# FREE VIBRATION RESPONSE ANALYSIS OF BURIED CYLINDRICAL STORAGE TANKS

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**ABSTRACT**

This study presents a reliable and effective idealization scheme for the free vibration analysis of buried cylindrical storage tanks. The three dimensional problem is transformed into a two dimensional one by using a semi analytical finite element procedure. Conical shell of revolution element is used. to represent the cylindrical wall, top plate, and bottom plate of the tank by an appropriate method. The Combined effect of structure-soil-fluid interaction is of primary importance as concluded in this work. The soil medium is idealized by the elastic half space model, that is, linear springs are assumed to represent the structure-soil interface, added masses and viscous dampers of soil are also included. The liquid region is treated analytically; also analytical integration is used to got the added stillness and mass matrices for hydrostatic and hydrodynamic pressure effects, respectively. The free vibration characteristics of the liquid storage tank are validated against experimen.tal and theoretical results available in literatures.

**الخلاصة**

**تتضمن الدراسة تحليل الاهتزاز الحر للخزانات الاسطوانية المدفونة بأستخدام طرق ملائمة ومبسطة بحيث تم تحويل المسألة ثلاثية البعد الى مجموعة من المسائل ثنائية البعد بأستخدام طريقة شبه تحليلية ( Semi Analytical Method) .**

**لقد تم انتخاب تحليلي للخزان عبارة عن عنصر صدفي بشكل مخروط ناقص . أن من فوائد هذا العنصر هي أمكانية استخدامه لتمثيل كل من جدران الخزان الآسطواني والقاعدتين العليا والسفلى بأستخدام زاويتي تدوير مناسبة للعنصر الصدفي .**

**تم الاخذ بنظر الاعتبار التاثير المتبادل بين جدران الخزان والسائل الموجود داخله من جهه والتربة المحيطة به من جهه أخرى بحيث تم نمذجة الوسط الترابي أعتمادا على مبدأ نصف الفضاء المرن (Elastic Half Space) يفرض وجود نوابض خطية مرنة (Elastic Springs) مع كتل ومخمدات اضافية للتربة ( Added Masses and Dampers) , أما السائل الموجود داخل الخزان فقد تم تحليله رياضيا أخذا بنظر الاعتبار تاثير الضغط الهيدروستاتيكي والهيدروديناميكي للسائل أثناء الاهتزازات حيث تم اشتقاق مصفوفات جساءة وكتلة اضافية ( Added Stiffness and Masses) لتمثيل هذا التاثير وتضاف الى مصفوفات الجساءة والكتلة لعناصر جدران الخزان .**

خصائص الاهتزاز الحر للخزان تم أثباتها مقارنة مع عدد نتائج عملية ونظرية سابقة . تم أجراء دراسة موسعة تأخذ بنظر الاعتبار اهم العوامل الموثرة على التصرف الديناميكي للخزانات أثناء الاهتزاز الحر ....

## INTRODUCTION

Liquid storage tanks are important elements of lifeline and industrial facilities. These structures come in a variety of configurations; they might be elevated, ground-supported, partly or completely buried. In general, cylindrical storage tanks are widely used in practice as compared to other types because they are simple in design, efficient in resisting primary hydrostatic pressure, and can be easily constructed. The free vibration characteristics of liquid storage tanks have been studied by using various methods such as finite element methods, boundary integral techniques and variational methods. As the number and the sizes of these tanks increased, their behavior under free vibration become a matter of concern and led to investigations of their vibrational characteristics. The standard finite element models were shown to be capable, in principle, of dealing with any two or three-dimensional problems. Nevertheless, in cases where the geometry, and elastic, properties of the structure remain independent of the circumferential coordinate, semi-analytical finite element technique can be used to exchange the original Three-dimensional problem into several separate two-dimensional problems by making use of Fourier series expansion in the circumferential direction, The exchange is worthwhile because a single three-dimensional solution is usually mach more expensive than several two-dimensional solutions. In buried tank analysis, there are two aspects of interaction that must be considered:

- Interaction between the tank and the contained liquid.

- Interaction between the tank-liquid system and the surrounding medium.

The contained liquid is treated analytically as a continuum by the boundary solution technique, where the number of unknowns is substantially less than in those analyses where both tank components and liquid are subdivided into finite elements. A complete analysis of the soil-tank system by the finite element method is expensive and complicated; however, an elag.tio half-space model of the soil is employed with a finite clement model of the shell to exhibit the fundamental dynamic characteristics of the overall system

and to assess the significance of the interaction on the free vibration response of tanks.

## TANK GEOMETRY AND COORDINATE SYSTEM

A typical shape of tanks under consideration is shown in Fig. (1), it consists of a circular cylindrical, thin-walled liquid containers of radius $R$, length $L_T$, and thickness $t$. The tank is partly filled with liquid to a height, H.A cylindrical coordinate system is used with the center of the base being tile origin. The radial, circumferential, and axial coordinates are denoted $r$, $\theta$, and $z$, respectively, mid the corresponding displacement components of a point on the shell middle surface are denoted by $w$, $v$, and $u$, respectively. To describe the location of a point on the free surface during vibration, let $\xi$ measure the superelevation of that point from the quiescent liquid free surface.

## TYPES OF VIBRATIONAL MODES

The natural free vibrational modes of a circular cylindrical liquid storage tank can be viewed as a combination of four distinct types of modes [(5)], and as follows:

- Lateral vibrational modes of the tank wall itself under the action of hydrodynamic pressure, Fig.(2)

- Circumferential vibrational modes involving ($sin(n\theta)$ & $cos(n\theta)$) type modes as shown in Fig. (3).

- Low frequency sloshing modes of the contained liquid, Fig (4)

- Natural modes associated with the motion of a compressible liquid.

The effects of sloshing motion and liquid compressibility are weak [(7)] therefore, these effects are assumed to be negligible for the purpose of present work.

