FATIGUE EQUATION PARAMETERS RELATIONSHIPS

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ABSTRACT

Substantial amounts of data concerning the fatigue performance tests are presented. These data contain the information about results that are expressed by means of the fatigue equation parameters. Different fatigue tests are included using different types of mixes. The collected information was not similar in all aspects. Some studies included variables, which were not considered by others. The different representations of the fatigue equation makes it necessary to find the relations between the parameters of these different equations in order to study all the data together. The collected data were examined statistically to check its consistency; that is to say, they belong to the same population. The Kolmogorov-Smirnov test for goodness of fit was adopted for this purpose.

الخلاصة

تم تجميع كمية كبيرة من البيانات الخاصة بفحوص اداء الكلل. هذه البيانات تتكون من نتائج الفحوص المختلفة وتمثل عادة بمعاملات معادلة الكلل. تخص البيانات مجموعة مختلفة من فحوص الكلل والتي تتم باستخدام خلطات اسفلتية مختلفة كما تركز هذه الفحوص على تأثير عوامل مختلفة وحسب الغاية من كل فحص. ونتيجة لذلك اصبح من الواجب ايجاد العلاقات اللازمة لتوحيد المعاملات لمعادلات الكلل المختلفة وذلك لدراسة العلاقة التي تحكم هذه البيانات. تم فحص البيانات احصائيا لثبوت انتماء كافة المعاملات الى نفس المجتمع وذلك باستخدام طريقة كولموجوروف- سميرنوف لتحليل البيانات.

KEY WORDS
Fatigue, flexible pavement, pavement performance

INTRODUCTION

According to the previous studies, there exist a variety of fatigue equations, which were used for the prediction of fatigue performance of flexible pavements. For the controlled strain mode of loading test, the general fatigue equation is (Pell S. F. 1973):

$$N_f = K\ (\Box)^{-A} \qquad (1)$$

Where:

N_f: the number of repetitions of loading till failure at a particular level of initial strain.
$\Box$: the strain that is repeatedly applied on the specimen.
K, A: material coefficients.

For the controlled stress mode of loading test, the equation given below is used:

$N_f = k \ (\square)^{-a}$ (2)

Where:
N_f: the number of repetitions of loading till failure.
$\square$: the stress that is repeatedly applied on the specimen
k, a : the material coefficients.
The importance of the tensile strain in the fatigue cracking makes it necessary to express both the strain and stress controlled tests in the log strain against log number of repetitions to failure [Equation (1)]. Accordingly, the first equation was adopted by most fatigue studies for both controlled strain and stress modes of loading. Noting that the strain ($\square$value for the case of the controlled stress is the initial strain due to the repeatedly applied stress on the specimen.
It is worthwhile mentioning that the fatigue equations were also found in literature in an inverse presentation (strain as a function of the number of load application) as follows:

$\square = K1 \ (N_f)^{-A1}$ (3)

Furthermore, they are found similar to Equation (1) but the strain value is measured in microns as follows:

$N_f = K2 \ (\square\square)^{-A2}$ (4)

Fatigue equation parameters

The constants for all the mentioned above equations depend on the material properties. Since the first equation is widely adopted in fatigue studies, it will be adopted for the study. Thus, the constants for the Equations (3) and (4) are related to those of Equation (1) in order to use all the available fatigue performance data in one form. The relations between constants are obtained as follows:

From Equation (1):
$\mathrm{Log}\,(N_f) = \mathrm{Log}\,(K) - A\,\mathrm{Log}\,(\square)$ (5)
From Equation (3):
$\mathrm{Log}\,(\square) = \mathrm{Log}\,(K1) - A1\,\mathrm{Log}\,(N_f)$ (6)
$\mathrm{Log}\,(N_f) = (1/A1)\,\mathrm{Log}\,(K1) - (1/A1)\,\mathrm{Log}\,(\square)$ (7)
Comparing Equation (5) with (7) indicates that:
$A = 1/A1$ (8)
In a similar way, the value of (K) can be related to (K1) as shown below:
$\mathrm{Log}\,(K) = (1/A1)\,\mathrm{Log}\,(K1)$ (9)
$\mathrm{Log}\,(K) = \mathrm{Log}\,(K1)^{1/A1}$ (10)
$K = K1^{1/A1}$ or (11)
$K = K1^{A}$ (12)

The variable reduction procedure described as above is used to find the relations between (A) and (A2) and (K) and (K2):
From Equation (4):
$\mathrm{Log}\,(N_f) = \mathrm{Log}\,(K2) - A2\,\mathrm{Log}\,(\square\square)$ (13)
$\mathrm{Log}\,(N_f) = \mathrm{Log}\,(K2) - A2\ (\mathrm{Log}(10^6) + \mathrm{Log}\,(\square))$ (14)
$\mathrm{Log}\,(N_f) = \mathrm{Log}\,(K2) - \mathrm{Log}(10^{6A2}) - A_2\ \mathrm{Log}\,(\square)$ (15)
$\mathrm{Log}\,(N_f) = \mathrm{Log}\,(K2/10^{6A2}) - A2\ \mathrm{Log}\,(\square)$ (16)

$\text{Log}(N_f) = \text{Log}(K2 * 10^{-6A2}) - A2\ \ \text{Log}(\square)$ (17)

Comparing Equation (5) with (17):

$A = A_2$ and (18)

$\text{Log}(K) = \text{Log}(K_2\ \ 10^{-6A}{}_2)$ (19)

$K = K2\ \ * 10^{-6A2}$ or (20)

$K = K_2\ \ * 10^{-6A}$ (21)

Depending on the presented as above conversions, all the collected data are presented in a unified mathematical form as given by Equation (1) in order to use the parameters of the equation as the dependent variables in any regression analysis.

