



## USING MARKOV MODEL IN RELIABILITY ASSESSMENT OF AN ELECTRICAL POWER PLANT

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### ABSTRACT

The aim of this work is to make an evaluation to the reliability and availability of an electrical power plant as special kind of production systems to assess its ability in providing power with acceptable quality at a given period of time.

Markov model was used as an analytical tool in assessment of the reliability and availability of a production power plant in Iraq, and especially for its four new steam-power units, which considered as basic power units of this plant.

This model (Markov) was used for the first once in assessment of power production system in Iraq, and prove its ability to provide a general evaluation for the performance of the power plant during a period of time.

Since the method has too extensive mathematical operations, Matlab system (version 6.5) was used to formulize two computerized programs, once for define the mathematics model of the problem, and the other for the analysis and plot the curves.

### الخلاصة

يهدف البحث الى تقييم أداء محطة إنتاج الطاقة الكهربائية من حيث ما يعرف بالوثوقية والاتاحية لتقدير مدى قابلية المحطة كنظام إنتاجي على تجهيز الطاقة الكهربائية وبالنوعية المطلوبة خلال فترة زمنية محددة.

لقد استخدم النموذج الرياضي لماركوف ولأول مرة في تقييم الأداء الشامل لمحطة إنتاج الطاقة الكهربائية في العراق، ولقد تم اختيار إحدى محطات إنتاج الطاقة الكهربائية في بغداد كنموذج للتطبيق، حيث أثبتت هذه الطريقة إمكانية تحليل وتقييم أداء النظام لمحطة إنتاج الطاقة وخلال فترة زمنية معينة. وهذه الطريقة استلزمت بناء برنامج حاسوبي لحل المسألة لما تتطلبه من إجراءات تحليلية مطولة، ولقد استخدم نظام (Matlab version 6.5) في صياغة برنامجين أحدهما يختص بتعريف النموذج الرياضي للمسألة والآخر يقوم بالتحليل ورسم المنحنيات.

### KEY WORDS

Reliability, Availability, Fault Tree Analysis, Markov process model, Steam-Electrical power unit

### INTRODUCTION

A System in Markov model is looking to be in one of several states. One possible state, for example, is that, in which all the units composing the system are operating. Another possible state is

that in which one unit has failed but the other units continue to operate (G. Apostolakis, 1984) The main assumption in Markov process model is that the probability of a system will submit a transition from one state to another one depends only on the current state of the system and not on any previous states the system may have experienced. (Norman J., 1981)

The methodology will be presented in this paper deals with homogeneous Markov process that can be used if the failure and repair rates are constants with the considered of normal conditions of operation. Although the assumptions of constant failure and repair rates are not always desirable, both assumptions are necessary in order to avoid extensive mathematical complications.

The input necessary data, (failure & repair rates of steam-power generation units), of the presented Markov models have been obtained through the application of FT analysis at the power plant. (Soroor, 2005)

### REPRESENTATION OF THE GENERAL METHOD

To illustrate the general representation of Markov process analysis for a system with s-independent units, we consider a parallel system with two units, each of which will be in one of two states, operating or failed.

The System State is then defined as be in one of the  $2^n$  possible combinations of operating and failed units.

For the two-unit system we define the following four system states:

State	Unit 1	Unit 2
1	operating	operating
2	failed	operating
3	operating	failed
4	failed	failed

Since the two units are in parallel (redundant), only state number 4, result failure of whole system.

It is important to describe the Markov process in graphic description called "Markov diagram" or "the state-space transition diagram", that the whole information of the system is contained in such a diagram.

By giving 1 and 0 digits to operating and failed states respectively, the two-unit system Markov diagram could be represented as shown in Fig. (1).