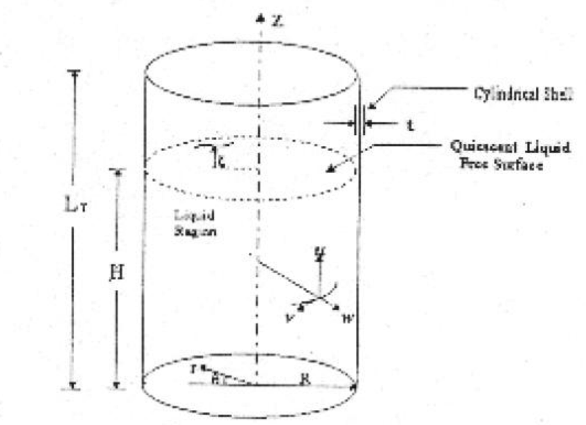



Fig.(1): Cylindrical wall tank and coordinate system

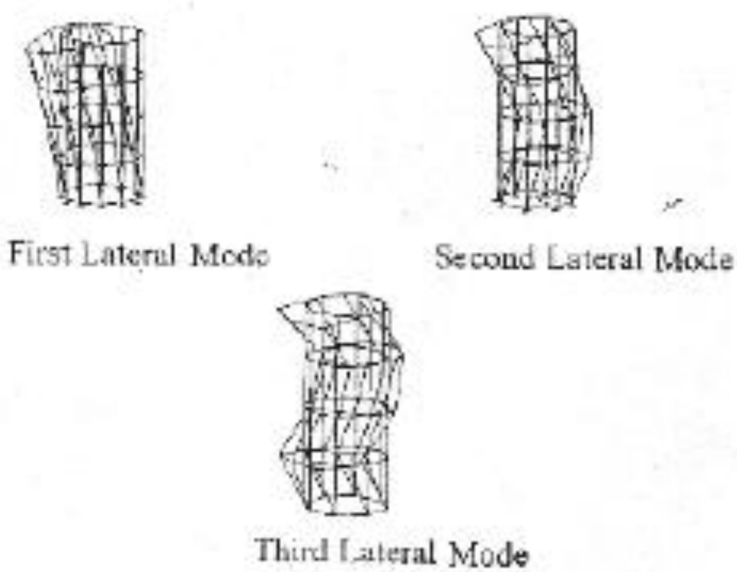



Fig.(2)Lateral Mode Shapes

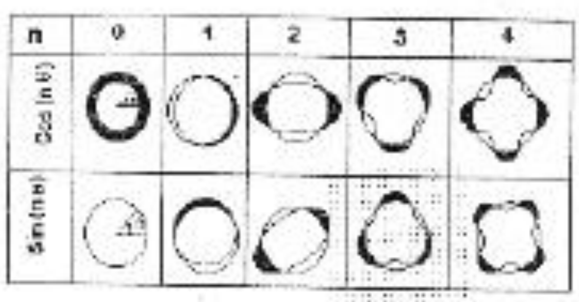
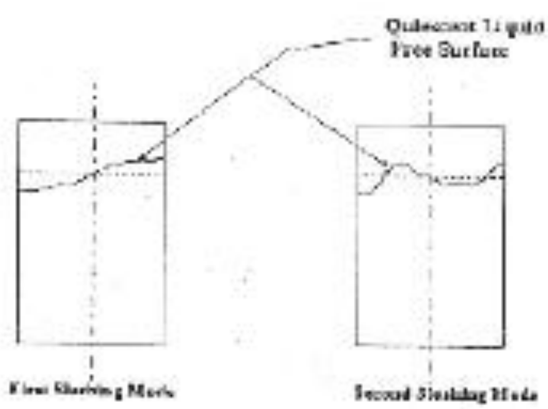



Fig (3) **Mode** Shapes of a **circular** cross
Section accordin to **Sinusoidal Functions**




Fig(4) :Sloshing modes

## FINITE ELEMENT FORMULATION

Consider the conical shell of Fig. (5). The deformation in the shell can be .expressed in terms of the middle Surface deformations $u, v$ and $w$, that is, the meridional (axial), tangential and normal displacements, respectively. It is assumed that, the displacement vector $U(r,\theta,z)$ to vary sufficiently smooth along the circumferential (i.e. 0) direction, such that $U$ may he expanded in a finite number of terms of Fourier series along the $\theta$ −ddirection [4,9], viz.

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{n=0}^{N} \begin{bmatrix} \cos(n\theta) & 0 & 0 & \sin(n\theta) & 0 & 0 \\ 0 & \sin(n\theta) & 0 & 0 & \cos(n\theta) & 0 \\ 0 & 0 & \cos(n\theta) & 0 & 0 & \sin(n\theta) \end{bmatrix} \begin{Bmatrix} u_n^s \\ v_n^s \\ w_n^s \\ u_n^a \\ v_n^a \\ w_n^a \end{Bmatrix} \ldots (1)$$

where each term of each series is called a harmonic (or a circumferential wave), $n$ is the order nf each harmonic. and $N$ is the number of Fourier terms. The subscript (s) refers to symmetric components of displacement while (a) applies to antsymmetric components; therefore, the solution becomes capable, of representing the general non-axsymmetric case. The shape functions associated with the axial and tangential displacements (i.e. $u$ and $v$) are taken to be linear between the nodal points. However, those associated with the radial displacement (i.e., Hermitian polynomials to assure slope continuity at the nodes.

For each harmonic $n$, the displacement field vector, $\{U_n\}$, in terms of the coordinate $z$ is:

$$\begin{Bmatrix} u_n^s \\ v_n^s \\ w_n^s \\ u_n^a \\ v_n^a \\ w_n^a \end{Bmatrix} = \begin{bmatrix} [f]_{3\times8} & [0] \\ [0] & [f] \end{bmatrix}_{6\times16} \{a\} \ldots\ldots (2)$$

$$\text{in which:} \quad [f] = \begin{bmatrix} 1 & \bar{z} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \bar{z} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \bar{z} & \bar{z}^2 & \bar{z}^3 \end{bmatrix} \ldots\ldots (3)$$

and the generalized coefficients vector $\{a\}$ is

$$\{a\}^T = \lfloor a_1^s \quad a_2^s \quad a_3^s \quad \ldots \quad a_8^s \quad a_1^a \quad a_2^a \quad a_3^a \quad \ldots \quad a_8^a \rfloor \ldots\ldots (4)$$

Introducing the boundary conditions at $z = 0$ and $z = L$ into Eq. (3) gives:

$$\begin{Bmatrix} \{U^{s}_{nc}\} \\ \{U^{d}_{nc}\} \end{Bmatrix} = \begin{bmatrix} [A]_{8x8} & [0] \\ [0] & [A] \end{bmatrix}_{16x16} \{a\} \quad \text{.......... (5)}$$

in which

$$\{U^{s}_{nc}\}^{T} = \begin{bmatrix} u^{s}_{n1} & v^{s}_{n1} & w^{s}_{n1} & \dfrac{\partial w^{s}_{n1}}{\partial z} & u^{s}_{n2} & v^{s}_{n2} & w^{s}_{n2} & \dfrac{\partial w^{s}_{n2}}{\partial z} \end{bmatrix} \quad \text{.......... (6-a)}$$

$$\{U^{d}_{nc}\}^{T} = \begin{bmatrix} u^{d}_{n1} & v^{d}_{n1} & w^{d}_{n1} & \dfrac{\partial w^{d}_{n1}}{\partial z} & u^{d}_{n2} & v^{d}_{n2} & w^{d}_{n2} & \dfrac{\partial w^{d}_{n2}}{\partial z} \end{bmatrix} \quad \text{.......... (6-b)}$$

and the submatrix $[A]$ is of the form:

$$[A] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & L & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L & L^{2} & L^{3} \\ 0 & 0 & 0 & 0 & 0 & 1 & 2L & 3L^{2} \end{bmatrix}_{8x8} \quad \text{.......... (7)}$$

Eq.(5) is now inverted to establish the generalized coefficients vector $\{a\}$ in terms of the generalized nodal displacement vector, $\{U_{8x8}\}$, as follows:

$$\{a\} = [AA]^{-1}\{U_{nc}\} \quad \text{.......... (8)}$$

$$\text{in which: } [AA]^{-1} = \begin{bmatrix} [A]^{-1}_{8x8} & [0] \\ [0] & [A]^{-1} \end{bmatrix}_{16x16} \quad \text{.......... (9)}$$

and the submatrix $[A]^{-1}$ is of the form:

$$[A]^{-1}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{-1}{L} & 0 & 0 & 0 & \frac{1}{L} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{L} & 0 & 0 & 0 & \frac{1}{L} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-3}{L^2} & \frac{-2}{L} & 0 & 0 & \frac{3}{L^2} & \frac{-1}{L} \\ 0 & 0 & \frac{2}{L^3} & \frac{1}{L^2} & 0 & 0 & \frac{-2}{L^3} & \frac{1}{L^2} \end{bmatrix} \quad \text{..........(10)}$$