Collection of data

The available studies for the fatigue performance of flexible pavements were taken under consideration. All the fatigue line equation parameters and test circumferences are considered input for fatigue analysis. The collected data consisted of many types of asphalt mixes including regular asphalt concrete (as the major part of the date), hot rolled asphalt, dense graded macadam and many others. Many types of fatigue tests, depending on the type of loading, are included within the data like normal bending, axial bending and wheel loading. The adopted data for the fatigue performance analysis, presented in **Table (1),** include the fatigue equation parameters from the different types mentioned earlier. These parameters of the adopted fatigue equation are calculated when other forms than the regular fatigue equation ($N=K*(\ \ ^{-A}$) is found depending on the conversion equations that was obtained previously.

The Kolmogorov-Smirnov test for data

In order to examine the similarity between the collected data, the Kolmogorov-Smirnov (K-S) test (Lindgren 1968, Gibbons 1971, Willemsen 1974 and Harnett 1975) is adopted which involved a comparison between cumulative relative frequency distributions of two samples. To make this comparison, the data is put into classes, which have been arrayed from the lowest class to the highest class. If the cumulative relative frequency for each category of sample (1) data distribution is (F_i) and (f_i) to denote the comparable value for the sample (2) frequency, then the (K-S) test is based on the maximum value of the absolute deference between (F_i) and (f_i). Accordingly:

$$D = \text{Max}_i\,(|F_i - f_i|) \tag{22}$$

Where:

D : the statistic of the Kolmogorov-Smirnov test.

F_i : the cumulative relative frequency of the ith class in sample (1)

f_i : the cumulative relative frequency of the ith class in sample (2)

The decision to reject the null hypotheses H_o (that the samples come from the same population) is based on the value of (D). The larger (D) is, the more confidence we have that H_o is false. The general procedure for the (K-S) test is to calculate the (D) values for each class and these values are compared with the appropriate critical values of (D). The critical values of (D) are obtained from the available tables in many statistical resources (Gibbons 1971 and Harnett 1975). If the maximum value of (D) is less than the critical value, then H_o is not rejected. In other words, the agreement between the data of the two samples is sufficiently close enough to believe that they come from the same distribution. It is worthwhile mentioning that the (K-S) test was adopted instead of the Chi-Square test because of many reasons.

The class width and number of classes have an effect on chi-square statistics (Gibbons 1971). Furthermore, (K-S) test can be applied for any sample size, while the chi-square statistic should be used only for large sample size and each expected cell frequency not too small. Figures 1 and 2 presents graphical presentation for the cumulative relative frequencies of the examined data and **Table (2)** shows the Kolmogorov-Smirnov test results. These results indicate that, the collected data are sufficiently close enough to believe that they come from the same distribution.

CONCLUSIONS

It is essential to convert the fatigue equations in the collected data to one type before the analysis process since many forms of fatigue equations are used for the presentation of the fatigue performance of flexible pavements. Although the required data for analysis are collected from different fatigue tests using different types of mixes, the results of the statistical test shows that the collected data are sufficiently close enough to believe that they come from the same distribution.

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Figure 1, Comparison of observed cumulative relative frequencies of examined data.

