SPLINE-APPROXIMATE METHODE FOR SOLUTION OF RVERSE PROFILE PROBLEM IN SPACE

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ABSTRACT.
Profiling is one of the important problems in mechanical engineering and industry because of its large applications in the field of cutting tool. Most papers and researches deal with the problem of profiling of involute profile. The problem of direct profile design consists in determination of profile tool if profile of gear is defined. This paper deal with problem of determination non-involute profile for helical surface in which profile of tool is defines and this problem called the reverse profile problem. There are very large applications of this problem in cutting tool, for example cutting of helical surface (flutes) of twist drill with disc (wheel) tool and generating of helical surface for hops with disc (wheel) tool. To solve this problem, we will use transformation method for determination profile of helical surface and spline – approximate method for describing initial profile of tool.

الخلاصة

تعتبر مسألة الجانبية(profile) واحده من أهم المسائل في الهندسة الميكانيكية والصناعة بسبب تطبيقاتها الواسعة في حقل عدد القطع. أن معظم البحوث والمقالات العلمية تناولت مسألة الجانبية(profile) في عدد القطع من وجه نظر الملفوف. المسألة المباشرة في تصميم الجانبية(profile) تتضمن حساب جانبية(profile) العدة في حالة جانبية(profile) الترس معرفة. هذا البحث يتعامل مع مسالة الجانبية غير الملفوفة (Non-involute) للسطوح الحلزونية والتي فيها جانبية(profile) العدة تكون معلومة والتي تعرف بالجانبية العكسية. هناك العديد من التطبيقات لهذه المسألة في عدد القطع على سبيل المثال قطع سطح لولبي لثاقب دوار مع قرص عدة و توليد سطح لولبي لعده من نوع hob بواسطة عدة قرصية . ولحل هذه المسائل نستخدم طريقة التحويل لاحتساب الجانبية للسطوح الحلزونية و طريقة المنحني-- التقريبي لوصف الجانبية الأولية.

KEY WARDS
Gear, involute profile, non-involute profile, profile, disk tool, cutting machine.

INTRODUCTION

Direct profile problem consists in determination of profile tool when profile of gear is defined and this problem we can study in plane such as generation profile of gear by rack cutter (Litvin1997). Our research deal with problem of determination profile for helical surface which generated by disk tool and to solve this problem we will study disk tool and helical surface in space.

The problem of the determination profiles for helical surface consists of two categories:

- Profile of helical surface: Research and determination of profile of helical surface by using transformation method (Novinkov1956). To solve this problem, we will study profile of helical surface in normal and axial plane. Also this method make possible to find profile in generation plane with angle λ. In which the ordinate axis of profile must be negative and this is the condition for proper generation of helical surface.

- Profile of disc tool: There are many methods to describing initial profile, these methods use explicit or implicit equations but have disadvantages, and therefore we will use spline – approximate method because it have many advantages in discretion of initial profile .

PROFILE OF HELICAL SURFACE:

Let's assume $X_b Y_b Z_b$ is the coordinate system of disk (wheel) tool S_b, as shown in **Fig. (1)**. We can write initial profile of tool as vector $\vec{r}_b = \begin{vmatrix} x_b \\ y_b \end{vmatrix}$ in its axial plane and unit vector of its normal as $\vec{e}_b = \begin{vmatrix} e_{xb} \\ e_{yb} \end{vmatrix}$.

Then the surface of disk (wheel) tool in the coordinate system $S_\Delta : \vec{r}_\Delta = M_{\Delta b}\vec{r}_b$, where $M_{\Delta b}$ -matrix transformation from coordinates system S_b to coordinates system S_Δ (Litvin1968).

$$M_{\Delta b} = \begin{vmatrix} 1 & 0 & 0 & \Delta x \\ 0 & \cos\varphi & -\sin\varphi & 0 \\ 0 & \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} . \qquad (1)$$

where: φ -Angle of parameter surface for disk tool;

Δx - Displacement from original point O_b in coordinates system S_b to original point O_Δ coordinates system S_Δ the displacement Δx of disk tool toward its axial will be first cutting machine adjustment to the profile of helical surface.

After solving: $\vec{r}_\Delta = M_{\Delta b}\vec{r}_b$, we obtain :

$$\left.\begin{aligned} x_\Delta &= x_b + \Delta x \\ y_\Delta &= y_b \cos\varphi \\ z_\Delta &= y_b \sin\varphi \end{aligned}\right\} \qquad (2)$$