Substituting Eq.(8) into Eq.(2) gives:

$$\{U_n\}=[N]\{U_{ne}\} \quad \text{..........(11)}$$

in which $[N]$ is a matrix of the interpolation functions given by

$$[N]=\begin{bmatrix} [NN]_{3x8} & [0] \\ [0] & [NN] \end{bmatrix}_{6x16} \quad \text{..........(12)}$$

where the submatrix $[NN]$ is of the form:

$$[NN]=\begin{bmatrix} S_1 & 0 & 0 & 0 & S_2 & 0 & 0 & 0 \\ 0 & S_1 & 0 & 0 & 0 & S_2 & 0 & 0 \\ 0 & 0 & N_1 & \hat{N}_1 & 0 & 0 & N_2 & \hat{N}2 \end{bmatrix} \quad \text{..........(13)}$$

and $S_1, S_2, N_1, \hat{N}_1, N_2$ $and$ $\hat{N}_2$

$$\left.\begin{aligned} S_1 &= 1-\frac{\bar{Z}}{L} \\ S_2 &= \frac{\bar{Z}}{L} \\ N_1 &= 1-\frac{3\bar{Z}^2}{L^2}+\frac{2\bar{Z}^3}{L^3} \\ \hat{N}_1 &= Z-\frac{2\bar{Z}^2}{L}+\frac{\bar{Z}^3}{L^2} \\ N_2 &= \frac{3Z^2}{L^2}-\frac{2\bar{Z}^3}{L^3} \\ \hat{N}_2 &= -\frac{\bar{Z}^2}{L}+\frac{\bar{Z}^3}{L^2} \end{aligned}\right\} \quad \text{.........................} (14)$$

Substitute Eq.(11) into Eq.(1) gives:

$$\{U\}=\sum_{n=0}^{N}[\theta_n][N]\{U_{no}\} \quad \text{.........................................} (15)$$

## Strain-Displacement Relations

The strain of the middle surface in terms of the middle surface displacements are given by [4,9]

$$\begin{bmatrix} \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{z\theta} \\ k_z \\ k_\theta \\ k_{z\theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial z} & 0 & 0 \\ \frac{1}{r}\sin\phi & \frac{1}{r}\frac{\partial}{\partial\theta} & \frac{1}{r}\cos\phi \\ \frac{1}{r}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial z}-\frac{1}{r}\sin\phi & 0 \\ 0 & 0 & -\frac{\partial^2}{\partial z^2} \\ 0 & \frac{1}{r^2}\cos\phi\frac{\partial}{\partial\theta} & -\frac{1}{r}\sin\phi\frac{\partial}{\partial z}-\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \\ 0 & \frac{2}{r}\cos\phi(\frac{\partial}{\partial z}-\frac{1}{r}\sin\phi) & \frac{2}{r}(\sin\phi\frac{\partial}{\partial\theta}-\frac{\partial^2}{\partial z\partial\theta}) \end{bmatrix} \begin{bmatrix} \mu \\ v \\ w \end{bmatrix} \quad \text{...} (16)$$

where $\emptyset$ is the angle shown in Fig.(5)

and $\varepsilon, \gamma$ *and* $k$ denote normal strain, shear strain and curvature, respectively.

Substituting Eq.(15) into Eq.(16) gives:

$$\{\varepsilon\}=\sum_{n=0}^{N}[p][\theta_n][N]\{U_{no}\} \quad \text{..................................} (17)$$

$$\text{or } \{\varepsilon\} = \sum_{n=0}^{N} [B]\{U_{no}\} = \sum_{n=0}^{N} \{\varepsilon_n\} \quad \text{.......(18)}$$

$$\text{in which, } \{\varepsilon\} = [B]\{U_{no}\} \quad \text{.......(19)}$$

$$\text{and } [B] = [p][\vartheta_n][N] = [\ [B_1]\ ,\ [B_2]\ ]_{6\times 16} \quad \text{.......(20)}$$

The submatrix $[B_1]$ refers to the symmetric part of the strain-displacement matrix and $[B_2]$ refers to antisymmetric part.

## CONSTITUTIVE RELATIONS

For linearly elastic homogenous and isotropic material, the force and moment resultants can be expressed in terms of the normal and shear strains in the middle surface $\varepsilon_z, \varepsilon_\theta$ $and$ $\varepsilon_{z\theta}$; in terms of midsurface changes in curvature $k_z$ and $k_\theta$ and in terms o fthe midsurface twist, $k_{z\theta}$, as follows [1,9]

$$\{\sigma\} = [D]\{\varepsilon\} \quad \text{.......(21)}$$

in which

$$\{\sigma\}^T = [N_z \quad N_\theta \quad N_{z\theta} \quad M_z \quad M_\theta \quad M_{z\theta}] \quad \text{.......(22)}$$

where $N_z$ and $N_\theta$ are referred to as the effective membrane shear force resultant and the effective twisting moment resultant, respectively,
and the matrix $[D]$ is of the form[4]:

$$[D] = \begin{bmatrix} C & \upsilon C & 0 & & & \\ \upsilon C & C & 0 & & [0] & \\ 0 & 0 & \frac{1-\upsilon}{2}C & & & \\ & & & D & D\upsilon & 0 \\ & [0] & & D\upsilon & D & 0 \\ & & & 0 & 0 & \frac{1-\upsilon}{2}D \end{bmatrix} \quad \text{.......(23)}$$

Where $[D]$ is the element constitutive matrix

$$\text{and } C = \frac{Et}{1-\upsilon^2}, \quad D = \frac{Et^3}{12(1-\upsilon^2)}$$

in which $E, t, \upsilon$ denote the modulus of elasticity; thickness of the shell element and Poisson's ratio, respectively.
Substituting Eq.(18) into Eq.(21) gives:

$$\{\sigma\} = \sum_{n=0}^{N} [D][B]\{U_{no}\} \quad \text{.......(24)}$$

## STIFFNESS & MASS MATRIX FORMULATION

The strain energy of any system is given by [1,4,5]

$$U_{1(t)} = \frac{1}{2}\int_0^{L}\int_0^{2\pi}\{\varepsilon\}^T\{\sigma\}\, r\, d\theta\, dz \quad \ldots\ldots\ldots\ldots(25)$$

Substituting Eq.(18) and Eq.(24) into Eq.(25) gives:

$$U_{1(t)} = \frac{1}{2}\int_0^{L}\int_0^{2\pi}\sum_{n=0}^{N}\sum_{n=0}^{N}\{U_{no}\}^T[B]^T[D][B]\{U_{no}\}\, r\, d\theta\, dz \quad \ldots(26)$$

For the $n^{th}$ term, Eq.(26) may be expressed conviently in terms of the element stiffness matrix $[K]_e$ as:

$$U_{1(t)} = \frac{1}{2}\sum_{e=1}^{NEL}\{U_{no}\}^T[K]_e\{U_{no}\} \quad \ldots\ldots\ldots\ldots(27)$$

In which, e is the subscript including "element"; NEL is the number of shell element along the shell length.
and $[K]_e$ is of the form

$$[K]_e = \int_0^{L}\int_0^{2\pi}[B]^T[D][B]\, r\, d\theta\, d\bar{z} \quad \ldots\ldots\ldots\ldots(28)$$

The kinetic energy of the shell, neglecting rotary inertia can be expressed as [1,4,5]

$$T(t) = \frac{1}{2}\int_0^{L}\int_0^{2\pi}\rho\, t\{\dot{U}_n\}^T\{\dot{U}_n\}\, r\, d\theta\, dZ \quad \ldots\ldots\ldots\ldots(29)$$

In which $\rho$ is the mass of the shell per unit volume; $\{U_n\}$ is the displacement vector defined by Eq.(11) and the dots ( ˙ ) denotes differentiation with respect to the time $t$.
sub. Eq.(11) into Eq.(29) results in:

$$T(t) = \frac{1}{2}\sum_{e=1}^{NEL} r\rho t\int_0^{L}\int_0^{2\pi}\{\dot{U}_{no}\}^T[N]^T[N]\{\dot{U}_{no}\}\, d\theta\, d\bar{z} \quad \ldots\ldots(30)$$