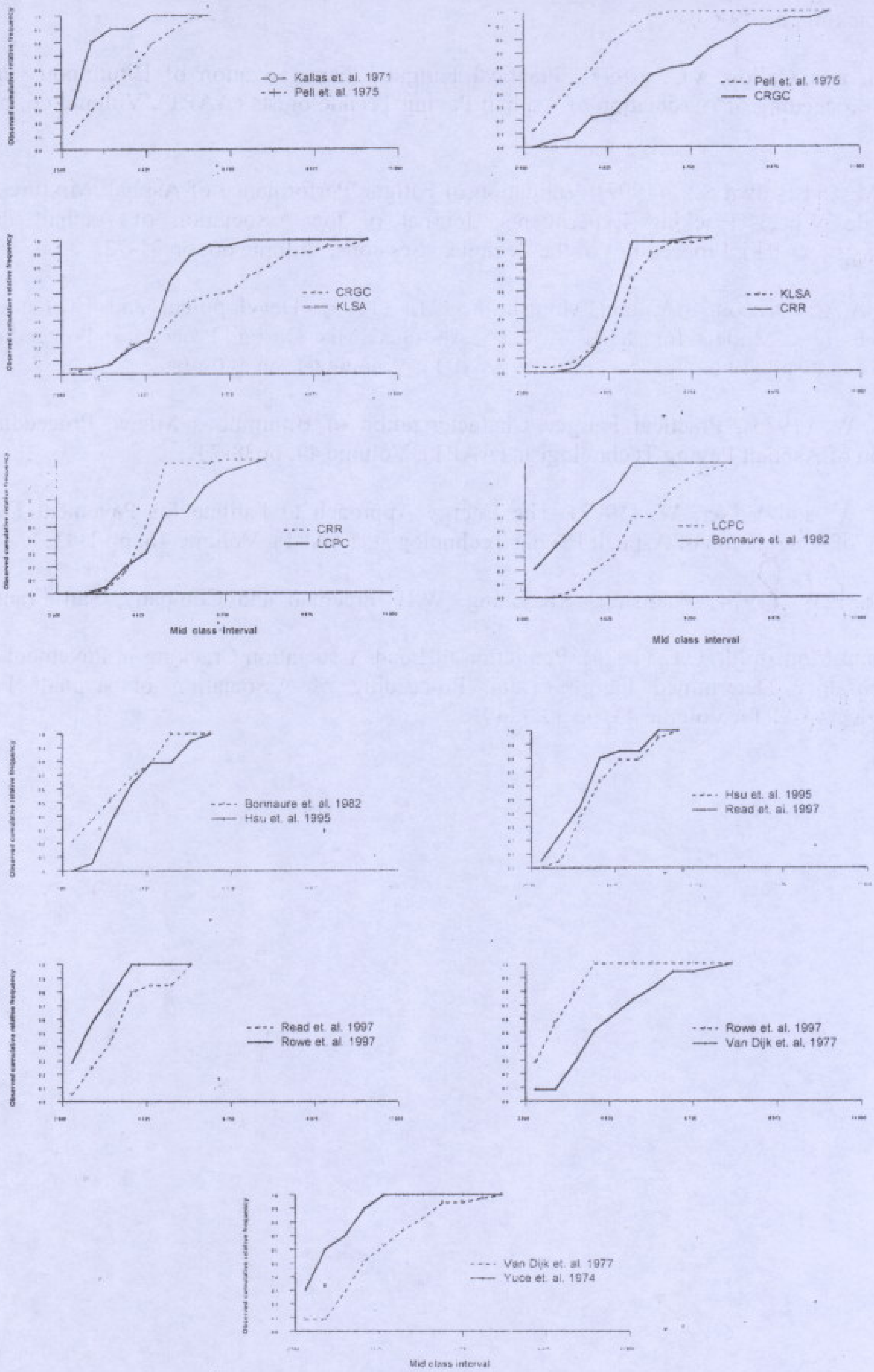

Table (1) The collected data from previous fatigue performance studies.

Ref.	Mode	K1	A1	K	A
Epps et al. 1971	C. Stress	1.30E-03	0.19	1.19E-15	5.17
	===	1.70E-03	0.21	3.19E-14	4.87
	===	3.10E-03	0.26	2.42E-10	3.83
	===	7.90E-03	0.29	7.43E-08	3.39
Kallas et al. 1971	C. Stress	-----	-----	2.73E-07	3.25
	===	-----	-----	2.01E-05	2.69
	===	-----	-----	1.37E-06	3.27
	===	-----	-----	1.66E-05	2.72
	===	-----	-----	6.52E-05	2.50
	===	-----	-----	2.32E-09	3.99
	===	-----	-----	4.00E-06	3.08
	===	-----	-----	1.40E-06	3.45
	===	-----	-----	8.19E-07	3.15
	===	-----	-----	2.52E-09	3.58
Pell et al. 1975	C. Stress	-----	-----	3.20E-15	5.10
	===	-----	-----	1.30E-13	4.60
	===	-----	-----	1.30E-10	3.90
	===	-----	-----	2.50E-13	4.60
	===	-----	-----	4.00E-17	5.50
	===	-----	-----	1.30E-13	4.50
	===	-----	-----	7.90E-12	4.20
	===	-----	-----	1.60E-14	4.70
	===	-----	-----	3.20E-08	3.20
	===	-----	-----	1.00E-07	3.10
	===	-----	-----	1.00E-15	5.20
	===	-----	-----	2.50E-12	4.00
	===	-----	-----	1.50E-10	3.80
	===	-----	-----	1.00E-09	3.50
	===	-----	-----	7.90E-13	4.20
	===	-----	-----	2.00E-10	3.60
	===	-----	-----	1.00E-09	3.60
	===	-----	-----	5.00E-20	6.30
	===	-----	-----	2.50E-14	4.90
	===	-----	-----	2.50E-14	4.90
	===	-----	-----	1.30E-14	5.10
	===	-----	-----	1.00E-15	5.50
	===	-----	-----	5.00E-13	4.70
	===	-----	-----	5.00E-10	3.40
	===	-----	-----	1.00E-07	2.70
	===	-----	-----	3.10E-09	3.00
	===	-----	-----	1.30E-10	3.40
	===	-----	-----	2.00E-11	3.80
	===	-----	-----	5.00E-07	2.50

Ref.: Data reference.

Mode: Loading mode (controlled strain or controlled stress).

K, A: Parameters of the general fatigue equation form ($N=K*\square^{-A}$).

K1, A1: Parameters of the fatigue equation form ($\square=K1*N^{-A1}$).

Table (1) Continued.