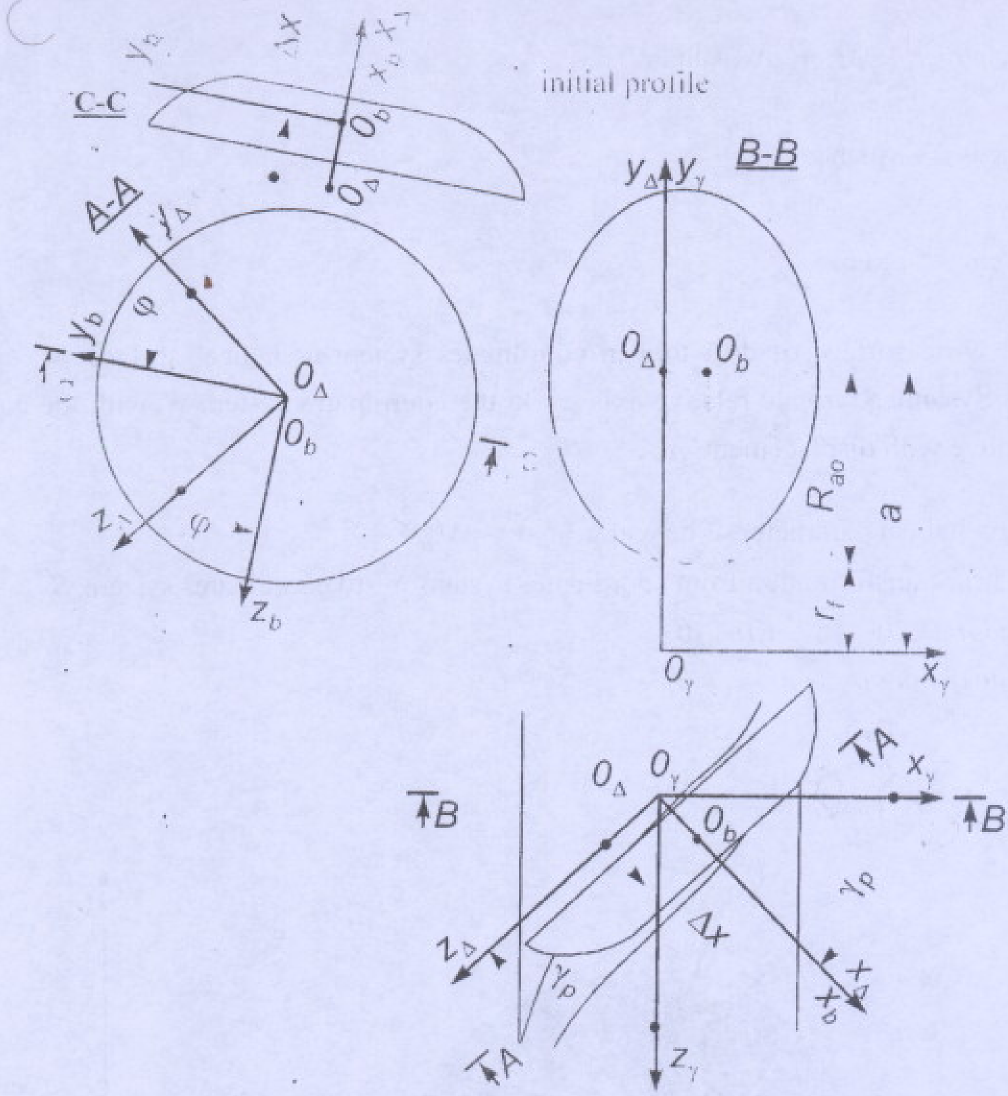

Fig.(1) Disk tool in coordinate system S_b with adjustment of cutting machine for helical surface.

The surface of disk tool in coordinates system $S_\gamma : \vec{r}_\gamma = M_{\gamma\Delta} \vec{r}_\Delta$

where $M_{\gamma\Delta}$ -matrix transformation from coordinates system S_Δ to coordinates system S_γ .

$$M_{\gamma\Delta} = \begin{vmatrix} \cos\gamma & 0 & -\sin\gamma & 0 \\ 0 & 1 & 0 & a \\ \sin\gamma & 0 & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \qquad (3)$$

Where γ- is the turn angle of disk tool relative to normal plane of helical surface . This angle will be the second cutting machine adjustment of the profile of helical surface;

a- The distance between center of disk tool and center of helical surface can be written as:

$a = R_{ao} + r_f$

Where R_{ao} - External radius of the disk tool;

r_f - Internal radius of the helical surface.

Multiplying: $\vec{r}_\gamma = M_{\gamma\Delta}\vec{r}_\Delta$, we obtain :

$$\left.\begin{aligned} x_\gamma &= x_\Delta \cos\gamma - z_\Delta \sin\gamma \\ y_\gamma &= y_\Delta + a \\ z_\gamma &= y_\Delta \sin\gamma + z_\Delta \cos\gamma \end{aligned}\right\} \quad (4)$$

We will write surface of disk tool in coordinates system of helical surface S_υ as indicated in **Fig. (2).** System S_γ rotate relative axis z_υ in the coordinates system S_υ with the parameter angle υ and move with displacement $p\upsilon$.

Where: p - helical parameter of helical gear $\vec{r}_\upsilon = M_{\upsilon\gamma}\vec{r}_\gamma$.