Eq.(30) can be written as:

$$T(t) = \frac{1}{2}\sum_{e=1}^{NEL}\{\dot{U}_{no}\}^T[M]_e\{\dot{U}_{no}\} \quad \ldots\ldots\ldots\ldots(31)$$

In which, $[M]_e$ is the consistent mass matrix of the element which can be defined by:

$$[M]_e = r\rho t\int_0^{L}\int_0^{2\pi}[N]^T[N]\, d\theta\, d\bar{z} \quad \ldots\ldots\ldots\ldots(32)$$

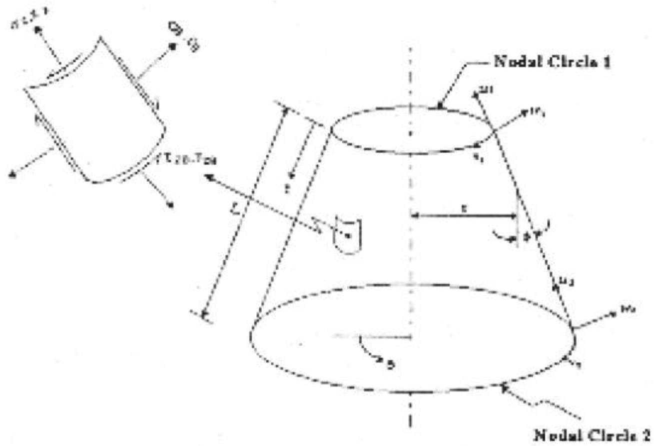



Fig.(5):Conical shell element geometry and internal forces

## MATHEMATICAL MODEL

The components of a typical buried cylindrical tank arc explained briefly as below:

- Tank wall: The geometrically axisymmetric shell is discretized as a series of frustums of connected at their modal point circles. In the case of a cylindrical tank, the cone shall be changed to a cylinder since the radius has a constant value. This means that, the angle $\phi$ indicated in Fig.(5), becomes equal to zero. See Fig(6-a).
- Top and bottom plates: making use of the conical shell element, these plates shall be represented by several elements after putting $\phi = 90$. This concept is shown in Fig.(6-a). A small hole at the center of plate is used to avoid the singularity at the center r=0.
- Contained liquid: the liquid region is treated by an analytical model taking into a count the effect of the hydrodynamic pressure and the initial hoop stresses due to the hydrostatic pressure.

These effect are estimated by considerable details and will be discussed later.

- Surrounding soil. An elastic half space model is used to represent the soil-tank interaction. The interaction system represented by a set of discrete 9lump0 mass springs, and dashpots as shown in Fig.(7). The coefficient m, K and C, for this model are evaluated by the method of continuum mechanics[3].

## IDEALIZATION OF LIQUID

**Equation Governing Liquid Motion**

For the irrotational flow of an incompressible inviscid liquid, the velocity potential function, $\Phi(r,\theta,Z,t)$ satisfying Laplace equation is giving by [5,9]:

$$\nabla^2 \Phi = 0 \quad \text{.......} \quad (33\text{-a})$$

In the region occupied by the liquid $(0 \le r \le R, 0 \le \theta \le 2\pi, 0 \le Z \le H)$

in which

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \quad \text{.......} \quad (33\text{-b})$$

In addition to being a harmonic function, $\Phi$ must satisfy the proper boundary conditions. The velocity vector of the liquid is the gradient of the velocity potential, and consequently, the liquid-container boundary conditions can be expressed as follows [5]:

- At the tank bottom (assumed rigid), Z=0, the liquid velocity in the vertical direction is zero.

$$\frac{\partial \Phi}{\partial z}(r,\theta,0,t)=0 \quad \ldots\ldots (34)$$

- The liquid adjacent to the wall of the elastic shell r = R, must move radially with the same velocity of the shell:

$$\frac{\partial \Phi}{\partial r}(R,\theta,Z,t)=\frac{\partial w}{\partial t}(0,Z,t) \quad \ldots\ldots (35)$$

In which $w(\theta,Z,t)$ is the shell radial displacement of the tank shell.

- At the liquid free surface, $Z=H+\xi(r\theta,t)$ two boundary conditions must be imposed. If the sloshing free surface are neglected [5,6,7], only one condition need to be specified at the surface namly:

$$\frac{\partial \Phi}{\partial t}(r,\theta,H,t)=0 \quad \ldots\ldots (36)$$

The solution $\Phi(r,\theta,Z,t)$ which satisfies the boundary conditions at the rigid bottom plate Eq.(43), and the quiescent liquid free surface Eq.(36), can be expressed as:

$$\Phi(r,\theta,z,t)=\sum_{n=1}^{\infty}\sum_{i=1}^{\infty}\left[A_{ni} I_n(\alpha_i r)\cos(\alpha_i z)\cos(n\theta)\right] \quad \ldots\ldots (37)$$

in which $\alpha_i$ is given by :

$$\alpha_i=\frac{(2i-1)\pi}{2H} \quad ; \quad (i=1,2,3,\ldots\ldots) \quad \ldots\ldots (38)$$

And $I_n$ is the modified Bessel function of the first kind of order $n$

The remaining boundary condition at the liquid shell interface, Eq.(35), can be written as:

$$\sum_{i=1}^{\infty}\left[A_{ni}\alpha I'_n(\alpha_i R)\cos(\alpha_i z)\right] = \dot{w}_n(z,t) \quad \ldots\ldots (39)$$

After the appropriate algebraic manipulations of Eq.(39), the following expression for $A_{ni}(t)$ result:

$$A_{ni}=\frac{2\int_0^H \dot{w}_n(z,t)\cos(\alpha_i z)\,dz}{\alpha_i H \; I'_n(\alpha_i R)} \quad ; (i=1,2,3,\ldots) \quad \ldots(40)$$

in which $I'_n(\alpha,R)$ is the relative of the modified Bessel function

the pressure distribution, $P(r,\theta,Z,t)$ can be determined from the Bernoulli equation and is given by:

$$P(r,\theta,Z,t)=-\rho_L\frac{\partial \Phi}{\partial t}+\rho_L g(H-Z) \quad \ldots\ldots (41)$$

in which $\rho_L$ is the mass density of the liquid and g is the acceleration of gravity.

The hydrodynamic pressure can therefore be expressed as:

$$P_d(R,\theta,Z,t) = -\rho_L \frac{\partial \Phi}{\partial t}(R,\theta,Z,t)$$
$$= \frac{-2\rho_L}{H} \sum_{i=1}^{\infty} \frac{\int_0^H \ddot{w}_n(z,t)\cos(\alpha_i z)dz}{\alpha_i I_n'(\alpha_i R)} I_n(\alpha_i R).\cos(\alpha_i R).\cos(n\theta) \quad \ldots(42)$$

## HYDRODYNAMIC PRESSURE

**Evaluation of the Added Mass Matrix**

As a consequence of neglecting the free surface oscillation methods, the motion of the tank wall can be analyzed by introducing an additional matrix in the matrix equation of shell motion, such addition represents the effect of liquid dynamic pressure during vibration. The hydrodynamic pressure exerted on the wall of the tank is given by Eq.(42) and therefore, the work done by such pressure though an arbitrary virtual displacement, $\delta\omega_n \cos(n\theta)$ refers to symmetric component of displacement, can be written as:

$$\delta W = \int_0^H \int_0^{2\pi} \{P_d(R,0,Z,t)\,\delta w_n \cos(n\theta)\} R\, d\theta\, dZ$$
$$= -\sum_{i=1}^{\infty} b_m \left(\int_0^H \delta w_n \cos(\alpha_i z) dz\right)\left(\int_0^H \ddot{w}_n \cos(\alpha_i z) dz\right) \quad \ldots.(43)$$

in which, $b_m = \frac{2\pi R \rho_L I_n(\alpha_i R)}{H \alpha_i I_n'(\alpha_i R)}$ ....................(44)

The work expression, Eq.(43), gives rise to the definition of the added mass matrix [DM]. In order to compute its elements, one has to express the integral in Eq.(43) in terms of the nodal displacement vector $\{U_{no}\}$. With the aid of the finite element displacement modal, one can write:

$$\int_0^H \ddot{w}_n(z,t)\cos(\alpha_i z)dz = \sum_{e=1}^{NEH} \int_0^L \{Nd(\bar{z})\}^T \{\ddot{U}_{no}(t)\}_e \cos\{\alpha_i[\bar{z} + (e-1)L]\} d\bar{z} \ldots(45)$$

Where NEH is the number of shell elemnt id contact with liquid.