To assess system availability, all the possible evolutions of the system have to be taken into account, so, the Markov diagram of Fig. (1), can be used without any modifications and the transition matrix [A] that corresponding to this diagram will be: <sup>(1)</sup>

$$A = \begin{bmatrix} -(\lambda_1 + \lambda_2) & \lambda_1 & \lambda_2 & 0 \\ \mu_1 & -(\mu_1 + \lambda_2) & 0 & \lambda_2 \\ \mu_2 & 0 & -(\lambda_1 + \mu_2) & \lambda_1 \\ 0 & \mu_2 & \mu_1 & -(\mu_1 + \mu_2) \end{bmatrix}$$

Where:

$\lambda$  = Failure Rate in ( $\text{hr}^{-1}$ ),

$\mu$  = Repair Rate in ( $\text{hr}^{-1}$ ),

and the sum of the elements of each row is zero, this is a feature of Markov Matrix.

This matrix deals with the differential equations of the form: <sup>(4)</sup>

$$\frac{d}{dt} P(t) = A \cdot P(t) \tag{1}$$

This equation can be written as:

$$[P_s(t)] = [A] \cdot [P_s'(t)] \tag{2}$$

where:

$[P_s(t)]$  = is the column vector of the system probability function (availability) that consists of:

$$[P_s(t)] = \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{bmatrix}$$

$[P_s(t)]$  = is the column vector of the system differential function that consists of:

$$[P(t)] = \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{bmatrix}$$

The solution of eq. (2) beginning with an initial condition represented by the probability of zero for each state matrix unless given one to the first state.

In starting of solution, a matrix of  $[P(0)]$  should be considered as the initial or boundary conditions column vector which contains the probabilities to be in any state  $i$  at time  $t = 0$ , where in our example  $i = 1, 2, 3, 4$ .

The method used to solve eq. (2) is the numerical solution of differential equations to resolve the set of differential equations that related to the problem under study in this paper and by using computer programs, which were constructed in Matlab system.

### FUNCTIONAL DESCRIPTION OF THE POWER PLANT SYSTEM

The plant has six power units and four of them denoted by the new units, which they are the basic steam-power units of the plant and they are working independently to supply the Super-Grid across 132 kV bus bar.

Through the studying of work nature of each power unit it will be found that there are three systems that affect on the reliability and operating efficiency (that represented by unit availability at any given time), these systems are the mechanical, electrical, and control systems.

Each one of the above systems has a combination of components and subsystems that connected with each other in a specified manner to give the required output power of unit.

The input data used in this paper depend on the results that obtained from Fault Tree Analysis and Minimal Cut-Sets method in analyzing the failure and repair rates for each of the three systems within the power unit.

The probability functions of the three systems within the unit are connected with each other by OR gate since the failure of any system leads to the unit failure to give its required mission.

The unit shutdown is the Top-Event that finally reached by the Fault Tree analysis of each unit as shown in **Fig. (2)**.

The Minimal Cut-Sets of mechanical system failures (events) are connected with each other by OR gate leading to the top event of the mechanical system, (MF1=Turbine Failure), as shown in the following probability function:

$$MF 1 = MF 2 Y MF 4 Y MF 5 Y MF 6 Y MF 7 Y MF 8 Y MF 9 Y MF 10 Y MF 11 Y MF 13 Y MF 14 Y MF 15 Y MF 16 Y MF 18 Y MF 19 Y MF 21 Y MF 22 \quad (3)$$

where:

MF = a type of Mechanical Components Failures, which described in **Table (1)**.

Y =Union symbol which means logically OR, and mathematically represented by "+".

The Minimal Cut-Sets of the electrical system failures (events) which lead to the top event of the system (EF1=Electrical System Failure) are related with each other by the following probability function:

$$EF 1 = EF 3 Y EF 5 Y EF 6 Y EF 7 Y EF 8 Y (EF 10 I EF 13) Y (EF 10 I EF 14) Y (EF 11 I EF 13) Y (EF 11 I EF 14) Y EF 16 Y EF 18 Y EF 19 \quad (4)$$

where:

EF= a type of Electrical Components Failures which described in **Table (2)**.

