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Transient Response Investigation of Cross-Ply Plates Using Refined Theory

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ABSTRACT

Response of cross-ply plates subjected to transient load is obtained using five variables refined plate theory, and four variables plate theory. Equations of motion are derived through the principle of virtual work. Navier series used for simply supported laminated plates. The results of this work are presented for different parameters, such as the ply number, thickness, and modulus ratio with mechanical load (sinusoidal and step pulses), which are compared with those obtained using high-order shear plate theory. Five variables of refined plate theory give results that are considerably different from the four variables of refined plate theory and higher-order theory. The obtained results from the four variables theory have the same behavior as those given by higher order theory, but are under-predicted with small time shifting.

Keywords: Refined plate theory, Composite laminated plate, Transient vibration analysis, Higher-order shear deformation theory.

1. INTRODUCTION

Laminated composite shells and plates are the main structural components that are used in different applications like supersonic flight vehicles, marine, and space structures that are made up of fiber-reinforced plastics. Dynamic loadings are subjected to some structures that induce mechanical vibrations and damage; therefore, many vibration researchers have studied the natural frequency and responses of composite plates.

For higher theory, exact solutions of a higher-order shear deformation plate are studied for the dynamic response of symmetric cross-ply laminated plates and anti-symmetric **(Khdeir and Reddy, 1988).** Also, angle-ply rectangular plates are subjected to arbitrary loading **(Khdeir and Reddy, 1989),** while **v**arious boundary conditions are investigated to obtain the response of cross-ply laminate composite plates. by using state variables, equations of

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the classical, first-order, and third-order theories are converted into a single order of equations **(Khdeir, 1995).** Transient analysis of smart laminated composite plates is investigated using higher-order shear deformation theory, shear deformation, and degree of orthotropy effects on the response of the plate are considered by **(Kumar et al., 2016)**. A new higher-order displacement function is used to derive an equation of motion of the thick and thin cross-ply composite plate and investigate transient response under different loadings, such as triangle, step, and sinusoidal distribution with different design parameters **(Ali and Majeed, 2021)**. A new displacement function is used to solve equations under the combined load of laminated plates using thermal buckling and transient mechanical loads. The transient response of thick and thin plates is considered with different parameters **(Sadiq and Majeed, 2024)**. The natural frequency and fatigue analysis of simply supported shells using the general third shell theory are studied by **(Jweeg and Alazzawy, 2010)**.

For classical and first-order theory, **(Thinh et al., 2014)** investigated the natural frequencies and the harmonic response of cross and angle ply thick plates and computed to construct dynamic stiffness matrices using a new element for thick plates, which is based on first shear deformation theory. And general boundary of cross-ply composite laminated plate is presented for transient analysis using the first-order theory by reverberation ray matrix **(Shao et al., 2016),** while **(Shao et al., 2019)** adopted **c**lassical laminated and simple order theory using reverberation ray matrix method to analyze dynamic characteristic of composite laminated plate for boundary and coupling condition. **(Majeed and Tayeh, 2015)** studied the dynamic analysis of laminated plates using classical laminated plate theory by the Ritz method, while **(Ibrahim and Ghani, 2017)** used the same method to study vibration analysis of a composite plate with different boundary conditions.

For refined theory: **(Ta and Noh, 2015)** developed an analytical solution for the dynamic response under transverse loading using a new refined theory for Functionally Graded Material (FGM) with the state space method. Dynamic analysis using 4-variable refined plate theory is developed for $[0/90]_n$ composite plate with piezoelectric composite actuator on the upper surface analytically and gives good agreement when using finite element analysis **(Rouzegar et al., 2020)**. A refined three-dimensional trigonometric shear deformation theory is discussed for stability responses of $[0/90]_n$ composite plates. Transverse displacements of thickness and in-plane trigonometric variation are proposed for shear deformation **(Belbachir et al., 2023).** Five-variable and four-variable refined plate theories are investigated for the thermal buckling of simply supported laminated plates **(Hashim and Sadiq, 2022; Yahea and Majeed, 2021)**.