By definition the vector $\{f^{(i)}\}$, as the integrals

$$\{f^{(i)}\} = \int_0^L \{Nd(\bar{z})\}^T \cos\{\alpha_i[\bar{z} + (e-1)L]\} d\bar{z}$$
$$= [0,\ 0,\ f_3^{(i)},\ f_4^{(i)},\ 0,\ 0,\ f_7^{(i)},\ f_8^{(i)}]_e \quad \ldots\ldots\ldots\ldots.(46\text{- a})$$

and $\{Nd(\bar{z})\}^T = [0\quad 0\quad N_1(\bar{z})\quad \hat{N}_1(\bar{z})\quad 0\quad 0\quad N_2(\bar{z})\quad \hat{N}_2(\bar{z})] \ldots.(46\text{-b})$

The vector $\{f^{(i)}\}$ can be defined as:

$$\{F^{(i)}\} = \sum_{e=1}^{NEH} \{f^{(i)}\} \quad \ldots\ldots\ldots(47)$$

one can rewrite Eq.(45) as follows:

$$\int_0^H \ddot{w}_n \cos(\alpha_i z)dz = \{F^{(i)}\}^T \{\ddot{U}_{no}\} \quad \text{......} (48)$$

Therefore the expression of the work done, Eq.(43), becomes

$$\delta W = -\sum_{i=1}^{\infty} b_{ni} \{\delta U_{no}\}^T \{F^{(i)}\} \{F^{(i)}\}^T \{\ddot{U}_{no}\}$$

$$= -\{\delta U_{no}\}^T (\sum_{i=1}^{\infty} b_{ni} \{F^{(i)}\}\{F^{(i)}\}^T)\{\ddot{U}_{no}\} \quad \text{......} (49)$$

Eq.(49) leads to the definition of the added mass matrix, [DM], as:

$$[DM] = \sum_{i=1}^{\infty} b_{ni} \{F^{(i)}\}\{F^{(i)}\}^T \quad \text{......} (50)$$

## HYDRODYNAMIC PRESSURE

### Effect of Initial Hoop Stress

Due to presence of the liquid, tank walls are subjected to hydrostatic pressure which cause hoop tensions. The pressure of such stresses affects the vibrational characteristics of the shell, especially the $\cos(n\theta)$ *and* $\sin(n\theta)$ mode types, these mode types are shown in Fig.(3).

To incorporate these effects, it is necessary to modify the strain energy expression of the shell and to generalize accordingly the equations of motion. Upon using the finite element modal, the matrix equation can be easily derived, and takes the familiar form with an added stiffness matrix due to the presence of the stress field.

## MODIFICATION OF POTENTIAL ENERGY OF SHELL

Consider a circular cylindrical shell acted upon by a static initial stress field which is in equilibrium. The initial stresses in the shell result from the hydrostatic pressure. During vibration, the shell stresses consist of the initial stresses plus additional vibratory stresses. In the subsequent analysis, the bending stresses produced by the initial loading are neglected, i.e., only the initial membrane stresses are considered

Since the initial stress state is in equilibrium, the potential energy of the system in this state may be taken as the reference level. Thus, the internal strain energy of the shell can be written as:

$$\bar{U}(t) = U_1(t) + U_2(t) \quad \text{......} (51)$$

in which $U_1(t)$ is the strain energy employed in deriving the stiffness matrix [K], of the shell and it is defined by Eq.(25) and $U_2(t)$ is given by

$$U_2(t) = \int_0^H \int_0^{2\pi} (N_\theta \varepsilon_\theta) R \, d\theta \, dZ \quad \text{......} (52)$$

in which $N_o$ is the initial membrane force resultant in the circumferential direction, and $\varepsilon_\theta$ is the midsurface strain

Since the initial hoop stress may be large, it is necessary to use the second-order nonlinear strain-displacement equation in the second term of Eq.(51) while using only the linear relationship in the first term [6]

This maintains the proper homogeneity in the order of magnitude of the terms in the integrands. The midsurface strain in Eq.(52), therefore, it can be expressed as

$$\varepsilon_\theta = \frac{1}{R}\left[\frac{\partial v}{\partial \theta} + w\right] + \frac{1}{2}\left\{\left(\frac{1}{R}\frac{\partial u}{\partial \theta}\right)^2 + \left[\frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right)\right]^2 + \left[\frac{1}{R}\left(v - \frac{\partial w}{\partial \theta}\right)\right]^2\right\} \quad ..(53)$$

The initial force resultant, $N_\theta$ and the liquid hydrostatic pressure, $P_s$, are in equilibrium, and therefore, it satisfies

$$N_\theta = \rho_1 g R(H - Z); \quad \frac{\partial N_\theta}{\partial \theta} = 0 \quad \ldots\ldots(54)$$

**EVALUATION OF THE ADDED STIFFNESS MATRIX**

Since $N_\theta$ is not a function of $\theta$, the strain energy $U_2(t)$ can be written as:

$$U_2(t) = R\int_0^H \left[N_\theta \left(\int_0^{2\pi} \varepsilon_\theta d\theta\right)\right] dz \quad \ldots\ldots (55\text{-a})$$

The strain-displacement relation Eq.(53) is the inserted into the strain energy expression, Eq.(55-a). however, the linear terms of Eq.(53) do not contribute to $U_2(t)$ since

$$\int_0^{2\pi} \cos(n\theta)d\theta = 0 \qquad (n \geq 1) \quad \ldots\ldots (55\text{-b})$$

Furthermore, the terms can be expressed more conveniently in the following matrix form:

$$\varepsilon_\theta^{nL} = \frac{1}{2}([\bar{p}]\{U\})^T([\bar{p}]\{U\}) \quad \ldots\ldots (56)$$

in which {U} is the displacement vector, Eq.(1) [P] is a differential matrix given by:

$$[\bar{P}] = \frac{1}{R}\begin{bmatrix} \frac{\partial}{\partial \theta} & 0 & 0 \\ 0 & \frac{\partial}{\partial \theta} & 1 \\ 0 & 1 & -\frac{\partial}{\partial \theta} \end{bmatrix} \quad \ldots\ldots (57)$$

and the superscript nL indicates "nonlinear:. With the aid of Eq.(1) and (57), Eq.(56) can be expressed as:

$$\varepsilon_\theta^{nL} = \frac{1}{2}([\bar{p}][\theta_n]\{U_n\})^T([\bar{p}][\theta_n]\{d_n\}) = \frac{1}{2}\{U_n\}^T[\bar{p}_n]^T[\bar{p}_n]\{U_n\} \quad \ldots.. (58)$$

in which,

$$[\bar{p}_n] = [\bar{p}][\theta_n]$$

$$= \frac{1}{R}\begin{bmatrix} -n\sin(n\theta) & 0 & 0 & n\cos(n\theta) & 0 & 0 \\ 0 & n\cos(n\theta) & \cos(n\theta) & 0 & -n\sin(n\theta) & \sin(n\theta) \\ 0 & \sin(n\theta) & n\sin(n\theta) & 0 & \cos(n\theta) & -n\cos(n\theta) \end{bmatrix} \quad \ldots (59)$$

