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DIRECTION OF ARRIVAL USING PCA NEURAL NETWORKS

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ABSTRACT
This paper adapted the neural network for the estimating of the direction of arrival (DOA). It uses an unsupervised adaptive neural network with APEX algorithm to extract the principal components that in turn, are used by Capon method to estimate the DOA, where by the PCA neural network we take signal subspace only and use it in Capon (i.e. we will ignore the noise subspace, and take the signal subspace only).

الخلاصة

في هذا البحث تم تصميم شبكة عصبية لإيجاد زاوية الوصول . حيث استعملنا الشبكة العصبية ذات التعليم الذاتي مع خوارزمية APEX لانتزاع المركبات الأساسية للإشارة المستلم من قبل الهوائيات ذات الترتيب الدائري التي بدورها(المركبات الأساسية)تستخدم بطريقة كابون لإيجاد زاوية الوصول . حيث من الشبكة العصبية أل PCA نأخذ فقط أجزاء من فضاء الإشارة الذي بدوره يستخدم بطريقة كابون (أي سنهمل فقط أجزاء من فضاء الضوضاء ونأخذ فقط أل أجزاء من فضاء الإشارة) .

KEY WORDS
Direction of arrival (DOA), Adaptive principal component extraction (APEX), Principal component analysis (PCA), Capon.

INTRODUCTION
Estimate the DOA of sources is a central problem in array signal processing. Many methods for estimation the DOA have been proposed, including the Maximum Likelihood (ML) technique [Ble86], the minimum variance method of Capon [Cap69], and the MUSIC method of Schmidt [Sch86]. The ML method has the best performance. Nonetheless, because of high computational load of the multivariate nonlinear maximization problem involved, the ML technique did not becomes popular. Suboptimal methods are more prevalent than the ML technique when the signal_to_noise ratio and number of samples are both not too small, because the suboptimal methods involve solving only a one-dimensional maximization problem and subspace (signal subspace or noise subspace).
This paper uses adaptive algorithm for extracting the subspace information based on the PCA neural network. Where we extract the principal component (i.e. the signal subspace only) by using of APEX algorithm with adaptive learning rate, which then used by a Capon to find the DOA.
The mapping of this paper is as follows: Section two provides some background information. Where the data model and the subspaces are present.
In section three an expression for the Capon method is presented.
In section four an expression for neural estimator is presented.

In section five a computer simulation using matlab 6.0 is provided to support the theoretical observations.

GENERAL CONSIDERATION

Assume that plane waves emitted by D[1] narrow band sources impinged on a uniform circular array (UCA) consisting of M sensors, and the DOAs of these sources (the azimuth angle is measured with respect to reference sensor, and elevation is measured with respect to z-axis as shown in **Fig. (1)** are $[(\theta_1, \Phi_1), (\theta_2, \Phi_2),\ldots, (\theta_D, \Phi_D)]$.The array output vector at the k_th snapshot can then be expressed as

$$x(k) = A*s(k)+n(k) \quad (1)$$

Where s(k) is the D×1 vector of incident signals which are assumed to be zero mean stationary, complex and Gaussian random processes n(k) is M×1 vector of additive noises which assumed to be zero mean random processes that are uncorrelated with each other and with the signals, and A is the steering (or direction) matrix given by $A= [a(\theta_1,\Phi_1), a(\theta_2,\Phi_2),\ldots a(\theta_D,\Phi_D)]$.The steering vector corresponding to the i_th DOA(θ,Φ) is given by

$$a(\theta i,\Phi i)=[\, a_1(\theta_1,\Phi_1)*e^{-j*w_0*\tau 1(\theta i,\Phi i)},\ldots, a_M(\theta_D,\Phi_D)*e^{-j*w_0*\tau M(\theta i,\Phi i)}] \quad (2)$$

Where $a_m(\theta i,\Phi i)$ denotes the complex gain response of m_th sensor to a wave front arriving from direction $(\theta i,\Phi i)$,w_0 denotes the center frequency of the signals ,and $\tau_m(\theta i,\Phi i)$ denote the propagation delay between the sensors for a wave front impinging from direction $(\theta i,\Phi i)$ which is given by

$$\tau i=j*2\Pi*R/\lambda*\sin(\Phi i)*\cos(2\Pi m/M-\theta i) \quad (3)$$

Where R is the radius of the circular array, and λ is the wavelength.

