PREDICTION OF TEMPERATURE DISTRIBUTION IN FLEXIBLE PAVEMENTS

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ABSTRACT

In the present study, finite difference method is proposed to use for the development of mathematical model for the prediction of the pavement temperature distribution (PPTD) in flexible pavement.

To simplify the mathematical calculation, Visual Basic language was selected to perform a program that has a capability for the Prediction of Pavement Temperature as a function of depth and time. To test its accuracy, the program was applied to predict the pavement temperature distribution on specific location in section (R12) from the Expressway No.1 in Iraq. The output of this program appears to be in good agreement with the (R12) observed data. Therefore the program can be recognized for use in the prediction of the asphaltic pavement temperature distribution.

الخلاصة

في الدراسة المقدمة تم اقتراح طريقة الفوارق المحددة لاعداد نموذج رياضي للتنبؤ بتوزيع درجات الحرارة في التبليط المرن و لغرض تسهيل الحسابات الرياضية تم إعداد برنامج بلغة Visual Basic وسمي PPTD وللبرنامج القابلية للتنبؤ بدرجات الحرارة مع العمق والزمن.

لمعرفة دقة نتائج هذا البرنامج تم تطبيقه للتنبؤ بتوزيع درجات الحرارة لموقع محدد في المقطع ط12 من طريق المرور السريع رقم 1 في العراق ولوحظ بان هنالك تطابق جيد بين نتائج هذا البرنامج والقراءات الموقعية ولذا فانه بالإمكان استخدام هذا البرنامج للتنبؤ بتوزيع درجات الحرارة في التبليط المرن.

KEY WORDS

Pavement Temperatture Distribution, Heat Transfer Theory, Finite Difference Modeling, Flexible Pavement Temperature.

INTRODUCTION

Pavement temperature is an important factor that strongly affects thermal distress. Based on this effect, it is worth noting that the pavement temperature draws the main interest in the most of the previous researches.

Barber [1957] is the pioneer researcher in this specific approach. He applied a basic heat equation in pavement structure to predict the pavement temperature as a function of depth. Conversely Shahin and McCllough [1972] presented a regression equation to predict the temperature distribution in rigid pavement. This equation is based on the air temperature and has the ability to predict the pavement distribution that matched the field observations. Solaimanian and Kennedy [1993], as

SHRP researchers, used the heat energy approach to find a maximum temperature in the asphalt concrete layer. A mathematical model for the prediction of pavement temperature on the asphalt concrete layer that overlay the rigid pavement was suggested by Cho et al. [1998].

MATHEMATICAL MODEL FOR THE PREDICTION OF PAVEMENT TEMPERATURE DISTIRIBUTION

Based on the requirement of the PPTD model development, three main items may be drawn. These are; theoretical consideration, performing of PPTD program and validation process.

Theoretical Consideration

To explain the theoretical consideration of the above-mentioned model, three aspects will be presented. These are; problem definition, heat transfer theory and finite difference method.

Problem Definition

The objective from the development of PPTD model is to predict the pavement temperature distribution in flexible pavement. The prediction of pavement temperature is an important factor for searching on thermal distress. The pavement structure that simulates the field performance for the Expressway No.1 in Iraq was proposed for the purpose of this development. This pavement structure can be seen in **Fig. (1).**

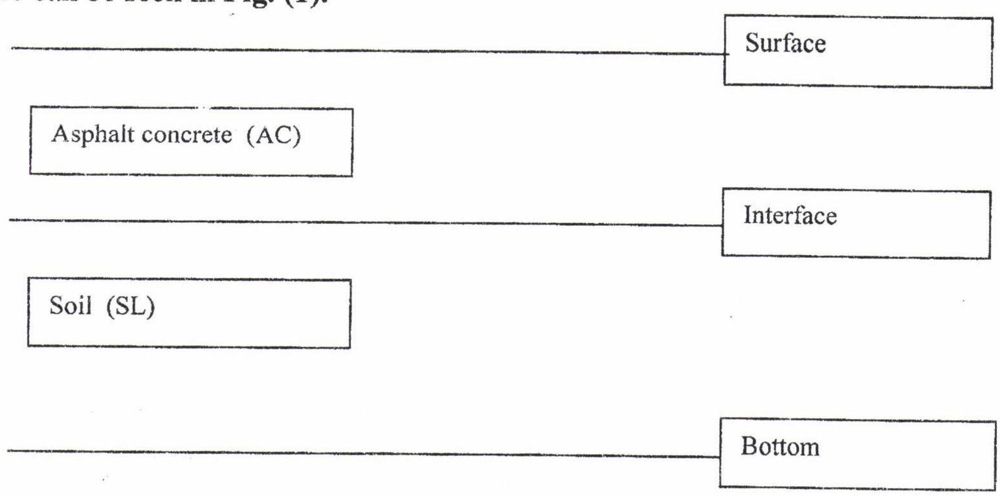


Fig. (1) The pavement structure that proposed to simulate the field performance.

Heat Transfer Theory

The basic formula of the heat transfer in homogenous isotropic materials is;

$$\frac{\partial U}{\partial t} = c\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2}\right) \quad (1)$$

where,
U= Temperature of the mass,
t = time,
X,Y,Z= Three dimension coordinates ,and
c= Diffusivity.

The diffusivity is defined by the ratio of conductivity and a value that is a multiple of density and specific heat. Thermal conductivity relates to the materials capacity for transferring heat. Specific heat is the amount of heat required for a unit mass of material to increase its temperature by one degree [Cho and McCullough 1994].

In this study the heat flow is assumed to be one-dimensional that conform to the assumption of many researchers [Monismith 1965,Hill and Brien 1966 and Cho and McCullough 1994]. Thus equation (1) can be abbreviated as follows:

$$\frac{\partial U}{\partial t} = c\frac{\partial^2 U}{\partial X^2} \quad (2)$$

where X represents the depth from the pavement surface.

Finite Difference Modeling

In pre-computer times the finite difference method was an area of great importance. In this study the Crank-Nicolson finite difference method is employed to solve the present problem. The finite difference method uses approximately differential increment in the temperature and depth coordinate. More details of this method can be seen elsewhere [Kreyszig 1983].

The boundary conditions of the problem are briefly described as follows;

Initial Condition

The pavement temperature is assumed to follow Barber equation [Barber 1957], This equation can be found in the following form;

$$U = U_M + U_V \frac{He^{-XC}}{\sqrt{(H+C)^2 + C^2}} \sin(0.262t - XC - \arctan\frac{C}{C+H}) \quad (3)$$

where,

U= Temperature of the mass in (^oF),

U_M= Mean effective for air temperature in (^oF),

U_v= Maximum variation in temperature from mean effective in (^oF),

H= h/k,

h= Surface coefficient in (BTU / ft^2),

K= Conductivity in (BTU / ft^2),

c= Diffusivity in (ft^2/hr),

c= K/sw,

s= Specific heat in (BTU / Ib),

w= Density in (Ib/ft^3),

C= 0.131/c,

t= Time from the beginning of cycle in (hours), (One cycle = one day),and

X=Depth below the surface in (ft).

Cho and McCullough [1994] postulated that the assumption of Barber could be a problem in an actual pavement in a multilayer system. Thus, the present model is proposed to identify the temperature distribution in the actual field performance.

In this study and referring to the finite difference mesh, the depth and time will be presented by i and j respectively.