$M_{\upsilon\gamma}$ - matrix transformation from coordinates system S_γ to coordinates system S_υ.

$$M_{\upsilon\gamma} = \begin{vmatrix} \cos\upsilon & 0 & -\sin\upsilon & 0 \\ \sin\upsilon & \cos\upsilon & 0 & 0 \\ 0 & 0 & 0 & p\upsilon \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad (5)$$

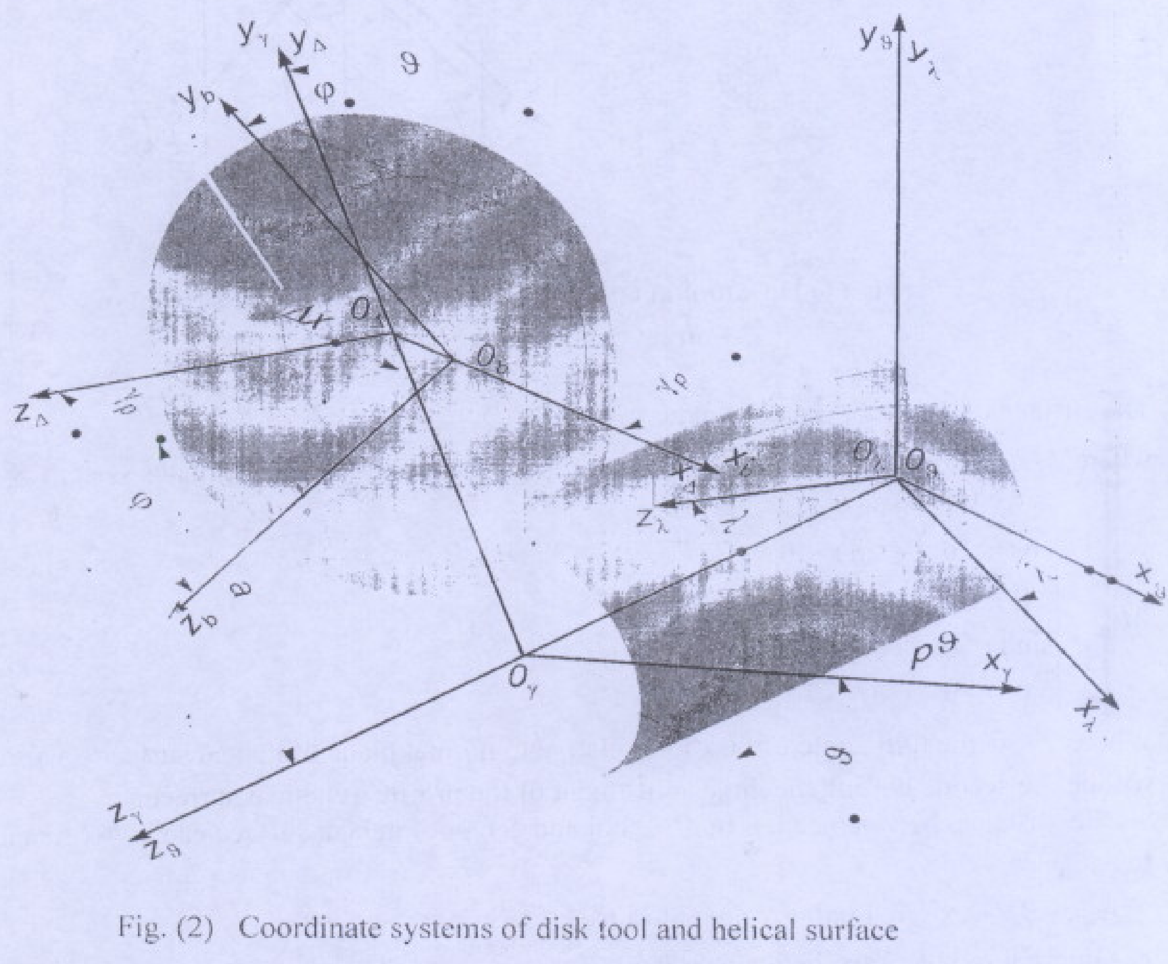

Fig. (2) Coordinate systems of disk tool and helical surface

Multiplying obtain:

$$\left.\begin{aligned} x_\upsilon &= x_\gamma \cos\upsilon - y_\gamma \sin\upsilon \\ y_\upsilon &= x_\gamma \sin\upsilon + y_\gamma \cos\upsilon \\ z_\upsilon &= z_\gamma + p\upsilon \end{aligned}\right\} \quad (6)$$

Finally we will transfer the surface of disk tool to find the generation plane with angle λ in the coordinate system S_λ and obtain: $\vec{r}_\lambda = M_{\lambda\upsilon}\vec{r}_\upsilon$.

