



SUGGESTED ENERGY EQUIVALENCE APPROACH FOR THE DYNAMIC ANALYSIS OF TAPERED CHIMNEYS

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ABSTRACT

In this paper an energy equivalence approach is suggested for the dynamic analysis of tapered chimneys. For any mode of vibration; by equating the kinetic energy of the actual chimney to that of an equivalent prismatic chimney, a hypothetical equivalent uniform mass has been obtained for the equivalent chimney. Also, by equating the strain energy of the actual and equivalent prismatic chimney, a hypothetical equivalent constant moment of inertia has been derived for the equivalent chimney.

The accuracy of the proposed equivalent prismatic model has been checked by comparing the results with that of the conventional segmented model using the stiffness method, which is deemed to be the more rigorous solution. Free and forced dynamic analyses have been carried out and the results indicated that the proposed equivalence energy model is in a good agreement with the stiffness (segmented chimney) model. The maximum discrepancy in the fundamental natural frequency ranges between 1.2% and 1.44%. The difference in the maximum dynamic bending moment due to wind vortex shedding is about 3.0%

الخلاصة

تم في هذا البحث اقتراح طريقة تعتمد على الطاقة لتحليل المداخل المستدقة، فبمساواة الطاقة الحركية لكل من المدخنة المستدقة ومدخنة مكافئة منتظمة المقطع تم حساب الكتلة المنتظمة لوحدة الطول ولكل طور من أطوار الاهتزاز. كذلك عند مساواة طاقة الانفعال لكل من المدخنة الحقيقية وتلك المقترحة تم حساب العزم الثاني للمساحة الثابتة على طول المدخنة.

ولمعرفة مدى دقة هذه الطريقة المقترحة تم مقارنتها مع طريقة الجساءة والتي تعتبر من الحلول المعروفة. لقد تم تحليل المدخنة وحالاتي الاهتزاز الحر والمقيد وقد بينت النتائج ان الطريقة المقترحة ذات دقة جيدة مقارنة مع طريقة الجساءة حيث كان اكبر تباين في الاهتزاز الطبيعي الاساسي (ω_1) هو بين (1.2% و 1.44%) وان التباين في اكبر عزم عند قاعدة المدخنة هو (3.0%).

KEY WORDS.

Chimneys ,dynamic analysis ,energy method ,stiffness method, wind loads.

INTRODUCTION

The wind and seismic analyses procedures for chimneys are outlined in the codes of practice (DIN 1056(1969), ACI. 307(1998). A considerable economy can be achieved when using tapered chimneys because the wind and seismic responses are generally reduced.

In most cases, structural engineers follow the static approach for wind and seismic analysis of chimneys. However, depending on the seismic and wind activities of the region, the dynamic analysis may be necessary.

The design wind speed in the central and southern parts of Iraq may be taken as (45 m/sec.) (D. B. Ghailan, (1993)). However, lower wind speed may cause resonance if the vortex shedding frequency is equal to any one of the modal natural frequencies. This may make the static analysis and design questionable under these circumstances.

The beam model or the conventional stiffness method may be used for the dynamic analysis of prismatic chimneys. A more elaborated (3-D) dynamic analysis of towers using shell type finite elements had been carried out (I. A. Muhammed, (1997)). The results indicated a maximum difference of 4% between the (3-D) and (2-D) (beam model) deflections.

For tapered chimneys the beam model of the dynamic analysis cannot be used directly and the beam stiffness method is to be recommended. The latter needs special care in segmental idealization of the tapered chimneys. Otherwise tapered elements may be used for a more accurate idealization.

To simplify the dynamic analysis of tapered chimneys an energy equivalence approach is suggested in the present study to obtain an equivalent uniform mass per unit length and an equivalent constant moment of inertia for the tapered chimney for any mode of vibration. The derivation of the equivalent hypothetical uniform properties for the tapered chimney makes it possible to use the well-known prismatic cantilever beam model in the dynamic analysis. Such analysis may be carried out simply for both the free and forced vibrations by coding the basic steps through a relatively short computer programs or even using any available software.

DYNAMIC ANALYSIS

The dynamic analysis considered in the present study includes the use of a beam model for prismatic section with the stiffness analysis model, the latter is used to discretize the nonprismatic (tapered) chimneys into a number of finite beam element for the purpose of the dynamic analysis.

Beam Model

The equation of dynamic equilibrium of a prismatic beam is (Paz, M., 1986), Fig. (1)

$$m \frac{\partial^2 y}{\partial t^2} + c \frac{\partial y}{\partial t} + EI \frac{\partial^4 y}{\partial x^4} = p(x,t) \quad (1)$$

where

m : Mass/Unit length.

Y : Beam deflection at (x) from origin.

T : Time (sec.).

c : Damping coefficient (N.sec./m²).

E : Modulus of elasticity (N/m²).

I : Moment of inertia (m⁴), and.

$P(x,t)$: Dynamic force intensity per unit length (N/m) at time t .

For undamped free vibration the equation will be

$$m \frac{\partial^2 y}{\partial t^2} + EI \frac{\partial^4 y}{\partial x^4} = 0 \quad (2)$$

For any mode of vibration (n)

$$y_n(t,x) = \beta_n(x) \sin(\omega_n t + \alpha_n) \quad (3)$$

where

$$\beta_n(x) = A_1 \sin \lambda_n x + A_2 \cos \lambda_n x + A_3 \sinh \lambda_n x + A_4 \cosh \lambda_n x \quad (4)$$

in which

$\beta_n(x)$: Mode shape.