- At time = 0 Barber equation can be seen in the following form;

$$U_{i,1} = U_M + U_V \frac{He^{-XC}}{\sqrt{(H+C)^2 + C^2}} \sin(-XC - \arctan\frac{C}{C+H}) \quad (4)$$

- At depth=0 (surface of pavement) equation (3) can be drawn as;

$$U_{1,j} = U_M + U_V \frac{H}{\sqrt{(H+C)^2 + C^2}} \sin(0.262t - \arctan\frac{C}{C+H}) \quad (5)$$

Interface Condition

The condition at the surface between asphalt concrete and soil can be expressed as;

$$Ka\frac{\partial U}{\partial X} = Ks\frac{\partial U}{\partial X} \quad (6)$$

where,

Ka= Conductivity of asphalt concrete in (BTU/ft^2), and

Ks= Conductivity of soil in (BTU/ft^2).

Bottom Boundary

At the specific depth of pavement, the variations of temperature as a function of depth approach to zero.

$$At .... U = U_{\geq L,j}$$

$$Ks\frac{\partial U}{\partial X} = 0 \quad (7)$$

Finite Difference (Crank-Nicolson) Method

Based on the above-mentioned boundary conditions and using the finite difference approach, equation (2) can be simplified as;

$$\frac{\partial^2 U}{\partial X^2} = \frac{ws}{K}\frac{\partial U}{\partial t} \quad (8)$$

The second derivative in X-direction can be determined at the mid point by applying Crank-Nicolson method;

$$\frac{\partial^2 U}{\partial X^2} = \frac{1}{2}\left(\frac{U_{i+1,n+1} - 2U_{i,n+1} + U_{i-1,n+1}}{\Delta X^2} + \frac{U_{i+1,n} - 2U_{i,n} + U_{i-1,n}}{\Delta X^2}\right) \quad (9)$$

The first derivative of temperature can be drawn as;

Then,

$$\frac{\partial U}{\partial t} = \frac{U_{i,n+1} - U_{i,n}}{\Delta t} \quad (10)$$

$$Let \Rightarrow KK = \frac{\Delta t}{\Delta X^2} .... FF = \frac{ws}{K}$$

By substituting equations (9) and (10) into equation (2), the following form can be found;

$$\frac{KK}{2}(U_{i+1,n+1} - 2U_{i,n+1} + U_{i-1,n+1} + U_{i+1,n} - 2U_{i,n} + U_{i-1,n}) = FF(U_{i,n+1} - U_{i,n}) \quad (11)$$

Applying central difference on equation (6) at the interface, then

$$Ka\frac{U_{i+1,n}-U_{i-1,n}}{2\Delta X}=Ks\frac{U_{i+1,n}-U_{i-1,n}}{2\Delta X} \qquad (12)$$

$$\Rightarrow U_{i+1,n}=U_{i-1,n} \qquad (13)$$

At the specific depth, it is assumed that there is no variation in temperature. In other words, the heat flow would be beyond a certain depth; this depth (in this study) is assumed to be 2.0 m.

Then equation (7) can be simplified as;

$$U_{i+1,n}=U_{i-1,n} \qquad (14)$$

Fig. (2) presents the proposed arrangement of nodes that will be used in solving this problem. Generally, and for the purpose of the grids configuration, the smaller the increment, the better is the solution [Kreyszig 1983].

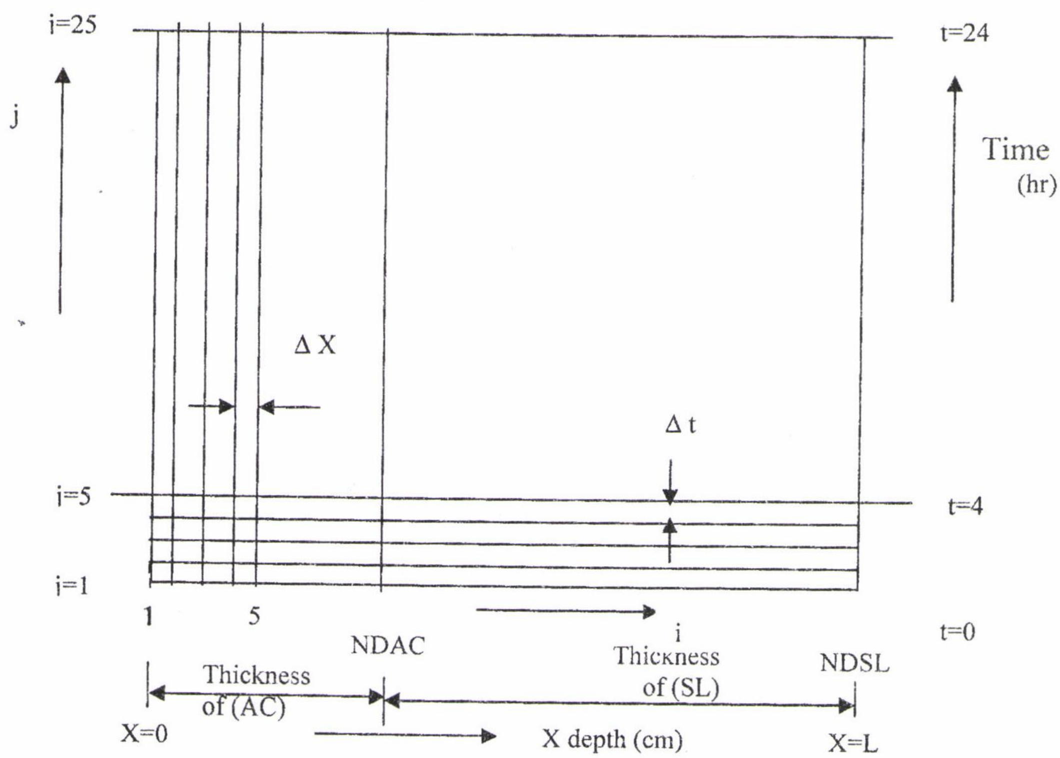


Fig. (2) Grids and nodes configuration .

Concerning the previously mentioned equations and the mesh arrangement, linear equations are drawn as follows;

Referring to equation (11), initial condition, boundary condition and the linear equations at initial depth, j=2,25 and i=2 can be found as;

$$\frac{KK}{2}(U_{3,n+1}-2U_{2,n+1}+U_{1,n+1}+U_{3,n}-2U_{2,n}+U_{1,n})=FF(U_{2,n+1}-U_{2,n}) \qquad (15)$$

Then, the previous equation can be simplified as;

$$\frac{KK}{2}U_{3,n+1}+(-KK-FF)U_{2,n+1}=(-FF+KK)U_{2,n}-\frac{KK}{2}U_{3,n}-\frac{KK}{2}U_{1,n}-\frac{KK}{2}U_{1,n+1} \quad (16)$$

$n=2$

$$\frac{KK}{2}U_{3,3}+(-KK-FF)U_{2,3}=(-FF+KK)U_{2,2}-\frac{KK}{2}U_{3,2}-\frac{KK}{2}U_{1,2}-\frac{KK}{2}U_{1,3}$$

$n=3$

$$\frac{KK}{2}U_{3,4}+(-KK-FF)U_{2,4}=(-FF+KK)U_{2,3}-\frac{KK}{2}U_{3,3}-\frac{KK}{2}U_{1,3}-\frac{KK}{2}U_{1,4}$$

....

....

....

Intermediate depth condition, j=2,25 and i=3,L-1 can be shown as;

$$\frac{KK}{2}U_{i-1,n+1}+(-KK-FF)U_{i,n+1}+\frac{KK}{2}U_{i-1,n+1}=(-FF+KK)U_{i,n}-\frac{KK}{2}U_{i+1,n}-\frac{KK}{2}U_{i-1,n} \quad (17)$$

$i=3\, and\, n=2$

$$\frac{KK}{2}U_{4,3}+(-KK-FF)U_{3,3}+\frac{KK}{2}U_{2,3}=(-FF+KK)U_{3,2}-\frac{KK}{2}U_{4,2}-\frac{KK}{2}U_{2,2}$$

$i=3\, and\, n=3$

$$\frac{KK}{2}U_{4,4}+(-KK-FF)U_{3,4}+\frac{KK}{2}U_{2,4}=(-FF+KK)U_{3,3}-\frac{KK}{2}U_{4,3}-\frac{KK}{2}U_{2,3}$$

....