Transient responses and free vibration are derived analytically using the trigonometric zigzag theory of laminated composites and sandwich plates, the results are more accurate than shear theory **(Chanda and Sahoo, 2021)**. The inverse hyperbolic zigzag theory is used to present the transient response of smart laminated plates analytically. Electromechanical load and time dependence are used to derive transient responses and compare the results with other theories **(Sahoo and Chanda, 2021).** Analytical deprived of free and forced vibration of sandwich and cross-ply laminated composite plates with trigonometric zigzag theory. This kinematic field gives results that are more accurate when compared with other theories **(Chanda and Sahoo, 2021).** Free and forced vibration of the composite plate using a five-variable displacement field with non-polynomial zigzag theory, the responses of composite plates subjected to different time-dependent loads and blast loads, which give more accurate results when compared with those obtained by other theories **(Chanda and Sahoo, 2021).** Evaluated transient response analytically using the trigonometric zigzag theory of smart laminated plate coupled with piezoelectric actuators and sensors. The dynamic behavior of plates under different electromechanical excitations with different geometries and materials is considered **(Chanda and Sahoo, 2021)**.

(Kant et al., 1992) used different loads to determine the transient dynamic response of composite and sandwich plates using refined theory and superposition technique. The dynamic response is developed using the Jaeobi method with a subspace iteration technique, and then the mode shapes, while **(Khante et al., 2007)** developed a higher-order shear deformation theory using a finite element model to investigate the damped transient dynamic elastoplastic analysis of a plate. The finite element method is proposed to study static and dynamic analyses of shells. Numerical solutions of laminated composite shell response compared with that developed by first-order shear deformation theory are studied by **(Pham et al., 2018).** Isoperimetric elements with five degrees of freedom are used to model the plate and study the displacement for composite plates subjected to different time-dependent loads and blast loads, which gives more accurate results when compared with other theories **(Saha and Mandal, 2021)**.

In the present work analytical solutions for the transient response of cross play plates are studied using different plate theories, a higher-order theory which gives results that closer to three dimensions elasticity theory, five variables refined plate theory, and four variables plate theory, which gives results closest to those obtained from higher order shear deformation theory. Many design parameters have been investigated, such as thickness ratio, modulus ratio, and number of layers.

2. DISPLACEMENT FIELD

Five and four variables Refined Plate Theory (RPT) were used to investigate the response of a simply supported rectangular plate of total thickness (h) of (n) orthotropic layers as shown in **Fig. 1**. Three components of displacement are extension. w_a, bending w_b, and shear w_s which are functions of x, y, and t, represent total transverse displacement (w). **(Matsunaga, 2001)**. Based on the RPT assumption, the displacement field is expressed as **(Kim et al., 2009):**

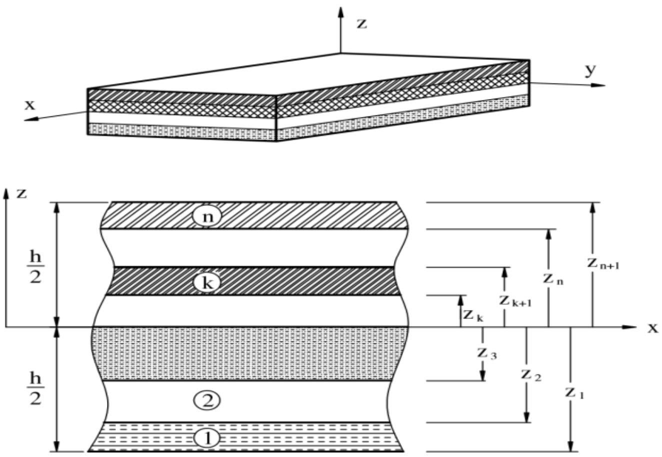


Figure1. Coordinate system of laminated plates

$$u(x,y,z,t) = u_o(x,y,t) - z\left(\frac{\partial w_b}{\partial x}\right) + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial x} \tag{1-a}$$