The covariance matrix of the array signal vector is given by

$$\begin{aligned}\mathbf{R} &= E[x(k)\times x(k)^H] \\ &= ASA^H + \acute{o}_n^{\,2} I \\ &= \mathbf{R}s + \mathbf{R}n\end{aligned} \quad (4)$$

Where the superscript "H" denote the conjugate transpose .$S=E[s(k)\times s(k)^H]$ and $\acute{o}_n^{\,2}$ is the variance of the additive noise, let $\lambda_1\geq \lambda_2\geq \ldots \lambda_D\geq \lambda_{D+1}= \ldots \lambda M= \acute{o}_n^{\,2}$, denote the eigenvalues of **R** and $e_1, e_2, \ldots e_M$ denotes the corresponding eigenvectors. If the matrices Es and En are formed as

$$Es= [e_1, e_2,\ldots, e_D] \quad (5)$$

And $En=[e_{D+1}, e_{D+2}\ldots e_M]$ (6)

Then the linear span of Es, known as the signal space, is same as spanned by the columns of A. The linear span of En, known as the noise subspace, is the orthogonal component of the signal subspace, then

$$En^H\times a(\theta_k,\Phi_k)=0, k=1,2,\ldots,D.$$

THE CAPON METHOD (MINIMUM VARIANCE)

The Capon method tries to optimize the beamforming process according to the time varying covariance matrix .Its spectrum is given by

$$P_{MV}(\theta, \Phi) = 1/(a^H(\theta, \Phi)*R^{-1}*a(\theta, \Phi)) \quad (7)$$

The method minimizes the power contributed by the noise and signals originating from other direction the current steering direction.

Because R is consisting of a signal subspace and noise subspace, then we will take only the signal subspace R_S, which is equal to

$$E_S \Lambda_S E_S \quad (8)$$

Where Λ_S is a diagonal matrix of the signal eigenvealues, and E_S is the corresponding eigenvectors as stated in (5).This feature of selection the signal subspace can be obtained by applying the Principal Component Analysis (PCA) Neural Network which extract the principal component i.e. $\lambda_1, \lambda_2,\ldots, \lambda_D$ and its corresponding eigenvectors $e_1,e_2,\ldots,e_D$ as illustrated in the next section.

[1] The number of sources is assumed known or we can find it by AIC, or MDL [Wax85].

THE NEURAL ESTIMATOR

In the last years several papers dealing with PCA neural networks [cha2000],[Chc96],[Che98],[Chi96],[Hay94],[Kun93],[Wei2000]and [Wei96] have discussed advantages ,problems, and difficulties of such neural network (which is shown in **Fig. (2)** .In what follow we make use of an APEX algorithm with adaptive learning rate .

Our neural estimator can be summarized as follows

FOR j=1 to M set the forward weight W_j (0), and lateral weight C_j (0) to random values.

1- FOR each signal samples x(k)

set j=1,k=0,$\eta_j(0)$= $\dot{o}[x(n)]/M$, $\dot{o}[x(n)]=\sum_n x^2(n)$
While stable is false do
$y_j=x(k)^H \times w_j(k-1)$
$w_j(k)= w_j(k-1)+(y_j/ \eta_j(k-1)) \times(x(n)-w_j(k-1) \times y_j)$
$\eta_j (k) = \eta_j(k-1)+ y_j^2$.
If $| w_j(k)- w_j(k-1) | < \epsilon$ (where ϵ is very small value)
Stable = true
Else
K=k+1
End

Set j=2, k=0
While stable false
$y_j=x(n)^H \times w_j(k-1)-c_j(k-1) \times y_{j-1}$
$w_j(k)= w_j(k-1)+(y_j/ \eta_j(k-1)) \times(x(n)-w_j(k-1) \times y_j)$
$c_j(k)= c_j(k-1)-y_j/ \eta_j(k-1)(y_j \times c_j(k-1)+y_{j-1})$
$\eta_j(k)= \eta_j(k-1)+ y_j^2$.
If $| w_j(k)- w_j(k-1) | < \epsilon$ (where ϵ is very small value)
Stable = true
Else
K=k+1
End

Increase j by 1 and go to stable ,and continue until j=D.