I =Intersection symbols of two events logically means AND, mathematically represented by multiplication symbol “\*”.

For control system failure (events) that leading to the top event of this system (CF1=Control System Failure) are related with each other by the following probability function:

$$CF 1 = CF 5 Y CF 6 Y CF 7 Y CF 8 Y CF 9 Y CF 10 Y CF 11 Y CF 13 Y CF 14 Y CF 15 Y CF 16 Y CF 17 Y CF 18 Y CF 19 Y CF 20 \quad (5)$$

CF= a type of control system failure which described in **Table (3)**.

The probability of top event (TE) of the power unit could be found from the probability function that resulted from **Fig. (2)** as follows:

$$P(TE) = P(MF 1) Y P(EF 1) Y P(CF 1) \quad (6)$$

The reliability expression of function (6) is expressed as follows:

$$RU = RM Y RE Y RC \quad (7)$$

where:

RM, RE, & RC = are the reliabilities of Mechanical, Electrical, & Control systems respectively for the unit to perform its function during the production cycle time (t).

By assuming a constant failure rate with time t, then: (Norman J., 1981), (Patr)

$$R(t) = e^{-\lambda t} \quad (8)$$

where:

R(t)= reliability of a component or (system) as a function of time t.

t= production cycle time of a component or (system), in (hr).

Since the unit operates daily along 24 hours, this time includes the idle times for repairing and maintenance.

The availability function of each component or (system) in a unit is obtained from the steady state equation as follows: (Norman J., 1981), (Charles E., 1997)

$$A(t) = \frac{\mu}{\mu + \lambda} \quad (9)$$

A(t) = the availability of a system with constant time (t).

The Fault Tree Analysis results of the power unit system are given by the following values: (Soroor., )

RU = 0.6051 = reliability of each unit

AU = 0.6731 = availability of each unit

For t = 24 hr, then, the resulted failure and repair rates of the unit are: (Soroor, 2005)

$\lambda = 0.0091 \text{ hr}^{-1}$

$\mu = 0.0187 \text{ hr}^{-1}$



Therefore we take the above values of failure and repair rates for each units as input data that required to the Markov Model analysis.

The whole system of the power plant is has four basic power units (denoted by the new steam-power units) are related in parallel with each other and connected with the Super-Grid across 132kV Bus-Bar.

The system of power plant could be represented as shown in **Fig. (3)**.

### THE APPLICATION OF MARKOV MODEL IN AVAILABILITY AND RELIABILITY ASSESSMENT FOR THE PRODUCTION POWER PLANT

During the normal operation of the plant, there are two basic states-of each unit, which they are:

- The unit in operating state, this state represented by "1" digit.
- The unit in a failed-state, this state represented by "0" digit.

To assess the power plant availability, all the possible evolutions of the system have to be taken into account, so that, the Markov State-Transition diagram of the availability assessment will have all the 16-transition states as shown in **Fig. (4)**.

To assess the plant reliability, only the possible evolutions of the system which will be never reach the state of failure  $\{0\ 0\ 0\}$ , have to be taken into account.

From Markov diagram, one can find the set of the first order differential equations which form the mathematical representation of the power plant system as follows:

$$\begin{aligned}
 dp_1(t)/dt &= (-4\lambda)p_1(t) + (3\mu)[p_1(t) + p_2(t) + p_3(t) + p_4(t)] \\
 dp_2(t)/dt &= \lambda p_1(t) - (\mu + 3\lambda)p_2(t) + (3\mu)[p_6(t) + p_7(t) + p_8(t)] \\
 dp_3(t)/dt &= \lambda p_1(t) - (\mu + 3\lambda)p_3(t) + (3\mu)[p_6(t) + p_9(t) + p_{10}(t)] \\
 dp_4(t)/dt &= \lambda p_1(t) - (\mu + 3\lambda)p_4(t) + (3\mu)[p_7(t) + p_9(t) + p_{11}(t)] \\
 dp_5(t)/dt &= \lambda p_1(t) - (\mu + 3\lambda)p_5(t) + (3\mu)[p_8(t) + p_{10}(t) + p_{11}(t)] \\
 dp_6(t)/dt &= \lambda p_2(t) - (\mu + 3\lambda)p_6(t) + (2\mu)[p_{12}(t) + p_{13}(t)] \\
 dp_7(t)/dt &= \lambda p_2(t) - (\mu + 3\lambda)p_7(t) + (2\mu)[p_{12}(t) + p_{13}(t)] \\
 dp_8(t)/dt &= (2\lambda)[p_2(t) + p_4(t)] + 2(\mu + \lambda)p_8(t) + (2\mu)[p_{12}(t) + p_{14}(t)] \\
 dp_9(t)/dt &= (2\lambda)[p_3(t) + p_4(t)] - 2(\mu + \lambda)p_9(t) + (2\mu)[p_{12}(t) + p_{15}(t)] \\
 dp_{10}(t)/dt &= (2\lambda)[p_3(t) + p_5(t)] - 2(\mu + \lambda)p_{10}(t) + (2\mu)[p_{13}(t) + p_{15}(t)] \\
 dp_{11}(t)/dt &= (2\lambda)[p_4(t) + p_5(t)] - 2(\mu + \lambda)p_{11}(t) + (2\mu)[p_{14}(t) + p_{15}(t)] \\
 dp_{12}(t)/dt &= (3\lambda)[p_6(t) + p_7(t) + p_9(t)] - 2(\mu + \lambda)p_{12}(t) + \mu p_{16}(t) \\
 dp_{13}(t)/dt &= (3\lambda)[p_6(t) + p_8(t) + p_{10}(t)] - (3\mu + \lambda)p_{13}(t) + \mu p_{16}(t) \\
 dp_{14}(t)/dt &= (3\lambda)[p_7(t) + p_8(t) + p_{11}(t)] - (3\mu + \lambda)p_{14}(t) + \mu p_{16}(t) \\
 dp_{15}(t)/dt &= (3\lambda)[p_9(t) + p_{10}(t) + p_{11}(t)] - (3\mu + \lambda)p_{15}(t) + \mu p_{16}(t) \\
 dp_{16}(t)/dt &= (4\lambda)[p_{12}(t) + p_{13}(t) + p_{14}(t) + p_{15}(t)] - (4\mu)p_{16}(t)
 \end{aligned}
 \tag{10}$$

The above set of equations are related to the availability estimation, and for reliability estimation the same set of equations can be used, but the probability function sixteen and each factors that related to the state (16) are negligible, since the failure states are considered as absorbing states. By using the numerical solution of differential equations in Matlab system, we were reached to the output probabilities as the availability and reliability of each system states of the power plant, as shown in the output results of the programs.

## RESULTS AND DISCUSSION

The output results of Matlab programs represent the availability and reliability of each state in the Power Plant as a whole production system of electrical power.

The availability-state space diagram (Markov State-Transition diagram) consists of 16 states as shown in **Fig. (4)**, therefore, this diagram leads to 16 probabilistic differential equations, which will be resolved by using the numerical solution of differential equations in Matlab programming.

The state curves are plotted as a function of time in **Fig. (5)** to show the variation of the probabilistic relationships with time along the 24 hours of working day.

**Fig. (5-A)** shows the system availability for the states (1 to 4). Curve no.1 of this figure shows that, the state (1) is beginning with probability of one at the start of operation life and then decreased slowly with the increasing of time. And for the other states, their probabilities increased with time in same manner as also shown in **Figs. (5-B, C, & D)**. So that, it is clarified from this study, the degradation of the system with time, since the system tend to be transferred to failed state gradually as time goes up.

To assess the reliability of the power plant, the state space diagram should be consisted of 15 states only, since the failed state, (state no. 16), should be neglected. Therefore, this leads to the reliability differential equations, same as those formed for the availability assessment, but all factors that related to the state no.16 should be neglected. These equations are also resolved by using the numerical solution of differential equations with the aid of Matlab programming.