Ref.	Mode	K1	A1	K	A
Pell et al.	C. Stress	-----	-----	1.30E-09	3.20
1975	===	-----	-----	1.60E-15	4.40
	===	-----	-----	5.00E-08	2.80
	===	-----	-----	5.00E-18	5.80
	===	-----	-----	1.60E-15	5.30
	===	-----	-----	2.50E-17	5.80
	===	-----	-----	1.60E-12	4.50
	===	-----	-----	3.37E-12	4.10
	===	-----	-----	4.26E-09	3.40
	===	-----	-----	2.26E-11	3.90
	===	-----	-----	1.94E-07	2.90
	===	-----	-----	2.38E-05	2.50
Van Dijk	C. Stress	-----	-----	5.75E-05	2.71
1975	===	-----	-----	6.92E-05	2.66
	===	-----	-----	5.49E-04	2.53
Bonnaure	C. Stress	1.24E-03	0.19	5.04E-16	5.26
et al. 1980	===	1.65E-03	0.20	1.22E-14	5.00
CRGC	===	1.24E-02	0.32	1.10E-06	3.13
	===	1.75E-03	0.23	1.03E-12	4.35
	===	1.14E-03	0.18	4.46E-17	5.56
	===	3.09E-03	0.29	2.21E-09	3.45
	===	7.70E-04	0.12	1.13E-26	8.33
	===	5.90E-03	0.24	5.15E-10	4.17
	===	5.60E-04	0.10	3.03E-33	10.00
	===	1.19E-03	0.13	3.19E-23	7.69
	===	3.60E-03	0.23	2.37E-11	4.35
	===	7.50E-04	0.14	4.78E-23	7.14
	===	7.40E-04	0.18	4.04E-18	5.56
	===	5.70E-04	0.11	3.22E-30	9.09
	===	6.00E-04	0.13	1.65E-25	7.69
	===	3.10E-04	0.08	1.39E-44	12.50
CRGC	C. Strain	1.31E-03	0.18	9.66E-17	5.56
	===	4.75E-03	0.23	7.92E-11	4.35
	===	8.63E-03	0.23	1.06E-09	4.35
	===	2.33E-03	0.22	1.08E-12	4.55
	===	1.38E-03	0.18	1.29E-16	5.56
	===	1.68E-03	0.20	1.34E-14	5.00
	===	2.07E-02	0.26	3.33E-07	3.85
	===	3.50E-03	0.19	1.19E-13	5.26
	===	6.30E-04	0.12	2.13E-27	8.33
	===	1.08E-03	0.20	1.47E-15	5.00
	===	4.40E-04	0.13	1.51E-26	7.69
	===	5.10E-04	0.11	1.17E-30	9.09

Ref.: Data reference.

Mode: Loading mode (controlled strain or controlled stress).

K, A: Parameters of the general fatigue equation form ($N=K*\square^{-A}$).

K1, A1: Parameters of the fatigue equation form ($\square=K1*N^{-A1}$).

CRGC: Grand-Couronne Shell Research Center

Table (1) Continued.

Ref.	Mode	K1	A1	K	A
Bonnaure	C. Strain	7.40E-04	0.14	4.34E-23	7.14
et al. 1980	===	1.38E-03	0.16	1.33E-18	6.25
CRGC	===	1.04E-03	0.14	4.93E-22	7.14
	===	8.00E-04	0.14	7.57E-23	7.14
	===	7.90E-04	0.15	2.08E-21	6.67
	===	1.26E-03	0.18	7.78E-17	5.56
	===	9.50E-04	0.16	1.29E-19	6.25
	===	6.80E-04	0.12	4.02E-27	8.33
	===	9.50E-04	0.12	6.52E-26	8.33
	===	7.50E-04	0.14	4.78E-23	7.14
	===	3.62E-03	0.23	2.43E-11	4.35
	===	1.42E-03	0.19	1.03E-15	5.26
	===	1.40E-03	0.17	1.63E-17	5.88
Bonnaure	C. Strain	3.11E-03	0.19	6.37E-14	5.26
et al. 1980	===	3.19E-03	0.19	7.28E-14	5.26
KSLA	===	3.96E-03	0.18	4.51E-14	5.56
	===	3.11E-03	0.14	1.23E-18	7.14
	===	2.54E-03	0.20	1.06E-13	5.00
	===	2.06E-03	0.17	1.58E-16	5.88
	===	7.04E-03	0.19	4.69E-12	5.26
	===	2.14E-03	0.18	1.48E-15	5.56
	===	1.26E-03	0.16	7.54E-19	6.25
	===	9.09E-03	0.24	3.12E-09	4.17
	===	5.62E-03	0.27	4.63E-09	3.70
	===	3.35E-03	0.20	4.22E-13	5.00
	===	4.37E-03	0.20	1.59E-12	5.00
	===	1.73E-03	0.16	5.47E-18	6.25
	===	2.85E-03	0.22	2.70E-12	4.55
	===	6.61E-03	0.23	3.33E-10	4.35
	===	2.09E-03	0.18	1.29E-15	5.56
	===	2.47E-03	0.19	1.89E-14	5.26
	===	3.85E-03	0.20	8.46E-13	5.00
	===	5.02E-03	0.19	7.91E-13	5.26
	===	1.43E-03	0.17	1.85E-17	5.88
	===	3.65E-02	0.36	1.01E-04	2.78
	===	2.81E-03	0.19	3.73E-14	5.26
	===	9.60E-03	0.24	3.92E-09	4.17
	===	3.64E-03	0.20	6.39E-13	5.00
	===	3.61E-03	0.22	7.90E-12	4.55
CRR*	C. Stress	1.58E-03	0.20	9.85E-15	5.00
	===	2.08E-03	0.22	6.45E-13	4.55
	===	1.10E-03	0.19	2.68E-16	5.26
	===	2.02E-03	0.22	5.64E-13	4.55

Ref.: Data reference.

Mode: Loading mode (controlled strain or controlled stress).

K. A: Parameters of the general fatigue equation form (N=K* $^{-A}$).

K1. A1: Parameters of the fatigue equation form (=K1* N $^{-A1}$).