....

....

Based on the boundary condition at the interface between the asphalt concrete and the soil at j=2,25 and i= NDAC (number of nodes in asphalt concrete layer) and NDSL (number of nodes in soil layer) from equation (13) and mesh configuration, the following equation can be drawn;

$$(-KK-FF)U_{NDAC,n+1}+KKU_{NDAC-1,n+1}=(-FF+KK)U_{NDAC,n+1}-KKU_{NDAC-1,n+1} \quad (18)$$

$n=2$

$$(-KK-FF)U_{NDAC,3}+KKU_{NDAC-1,3}=(-FF+KK)U_{NDAC,3}-KKU_{NDAC-1,3}$$

$n=3$

$$(-KK-FF)U_{NDAC,4}+KKU_{NDAC-1,4}=(-FF+KK)U_{NDAC,4}-KKU_{NDAC-1,4}$$

$n=4$

$$(-KK-FF)U_{NDAC,5}+KKU_{NDAC-1,5}=(-FF+KK)U_{NDAC,5}-KKU_{NDAC-1,5}$$

....

....

....

The bottom boundary
j=2,25 and i=L can be found as per the following form;

$$(-KK-FF)U_{L,n+1}+KKU_{L-1,n+1}=(-FF+KK)U_{L,n}-KKU_{L-1,n} \tag{19}$$

$n=2$

$$(-KK-FF)U_{L,3}+KKU_{L-1,3}=(-FF+KK)U_{L,2}-KKU_{L-1,2}$$

$n=3$

$$(-KK-FF)U_{L,4}+KKU_{L-1,4}=(-FF+KK)U_{L,3}-KKU_{L-1,3}$$

$n=4$

$$(-KK-FF)U_{L,5}+KKU_{L-1,5}=(-FF+KK)U_{L,4}-KKU_{L-1,4}$$

....

....

....

Referring to the previous equations (15 to 19), a tridiagonal matrix can be shown in **Fig. (3).**

$$[A][B]=[C]$$

$$A=\begin{bmatrix}
(-KK-FF) & \frac{KK}{2} & & & & & & \\
\frac{KK}{2} & (-KK-FF) & \frac{KK}{2} & & & & & \\
 & & \cdot & \cdot & \cdot & & & \\
 & & \frac{KK}{2} & (-KK-FF) & \frac{KK}{2} & & & \\
 & & & KK & (-KK-FF) & 0 & & \\
 & & & & \frac{KK}{2} & (-KK-FF) & \frac{KK}{2} & \\
 & & & & \cdot & \cdot & \cdot & \\
 & & & & & \frac{KK}{2} & (-KK-FF) & \frac{KK}{2} \\
 & & & & & & KK & (-KK-FF)
\end{bmatrix}$$

$$B=\begin{bmatrix}
U_{2,n+1} \\
U_{3,n+1} \\
\cdot \\
\cdot \\
U_{NDAC-1,n+1} \\
U_{NDAC,n+1} \\
U_{NDAC+1,n+1} \\
\cdot \\
\cdot \\
U_{L-1,n+1} \\
U_{L,n+1}
\end{bmatrix}
\qquad
C=\begin{bmatrix}
(-FF+KK)U_{2,n} & -\frac{KK}{2}U_{3,n} & -\frac{KK}{2}U_{1,n} & -\frac{KK}{2}U_{1,n+1} \\
(-FF+KK)U_{3,n} & -\frac{KK}{2}U_{4,n} & -\frac{KK}{2}U_{2,n} & \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
(-FF+KK)U_{NDAC-1,n} & -\frac{KK}{2}U_{NDAC-2,n} & -\frac{KK}{2}U_{NDAC,n} & \\
(-FF+KK)U_{NDAC,n} & -KKU_{NDAC-1,n} & & \\
(-FF+KK)U_{NDAC+1,n} & -\frac{KK}{2}U_{NDAC+1,n} & -\frac{KK}{2}U_{NDAC-1,n} & \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
(-FF+KK)U_{L-1,n} & -\frac{KK}{2}U_{L-1,n} & -\frac{KK}{2}U_{L+1,n} & \\
(-FF+KK)U_{L,n} & -KKU_{L-1,n} & &
\end{bmatrix}$$

Fig. (3) Tridiagonal Matrix.

PPTD Program

Visual Basic language is used to perform a program that has the capability to simplify the numerical calculation of the PPTD model.

It is worthy to mention that Thomas algorithm was used to solve the tridiagonal matrix system that is obvious at the end of the previous section. This program is based fully on the PPTD model and will be called PPTD program .The flow chart of the program is found in **Fig. (4).**

Main Input Parameters Affecting PPTD Program

The following main parameters are needed for the prediction of the pavement temperature distribution within the pavement layers;

- The pavement geometry and boundary conditions, the physical and thermal material properties, and the heat transfer coefficients.
- The average monthly temperature and the temperature daily range. The pavement temperature estimation is very sensitive to those temperatures.

Program Limitation

To use the PPTD program, the following limitation should be considered:

1- Asphalt concrete layers are suggested to have the same properties that are related to heat transfer and required as an input data.

2- The granular material layers are assumed to represent one deep layer.

RESULTS

The program was applied to predict the pavement temperature distribution on specific location in section R12.The input variables that are related to the investigated section can be seen in **Table (1).** The distribution of pavement temperature with time at different depths are presented in **Figs. (5) to (7),** while, The distribution of pavement temperature with depth at different time are shown in **Figs. (8) to (10).**

VALIDATION

For the purpose of validating the PPTD program the field performance data that were collected from the selected sections were used.