2- $w_j \longrightarrow q_j$,and $y_j \longrightarrow \sqrt{v_j}$

Where q_j are the eigenvectors corresponding to eigenvalues v_j.

3- DOA estimator, we use the modified Capon method where we take the synaptic weight as the eigenvector of the signal-subspace and the output of the PCA neural network as the squire roots of the output [Hay94].

Then DOA are obtained as the peak location of the function according to the equation

$$P_{MCapon}(\theta, \Phi) =1/(a^H (\theta, \Phi)*\check{R}_S^{-1}* a(\theta, \Phi)). \quad (9)$$

Where $\check{R}_S^{-1}$=inverse of $\check{R}_S$, and $\check{R}_S=\sum_{i=1}^{D} (y_j)^2 * w_i * w_i^H$.

SIMULATION

In our simulation we will use a uniform circular array consisting of eight sensors of radius $5\lambda/\Pi$ as shown in **Fig. (1).** Assuming a two noncoherent signals with the signal to noise ratio equal to one are impinging on the array and the first source have elevation angle θ_1=50,and azimuth angle Φ_1=30, while the second source have elevation angle θ_2=60, and azimuth angle Φ_2=40.Then by the use of the APEX algorithm we will find the signal subspace, that in turn are used by Capon method to find the DOA of impending sources as shown in figure 3,4, and 5,where figure 3 show the DOA(θi,Φi),figure 4 show the azimuth angle only, while figure 5 show the elevation angle only, where the angle will be corresponding to the peak locations in the spectrum of the Capon as shown in previous figures.

DISCUSSION AND CONCLUSIONS

This paper described a simple ,but efficient methods based on PCA Neural Network to find the DOA ,where by use of the PCA neural network we don't need to compute the correlation matrix R rather the first D eigenvectors of R are computed by the algorithm directly from the input data .the resulting computational saving can been enormous especially if the number of element M in the input vector is very large ,and the required number of the eigenvectors associated with the D largest eigenvalues of the correlation matrix R is small fraction of M..Then the DOA is achieved by incorporating the neural network with Capon method, with use of signal subspace only. Thus by the use of PCA neural network we neglect part of the noise, due to the neglecting of the noise subspace.

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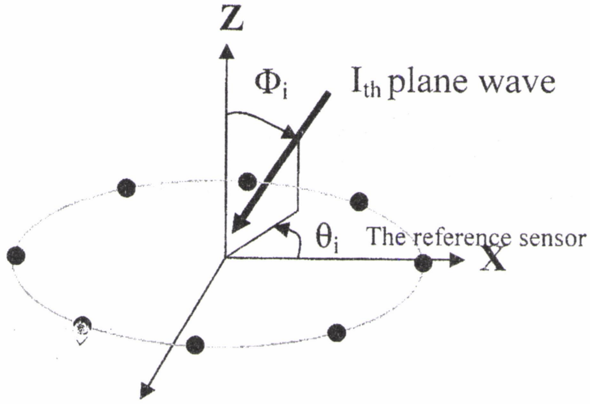