**Fig. (6)** related to system reliability of the power plant.

**Fig. (6-A)**, curve no.1, shows that the system reliability is one at the first hour of operation time, and then decreased with the passing of daily hours. In other states, the reliability of the system increased with the passing of time during a day, as also shown in **Figs. (6-B, C, & D)**. So that, this study shows the degradation of the system reliability with the passing of time, since the system tend to transfer to a less reliability running states without having passed through the failed state.

From the plotted curves of the reliability states, one can see that, the power plant system with less reliable state is, when it is reached to the states (12), (13), (14), and (15). Therefore to overcome the conditions of these states it will be needed for a high safety requirements to being in a more reliable state for a long interval of operating time. And especially the components and subsystems of power unit, which having less reliable factors should be more taken into account to increase the safety factors of the plant performance.

## CONCLUSIONS

The power generation plant could be considered as a special type of industrial systems, where the application of reliability algorithms have a respective importance, since the required functional objective of the power plant is to produce energy with reliable factors and available quality as planned as possible.

Due to the complexity of whole generation system of the power plant, there will be a need to applied some algorithms for the system reliability evaluation, such algorithm is the fault tree analysis to analyze the relationships among the failure events of whole system reaching the failure event of entire system, which is known by the top event. Then, the application of Markov analysis model is firstly suitable for description of all possible transition states in the system by the representation of state-space diagram. And secondly, to quantifying the reliability-availability at each state of system transition states, as probability values that are predicated for a specified period of time.

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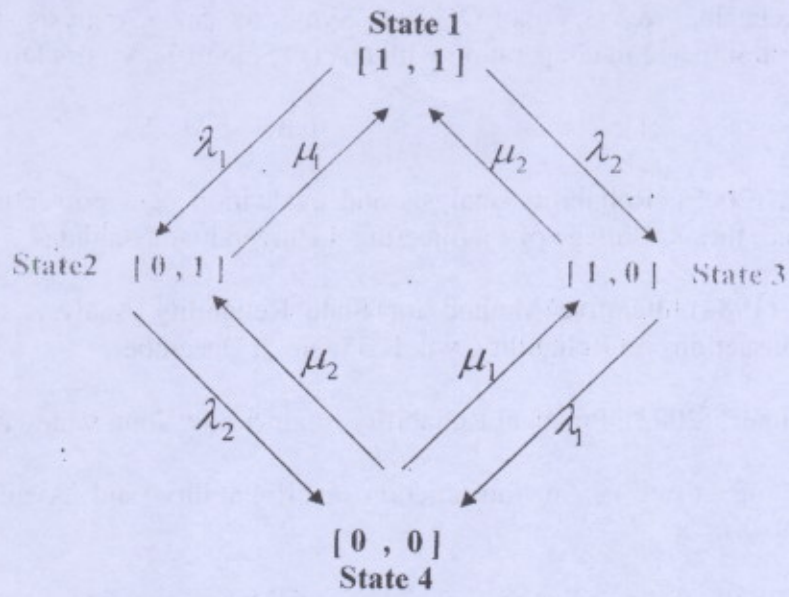