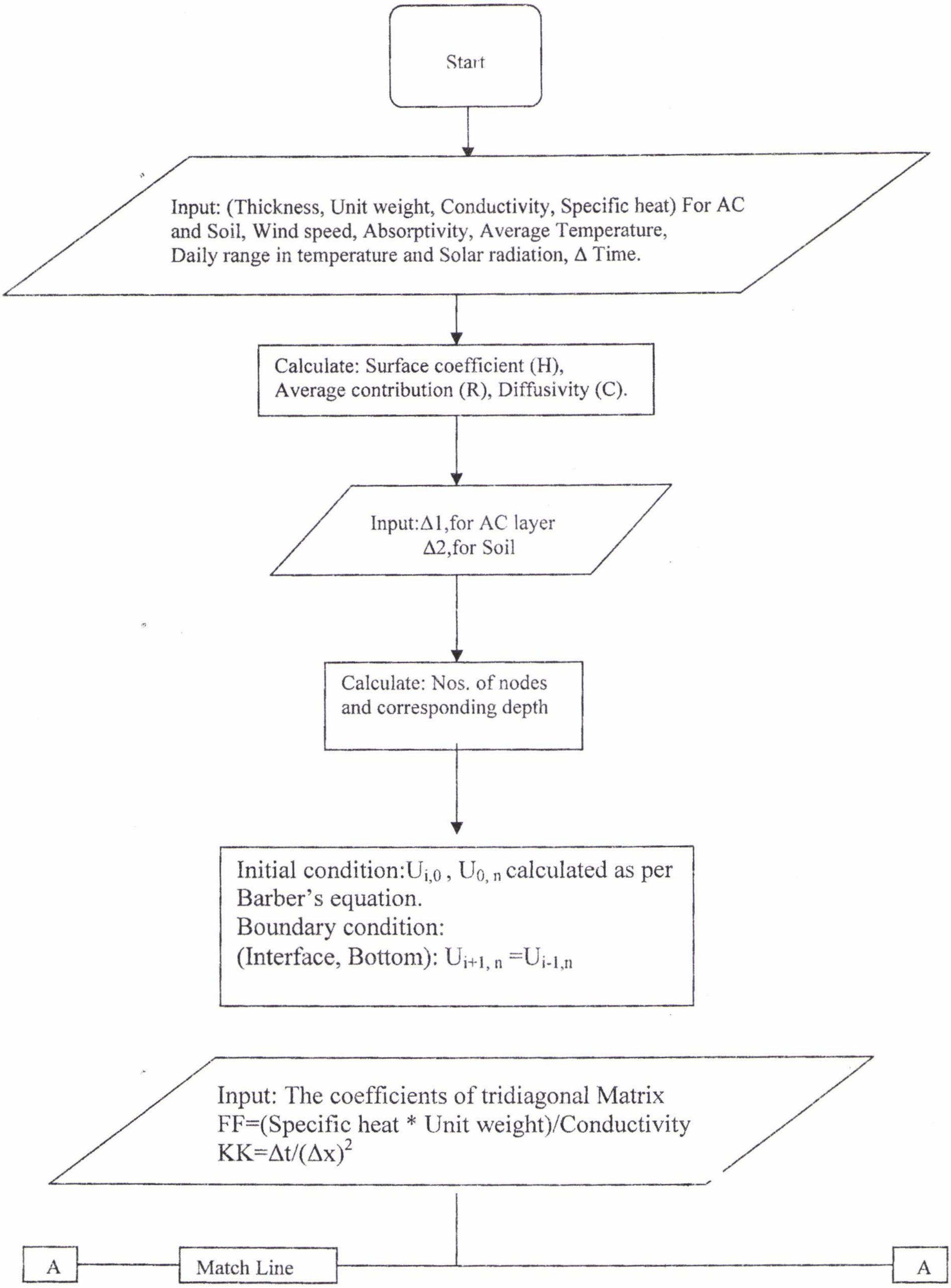
The field observed data for the pavement temperatures were plotted against these estimated by the program. This plotting is shown in **Figs. (11) and (12).** These Figures show that the predicted pavement temperatures as the output of the PPTD program fits the observed data very well.

To check the goodness of fit for the program output, Chi-square test was performed. It is noted from the results of this test that there is no significant difference between the PPTD output and the observed temperature distribution. The results of this test can be seen in **Table (2).**

CONCLUSION

The prediction of pavement temperature distribution (PPTD) is investigated by developing a finite difference model. To simplify the mathematical calculation a PPTD program was formulate.

As compare with the locally observed field data, the output of the program appears to be in good agreement and the program can be recognized for use in the prediction of the temperature distribution in flexible pavements.


Start
Input: (Thickness, Unit weight, Conductivity, Specific heat) For AC and Soil, Wind speed, Absorptivity, Average Temperature, Daily range in temperature and Solar radiation, Δ Time.
Calculate: Surface coefficient (H), Average contribution (R), Diffusivity (C).
Input:Δ1,for AC layer Δ2,for Soil
Calculate: Nos. of nodes and corresponding depth
Initial condition:$U_{i,0}$, $U_{0,n}$ calculated as per Barber's equation.
Boundary condition:
(Interface, Bottom): $U_{i+1,n} = U_{i-1,n}$
Input: The coefficients of tridiagonal Matrix
FF=(Specific heat * Unit weight)/Conductivity
$KK = \Delta t/(\Delta x)^2$
A
Match Line
A

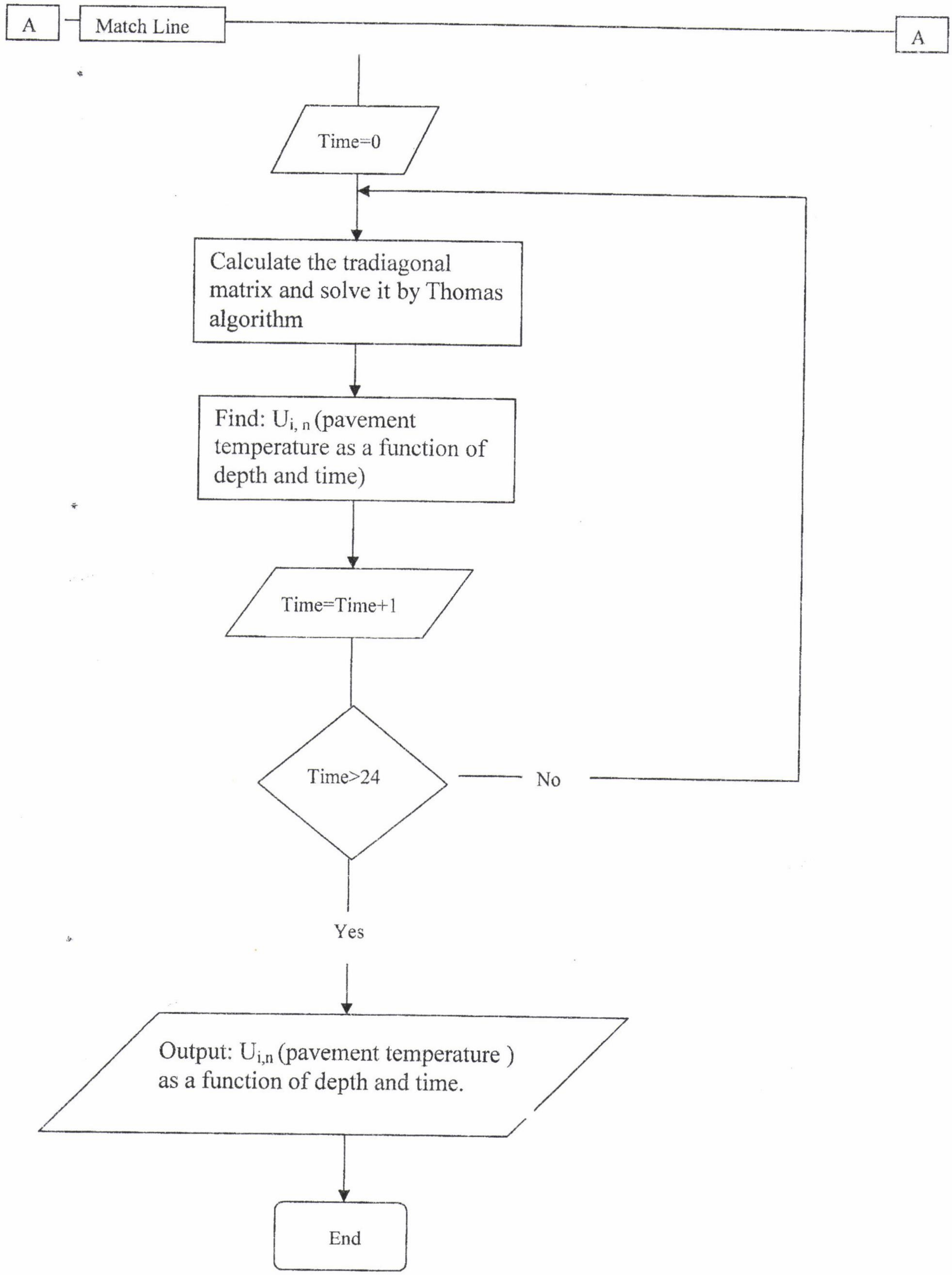


Fig. (4)Flow Chart of the PPTD Program.

Table (1) Input Variables for PPTD Program (Section R12).