Where: $M_{\lambda\upsilon}$ - matrix transformation from coordinates system S_υ to coordinates system S_λ.

$$M_{\lambda\upsilon} = \begin{vmatrix} \cos\lambda & 0 & \sin\lambda & 0 \\ 0 & 1 & 0 & a \\ -\sin\lambda & 0 & \cos\lambda & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \quad (7)$$

$$\left.\begin{aligned} x_\lambda &= x_\upsilon \cos\lambda - z_\upsilon \sin\lambda \\ y_\lambda &= y_\upsilon \\ z_\lambda &= -x_\upsilon \sin\lambda + z_\upsilon \cos\lambda \end{aligned}\right\} \quad (8)$$

To find the profile of helical gear in the plane $X_\lambda O_\lambda Y_\lambda$, we make $Z_\lambda = 0$ and for each angle we obtain:

$$-x_\upsilon \sin\lambda + \cos\lambda = 0 \quad (9)$$

For normal plane $\lambda = 0$. From formula (8) in which $Z_\lambda = 0$. Substituting the value of x_υ obtained by formula (6) we obtain:

$$x_\gamma \cos\upsilon - y_\gamma \sin\upsilon = 0 \quad (10)$$

This can be rearranged to determine υ as follows:

$$\tan\upsilon = \frac{x_\gamma}{y_\gamma} \quad (11)$$

For generation plane with $\lambda \neq 0$ and $\lambda \neq 90$ formula (6) can be used to obtain the following relationship:

$$-(x_\gamma \cos\upsilon + y_\gamma \sin\upsilon)\sin\lambda + (z_\gamma + p\upsilon)\cos\lambda = 0 \quad (12)$$

we can write general formula for transformation surface of disk tool relative surface of helical gear as following:

$$\vec{r}_\lambda = M_{\lambda\upsilon} M_{\upsilon\gamma} M_{\gamma\Delta} M_{\Delta b}\, \vec{r}_b \quad (13)$$

where:

$$M_{\Delta h} = \begin{vmatrix} 1 & 0 & 0 & \Delta x \\ 0 & \cos\varphi & -\sin\varphi & 0 \\ 0 & \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \;;\; M_{\gamma\Lambda} = \begin{vmatrix} \cos\gamma & 0 & -\sin\gamma & 0 \\ 0 & 1 & 0 & a \\ \sin\gamma & 0 & \cos\gamma & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$M_{\upsilon\gamma} = \begin{vmatrix} \cos\upsilon & 0 & -\sin\upsilon & 0 \\ \sin\upsilon & \cos\upsilon & 0 & 0 \\ 0 & 0 & 0 & p\upsilon \\ 0 & 0 & 0 & 1 \end{vmatrix} \;;\; M_{\lambda\upsilon} = \begin{vmatrix} \cos\lambda & 0 & \sin\lambda & 0 \\ 0 & 1 & 0 & a \\ -\sin\lambda & 0 & \cos\lambda & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$$\left.\begin{array}{l} x_\Delta = x_h + \Delta x \\ y_\Lambda = y_h \cos\varphi \\ z_\Lambda = y_h \sin\varphi \end{array}\right\} ;\; \left.\begin{array}{l} x_\gamma = x_\Lambda \cos\gamma - z_\Lambda \sin\gamma \\ y_\gamma = y_\Lambda + a \\ z_\gamma = y_\Lambda \sin\gamma + z_\Lambda \cos\gamma \end{array}\right\} ;\; \left.\begin{array}{l} x_\gamma = x_\Lambda \cos\gamma - z_\Lambda \sin\gamma \\ y_\gamma = y_\Lambda + a \\ z_\gamma = y_\Lambda \sin\gamma + z_\Lambda \cos\gamma \end{array}\right\} ;$$

$$\upsilon = -\frac{z_\gamma}{p} \qquad \{\lambda = 0 \text{ normal plane}\}$$

$$\tan\upsilon = \frac{x_\gamma}{y_\gamma} \qquad \{\lambda = 90 \text{ axial plane}\}$$

$$-x_\upsilon \sin\lambda + \cos\lambda = 0 \qquad \{\lambda \neq 90 \text{ and } \lambda \neq 0 \text{ generation plane}\}$$

Profile of helical surface in the generation plane can be written as:

$$\left.\begin{array}{l} x_\lambda = x_\upsilon \cos\lambda + z_\upsilon \sin_\lambda \\ y_\lambda = y_\upsilon \end{array}\right\} \qquad (14)$$

NUMERICAL RESULTS

The following steps can be employed to solve the problem of determination of the profile in space:

1- Solve the example of initial profile: by using spline – approximate method we can determine the initial profile which is defined as rectangle profile (Panakratov2000).
2- Determination profile for helical surface: by using transformation method we can determine the profile of helical surface for rectangle initial profile.
3- Progran. for design and calculation profile : to design initial profile of the disk tool and final profile of the helical surfacea software is build. The program is written in standard PASCAL 7.1 language. The program is developed to design profile of disk tool and helical surface.

Using the written program three examples with different parameters of disc tool and cutting operation conditions.