Fig. (1) Markov State-Transition Diagram of 2-Unit System

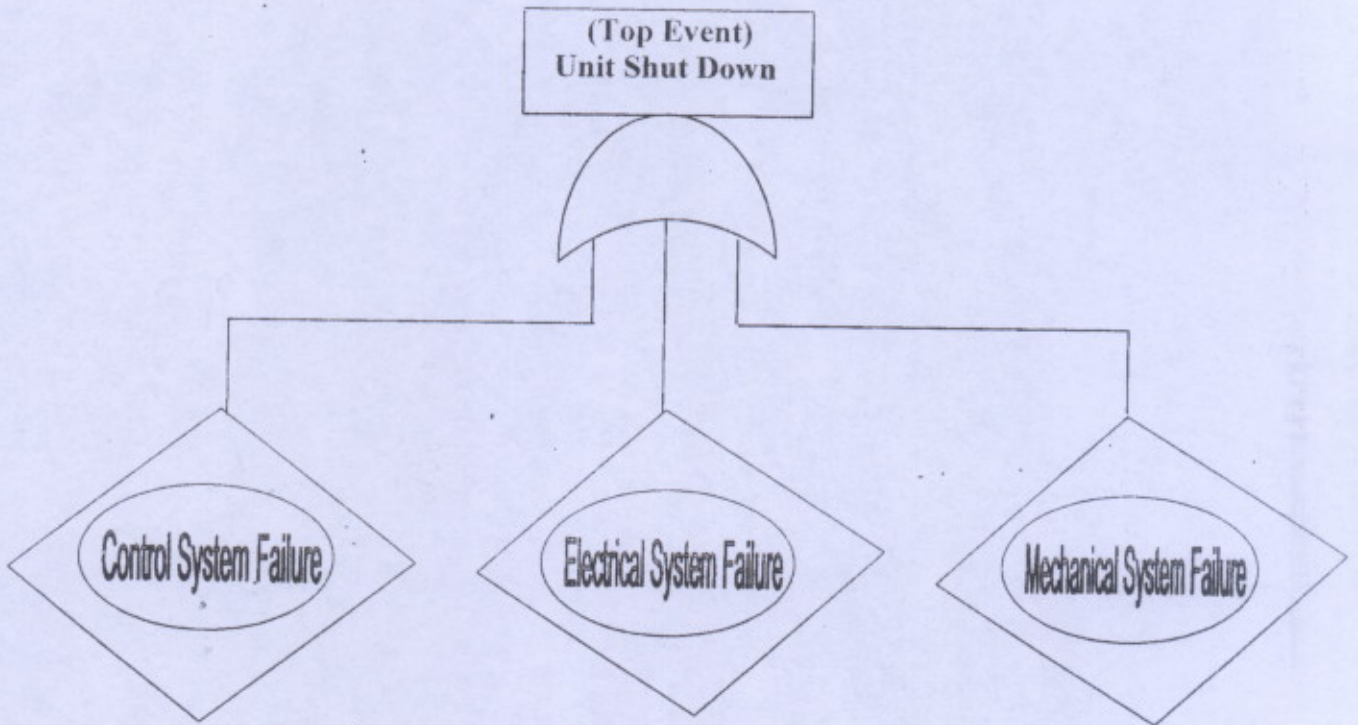


Fig. (2) The Fault Tree of Power Unit



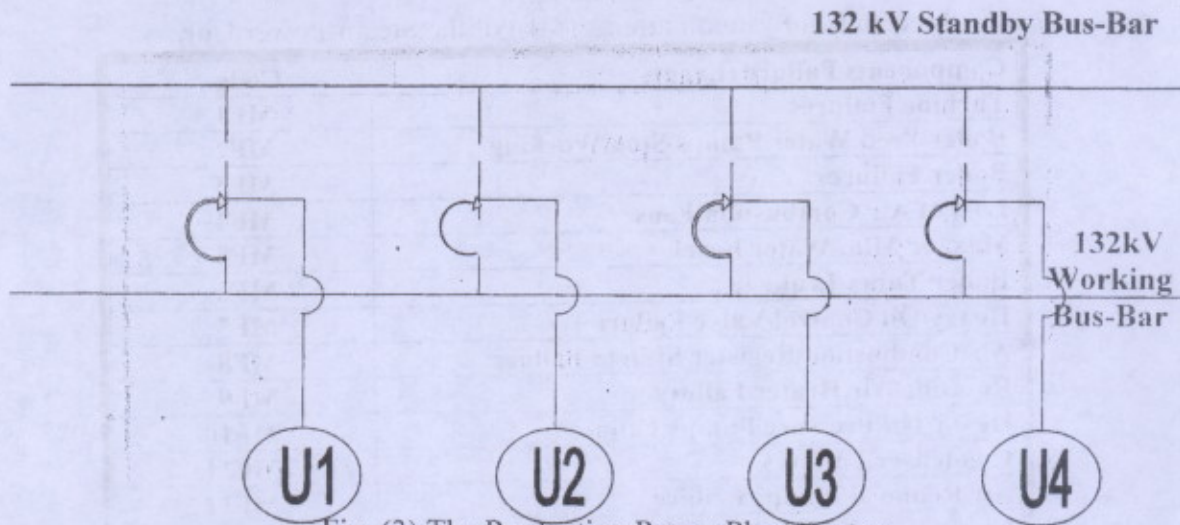


Fig. (3) The Production Power Plant System  
(The New 4-Steam Power Units)