A_i : Constants, $i = 1, 2, 3, 4$, that depend on the boundary conditions.

$$\lambda_n = \left(\frac{m\omega_n^2}{EI} \right)^{0.25} \tag{5}$$

ω_n : Natural frequency for mode (n).

α_n : Mode constants.

If the prismatic chimney is considered as a cantilever beam, then

$$\beta_n(x) = A_n \left[\cosh \lambda_n x - \cos \lambda_n x - \frac{(\cos \lambda_n l + \cosh \lambda_n l)(\sinh \lambda_n x - \sin \lambda_n x)}{(\sin \lambda_n l + \sinh \lambda_n l)} \right] \tag{6}$$

where

A_n : Mode constants.

l : Beam length.

The natural frequencies are

$$\omega_1 = \frac{(0.597)^2 \pi^2}{l^2} \sqrt{\frac{EI}{m}}, \quad n=1 \tag{7}$$

$$\omega_n = \frac{(n-0.5)^2 \pi^2}{l^2} \sqrt{\frac{EI}{m}}, \quad n > 1 \tag{8}$$

The dynamic response for forced vibration is

$$Y(x,t) = \sum_{n=1}^{\infty} \gamma_n(x) Y_n(t) \tag{9}$$

where

$$\gamma_n(x) = \frac{\beta_n(x)}{A_n} : \text{Normalized mode shape for mode } (n).$$

$Y_n(t)$: Generalized modal-time function for mode (n).

The $Y_n(t)$ time function for mode (n) is the solution of the modal equation

$$M_n \ddot{Y}_n + C_n \dot{Y}_n + K_n Y_n = F_n \tag{10}$$

where

$$M_n = \text{Modal mass} = \int_0^H m \gamma_n^2(x) dx \tag{11}$$

H : Chimney height.

$$C_n = \int_0^H c \gamma_n^2(x) dx = \text{modal damping.} \tag{12}$$

$$K_n = \text{Modal stiffness} = \omega_n^2 \times M_n \tag{13}$$

$$F_n = \text{Modal dynamic force} = \int_0^H p(x,t) \gamma_n(x) dx \tag{14}$$

Using Duhamel's integral, the $Y_n(t)$ function can be evaluated

$$Y_n(t) = \int_0^t \frac{F_n(\tau) \times d\tau}{m \times \omega_n} \exp[-\zeta \omega_n(t-\tau)] \sin[\zeta \omega_n(t-\tau)] \quad (15)$$

where ζ is the damping ratio.

Stiffness Method

The matrix differential equation for this method is

$$[m] \{\ddot{y}\} + [c] \{\dot{y}\} + [k] \{y\} = \{F(t)\} \quad (16)$$

In which $[m]$, $[c]$ and $[k]$ are the mass, damping and stiffness matrices respectively, $\{y\}$ is the nodal displacement vector and $\{F(t)\}$ is the nodal forces vector.

The natural frequencies and modal shapes are to be found by solving the undamped free vibration eigenvector equation⁽⁵⁾

$$|[K] - \omega^2[M]| = 0 \quad (17)$$

For the forced vibration the solution is analogous to that of the beam model. The generalized modal-time function $Y_n(t)$ may be obtained from the following normalized equation:

$$\ddot{Y}_n + 2\zeta \omega_n \dot{Y}_n + \omega_n^2 Y_n = F_n(t) \quad (18)$$

where

$$F_n(t) = \{\Phi_n\}^T \{F(t)\}$$

and $\{\Phi_n\}$ is the normalized eigenvector for mode (n), and $\{F(t)\}$ is the dynamic force vector.

PROPOSED ENERGY EQUIVALENCE APPROACH FOR THE ANALYSIS OF TAPERED CHIMNEY

To consider any change in the cross section of a tapered chimney so that the prismatic beam model can be applied, an equivalent uniform cross section may be derived as follows:

Equating the kinetic energy of the actual tapered chimney to the equivalent prismatic chimney for any mode of vibration (n)

$$\int_0^H 0.5m(x) \times [\dot{\gamma}_n(x)]^2 dx = \int_0^H 0.5m_{en} [\dot{\gamma}_n(x)]^2 dx \quad (19)$$

from which the equivalent uniform mass per unit length for mode (n) is

$$m_{en} = \frac{\int_0^H m(x) \gamma_n^2(x) dx}{\int_0^H \gamma_n^2(x) dx} \quad (20)$$

Also by equating the strain energy (due to bending only) of the actual tapered chimney to the equivalent prismatic chimney for any mode vibration (n)

$$\int_0^H \frac{(BM)_n^2(x) dx}{2EI(x)} = \int_0^H \frac{(BM)_n^2(x) dx}{2EI_{en}} \quad (21)$$

$$I_{en} = \frac{\int_0^H (BM)_n^2(x) dx}{\int_0^H [(BM)_n^2(x) / I(x)] dx} \quad (22)$$

where

$(BM)_n(x)$: Bending moment at any section (x) for mode (n) .

I_{en} : Equivalent constant moment of inertia for mode (n) .