Inserting Eq.(59) into the strain energy expression (Eq.55), one obtains

$$U_2(t)=\frac{R}{2}\int_0^H\left[N_\theta\{U_n\}^T\left(\int_0^{2\pi}[\overline{p}_n]^T[\overline{p}_n]d\theta\right)\{U_n\}\right]dz$$

$$=\frac{\pi}{2R}\int_0^H(N_\theta\{U_n\}^T[C_n]\{U_n\})\,dZ \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (60)$$

in which, $$[C_n]=\begin{bmatrix} n^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & n^2+1 & 2n & 0 & 0 & 0 \\ 0 & 2n & n^2-1 & 0 & 0 & 0 \\ 0 & 0 & 0 & n^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & n^2+1 & -2n \\ 0 & 0 & 0 & 0 & -2n & n^2+1 \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots \quad (61)$$

By using the displacement model, Eq.(11), one can write

$$U_2(t)=\frac{1}{2}\sum_{e=1}^{NEH}\{U_{no}\}_e^T[K_H]_e\{U_{no}\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (62)$$

In which $[K_H]_e$ is the element added stiffness matrix which given by:

$$[K_H]_e=\frac{\pi}{R}\int_0^{\ell}\left\{N_\theta(\overline{z})\left([N(\overline{z})]^T[C_n][N(\overline{z})]\right)\right\}d\overline{z} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (63)$$

## IDEALIZATION OF SOIL

The elastic half-space type is used to model the influence of the soil during vibration. The soil in this model is assumed to be homogenous, isotropic, and elastic, and characterized by shear modulus $(G_s)$ and Poisson's ratio $(\nu_s)$.

The soil is replaced by a set of an equivalent spring-mass-dashpot system as shown in Fig. (7).There various method used for estimating the spring constant (Ks) viscous damper (Cs) and added mass (mi) of the soil model. The most commonly used approach is to employ formulas from the theory of elasticity, which is used in this study. Formulas tire given in Table (1)

Table(1):Equivalent discrete properties for elastic half-space [3]

| Degree of freedom | Spring Constant ($K_s$) | Viscous Damper ($C_s$) | Added Mass ($m_s$) |
|---|---|---|---|
| Vertical | $\frac{4G_s r_0}{(1-\upsilon_s)}$ | $1.79\sqrt{K_s \rho_s r_0^3}$ | $1.5\rho_s r_0^3$ |
| Horizontal | $\frac{18.2G_s r_0(1-\upsilon_s^2)}{(2-\upsilon_s)^2}$ | $1.08\sqrt{K_s \rho_s r_0^3}$ | $0.28\rho_s r_0^3$ |
| Rocking | $\frac{2.7G_s r_0^3}{(1-\upsilon_s)}$ (Ref.49) | $0.47\sqrt{K_s \rho_s r_0^5}$ | $0.49\rho_s r_0^5$ |

Where

$r_0$ = radius of circular plate.

$\upsilon_s$ = Poisson's ratio of the soil

$\rho_s$ = mass density of the soil

$G_s$ = shear modulus of the soil which can be evaluated using following the equation [8]:

$$G_s = \rho_s V_s^2 \qquad \text{.......................... (64)}$$

in which $V_s$ = shear wave velocity

The radius ($r_0$) in Table (1) is directly used for calculations of the bottom circular plate, with equivalent radius is needed for calculations of the cylintlrical wall for the tank. On basis of equivalent areas, (surface area of the cylindrical wall is equal to the area of circular plate). The equivalent radius may be determined from the following relationship[6]

$r_0 = \sqrt{2RL}$ for horizontal and vertical degrees of freedom

$r_0 = 4\sqrt{\frac{2RL^3}{3}}$ for rocking degree of freedom

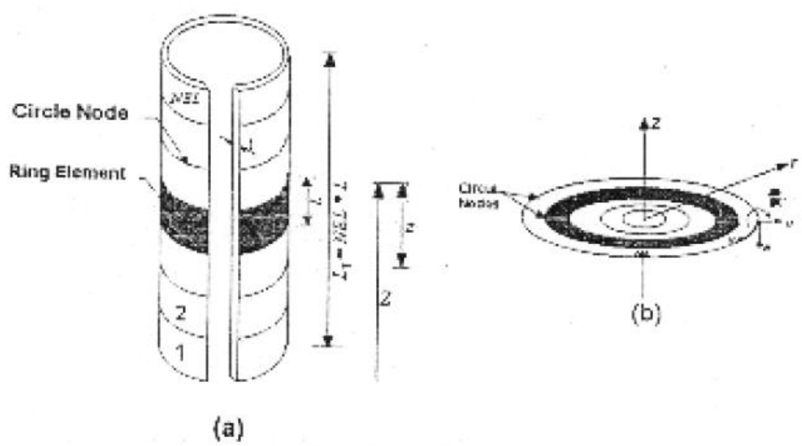



Fig.(6) : Mathematical Model

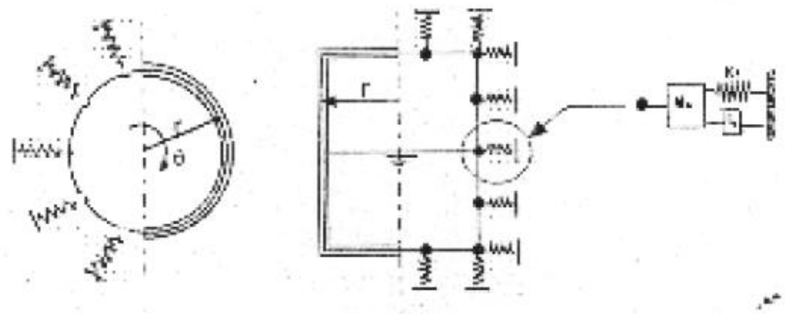


Fig.(7): Elastic half - space modeling for soil-tank interaction

## FREE VIBRATION RESPONSE ANALYSIS

A structure is said to be undergoing free vibration when it is disturbed from its static equilibrium position and then allowed external dynamic excitation or support motion.

The undamped free vibration response of any system can be obtained as:

$$[M]\{\ddot{U}\}+[K]\{U\}=\{0\} \quad \text{......................}(65)$$

Where [M] and [K] are the mass and the stiffness matrices of the system, respectively; and $\{U\}$ is the displaecment vector.

Therefore, the natural frequencies $(\omega)$ and the mode shape $(\phi)$ of any systcm governed by Eq.(65) are solution of the eigenvalue problem represented by

$$[[K]-\omega^2[M]]\{\phi\}=\{0\} \text{.................}(66)$$

For non trivial solution of Eq.(66),

$$|[K]-\omega^2[M]|=0 \text{.................}(67)$$

Equation (67) is called the frequency equation of the system. Expanding the determinant will give an algebraic equation of $m^{th}$ degree in thequmy parameter $\omega^2$ for a system having. m degrees or freedom. The roots of this equation $(\omega_1^2, \omega_2^2, \omega_3^2, \cdots \omega_m^2)$ represent the frequencies of the in mode of vibration which are possible in the system.