Variables	Value
Thickness of asphalt concrete layer,cm	27
Thickness of granular material(soil),cm	173
Unit weight of the asphalt concrete layer,pcf	150
Unit weight of the granular material,pcf	110
Thermal coductivity of asphalt concrete,$BTU/ft^2/hr$	0.8*
Thermal coductivity of granular material,$BTU/ft^2/hr$	0.7*
Specific heat of asphalt concrete,BTU/Ib	0.22*
Specific heat of granular material,BTU/Ib	0.17*
Wind speed,m/sec	26
Absorptivity of pavement surface,	0.95*
Mean air Temperature,°C	21
Daily air temperature range,°C	10
Solar radiation,Mw/cm^2	449
(*):These default values are suggested by Lytton et al.[1990]	

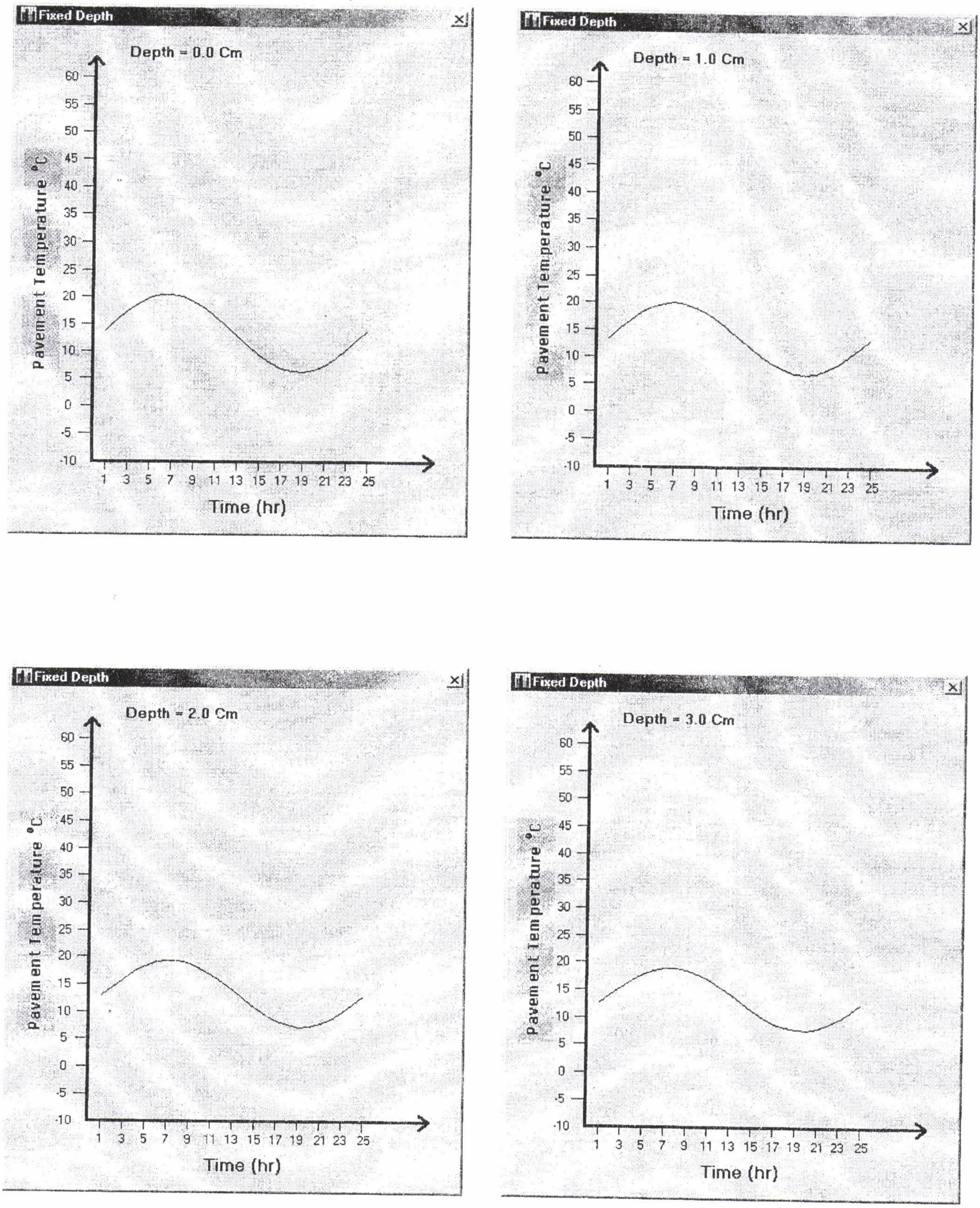


Fig. (5) Distribution of Pavement Temperature with Time at Depth = 0,1,2 and 3.

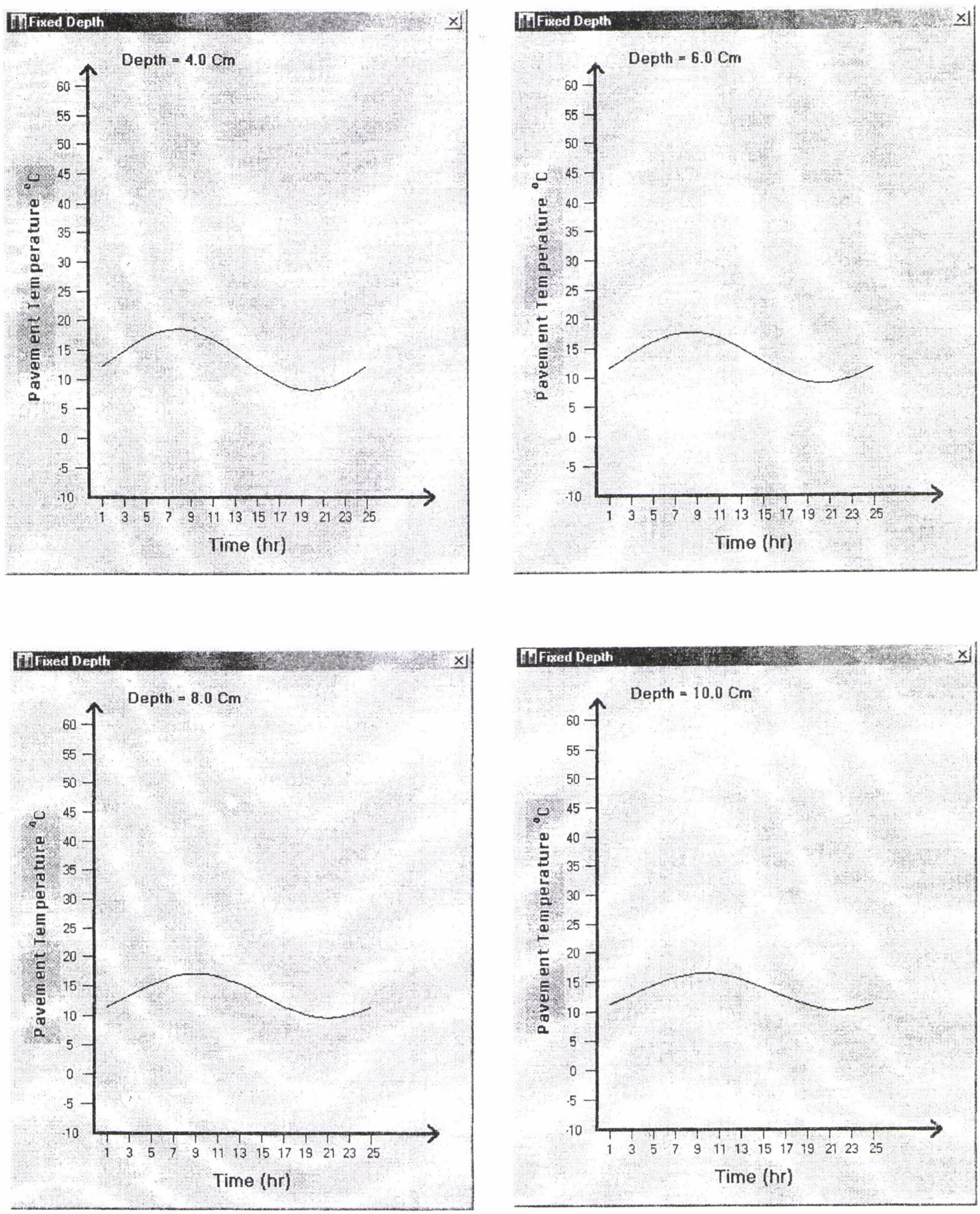


Fig. (6) Distribution of Pavement Temperature with Time at Depth=4, 6,8 and 10.