KSLA: Koninklijle Shekk Laboratorium Amsterdam

CRR: Road Research Center of Belgium

Table (1) Continued.

Ref.	Mode	K1	A1	K	A
CRR*	C. Stress	1.51E-03	0.20	7.85E-15	5.00
	===	1.18E-03	0.20	2.29E-15	5.00
	===	1.28E-03	0.19	5.95E-16	5.26
	===	2.29E-03	0.27	1.67E-10	3.70
	===	1.65E-03	0.23	7.98E-13	4.35
	===	1.87E-03	0.25	1.22E-11	4.00
	===	2.13E-03	0.22	7.18E-13	4.55
	===	1.21E-03	0.22	5.49E-14	4.55
	===	1.31E-03	0.19	6.73E-16	5.26
	===	1.47E-03	0.20	6.86E-15	5.00
	===	1.30E-03	0.19	6.46E-16	5.26
	===	1.69E-03	0.21	6.30E-14	4.76
	===	2.36E-03	0.21	3.09E-13	4.76
	===	2.28E-03	0.24	9.80E-12	4.17
CRR	C. Strain	1.35E-03	0.20	4.48E-15	5.00
	===	1.06E-03	0.19	2.21E-16	5.26
	===	2.00E-03	0.24	5.68E-12	4.17
	===	7.20E-04	0.15	1.12E-21	6.67
	===	1.20E-03	0.20	2.49E-15	5.00
	===	9.80E-04	0.19	1.46E-16	5.26
	===	1.19E-03	0.19	4.06E-16	5.26
	===	1.10E-03	0.19	2.68E-16	5.26
	===	1.63E-03	0.21	5.31E-14	4.76
	===	1.02E-03	0.21	5.69E-15	4.76
	===	1.56E-03	0.21	4.30E-14	4.76
	===	1.14E-03	0.19	3.24E-16	5.26
	===	1.94E-03	0.23	1.61E-12	4.35
	===	1.38E-03	0.22	9.99E-14	4.55
	===	1.36E-03	0.20	4.65E-15	5.00
	===	1.63E-03	0.22	2.13E-13	4.55
	===	1.84E-03	0.22	3.69E-13	4.55
	===	1.82E-03	0.19	3.80E-15	5.26
LCPC*	C. Strain	1.75E-03	0.19	3.09E-15	5.26
	===	1.17E-03	0.16	4.74E-19	6.25
	===	1.30E-03	0.19	6.46E-16	5.26
	===	7.40E-04	0.13	8.26E-25	7.69
	===	8.50E-04	0.15	3.38E-21	6.67
	===	1.57E-03	0.20	9.54E-15	5.00
	===	8.20E-04	0.16	5.14E-20	6.25
	===	8.40E-04	0.15	3.13E-21	6.67
	===	8.70E-04	0.16	7.45E-20	6.25
	===	1.21E-03	0.20	2.59E-15	5.00
	===	1.19E-03	0.14	1.29E-21	7.14

Ref.: Data reference.

Mode: Loading mode (controlled strain or controlled stress).

K, A: Parameters of the general fatigue equation form ($N=K*\square^{-A}$).

K1, A1: Parameters of the fatigue equation form ($\square=K1*N^{-A1}$).

LCPC: Laboratoire Central des Ponts et Chaussees-France

Table (1) Continued.

Ref.	Mode	K1	A1	K	A
LCPC*	C. Strain	1.77E-03	0.19	3.28E-15	5.26
	===	1.09E-03	0.19	2.56E-16	5.26
	===	1.41E-03	0.16	1.52E-18	6.25
	===	1.78E-03	0.19	3.38E-15	5.26
	===	4.29E-03	0.27	1.70E-09	3.70
	===	1.66E-03	0.22	2.31E-13	4.55
	===	2.81E-03	0.24	2.34E-11	4.17
	===	2.97E-03	0.25	7.78E-11	4.00
	===	1.02E-03	0.19	1.80E-16	5.26
Bonnaure et al. 1982	C. Strain	-----	-----	1.10E-06	3.10
	===	-----	-----	4.24E-07	3.31
	===	-----	-----	1.36E-08	3.82
	===	-----	-----	2.48E-05	2.86
	===	-----	-----	5.40E-05	2.78
	===	-----	-----	1.94E-09	3.77
	===	-----	-----	1.82E-09	3.85
	===	-----	-----	1.89E-07	3.31
	===	-----	-----	1.62E-13	5.07
	===	-----	-----	1.86E-13	5.11
	===	-----	-----	6.76E-03	2.18
	===	-----	-----	1.62E-12	5.11
	===	-----	-----	1.87E-03	2.27
	===	-----	-----	5.56E-12	4.56
	===	-----	-----	5.02E-11	4.30
	===	-----	-----	9.17E-13	4.83
	===	-----	-----	1.13E-13	5.14
	C. Stress	-----	-----	2.77E-11	4.47
	===	-----	-----	7.11E-11	4.34
Tayabafli et al. 1995	C. Strain	-----	-----	8.96E-08	3.57
Hsu et al. 1996	C. Stress	-----	-----	6.11E-10	4.40
	===	-----	-----	8.84E-11	5.50
	===	-----	-----	1.35E-12	5.50
	===	-----	-----	3.67E-13	5.50
	===	-----	-----	1.55E-12	6.20
	===	-----	-----	2.94E-06	3.50
	===	-----	-----	5.39E-07	3.50
	===	-----	-----	1.69E-07	3.50
	===	-----	-----	5.61E-07	4.10
	===	-----	-----	4.30E-08	4.10
	===	-----	-----	2.21E-08	4.10
	===	-----	-----	1.08E-08	4.80
	===	-----	-----	7.40E-11	4.80
	===	-----	-----	3.76E-11	4.80

Ref. : Data reference .