If all m eigenvalue are required and m is relatively small (roughly m < 200), the Jacobi method is a good choice(4) ; therefore, Eq.(66) is solved numerically by the Jacobi method given in Ref(3) for all cases considered in the present study,

## CYLINDRICAL STORAGE TANKS

Several cases of liquid storage tanks with widely different properties are presented to demonstrate the applicability of the proposed idealization developed herein and to cover the free vibration characteristics of these tanks. The analysis was applied to various case studies, which were considered by other investigators to serve as a validation procedure for the formulation proposed in the present work: and also to check the convergency of the solution. Examples of both broad and tall tanks are analyzed for each of three cases:

## - EMPTY STORAGE TANKS

The properties of the tall and broad tanks are as follows [5,7]:

a- Tall Tank: Radius = 7.32m; length = 21.96m.

b- Broad Tank: Radius. = 18.29m; Length = 12:19m.

Both tanks are assumed to be opened at the top: fixed at the base and have as uniform thickness of 0.0254m (1 in).

The tank's wall is made of steel having the following properties:

$E = 20.67 * 10^7 kN/m^2, v = 0.3 , and\ \rho = 78.4\ kN/m^3$

In Table (2), the three lowest natural frequencies of both tanks or the Fourier number (n=0 and n = 1) are presented along with those r tats obtained by Haroun and Housner [5] and by Haroun and Tayel [7]. Inspection of this Table shows excellent agreement between the values of these frequencies.

## - Partly Filled Tanks

The small tall and broad tanks described earlier are now assumed to be partly filled with water. Calculations of the natural frequencies for different types of liquid depths are presented in Table (3) along the results obtained by Haroun and Houner [5] for Fourier number (n = 1).

## PARAMETRIC STUDIES

To study the effect of tank geometry, liquid depth variation and hydrostatic pressure on natural frequencies, parametric studies for three different cases of soil types were carried out. The three types of soil arc classified as follows:

a) Soil type 1: dense sand and gravel, $\rho$ =17 kN/m$^3$,Vs=250 m/sec

b) Soil type 2: moist clay, $\rho$ =18 kN/m$^3$, Vs=150 m/sec

c) Soil type 3: fine - grained sand, $\rho$ =16.5 kN/m$^3$, Vs=110 m/sec

The properties of material that is used in all cases of the parametric studies are taken as follows:

a) Steel material is used fin both cover and walls of the tank whose properties are:
$E = 20.67 * 10^7 kN/m^2, v = 0.3 , and\ \rho = 78.4\ kN/m^3$. Both have a uniform thickness of 0.0508m.

b) Concrete material is used for base plate of the tank whose properties are ;
$E = 20 * 10^6 kN/m^2, v = 0.15 , and\ \rho = 78.424\ kN/m^3$. The thickness of the plate is 0.4m.

c) Water is used as a storage liquid o r the tank having density of 10 kN/m$^3$

## EFFECT OF TANK HEIGHT TO DIAMETER RATIO

For this purpose two cases of storage tanks were considered, empty tank and completely full tank. Diameter. D 20m and the tank height , $L_T$ , was varied from 5 to 30m at 5m increments to accommodate the aspect ratio ($L_T$/D) range of 0.25 to 1.5.

Results of natural frequencies are given in Plots of Figs.(8) and (9).for empty and completely fill tanks , respectively.

From these Plots, it is observed that as the soil medium becomes weaker (having low shear wave velocity) or as tank height increases, the natural frequency of the system, at all modes or vibration for both two cases of the tank., decreases.

It is also noticed by examination of these Tables and Plots, that the natural frequencies of the empty tank are much larger than those of the full tanks regardless of the type or soil. This can be explained by the fact that, the added liquid mass is much larger than that of the shell, and since the natural frequencies are proportional to the square root of

the inverse of the mass, the frequencies of the full tank are reduced appreciably as compared to those of the empty tank.

**EFFECT OF LIQUID HEIGHT TO TANK LENGTH RATIO VARIATION.**

To demonstrate the effect of liquid depth variation ($H/L_T$), two types of tanks (tall and broad) were considered for this purpose. Different values of liquid depths for the same three types of the soils were carried out of demonstrate the influence of liquid height on the dynamic characteristics. The resulting natural frequencies are given in Plots of Figs.(10) and (11) for broad and tall tanks, respectively.

It can be observed from these Plots that as the level of fluid increase, the natural frequencies decreases for both types of tanks and for all three types of the surouding soil. This behavior is obvious since the mass of the shell-fluid system increase with the level of fluid, while the structure stiffness properties remain unchanged.

**EFFECT OF INITIAL HOOP STRESS DUE TO HYDROSTATIC PRESSURE**

To investigate the effect of initial hoop tension due to hydrostatic pressure on the dynamic characteristics the. liquid storage tanks, two filled tanks: (broad and tall) were considered with one type of soil (soil type 1 is chosen for this purpose) with different values of Fourier terms number (n). The resulting natural frequencies for broad and tall tanks are given in Plots of Figs. (12) and (13), respectively.

It can be concluded from these Plots that the initial hoop stress effect has significant influence on the natural frequencies of vibration of tall tank while, the influence is almost insignificant and negligible for most practical purposes in broad tanks. It is of interest to note that the influence of the initial stress becomes more significant as the Fourier term number, n increases.

**EFFECT OF WALL THICKNESS VARIATION.**

To demonstrate the effect of wall thickness variation, an empty tank of 15m height and 15m diameter is studied for its free vibration characteristics when its wall thickness varies from 1cm to 4cm with one type of the surrounding soil (soil type 1 is also used herein). The resulting natural frequencies are given in Plot of Fig.(14).

It can be seen clearly from these results that the thicker the wall of the tanks is, the higher are the natural frequencies, since the wall's stiffness increases with increasing its thickness.

Table (2): Natural frequencies for empty broad and tall tanks

| Tank | Fourier term number | Mode number | Natural frequency (Hz) | | | |
|---|---|---|---|---|---|---|
| | | | Present analysis | By Haroun & Housner (5) | By Haroun & Tayle (7) | |
| | | | | | Analytical | Numerical |
| Broad Tank | 0 | 1 | 44.41 | — | 44.40 | 44.41 |
| | | 2 | 44.72 | — | 44.71 | 44.72 |
| | | 3 | 44.82 | — | 44.77 | 44.83 |
| | 1 | 1 | 34.02 | 34.04 | — | — |
| | | 2 | 43.81 | 43.86 | — | — |
| | | 3 | 44.50 | 44.54 | — | — |
| Tall tank | 0 | 1 | 57.79 | — | 57.72 | 57.80 |
| | | 2 | 109.11 | — | 108.97 | 109.14 |
| | | 3 | 111.19 | — | 111.04 | 111.23 |
| | 1 | 1 | 19.23 | 19.26 | — | — |
| | | 2 | 56.32 | 56.42 | — | — |

Table (3) : Natural frequencies for partly filled broad and tall tanks

| Filling ratio ($H/L_T$) | Natural frequency (Hz) , n=1 | | | | |
|---|---|---|---|---|---|
| | Mode number | Broad tank | | Tall tank | |
| | | Present analysis | By Haroun & Housner (5) | Present analysis | By Haroun &Housner (5) |
| 1 | 1 | 6.18 | 6.18 | 5.31 | 5.31 |
| | 2 | 11.3 | 11.28 | 15.63 | 15.64 |
| 0.8 | 1 | 7.23 | 7.24 | 7.04 | 7.05 |
| | 2 | 12.96 | 12.96 | 18.74 | 18.76 |
| 0.6 | 1 | 8.79 | 8.79 | 9.62 | 9.64 |
| | 2 | 15.38 | 15.37 | 22.2 | 22.45 |
| 0.5 | 1 | 9.88 | 9.88 | 11.41 | 11.42 |
| | 2 | 17.09 | 17.05 | 24.02 | 24.03 |
| 0.3 | 1 | 13.87 | 13.82 | 16.45 | 16.46 |
| | 2 | 24.20 | 24.00 | 25.59 | 25.61 |
| 0 | 1 | 34.04 | 34.04 | 19.24 | 19.26 |
| | 2 | 43.85 | 43.86 | 56.32 | 56.42 |