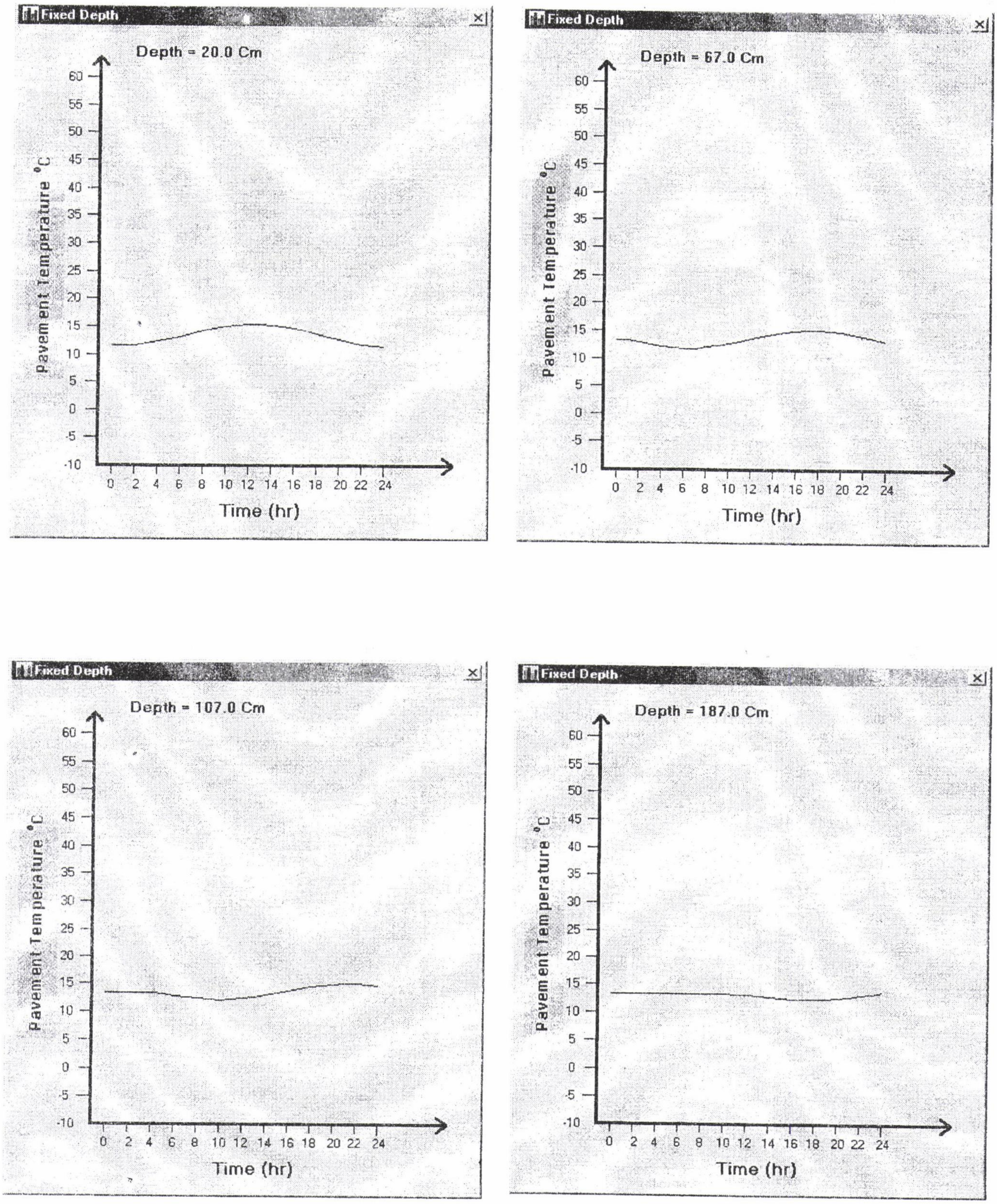


Fig. (7) Distribution of Pavement Temperature with Time at Depth = 20,67,107 and 187 cm.

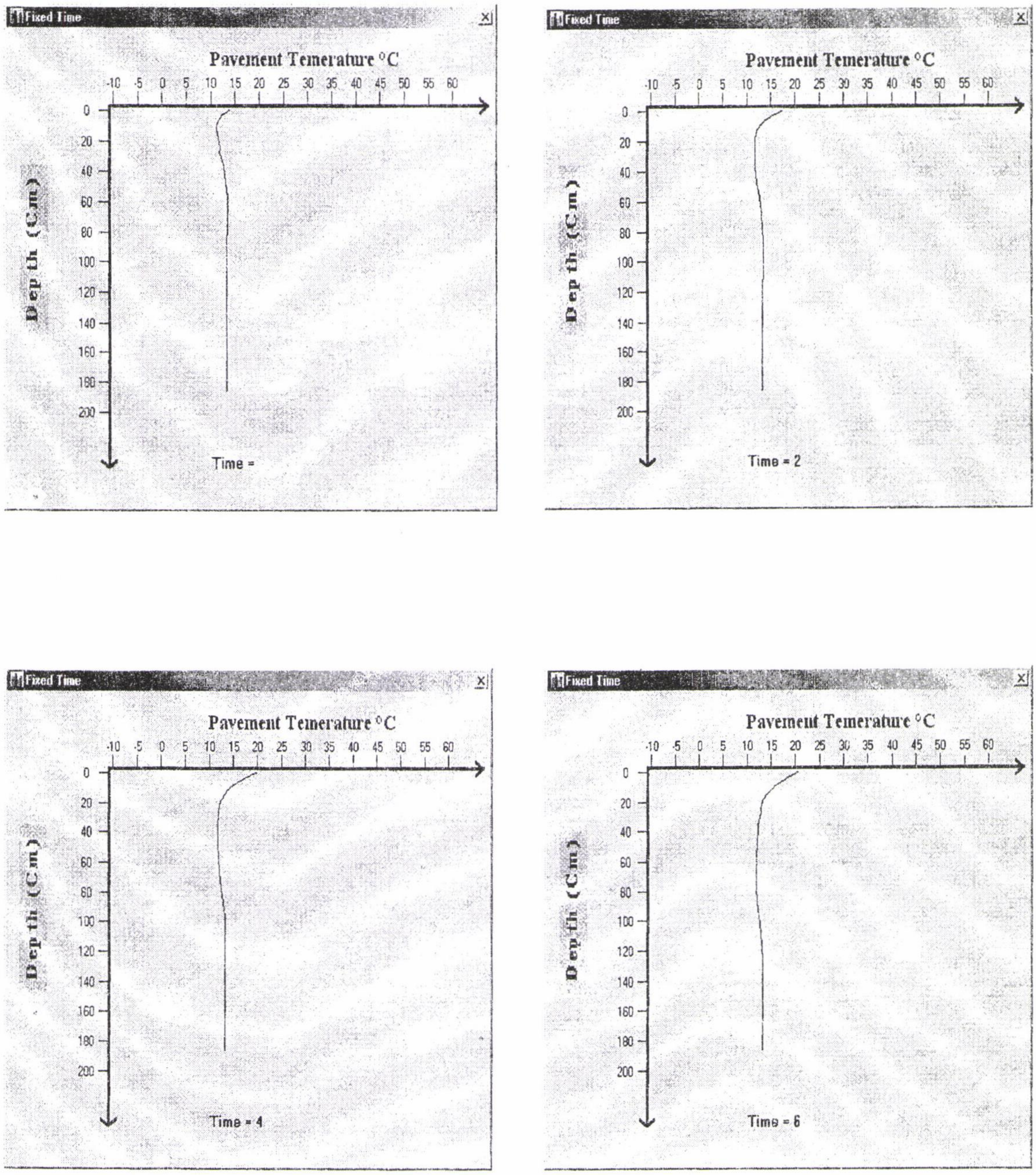


Fig. (8) Distribution of Pavement Temperature with Depth at Time = 0,2,4 and 6.