Mode : Loading mode (controlled strain or controlled stress).

K, A : Parameters of the general fatigue equation form ($N=K*\square^{-A}$).

K1, A1: Parameters of the fatigue equation form ($\square=K1*N^{-A1}$).

Table (1) Continued.

Ref.	Mode	K1	A1	K	A
Hsu et al. 1996	C. Stress	-----	-----	1.09E-05	3.10
	===	-----	-----	7.31E-07	3.80
	===	-----	-----	1.17E-07	3.80
	===	-----	-----	1.76E-08	3.80
	===	-----	-----	5.11E-08	4.20
Read et al. 1997	C. Stress	-----	-----	1.11E-09	3.79
	===	-----	-----	3.89E-11	4.03
	===	-----	-----	7.18E-11	3.92
	===	-----	-----	1.65E-10	4.15
	===	1.36E-03	0.24	1.15E-12	4.17
	===	1.53E-03	0.25	5.45E-12	4.00
	===	1.14E-03	0.24	5.54E-13	4.17
	===	5.52E-04	0.18	7.94E-19	5.56
	===	5.09E-04	0.17	4.24E-20	5.88
	===	5.52E-04	0.17	6.84E-20	5.88
	===	6.24E-03	0.31	8.12E-08	3.22
	===	6.01E-03	0.31	5.83E-08	3.26
	===	4.06E-03	0.32	3.18E-08	3.13
	===	3.78E-03	0.31	1.63E-08	3.22
	===	1.60E-03	0.26	1.77E-11	3.85
	===	1.39E-03	0.22	1.02E-13	4.55
	===	1.50E-03	0.23	5.32E-13	4.35
	===	3.52E-03	0.36	1.53E-07	2.78
	===	1.93E-03	0.27	8.79E-11	3.70
	===	1.33E-03	0.24	1.04E-12	4.17
Rowe et al. 1997	C. Stress	-----	-----	8.63E-07	2.64
	===	-----	-----	3.13E-06	2.65
	===	-----	-----	6.57E-05	2.23
	===	-----	-----	2.85E-07	2.75
	===	-----	-----	2.51E-07	3.16
	===	-----	-----	2.18E-07	3.38
	===	-----	-----	1.58E-07	3.39
	===	-----	-----	3.06E-09	3.78
	===	-----	-----	3.33E-10	4.30
	===	-----	-----	3.36E-09	4.04
	===	-----	-----	5.98E-10	3.89
	===	-----	-----	1.27E-09	3.67
	===	-----	-----	1.10E-11	4.27
	===	-----	-----	1.20E-07	3.12
Van Dijk et al. 1977	C. Strain	-----	-----	3.25E-14	5.38
	===	-----	-----	1.17E-13	5.16
	===	-----	-----	6.12E-14	5.50
	===	-----	-----	3.30E-19	7.37

Ref.: Data reference.

Mode: Loading mode (controlled strain or controlled stress).

K, A: Parameters of the general fatigue equation form ($N=K*\square^{-A}$).

K1, A1: Parameters of the fatigue equation form ($\square=K1*N^{-A1}$).

Table (1) Continued.

Ref.	Mode	K1	A1	K	A
Van Dijk et al. 1977	C. Strain	-----	-----	1.63E-13	4.93
	===	-----	-----	6.03E-17	6.02
	===	-----	-----	4.27E-12	5.26
	===	-----	-----	2.63E-15	5.41
	===	-----	-----	1.84E-18	6.12
	===	-----	-----	1.59E-09	4.14
	===	-----	-----	2.64E-06	2.96
	===	-----	-----	1.56E-08	3.97
	===	-----	-----	3.45E-14	5.07
	===	-----	-----	9.52E-11	4.07
	===	-----	-----	3.65E-22	7.59
	===	-----	-----	1.30E-16	6.34
	===	-----	-----	3.86E-09	3.74
	===	-----	-----	6.36E-13	4.93
	===	-----	-----	1.35E-12	5.03
	===	-----	-----	4.15E-07	3.80
	===	-----	-----	1.75E-17	6.07
	===	-----	-----	2.65E-12	4.55
	===	-----	-----	3.83E-10	4.38
	===	-----	-----	1.03E-15	5.52
	===	-----	-----	1.35E-14	5.38
	===	-----	-----	5.77E-13	5.07
	===	-----	-----	1.45E-12	5.13
	===	-----	-----	3.05E-12	4.57
	===	-----	-----	1.46E-17	5.92
	===	-----	-----	9.70E-05	2.79
	===	-----	-----	2.10E-14	5.38
	===	-----	-----	5.02E-09	4.11
	===	-----	-----	2.64E-01	2.02
	===	-----	-----	8.05E-13	4.94
	===	-----	-----	9.20E-12	4.62
Irwin 1977	C. Stress	-----	-----	8.56E-07	2.80
	===	-----	-----	3.11E-06	2.90
	===	-----	-----	5.60E-07	3.00
Yuce et al. 1974	C. Stress	-----	-----	6.61E-07	3.16
	===	-----	-----	2.19E-07	3.23
	===	-----	-----	2.70E-07	2.96
	===	-----	-----	1.72E-05	2.62
	===	-----	-----	9.33E-07	3.08
	===	-----	-----	8.73E-08	3.75
	===	-----	-----	1.20E-08	3.75
	===	-----	-----	1.35E-06	3.05
	===	-----	-----	3.79E-06	2.89
	===	-----	-----	6.32E-08	3.40

Ref.: Data reference.