In first example the profile of disk tool is defined as a linear_profile with turn angle $\gamma=20^0$, with internal radius of helical surface $r_f = 30\ mm$, and displacement from original point O_h in coordinates system S_b to original point O_A in coordinates system S_A $\Delta x = 10$ mm. Fig 3.a shows the initial proposed profile of disk tool in axial direction. The normal and axial sections of the final generated helical gear profile are show in **Fig (3,b)** and c.

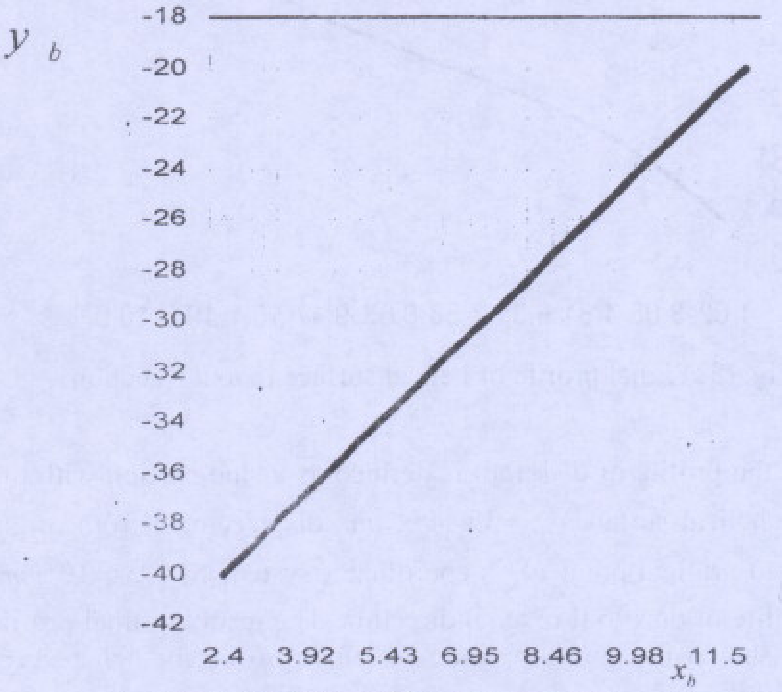

Fig. (3a) Initial profile of disk tool in axial section

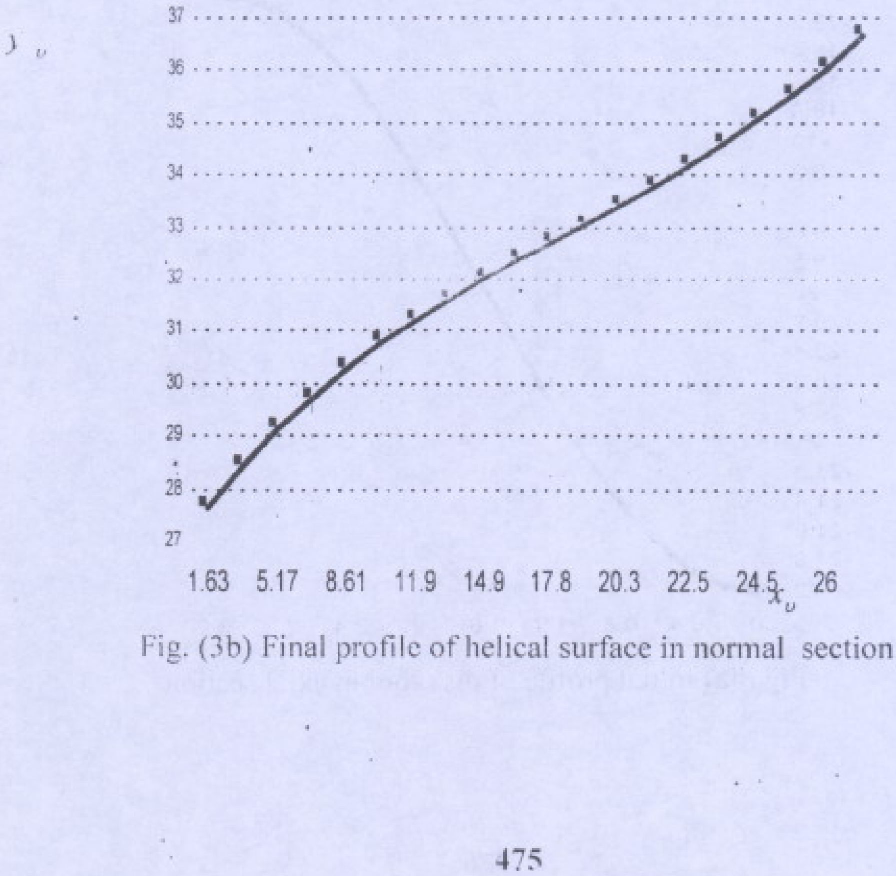

Fig. (3b) Final profile of helical surface in normal section.