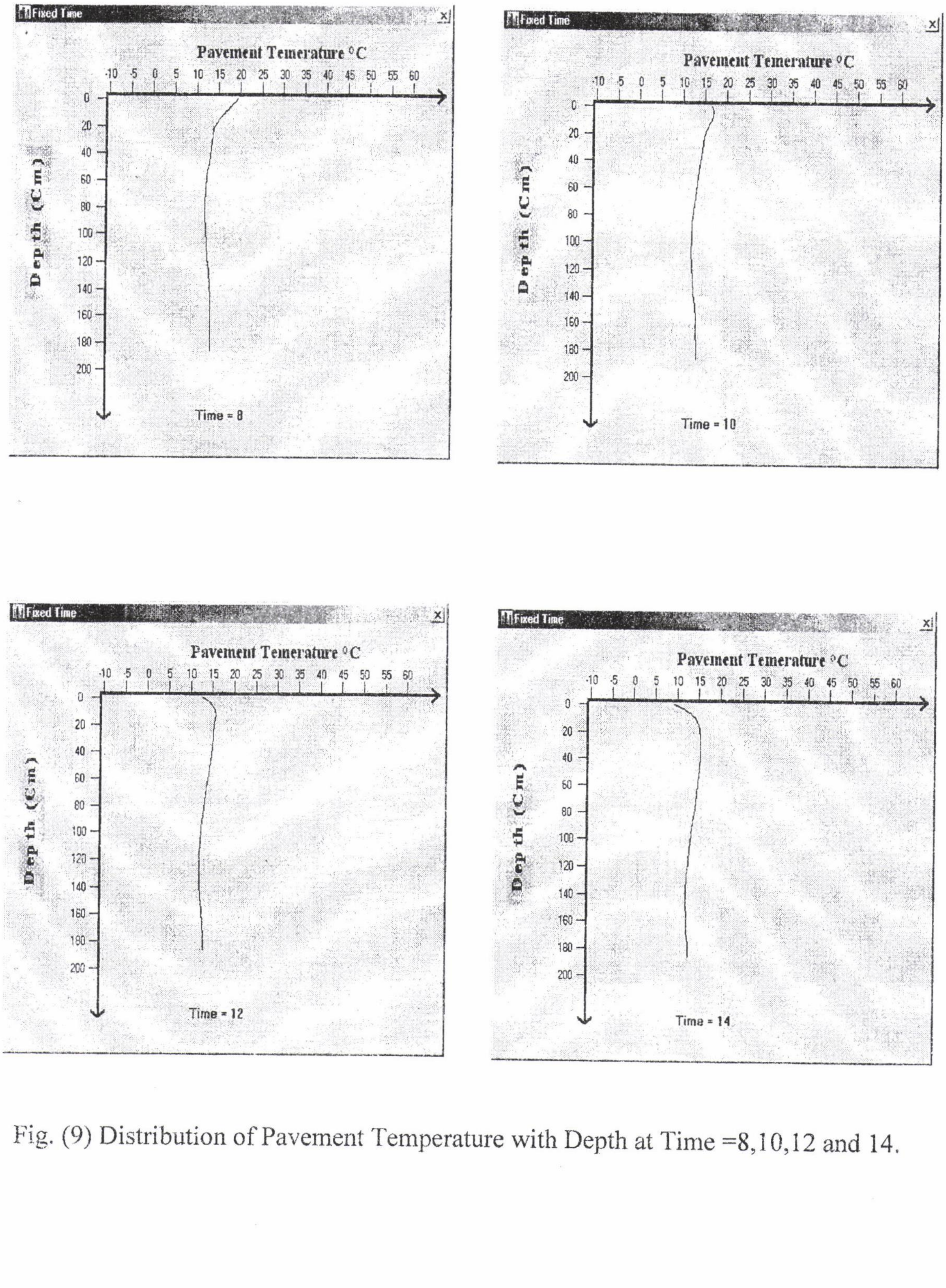


Fig. (9) Distribution of Pavement Temperature with Depth at Time =8,10,12 and 14.