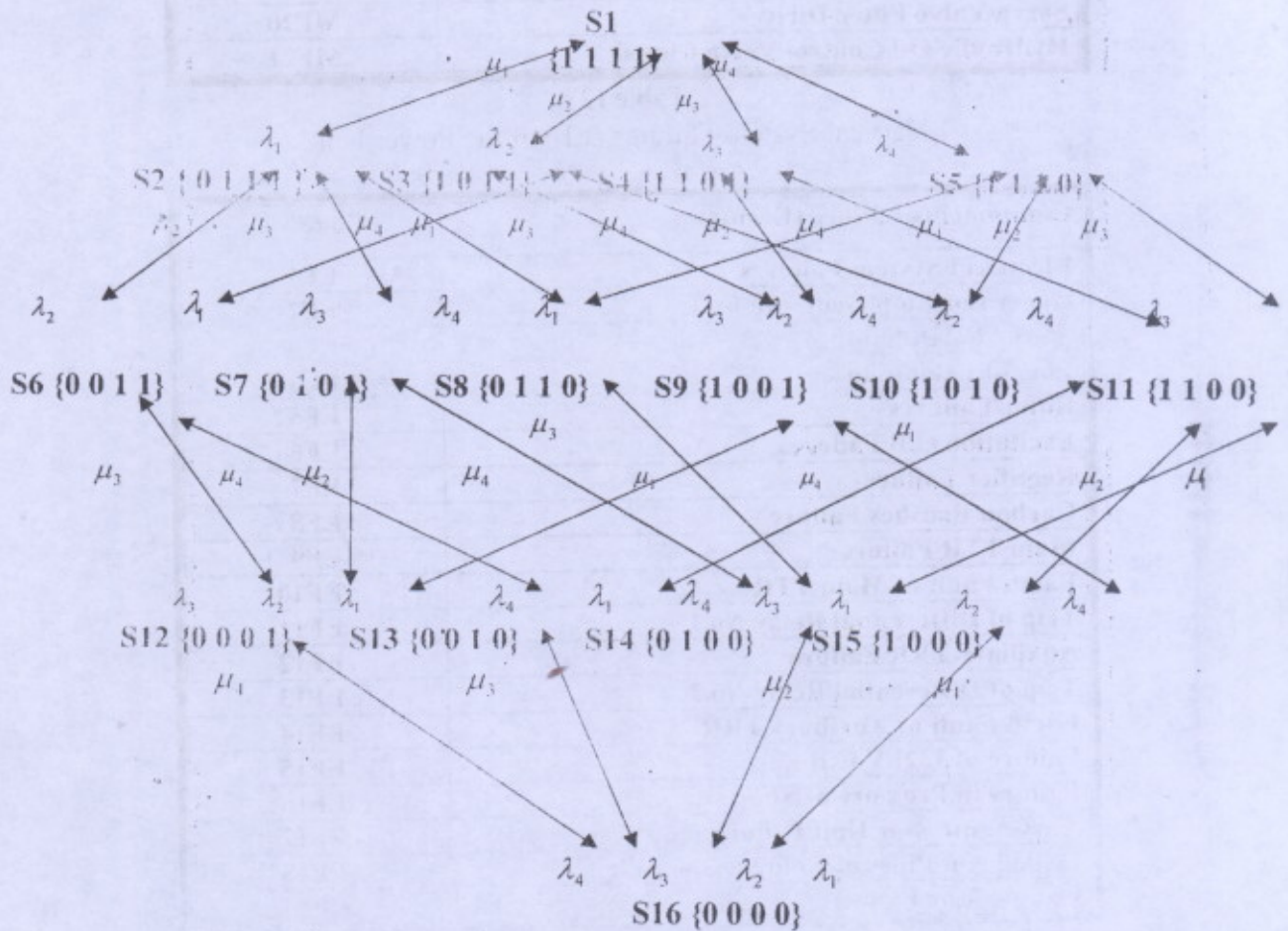


Fig. (4) Markov State -Transition Diagram of the Power Plant System

Table (1)  
Mechanical System Failures (MF) of the Steam-Power Unit:

Components Failure (Events)	Code
Turbine Failures	MF1
Boiler Feed Water Pumps Stop Working	MF2
Boiler Failures	MF3
Loss of Air Combustion Fans	MF4
Max. or Min. Water Level	MF5
Boiler Tubes Leakage	MF6
Heavy Oil Control Valve Failure	MF7
Air Combustion Register System Failure	MF8
Rotating Air Heater Failure	MF9
Heavy Oil Pressure Pumps Failure	MF10
Condenser Failures	MF11
Air Removal Pumps Failure	MF12
Circulating Water Pumps Failure	MF13
Condenser Tubes Dirty	MF14
Condensate Water Pumps Failure	MF15
Hydraulic System Failure	MF16
Hydraulic Oil Pressure Pumps Failure	MF17
Oil Level Low	MF18
Servo Valve Failures	MF19
Servo Valve Filter Dirty	MF20
Hydraulic Oil Control Valve Closed	MF21

Table (2)  
Electrical System Failures (EF) of the Power Unit

Components Failure (Events)	Code
Electrical System Failures	EF1
Generator Stop Generation	EF2
Stator Earth Fault	EF3
Loss of Excitation	EF4
Rotor Failures	EF5
Excitation C.B Failures	EF6
Rectifier Failure	EF7
Carbon Brushes Failure	EF8
Main TTR Failure	EF9
Earth Fault of Main TTR	EF10