$I(x)$: Moment of inertia at (x) from the base taken as the average between the gross and the cracked sections.

$$(BM)_n(x) = EI(x) \cdot \ddot{v}_n(x,t) = EI(x) \cdot \omega_n^2 \gamma_n(x) Y_n$$

Hence the natural frequencies for the equivalent chimney can now be evaluated by using Eqs. (7) and (8)

$$\omega_1 = (0.597\pi)^2 \left(\frac{E \cdot I_{en}}{m_{en} \cdot H^4} \right)^{0.5} \quad \text{for } n=1 \quad (23)$$

$$\omega_n = (n - 0.5)^2 \pi^2 \left(\frac{E \cdot I_{en}}{m_{en} \cdot H^4} \right)^{0.5} \quad \text{for } n > 1 \quad (24)$$

Knowing the natural frequencies ω_n and the mode shapes $\gamma_n(x)$ of different modes for the equivalent prismatic chimney, the prismatic beam model can be used, as outlined in Sec. 2.1.

DYNAMIC LOAD

The dynamic load due to wind is to be considered in the present study for the verification purposes. The wind load on chimneys, $F(t)$, Fig. (2), may be modeled as a sine wave of an amplitude that depends on the velocity, mass density of air and the chimney diameter. (Pinfeld, 1975, G. Piccardo, 2000)

Hence,

$$F(t) = 0.5 \times c_1 \times \rho \times D \times V^2 \times \sin \omega t \quad (25)$$

where

c_1 : a factor that depends on the cross-sectional shape = 0.66 for a circular shape.

ρ : air density (kg/m^3).

$$\Omega = 0.2V_r/D_c \text{ (cycles/sec.)} = \text{Vortex shedding frequency} \quad (26)$$

V_r : Wind resonance velocity (m/sec.).

D_c : Chimney's diameter at $(2H/3)$ from the base, (m).

Based on (V_r) value as obtained by considering $\omega_n = \dot{\omega}$ the wind velocity at (10 m) from the base can be estimated from the well-known velocity-height formula ((Pinfeld, G. M., 1975)

$$\frac{V(x)}{V_{10}} = \left(\frac{x}{10} \right)^{0.16} \quad (27)$$

where

x : Distance from the base, (m).

V_{10} : Wind velocity at 10m from the base (m/sec.).

$V(x)$: Wind velocity at x from the base (m/sec.).

Based on the above formulations the wind pressure $p(x,t)$ along the chimney due to vortex shedding will be

$$P(x,t) = 0.5 c_1 \times \rho \times D(x) \times V^2(x) \times \sin \omega t \quad (28)$$

For critical wind speed, the vortex shedding frequency (ω) is taken equal to any one of the natural frequencies for the chimney. Usually the first mode had been found is the dominant. (Pinfeld, G. M., 1975)

APPLICATIONS

To check the accuracy of the proposed energy equivalence analysis for tapered chimneys, a computer program has been coded in the present study to evaluate the equivalent uniform mass and the equivalent constant moment of inertia for each mode of vibration. Then the prismatic cantilever beam analysis, steps and equations outlined in Sec. 2.1, have been coded to evaluate the natural frequencies and to sum up the modal responses. The results have been compared with that of the stiffness method for a segmented chimney model using a ready computer program for frame analysis (Paz, M., 1986). The linearly tapered chimney was idealized by (20) equal length segments. This number has been decided by carrying out a convergency study for the natural frequency. The consistent mass approach was adopted to assemble the mass matrix for the chimney and the modal responses have been added.

For the evaluation of the natural frequencies, a parametric study for the effect of the thickness and diameter variation along the chimney has also been considered in the main program of the present study. The notations for the chimney's dimensions are as shown in **Fig. (3)**.

Application (No.1)

A 195 m high circular windshield having an outer diameter of 14.6 m and a thickness of 0.5 m at the base and an outer diameter of 7.3 m and a thickness of 0.2m at the top, the steel ratio (vertical) for any section is about 0.005. (Pinfeld, G. M., 1975)

Table (1) shows the results for the natural frequencies and the maximum dynamic response due to wind vortex shedding. The damping ratio was assumed to be 0.04. It is evident from these results that the differences in the natural frequencies between the stiffness and the proposed energy method are 1.44% and 17.40% for the first and second modes of vibration respectively. Although the difference for the second mode is greater than that for the first mode, the contribution of the second mode is negligible. This is attributed to the fact that the wind speed at the vortex shedding state for the second mode is greater than the maximum design wind speed (45 m/sec.) hence only the first mode contributes in the dynamic wind load. This is the most common result in dynamic wind analysis of chimneys. ((Pinfeld, G. M., 1975, D. B. Ghailan, 1993)

The proposed method gives 3% smaller maximum dynamic base moment and the tip deflection is 15.6% smaller than that obtained by using the stiffness method of analysis. The maximum dynamic bending moment at different levels of this chimney due to wind load is shown in **Fig. (4)**.

Application (No.2)

Al-Dora chimney in Baghdad is to be analyzed by using the proposed energy equivalence method. It is a reinforced concrete tapered chimney of 100m height, its diameter and thickness decrease linearly from bottom to top. The dimensions and reinforcement for this chimney are as shown in **Fig. (5)**.

The chimney was damaged during the 1991 war in Iraq by a direct hit causing a complete collapse. It had been rebuilt again starting from pile cap level as a new constructed chimney.

For this chimney, firstly the free vibration analysis has been carried out in the present study by using both the proposed energy equivalence method and the conventional stiffness method. Then a



wind dynamic analysis has been made considering the vortex shedding frequency being equal to the critical natural frequency.