Fig. (1) Uniform Circular Array

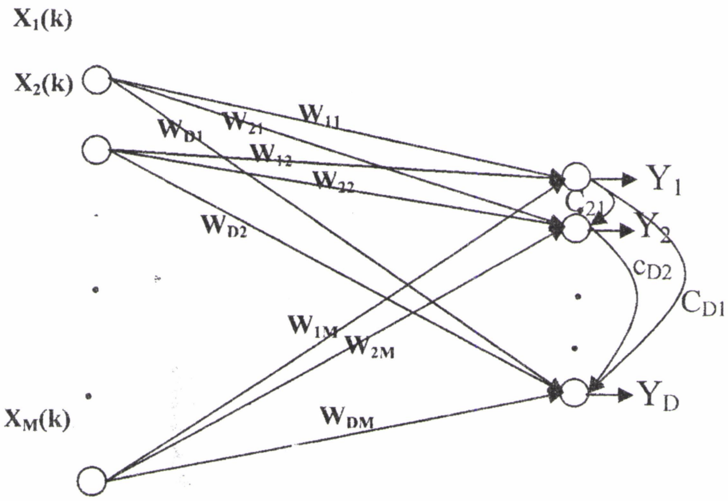

Fig. (2) PCA Neural Network

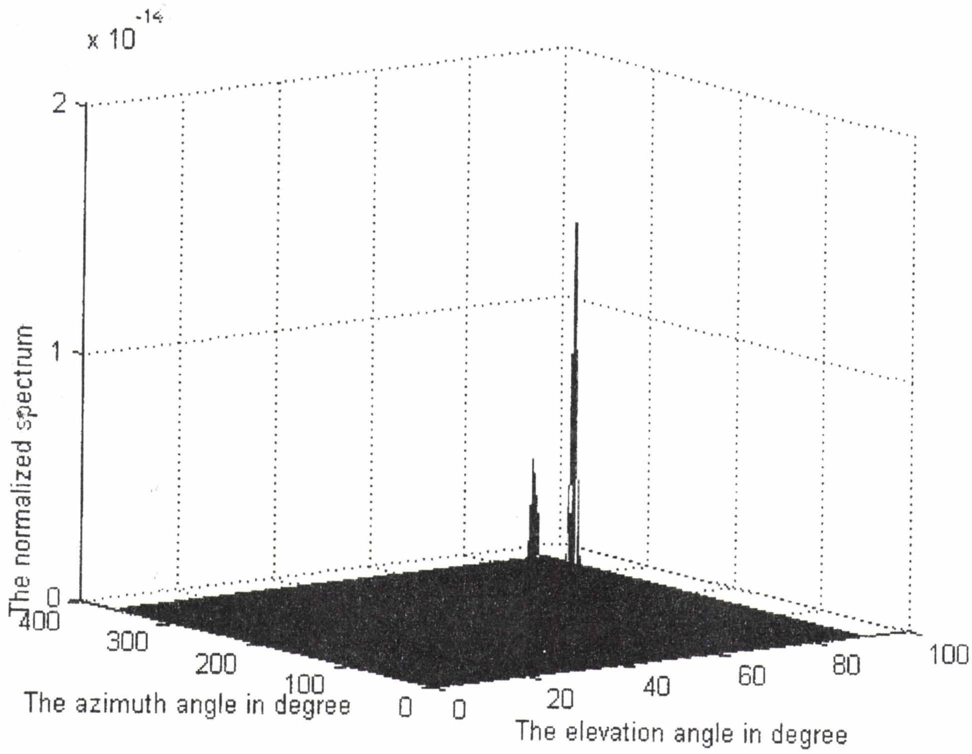

Fig. (3) The DOA(θ,ϕ).