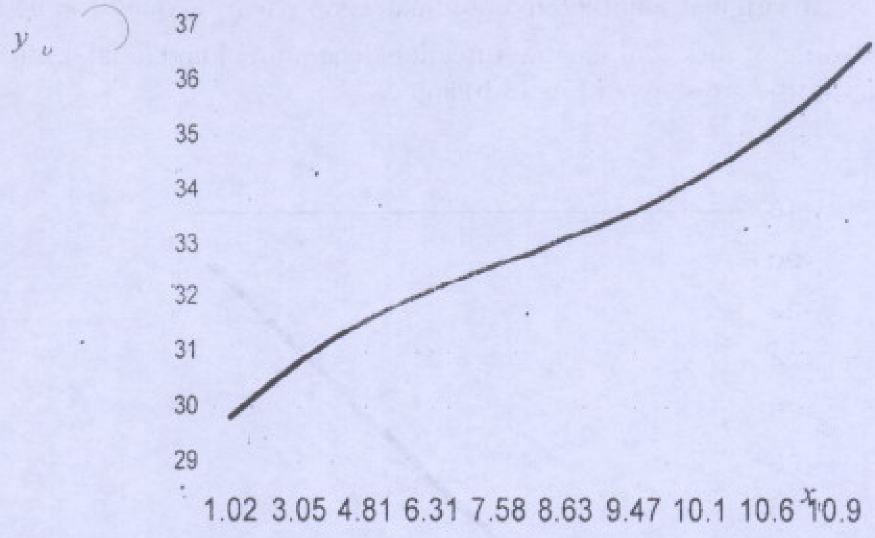

Fig. (3c) Final profile of helical surface in axial section.

In the second example the profile of disk tool is defined as a sine portion with turn angle $\gamma=30^0$, with internal radius of helical surface $r_i = 35$ mm, and displacement from original point O_b in coordinates system S_b to original point O_k in coordinates system S_k $\Delta x = 10$ mm. Fig 4.a shows the initial proposed profile of disk tool in axial direction. The resulting final profile of helical gear in the axial section is shown in figure 4b while the final profile for helical gear in the normal section is shown in Fig. (4c).