Table (2) gives the results for the natural frequencies of the first three modes for the stiffness method and for the proposed energy equivalence method. The difference in results ranges between 1.2% and 5.6%. The natural frequencies after building the 120mm brick lining are also given in this table.

For the dynamic analysis due to wind load, the critical wind velocity was $V_{r1} = 12.68$ m/sec for the first mode of vibration and $V_{r2} = 76.90$ m/sec for the second mode of vibration. Hence only the first mode was considered in the estimation of the dynamic wind load due to vortex shedding since V_{r2} is greater than the maximum design wind speed considered in the present study.

Figs. (6, 7a) and 7b show the maximum dynamic deflections, shear forces and bending moments at different levels of the chimney obtained by the proposed energy equivalence method.

PARAMETRIC STUDY FOR THE NATURAL FREQUENCIES

Using the computer program of the present study the natural periods for the chimney may be written in a general equation as

$$T_n = C \frac{H^2}{D_o} \sqrt{\frac{\rho}{E}} \quad (29)$$

where

C : Mode constant.

ρ , E : Density and Modulus of elasticity for concrete, respectively, and.

D_o : Top diameter of the chimney.

The constant C depends on two non-dimensional parameters T_o/T_b and D_o/D_b . These notations are as given in **Fig. (3)**. The results for C values for $T_o/T_b = 0.2$, to 1.0 and for D_o/D_b values of 0.2, to 1.0 are plotted in **Fig. (8)**.

CONCLUSIONS

Based on the results of the present study the following conclusions may be written:

- 1- The results of the proposed energy equivalence method for the dynamic analysis are in good agreement with that of the conventional stiffness method. The differences in natural frequencies are as follows:
 - a- For the fundamental natural frequencies the differences are 1.44% and 1.20% for the 195 m and 100 m chimneys respectively.
 - b- For the second mode frequencies the differences are 17.4% and 5.6% for 195m and 100 m chimneys respectively.

Since the first mode of vibration controls the dynamic wind load in vortex shedding state the accuracy of the proposed energy method is justified.

- 2- The proposed energy equivalence method gives a maximum dynamic bending moment due to wind, about 3% smaller than that obtained by using the stiffness method. However, this difference becomes 15.6% for the dynamic tip deflection.
- 3- The 120mm brick lining for a 100 m chimney reduces the natural frequencies by 16.6%, 16.3% and 10.5% for the first, second and third modes of vibrations respectively.
- 4- The 120mm brick lining for a 100 m chimney reduces the maximum dynamic tip deflection, base shear and base moment by a bout 25%.

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NOTATIONS

A_n = mode constants.

A_i = mode constants.

C_n = modal damping.

c = damping ratio.

c_1 = wind load factor.

D = chimney diameter.

E = modulus of elasticity.

$F(n)$ = modal force.

H = chimney height.

I = moment of inertia.

K = stiffness matrix.

Kn = modal stiffness.

l = beam length.

M_n = modal mass.

m = mass per unit length.

n = mode number.

$P(x,t)$ = dynamic wind force.

t = time.

T = chimney thickness.

y = deflection.

Y_n = generalized modal time function.

V = wind speed.

ω_n = Natural frequency for mode (n).

$\beta_n(x)$ = Mode shape.

$\gamma_n(x)$ = Normalized mode shape for mode (n).

α_n = Mode constants.

Table (1) Results for Application (1). 195 m Chimney.

Mode No. (n)	ω_n (rad/sec)(No lining)		ω_n (rad/sec)(With lining) Proposed Energy Method
	Proposed Energy Method	Stiffness Method	
1	2.82	2.94	2.45
2	16.75	17.76	14.86
3	45.94	46.52	41.66

Table (2) Natural Frequencies (Al-Dora Chimney).

Parameter	Stiffness Method	Present Study	Remarks
ω_1 (rad./sec.)	2.11	2.08	
V_{r1} (m/sec.)**	16.35	16.12	<< Design wind Speed = 45(m/sec.)
ω_2 (rad./sec.)	8.53	10.33	
V_{r2} (m/sec.)***	66.09	80.04	>> Design wind Speed = 45(m/sec.)
Max. Dynamic Deflection (mm)	282.35	237.12	
Max. Dynamic Base Moment (kN.m)	233301	225604	

V_{rn} = Critical wind speed, vortex shedding for mode (n).

** V_{r1} controls the dynamic wind load.

*** V_{r2} does not control the dynamic wind load.

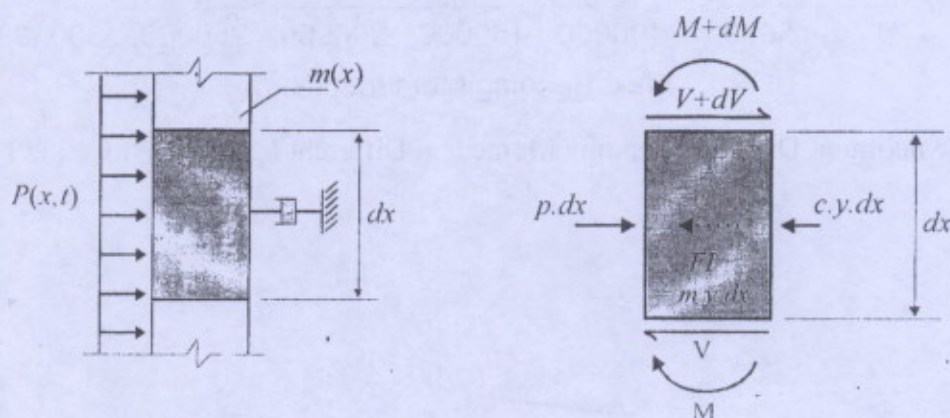


Fig. (1) Dynamic Equilibrium of Beam Element.