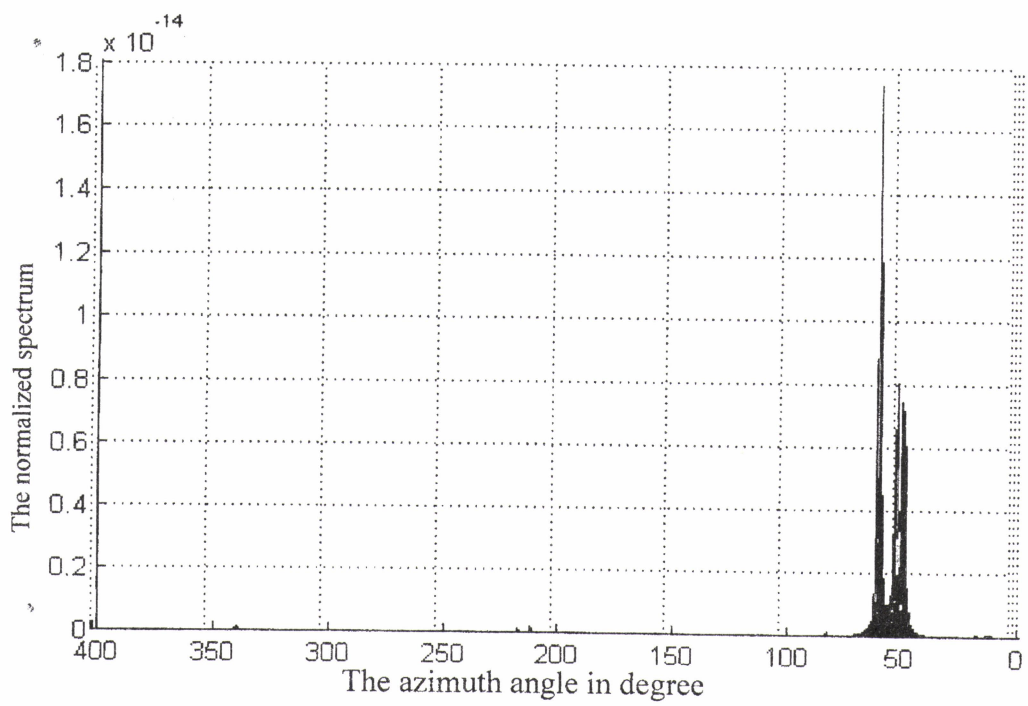

Fig. (4)The azimuth angle in degree

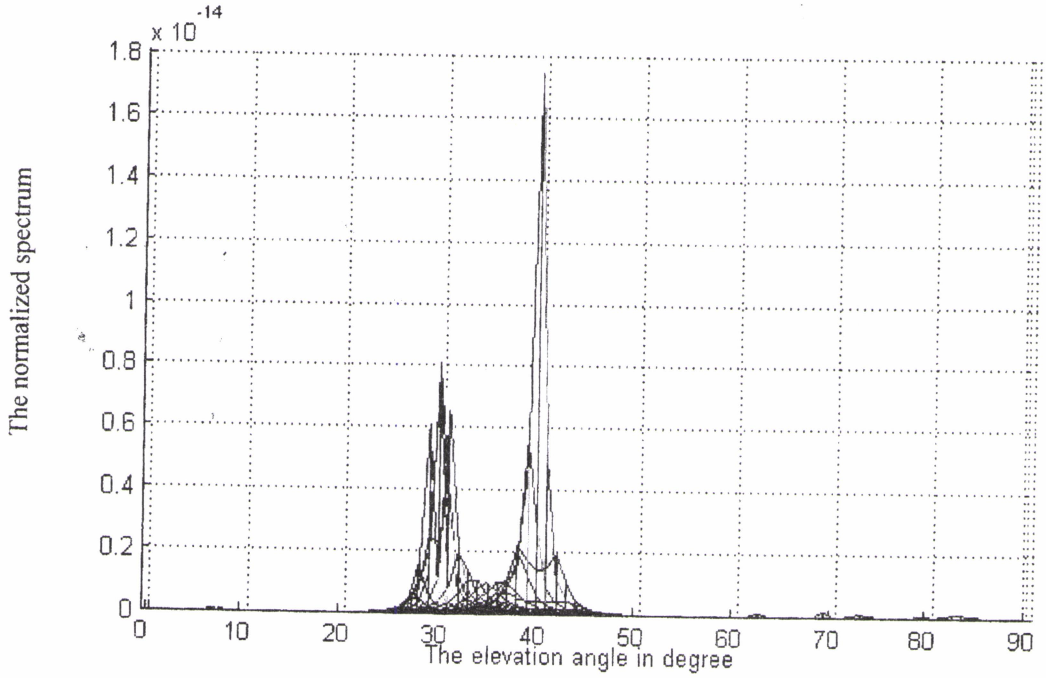

Fig. (5)The elevation angle in degree