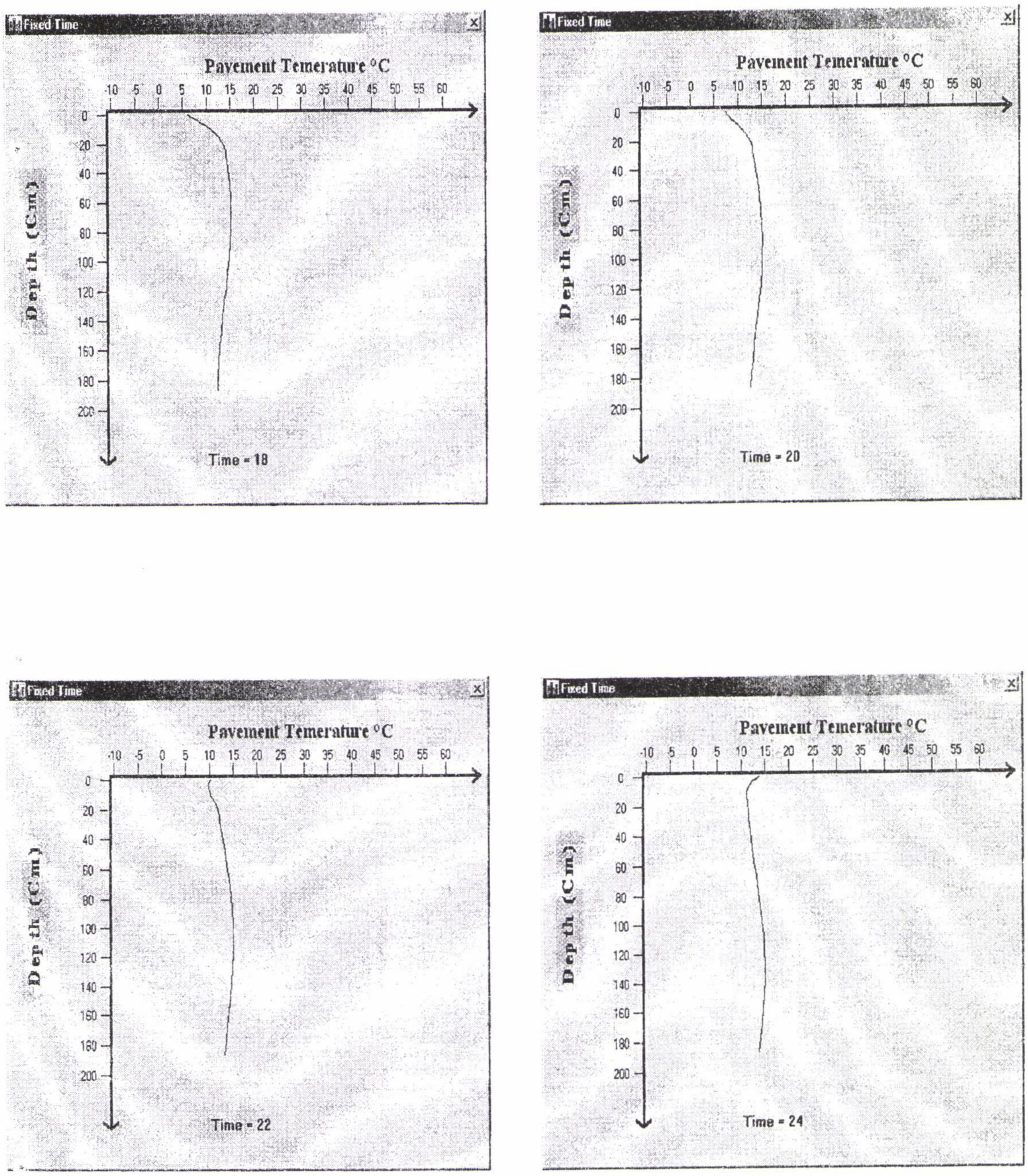
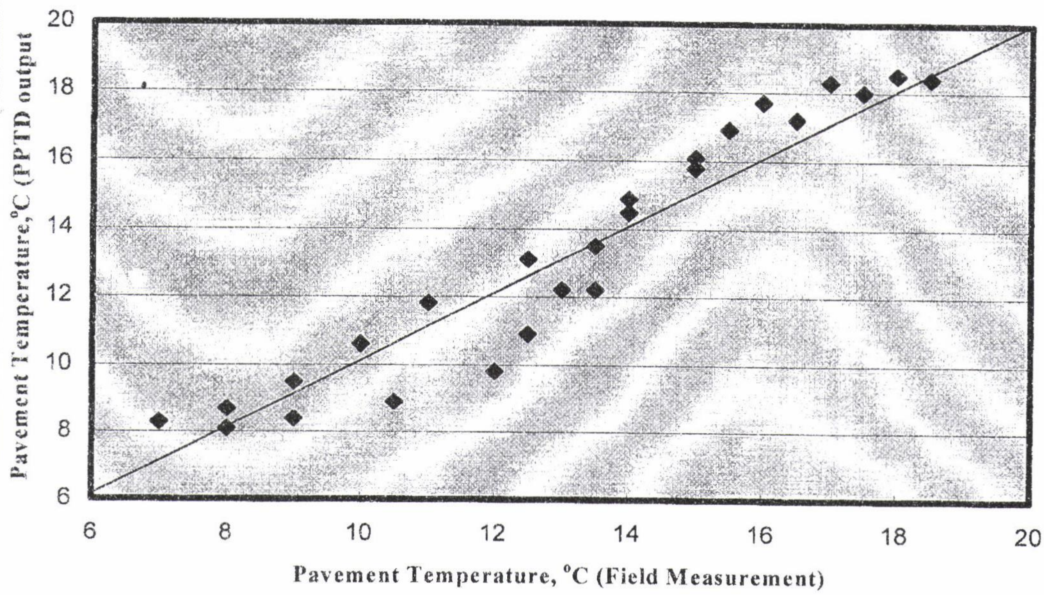
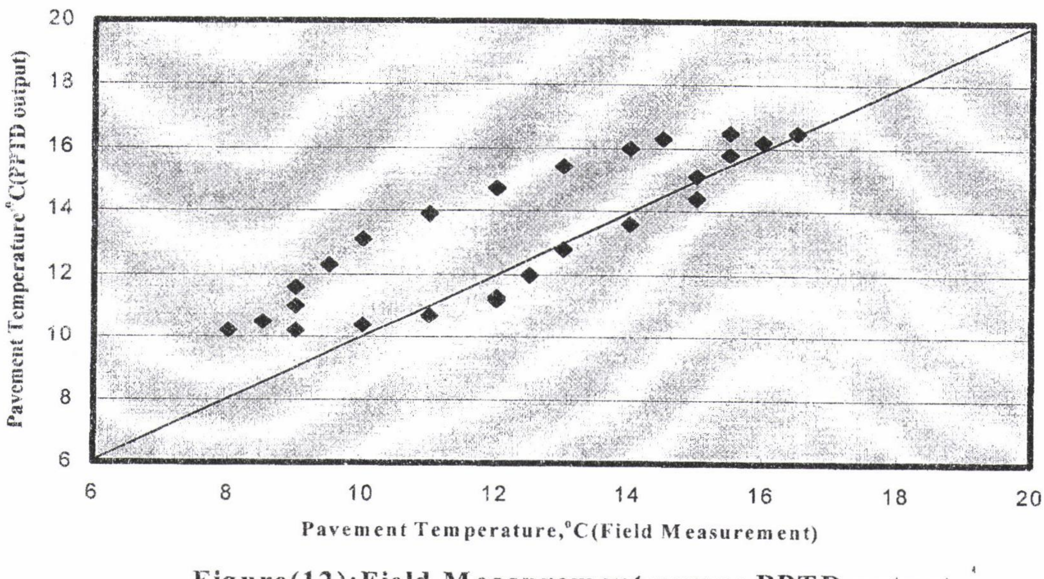


Fig. (10) Distribution of Pavement Temperature with Depth at Time = 18,,20,22 and 24.



Figure(11):Field Measurement versus PPTD output Pavement Temperature,Depth=4 cm.



Figure(12):Field Measurement versus PPTD output Pavement Temperature,Depth=10 cm.

Table(2)Chi-square Test Results for checking the Goodness of Fit of the PPTD Program output

Time hour	Pav.Temp.°C, Dep.=4cm (PPTD)output	Pav.Temp.°C, Dep.=4cm Measured	$\sum\frac{(o-e)^2}{e}$	Pav.Temp.°C, Dep.=10cm (PPTD)output	Pav.Temp.°C, Dep.=10cm Measured	$\sum\frac{(o-e)^2}{e}$
1	12.2	13	0.052	11.2	12	0.057
2	13.5	13.5	0.000	12	12.5	0.021
3	14.9	14	0.054	12.8	13	0.003
4	16.1	15	0.075	13.6	14	0.012
5	17.2	16.5	0.028	14.4	15	0.025
6	18	17.5	0.014	15.1	15	0.001
7	18.4	18.5	0.001	15.8	15.5	0.006
8	18.5	18	0.014	16.2	16	0.002
9	18.3	17	0.092	16.5	16.5	0.000
10	17.7	16	0.163	16.5	15.5	0.061
11	16.9	15.5	0.116	16.3	14.5	0.199
12	15.8	15	0.041	16	14	0.250
13	14.5	14	0.017	15.4	13	0.374
14	13.1	12.5	0.027	14.7	12	0.496
15	11.8	11	0.054	13.9	11	0.605
16	10.6	10	0.034	13.1	10	0.734
17	9.5	9	0.026	12.3	9.5	0.637
18	8.7	8	0.056	11.6	9	0.583
19	8.3	7	0.204	11	9	0.364
20	8.1	8	0.001	10.5	8.5	0.381
21	8.4	9	0.043	10.2	8	0.475
22	8.9	10.5	0.288	10.2	9	0.141
23	9.8	12	0.494	10.4	10	0.015
24	10.9	12.5	0.235	10.7	11	0.008
25	12.2	13.5	0.139	11.3	12	0.043

n=25,df=24,confidence level=95%

$\chi^2 = 2.268 < \chi_c^2 = 36.42$, for depth=4.0 cm.

$\chi^2 = 5.492 < \chi_c^2 = 36.42$, for depth=10.0 cm.

Thus, there is no significant difference between the estimated and measured values.

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