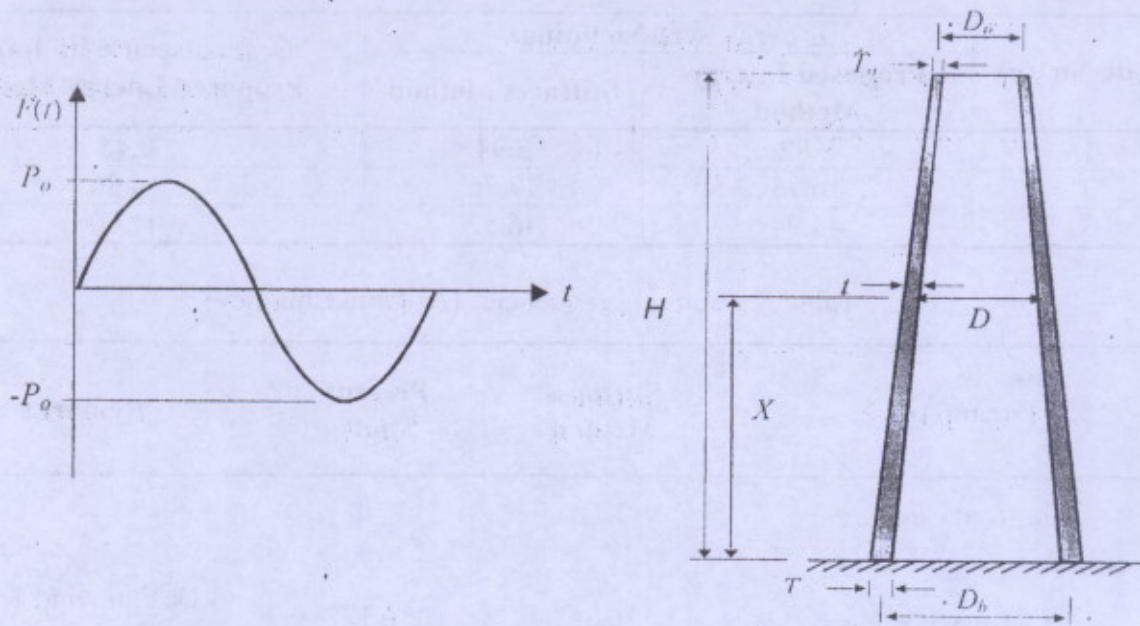


Fig. (2) Wind Load Model.

Fig. (3) Tapered Chimney Notations

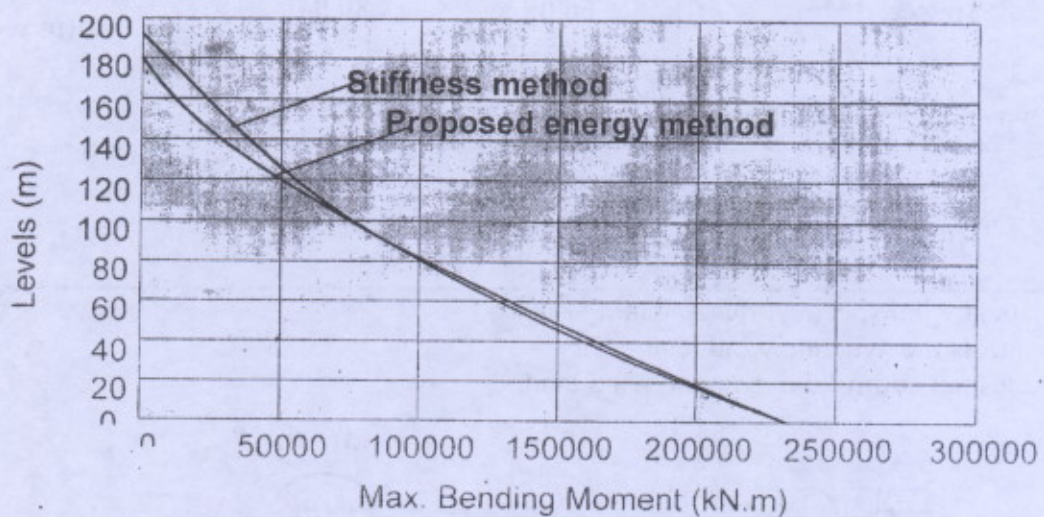


Fig. (4) Maximum Dynamic Bending Moment at Different Levels due to Wind Load.