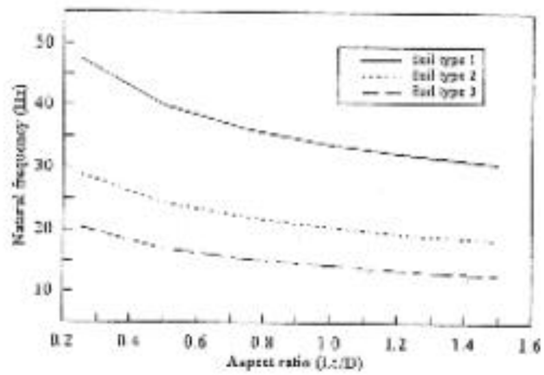



Fig.(8): Fundamental natural frequencies versus aspect ratio ($L_T/D$) variation of empty tank , n=1

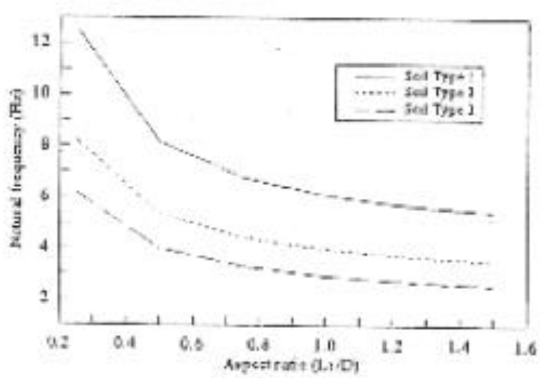



Fig.(9): Fundamental natural frequencies versus aspect ratio($L_T/D$) variation of filled tank , n=1

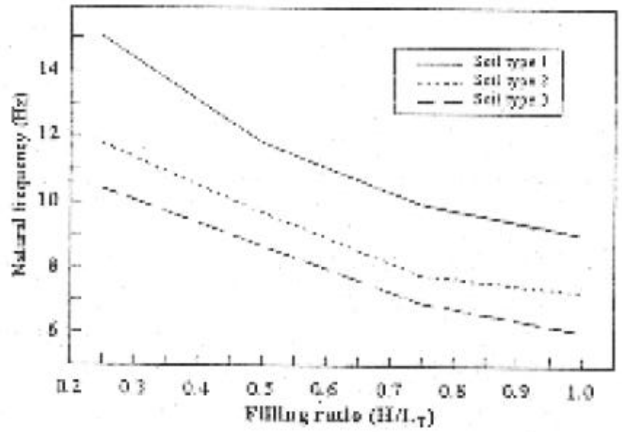



Fig.(10): Fundamental natural frequenies versus filling ratio ($H/L_T$) variation of broad tank , n=1

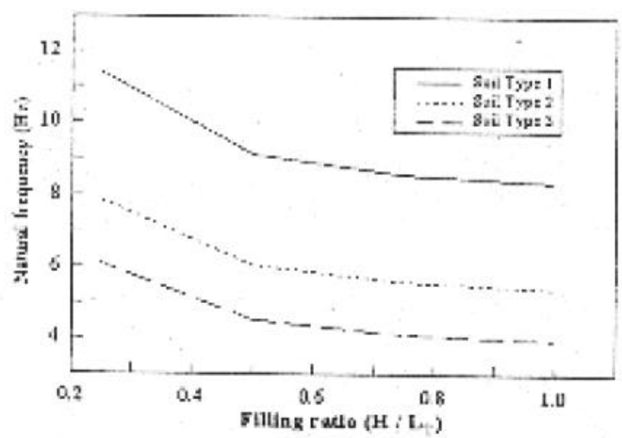



Fig.(11): Fundamental natural frequencies versus filling ratio($H/L_T$) variation of tall tank , n=1

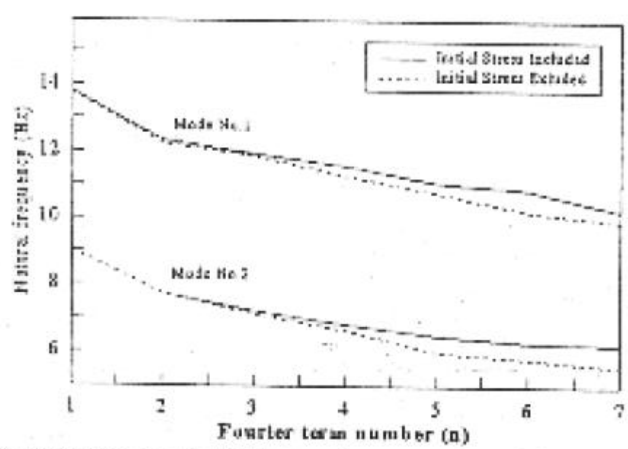



Fig.(12:Effect of initial hoop stress on natural frequencies of broad tank

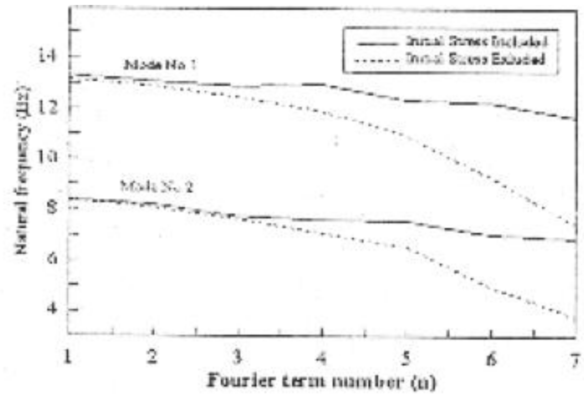



Fig.(13):Effect of Initial hoop stress on naturl frequencies of tall tank

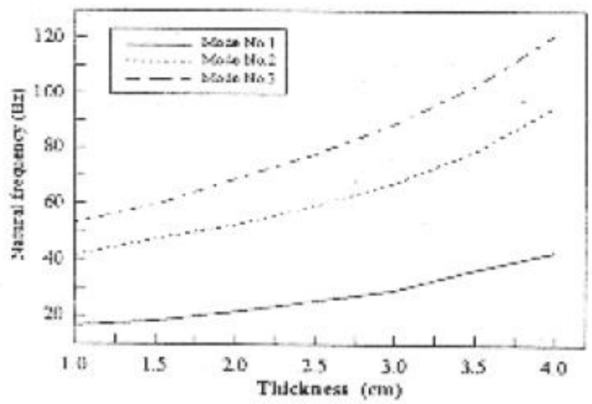



Fig.(14):Effect of thickness variation on natural frequencies of empty tank

## CONCLUSIONS

- The main conclusions that can he drawn from the present study are as follows:
- The change of the three-dimensional problem into several separate two dimensional problems using the semi-analytical technique is very useful to reduce the effort required, computer time and memory needed in solving the problem or storage tanks under dynamic loading_
- A conical shell finite element is derived in the present work, which is best suited for the analysis of circular plate, cylindrical arid conical shells. This element becomes more general after including the contribution of symmetric and anti-symmetric turns in the Fourier series expansion.
- The soil-tank interaction was represented by an elastic half-space medium. Variations of the properties of the. Surrounding soil medium are found to have an important influence oil the free and forced vibrational response (earthquake response) of the buried storage tanks.
- It was found that, the .initial hoop stress due to hydrostatic pressure, becomes more significant as the circumference wave number, n, increases, and these stresses have . more influence on the frequencies of vibration of tail tanks than broad tanks.

- It is also found that, the natural frequency is proportional to the wall thickness of the tank. This behavior is related to the fact that the dynamic stiffness of a tank is a function of its wall thickness.

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