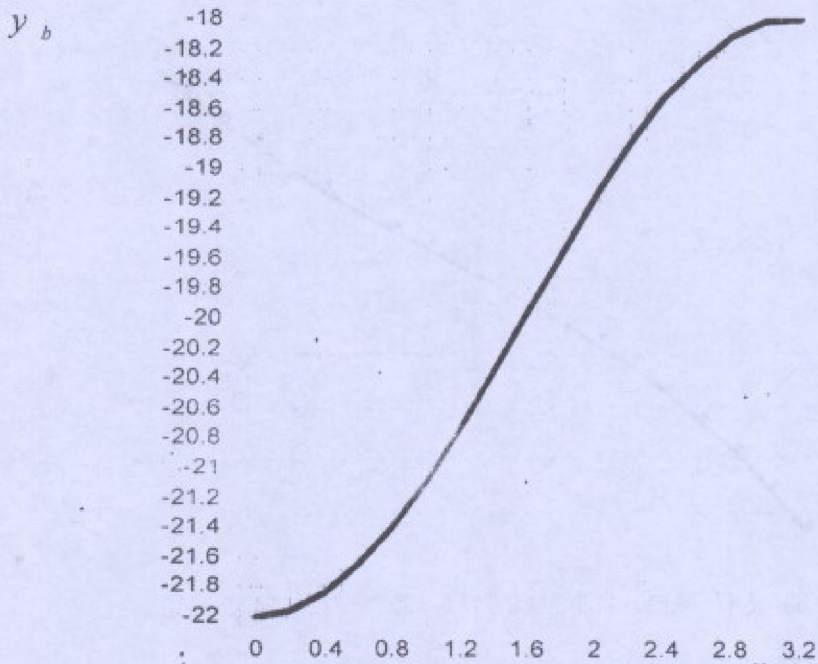

Fig.(4a) Initial profile of disk tool in axial section

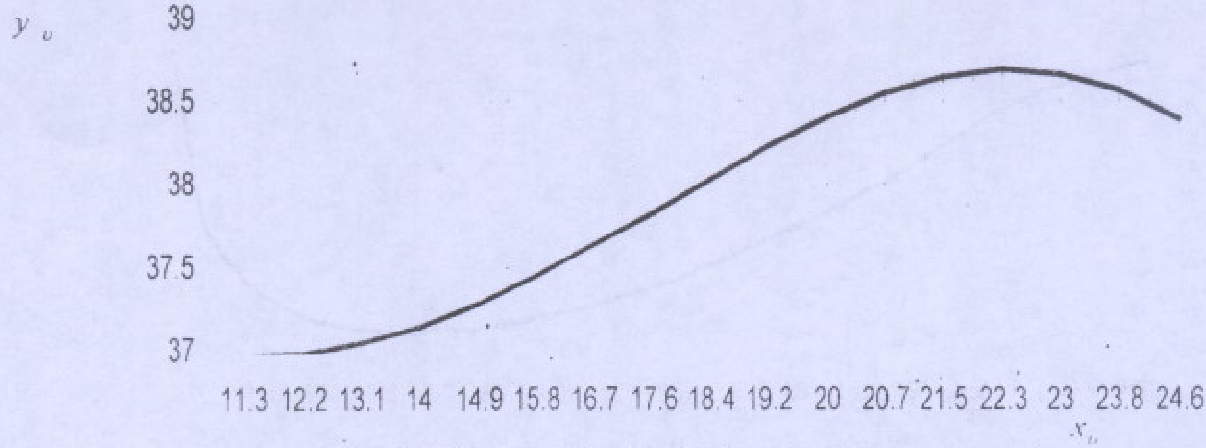

Fig. (4b0 Final profile of helical surface in normal section.

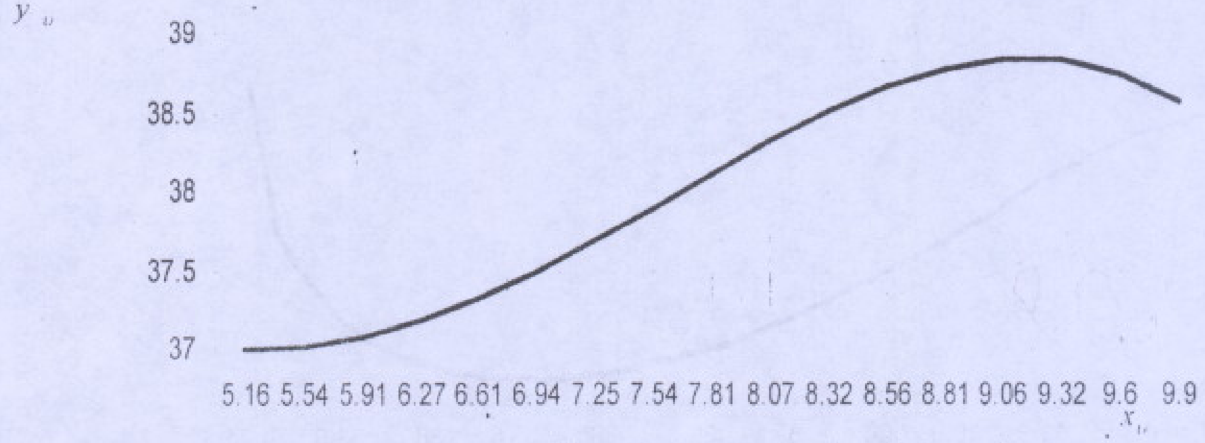

Fig. (4c) Final profile of helical surface in axial section

Fig. (5a) shows the profile of the third case studied which composed of three parts (line, arc, arc) With $\gamma=30^0, r_f = 35\ mm$ and $\Delta x = 10\ mm$. The final generated helical gear axial and normal sections are show in fig 6,b and c.