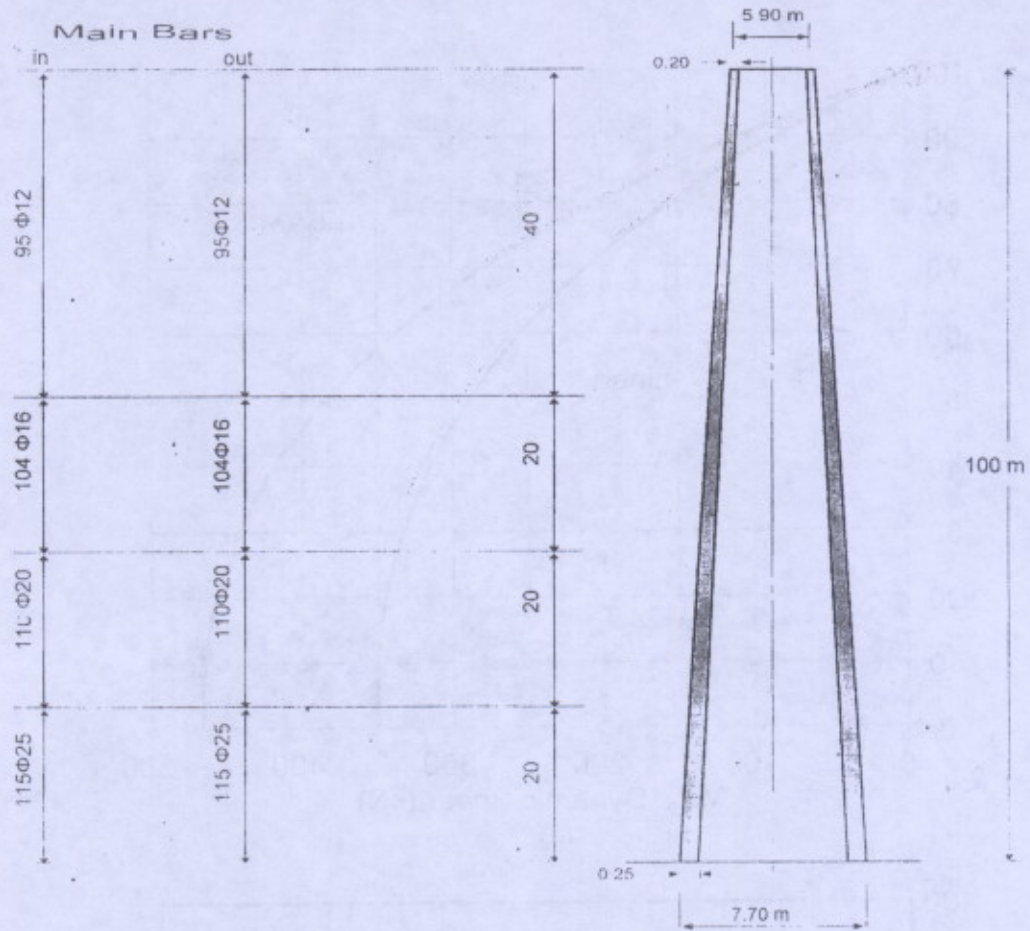


Fig. (5) Al-Dora Chimney Reinforcement.

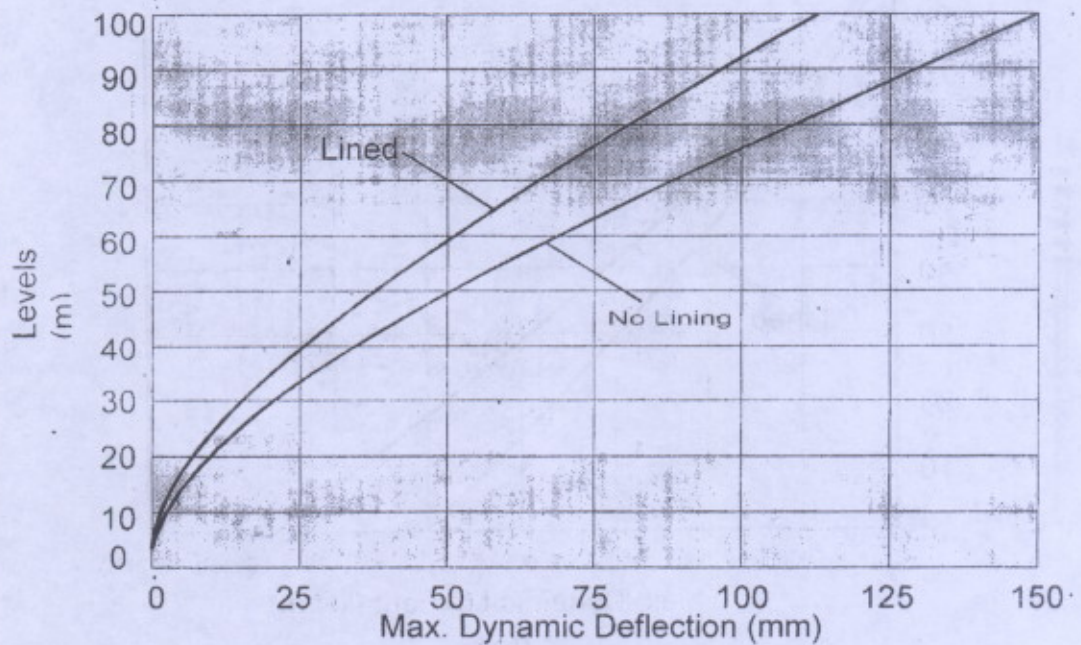


Fig. (6) Maximum Dynamic Deflection of Al-Dora Chimney Due To Wind Load.