$$v(x,y,z,t) = v_o(x,y,t) - z\left(\frac{\partial w_b}{\partial y}\right) + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial w_s}{\partial y} \tag{1-b}$$

$$w(x,y,z,t) = w_a(x,y,t) + w_b(x,y,t) + w_s(x,y,t) \tag{1-c}$$

For small strains, the relation of strain-displacement **(Reddy, 2003)**:

$$\varepsilon_x = \frac{\partial u_o}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial^2 w_s}{\partial x^2} \tag{2-a}$$

$$\varepsilon_y = \frac{\partial v_o}{\partial y} - z\frac{\partial^2 w_b}{\partial y^2} + z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial^2 w_s}{\partial y^2} \tag{2-b}$$

$$\gamma_{xy} = \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} - 2z\frac{\partial^2 w_b}{\partial x\,\partial y} + 2z\left[\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right]\frac{\partial^2 w_s}{\partial x\,\partial y} \tag{2-c}$$

$$\gamma_{yz} = \frac{\partial w_a}{\partial y} + \left[\frac{5}{4} - 5\frac{z^2}{h^2}\right]\frac{\partial w_s}{\partial y} \tag{2-d}$$

$$\gamma_{xz} = \frac{\partial w_a}{\partial x} + \left[\frac{5}{4} - 5\frac{z^2}{h^2}\right]\frac{\partial w_s}{\partial y} \tag{2-e}$$

The strain field is:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \tag{3-a}$$

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \gamma_{xz}^a \\ \gamma_{yz}^a \end{Bmatrix} + g\begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix}) \tag{3-b}$$

Where:

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial x} \\ \dfrac{\partial v_o}{\partial y} \\ \dfrac{\partial u_o}{\partial y} + \dfrac{\partial v_o}{\partial x} \end{Bmatrix} \tag{4-a}$$

$$\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x\,\partial y} \end{Bmatrix} \tag{4-b}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x\,\partial y} \end{Bmatrix} \tag{4-c}$$

$$\begin{Bmatrix} \gamma_{xz}^a \\ \gamma_{yz}^a \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_a}{\partial x} \\ \dfrac{\partial w_a}{\partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial x} \\ \dfrac{\partial w_s}{\partial y} \end{Bmatrix} \tag{4-d}$$

$$f = -\frac{1}{4}z + \frac{5}{3}z\left(\frac{z}{h}\right)^2, \quad g = \frac{5}{4} - 5\left(\frac{z}{h}\right)^2 \tag{4-e}$$

3. PRINCIPLE OF VIRTUAL WORK

Hamilton's principle is used to derive equations of motion, which depend on refined plate theory. **(Reddy, 2003)**.

$$0 = \int_0^t \delta U + \delta V - \delta T \tag{5}$$

The virtual strain energy δU is:

$$\delta U = [\int_{\frac{h}{2}}^{-\frac{h}{2}} \left(\int_{\Omega}^{k} [\sigma_x \delta\varepsilon_x^k + \sigma_y \delta\varepsilon_y^k + \sigma_{xy}\delta\gamma_{xy}^k + \sigma_{yz}\gamma_{yz}^k + \sigma_{xz}\gamma_{xz}^k]\, \partial x\, \partial y \right) \partial z] = 0 \tag{6}$$

Substituting Eq. (4) into Eq. (6) gives:

$$\delta U = \int \left\{ N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + Q_{yz}^a \delta\gamma_{yz}^a + Q_{xz}^a \delta\gamma_{xz}^a + Q_{yz}^s \delta\gamma_{yz}^s + Q_{xz}^s \delta\gamma_{xz}^s \right\} \partial x\, \partial y = 0 \tag{7}$$