Mode: Loading mode (controlled strain or controlled stress).

K, A: Parameters of the general fatigue equation form ($N=K*$[illegible]$^{-A}$).

K1, A1: Parameters of the fatigue equation form ([illegible]$=K1*N^{A1}$).

Table (2) The Kolmogorov-Smirnov (K-S) test results

1- Variable: Parameter (A). D_{max}= 0.57
Samples: (Kallas et al. 1971) and (Pell et al. 1975)

CI	SAMPLE [1]: (Kallas et al. 1971)			SAMPLE [2]: (Pell et al. 1975)			D
	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	
2.5—2.99	3	.3	.3	5	.121	.121	.178
3—3.49	5	.5	.8	7	.17	.292	.507
3.5—3.99	1	.1	.9	7	.17	.463	.436
4—4.49	0	0	.9	5	.121	.585	.314
4.5—4.99	1	.1	1	8	.195	.78	.219
5—5.49	0	0	1	4	.097	.878	.121
5.5—5.99	0	0	1	4	.097	.975	.024
6—6.49	0	0	1	1	.024	1	0

2- Variable: Parameter (A). D_{max}=0.56
Samples: (Pell et al. 1975) and (CRGC)

CI	SAMPLE [1]: (Pell et al. 1975)			SAMPLE [2]: CRGC			D
	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	
2.5—2.99	5	.121	.121	0	0	0	.121
3—3.49	7	.17	.292	2	.048	.048	.243
3.5—3.99	7	.17	.463	1	.024	.073	.39
4—4.49	5	.121	.585	6	.146	.219	.365
4.5—4.99	8	.195	.78	1	.024	.243	.536
5—5.49	4	.097	.878	6	.146	.39	.487
5.5—5.99	4	.097	.975	6	.146	.536	.439
6—6.49	1	.024	1	2	.048	.585	.414
6.5—6.99	0	0	1	1	.024	.609	.39
7—7.49	0	0	1	5	.121	.731	.268
7.5—7.99	0	0	1	3	.073	.804	.195
8-8.49	0	0	1	4	.097	.902	.097
8.5—8.99	0	0	1	0	0	.902	.097
9—9.49	0	0	1	2	.048	.951	.048
9.5—9.99	0	0	1	0	0	.951	.048
10—10.5	0	0	1	2	.049	1	0

3- Variable: Parameter (A). $D_{max}=0.41$
Samples: (CRGC) and (KLSA)

CI	SAMPLE [1]: CRGC			SAMPLE [2]: KLSA			D
	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	
2.5 – 2.99	0	0	0	1	.038	.038	.038
3--3.49	2	.048	.048	0	0	.038	.01
3.5—3.99	1	.024	.073	1	.038	.076	.003
4—4.49	6	.146	.219	3	.115	.192	.027
4.5—4.99	1	.024	.243	2	.076	.269	.025
5—5.49	6	.146	.39	11	.423	.692	.302
5.5—5.99	6	.146	.536	5	.192	.884	.348
6—6.49	2	.048	.585	2	.076	.961	.376
6.5—6.99	1	.024	.609	0	0	.961	.351
7—7.49	5	.121	.731	1	.038	1	.268
7.5 – 7.99	3	.073	.804	0	0	1	.195
8-8.49	4	.097	.902	0	0	1	.097
8.5—8.99	0	0	.902	0	0	1	.097
9--9.49	2	.048	.951	0	0	1	.048
9.5—9.99	0	0	.951	0	0	1	.048
10—10.5	2	.049	1	0	0	1	0

4- Variable: Parameter (A). $D_{max}=0.42$
Samples: (KLSA) and (CRR)

CI	SAMPLE [1]: KLSA			SAMPLE [2]: CRR			D
	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	
2.5—2.99	1	.038	.038	0	0	0	.038
3--3.49	0	0	.038	0	0	0	.038
3.5– 3.99	1	.038	.076	1	.027	.027	.049
4—4.49	3	.115	.192	5	.138	.166	.025
4.5—4.99	2	.076	.269	12	.333	.5	.23
5—5.49	11	.423	.692	17	.472	.972	.279
5.5—5.99	5	.192	.884	0	0	.972	.087
6—6.49	2	.076	.961	0	0	.972	.01
6.5—6.99	0	0	.961	1	.027	1	.038
7--7.49	1	.038	1	0	0	1	0

5- Variable: Parameter (A). D_{max}=0.45
Samples: (CRR) and (LCPC)

CI	SAMPLE [1]: CRR			SAMPLE [2]: LCPC			D
	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	
3.5—3.99	1	.027	.027	1	.05	.05	.023
4—4.49	5	.138	.166	3	.15	.2	.034
4.5 - 4.99	12	.333	.5	2	.1	.3	.2
5—5.49	17	.472	.972	6	.3	.6	.372
5.5—5.99	0	0	.972	0	0	.6	.372
6—6.49	0	0	.972	4	.2	.8	.172
6.5—6.99	1	.027	1	2	.1	.9	.099
7—7.49	0	0	1	1	.05	.95	.049
7.5—7.99	0	0	1	1	.05	1	0