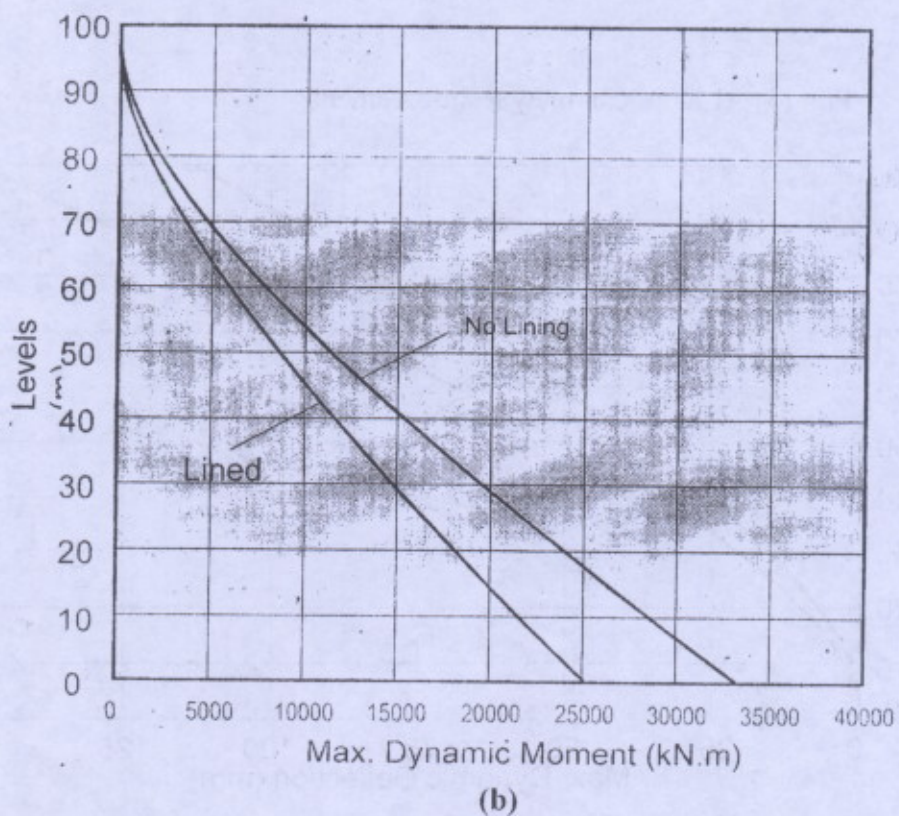
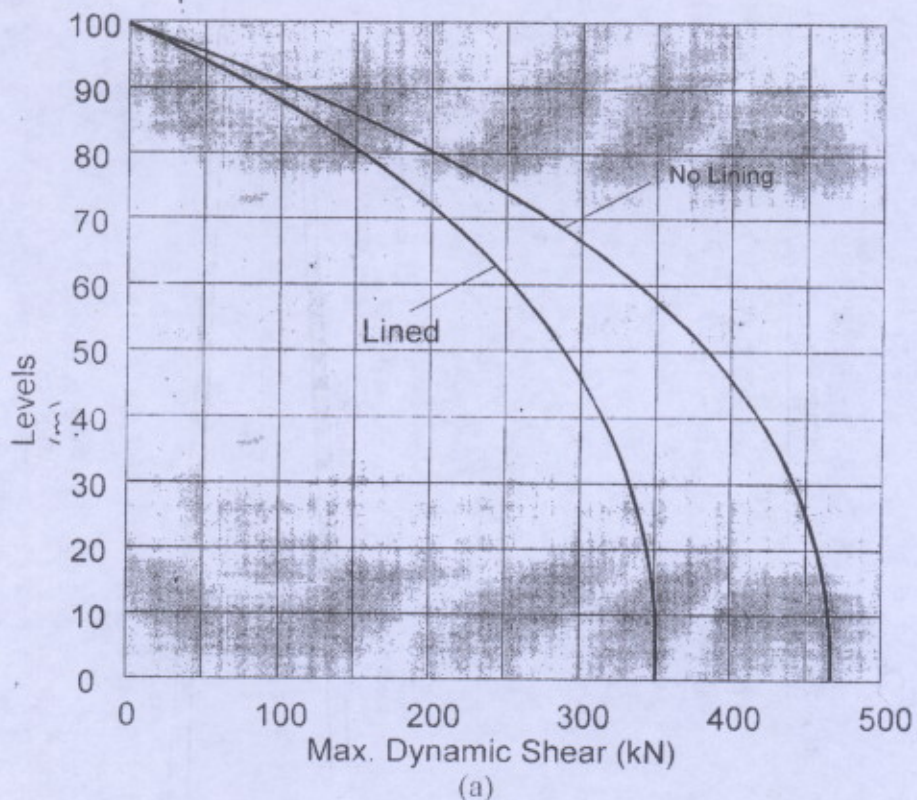
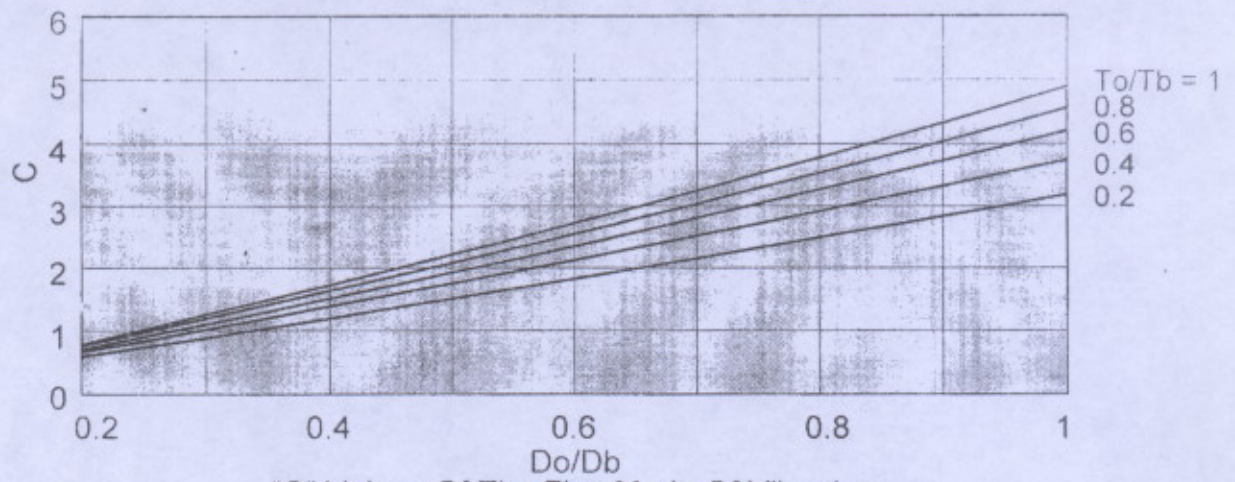
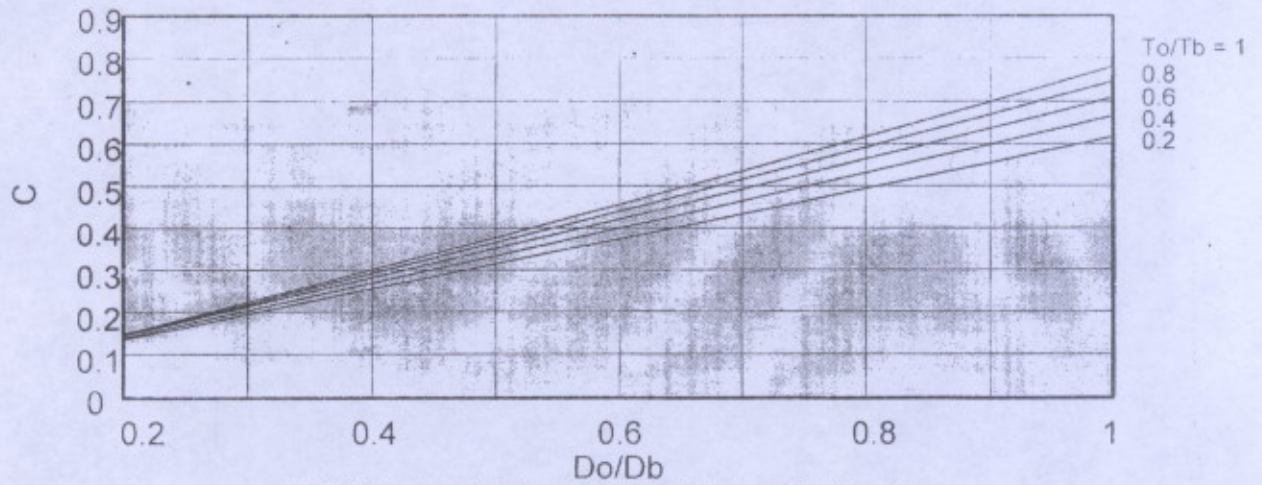