Where:

$$(N_x, N_y, N_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \sigma_{xy}) dz \tag{8-a}$$

$$(M_x^b, M_y^b, M_{xy}^b) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) z\, dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \sigma_{xy}) z\, dz \tag{8-b}$$

$$(M_x^s, M_y^s, M_{xy}^s) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) f\, dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \sigma_{xy}) f\, dz \tag{8-c}$$

$$(Q_{xz}^a, Q_{yz}^a, Q_{yz}^s, Q_{xz}^s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_{xz}, \sigma_{yz}, g\sigma_{xz}, g\sigma_{yz})\, dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} (\sigma_{xz}, \sigma_{yz}, g\sigma_{xz}, g\sigma_{yz})\, dz \tag{8-d}$$

Substituting the virtual strain in terms of virtual displacement, Eq. (4) in Eq. (7) and integrating, gives:

$$0 = \int [-\delta u_o \frac{\partial N_x}{\partial x} - \delta v_o \frac{\partial N_y}{\partial y} - \delta u_o \frac{\partial N_{xy}}{\partial y} - \delta v_o \frac{\partial N_{xy}}{\partial x} - \delta w_b \frac{\partial^2 M_x^b}{\partial x^2} - \delta w_b \frac{\partial^2 M_y^b}{\partial y^2} - 2\,\delta w_b \frac{\partial^2 M_{xy}^b}{\partial x \partial y} - \delta w_s \frac{\partial^2 M_x^s}{\partial x^2} - \delta w_s \frac{\partial^2 M_y^s}{\partial y^2} - 2\,\delta w_s \frac{\partial^2 M_{xy}^s}{\partial x \partial y} - \delta w_a \frac{\partial Q_{yz}^a}{\partial y} - \delta w_a \frac{\partial Q_{xz}^a}{\partial x} - \delta w_s \frac{\partial Q_{yz}^s}{\partial y} - \delta w_s \frac{\partial Q_{xz}^s}{\partial x}]\, \partial x \partial y \tag{9}$$

$$\delta V = \int_{\Omega} \{q * (w_a + w_b + w_s)\} dxdy \tag{10}$$

$$\delta T = \int \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho \{ \left[\dot{u}_o - z\frac{\partial \dot{w}_b}{\partial x} + f\frac{\partial \dot{w}_s}{\partial x}\right] \left[\delta\dot{u}_o - z\frac{\delta\,\partial \dot{w}_b}{\partial x} + f\frac{\delta\,\partial \dot{w}_s}{\partial x}\right] + \left[\dot{v}_o - z\frac{\partial \dot{w}_b}{\partial y} + f\frac{\partial \dot{w}_s}{\partial y}\right] \left[\delta\dot{v}_o - z\frac{\delta\,\partial \dot{w}_b}{\partial y} + f\frac{\delta\,\partial \dot{w}_s}{\partial y}\right] + [\dot{w}_a + \dot{w}_b + \dot{w}_s][\delta\dot{w}_a + \delta\dot{w}_b + \delta\dot{w}_s] \} dv \tag{11-a}$$

$$\delta T = \int \Big[\left(I_1\dot{u}_o - I_2\frac{\partial \dot{w}_b}{\partial x} + I_4\frac{\partial \dot{w}_s}{\partial x}\right)\delta\dot{u}_o + \left(-I_2\dot{u}_o + I_3\frac{\partial \dot{w}_b}{\partial x} - I_5\frac{\partial \dot{w}_s}{\partial x}\right)\frac{\delta\,\partial \dot{w}_b}{\partial x} + \left(I_4\dot{u}_o - I_5\frac{\partial \dot{w}_b}{\partial x} + I_6\frac{\partial \dot{w}_s}{\partial x}\right)\frac{\delta\,\partial \dot{w}_s}{\partial x} + \left(I_1\dot{v}_o - I_2\frac{\partial \dot{w}_b}{\partial y} + I_4\frac{\partial \dot{w}_s}{\partial y}\right)\delta\dot{v}_o + \left(-I_2\dot{v}_o + I_3\frac{\partial \dot{w}_b}{\partial y} - I_5\frac{\partial \dot{w}_s}{\partial y}\right)\frac{\delta\,\partial \dot{w}_b}{\partial y} + \Big(I_4\dot{v}_o -$$