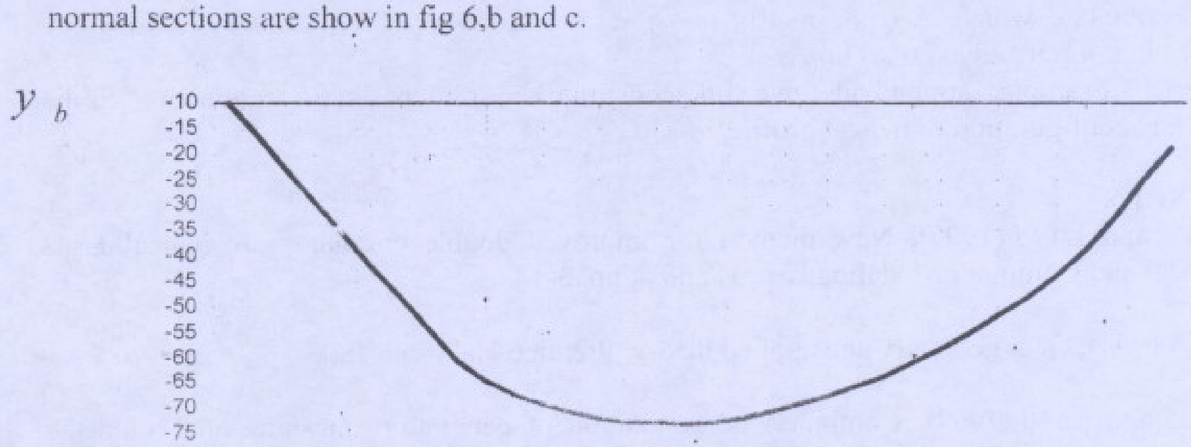

Fig.(5a) Initial profile of disk tool in axial section

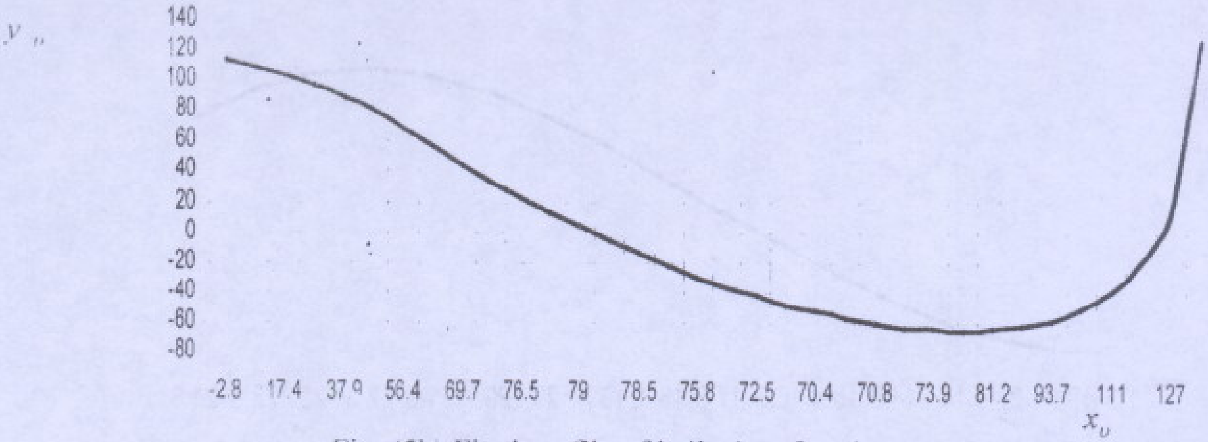

Fig. (5b) Final profile of helical surface in normal section.

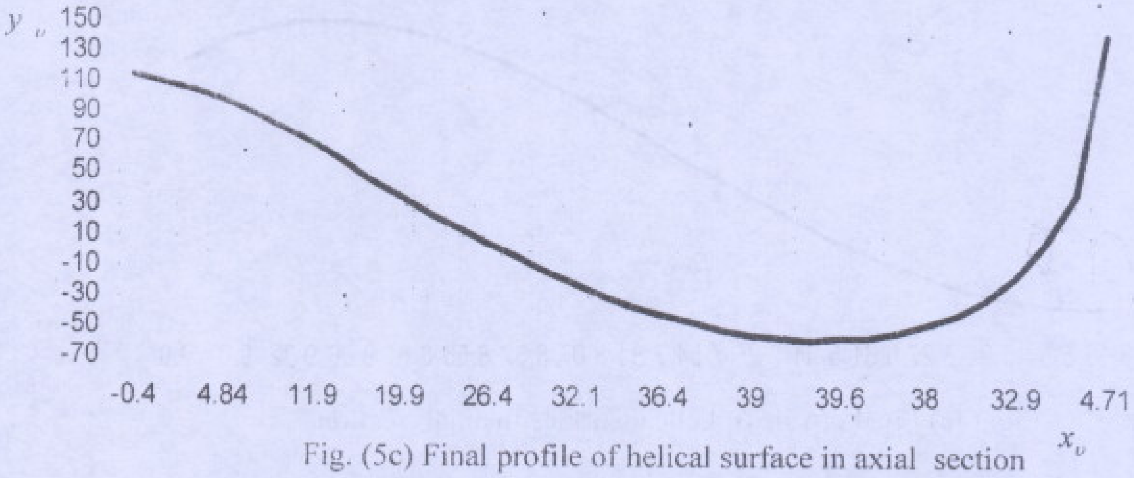

Fig. (5c) Final profile of helical surface in axial section

CONCLUSIONS

This paper essentially describes the construction of a general and reliable algorithmic method for dealing with the problem of determination a non involute profile of helical surfaces in which the profile of the tool is defined. Furthermore, to describe the initial profile, and the spline approximate method which used the matrix transformation to determine the generated disk and tool profiles in normal and axial sections.

The numerical examples applied in this work constitutes a solid basis for conducting further studies on any configuration of helical profile.

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NOMENCLATURE

$X_h Y_h Z_h$ - the coordinate system of disk (wheel) tool S_h;

$M_{\Delta h}$ - matrix transformation from coordinates system S_h to coordinates system S_Δ;

$M_{\gamma\Delta}$ - matrix transformation from coordinates system S_Δ to coordinates system S_γ;

$M_{\upsilon\gamma}$ - matrix transformation from coordinates system S_γ to coordinates system S_υ.

φ - angle of parameter surface for disk tool;

Δx - displacement from original point O_h in coordinates system S_h to original point O_Δ in coordinates system S_Δ;

γ - turn angle of disk tool relative to normal plane of helical;

a - distance between center of disk tool and center of helical surface;

R_{ui} - External radius of disk tool;

a - distance between center of disk tool. to the center of helical surface;

R_{ui} - external radius of disk tool;

r_f - internal radius of helical surface.