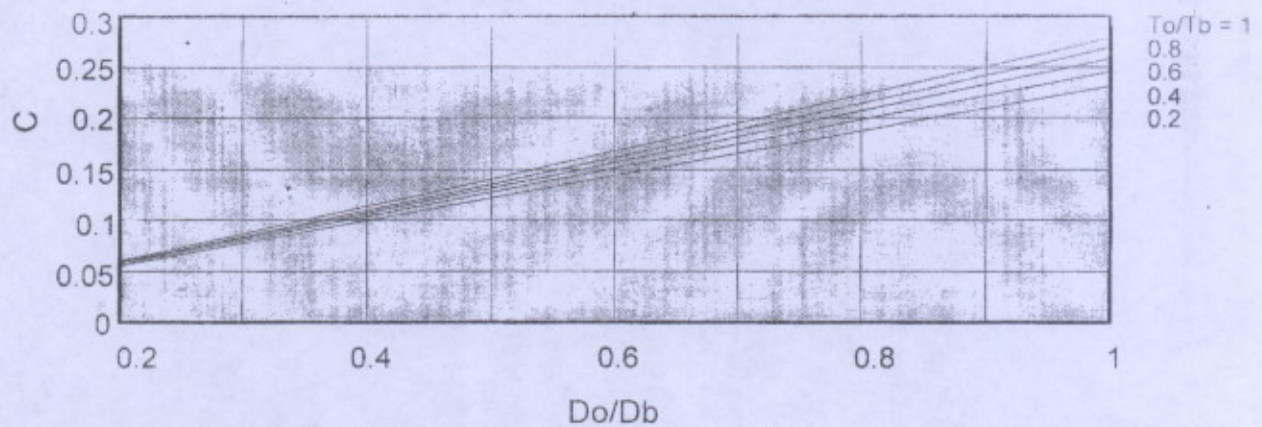
Fig. (7) (a) Maximum Dynamic Shear Forces; (b) Maximum Bending Moment, of Al-Dora Chimney Due To Wind Load.



"C" Values Of The First Mode Of Vibration



"C" Values Of The second Mode Of Vibration



"C" Values Of The third Mode Of Vibration

Fig: (8) Parametric Study For The Natural Periods.