Trip of Differential Relay No.1	EF11
Auxiliary TRR Failure	EF12
Trip of Differential Relay No.2	EF13
Earth Fault of Auxiliary TRR	EF14
Failure of 132kV C.B	EF15
Failure in Pressure of SF <sub>6</sub>	EF16
Air Compressor Unit Failure	EF17
Leakage in Pipes of Compressor Unit	EF18
Compressor Failure	EF19

TTR = Transformer  
C.B = Circuit Breaker



Table (3)  
Control System Failures (CF) of the Power Unit:

Type of Failures (Trip Signal)	Code
Control System Failure	CF1
STC System Failure	CF2
FSSS Failures	CF3
Spec 200 System Failure	CF4
Set Point Select Board Failure	CF5
Dual Set Point Board failure	CF6
Servo Amplifier Board Failure	CF7
3 kHz Board failure	CF8
Turbine Trip Signal	CF9
Flame Trip Signal	CF10
Loss of Fuel Trip Signal	CF11
High Furnace Pressure Trip Signal	CF12
Loss of FD Fan Trip Signal	CF13
High & Low Drum Level Trip Signal	CF14
Low Air Flow Trip Signal	CF15
Loss of 125V DC Trip Signal	CF16
Drum Level Control System Failure	CF17
Air Flow Control System Failure	CF18
Oil Pressure Control System Failure	CF19
DA Level Control System Failure	CF20

STC system = Steam Turbine Control system  
FSSS = Furnace Set & Supervisory System  
FD Fan = Forced Draft Fan  
DA = Dearator