6- Variable: Parameter (A). D_{max}=0.52
Samples: (LCPC) and (Bonnaure et al. 1982)

CI	SAMPLE [1]: LCPC,			SAMPLE [2]: Bonnaure et al. (1982)			D
	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	
2—2.49	0	0	0	2	.105	.105	.105
2.5—2.99	0	0	0	2	.105	.21	.21
3—3.49	0	0	0	3	.157	.368	.368
3.5—3.99	1	.05	.05	3	.157	.526	.476
4—4.49	3	.15	.2	3	.157	.684	.484
4.5—4.99	2	.1	.3	2	.105	.789	.489
5—5.49	6	.3	.6	4	.21	1	.399
5.5—5.99	0	0	.6	0	0	1	.399
6—6.49	4	.2	.8	0	0	1	.199
6.5—6.99	2	.1	.9	0	0	1	.099
7—7.49	1	.05	.95	0	0	1	.049
7.5—7.99	1	.05	1	0	0	1	0

7- Variable: Parameter (A). D_{max}=0.53
Samples: (Bonnaure et al. 1982) and (Hsu et al. 1995)

CI	SAMPLE [1]: Bonnaure et al. (1982)			SAMPLE [2]: Hsu et al. (1995)			D
	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	
2—2.49	2	.105	.105	0	0	0	.105
2.5—2.99	2	.105	.21	0	0	0	.21
3—3.49	3	.157	.368	1	.052	.052	.315
3.5—3.99	3	.157	.526	6	.315	.368	.157
4—4.49	3	.157	.684	5	.263	.631	.052
4.5—4.99	2	.105	.789	3	.157	.789	0
5—5.49	4	.21	1	0	0	.789	.21
5.5—5.99	0	0	1	3	.157	.947	.052
6—6.49	0	0	1	1	.052	.999	0

8- Variable: Parameter (A). D_{max}=0.52
Samples: (Hsu et al. 1995) and (Read et al. 1997)

CI	SAMPLE [1]: Hsu et al. (1995)			SAMPLE [2]: Read et al. (1997)			D
	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	
2.5—2.99	0	0	0	1	.05	.05	.05
3—3.49	1	.052	.052	4	.2	.25	.197
3.5—3.99	6	.315	.368	4	.2	.45	.081
4—4.49	5	.263	.631	7	.35	.799	.168
4.5—4.99	3	.157	.789	1	.05	.849	.06
5—5.49	0	0	.789	0	0	.849	.06
5.5—5.99	3	.157	.947	3	.15	1	.052
6—6.49	1	.052	.999	0	0	1	0

9- Variable: Parameter (A). $D_{max}=0.57$
Samples: (Read et al. 1997) and (Rowe et al. 1997)

CI	SAMPLE [1]: Read et al. (1997)			SAMPLE [2]:-Rowe et al. (1997)			D
	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	
2—2.49	0	0	0	1	.071	.071	.071
2.5—2.99	1	.05	.05	3	.214	.285	.235
3—3.49	4	.2	.25	4	.285	.571	.321
3.5—3.99	4	.2	.45	3	.214	.785	.335
4—4.49	7	.35	.799	3	.214	1	.2
4.5—4.99	1	.05	.849	0	0	1	.15
5—5.49	0	0	.849	0	0	1	.15
5.5—5.99	3	.15	1	0	0	1	0

10- Variable: Parameter (A). $D_{max}=0.52$
Samples: (Rowe et al. 1997) and (Van Dijk et al. 1977)

CI	SAMPLE [1]: Rowe et al. (1997)			SAMPLE [2]: Van Dijk et al. (1977)			D
	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	
2—2.49	1	.071	.071	1	.028	.028	.042
2.5—2.99	3	.214	.285	2	.057	.085	.2
3—3.49	4	.285	.571	0	0	.085	.485
3.5—3.99	3	.214	.785	7	.2	.285	.5
4—4.49	3	.214	1	8	.228	.514	.485
4.5—4.99	0	0	1	4	.114	.628	.371
5—5.49	0	0	1	4	.114	.742	.257
5.5—5.99	0	0	1	3	.085	.828	.171
6—6.49	0	0	1	4	.114	.942	.057
6.5—6.99	0	0	1	0	0	.942	.057
7—7.49	0	0	1	1	.028	.971	.028
7.5—7.99	0	0	1	1	.028	1	0

11- Variable: Parameter (A). $D_{max}=0.58$
Samples: (Van Dijk et al. 1977) and (Yuce et al. 1974)

CI	SAMPLE [1]: Van Dijk et al. (1977)			SAMPLE [2]: Yuce et al. (1974)			D
	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	Observed Frequency	Observed Relative Frequency	Observed Cumulative Frequency	
2—2.49	1	.028	.028	0	0	0	.028
2.5—2.99	2	.057	.085	3	.3	.3	.215
3—3.49	0	0	.085	3	.3	.6	.515
3.5—3.99	7	.2	.285	1	.1	.7	.415
4—4.49	8	.228	.514	2	.2	.9	.386
4.5—4.99	4	.114	.628	1	.1	1	.372
5—5.49	4	.114	.742	0	0	1	.257
5.5—5.99	3	.085	.828	0	0	1	.171
6—6.49	4	.114	.942	0	0	1	.057
6.5—6.99	0	0	.942	0	0	1	.057
7—7.49	1	.028	.971	0	0	1	.028
7.5—7.99	1	.028	1	0	0	1	0