiya earthquake occurred in February 20, 1990 is used in the analysis. It had a magnitude of 3.7 according to Richter magnitude scale with maximum acceleration equal to 0.018g. The location of its epicenter was estimated at about (85 km) south- west of Baghdad city. The velocity-time history record of this actual earthquake was recorded by Iraqi Seismic Monitoring Unit. The components of the 20 February, 1990 modified accelerogram of Al- Hindiya earthquake Fig. (3) is considered, in the longitudinal and lateral directions, sparately (Rasheed, 1998).

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**Fig. (2) Soil system**

- Dr = 25%
- \( \gamma_d = 14.4 \text{ kN/m}^3 \)
- Damping factor \( (\xi) = 0.05 \)

- Dr = 50%
- \( \gamma_d = 15.1 \text{ kN/m}^3 \)
- Damping factor \( (\xi) = 0.05 \)

- Dr = 75%
- \( \gamma_d = 15.9 \text{ kN/m}^3 \)
- Damping factor \( (\xi) = 0.05 \)
Fig. (3) Design acceleration response spectrum of Al- Hindiya earthquake.

From Fig. (3), it can be seen that the predominant period of Al- Hindiya earthquake is (0.6 sec.). Among the available strong records, the Pasadena record from the 1952 Kern Country earthquake seems to have characteristics most similar to those desired. The maximum acceleration was 0.057g and the predominant period was (0.63 sec.). Modification of this record to give a maximum acceleration 0.018g gives the desired characteristics for the motion. For safety, modification of the Pasadena record to give a maximum acceleration 0.1g.

RESULTS AND DISCUSSION:
Based on these material characteristics Fig. (2), the response of the deposit to the applied base motion and the time history of the ground surface acceleration are computed. For comparing the time history of the ground surface acceleration and the base acceleration, the acceleration response spectrum is calculated (see Figs. (4, 5) and (6)).

From these figures, it can be seen that the magnitude of the acceleration of the ground surface is more than the magnitude of the acceleration of the base, i.e., the acceleration of the base is duplicated because of the soil conditions.

For the sand with relative density equal to (25%) see Fig. (4b), the mean square frequency of the layer (1) is equal to (4 c/sec.) or the predominant period is equal to (0.25 sec.) and the acceleration at this period reached to 0.223g while the acceleration of the base at the same period is equal to (0.05g).

For the soil with relative density equal to (50%), the magnitude of the acceleration of the layer (1) at the same period reached to 0.2199 see Fig. (5b) and for the soil with relative density equal to (75%), the magnitude of the acceleration of the layer (1) at the same period reached to 0.208g (see Fig. (6b)).
CONCLUSIONS

From the previous results, data have been presented to show that the characteristics of ground surface motions during earthquakes, and the corresponding forms of the response spectra, vary with soil conditions. Because the change in the form of the response spectra can have significant effects on the lateral forces on structures. It would seem desirable that the nature of the soil condition underlying a site should be taken into account in evaluating the lateral forces for design purposes.

For Al-Rutba soil, the following conclusions are drawn:

1. The magnitude of the acceleration is increasing as the relative density is decreasing.
2. The natural frequency of the structures that would be built on Al-Rutba soil must not be equal to (4 e/sec.) or the predominant period of the structures must not be equal to (0.25 sec.) to avoid resonance condition.

REFERENCES


Schnable, P. B., Lysmer, J. and Seed, H. B., (1972), SHAKE, A computer Program for Earthquake Response Analysis of Horizontally Layered Site, Earthquake Engineering Research Center, Report No. EERC 72-12, University of California, Berkeley,.


Fig. 4b. The response spectra of the layer 1 (relative density = 25%).
Ug : Acceleration at the base,
Sa : Acceleration at the top of the layer 1.

Fig. 5b The response spectra of the layer (1) (relative density = 50%)
Fig. 6(a)

<table>
<thead>
<tr>
<th></th>
<th>sand</th>
<th>sand</th>
<th>pluck</th>
<th>sand</th>
<th>rock</th>
</tr>
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<tbody>
<tr>
<td>(1)</td>
<td>88</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2)</td>
<td></td>
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<td>(3)</td>
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<td>(4)</td>
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<td>(5)</td>
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<td></td>
<td>12.6</td>
</tr>
</tbody>
</table>

Shear Strain (%) vs. Depth (m)

Shear Strength (kN/m)

564
Ug: Acceleration at the base.
Sa: Acceleration at the top of the layer 1.

Fig. 6b The response spectra of the layer (1) (relative density = 75%)