$I_5 \frac{\partial \dot{w}_b}{\partial y} + I_6 \frac{\partial \dot{w}_s}{\partial y} \Big) \frac{\delta\, \partial \dot{w}_s}{\partial y} + (\dot{w}_a + \dot{w}_b + \dot{w}_s)\delta\dot{w}_a + (\dot{w}_a + \dot{w}_b + \dot{w}_s)\delta\dot{w}_b + (\dot{w}_a + \dot{w}_b +$
$\dot{w}_s)\delta\dot{w}_s \Big]\, dx\, dy$ (11-b)

Where:

$$\left(I_1, I_2, I_3, I_4, I_5, I_6\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho\, (1, z, z^2, f(z), zf(z), [f(z)]^2)\; dz \qquad (11\text{-c})$$

4. EQUATIONS OF MOTION

When substituting Eq. (7) into Eq. (11) into Eq. (5), the Euler-Lagrange equation is obtained. The equations of motion are calculated by setting the coefficient of (δu, δv, δw_a, δw_b, δw_s) of Eq. (6) to zero.

$$\delta u : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1 \ddot{u} \qquad (12\text{-a})$$

$$\delta v : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_1 \ddot{v} \qquad (12\text{-b})$$

$$\delta w_{be} : \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\, \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q * w_b = I_1(\ddot{w}_a + \ddot{w}_b + \ddot{w}_s) - I_3 \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_b}{\partial y^2}\right) \qquad (12\text{-c})$$

$$\delta w_{sh} : \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x\, \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y} + q * w_{sh} = I_1(\ddot{w}_a + \ddot{w}_b + \ddot{w}_s) - I_6 \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w_s}{\partial x^2} + \frac{\partial^2 w_s}{\partial y^2}\right) \qquad (12\text{-d})$$

$$\delta w_a : \frac{\partial Q_{xz}^a}{\partial x} + \frac{\partial Q_{yz}^a}{\partial y} + q * w_a = I_1(\ddot{w}_a + \ddot{w}_b + \ddot{w}_s) \qquad (12\text{-e})$$

The forces and moments results are given by: **(Reddy, 2003).**

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} dz \qquad (13\text{-a})$$

$$\begin{Bmatrix} M_x^b \\ M_y^b \\ M_{xy}^b \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} z\, dz \qquad (13\text{-b})$$

$$\begin{Bmatrix} M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} f\, dz \qquad (13\text{-c})$$

$$\begin{Bmatrix} Q_{xz}^a \\ Q_{yz}^a \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} dz \qquad (13\text{-d})$$

$$\begin{Bmatrix} Q_{xz}^s \\ Q_{yz}^s \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} g\, dz \qquad (13\text{-e})$$

The plane stress stiffness Q_{ij} are:

$$Q_{11} = \frac{E_1}{1 - v_{12} v_{21}}, \quad Q_{21} = \frac{v_{12} E_2}{1 - v_{12} v_{21}}$$
$$Q_{22} = \frac{E_2}{1 - v_{12} v_{21}}, \quad Q_{66} = G_{12},$$

$Q_{44} = G_{23}$, $Q_{55} = G_{13}$

From the constitutive relation of k^{th} layer lamina, the transformed stress-strain relation is:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{21} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (14)$$

The forces and moments results are related to the strains by relations:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + \begin{bmatrix} B_{11}^s & B_{12}^s & B_{16}^s \\ B_{12}^s & B_{22}^s & B_{26}^s \\ B_{16}^s & B_{26}^s & B_{66}^s \end{bmatrix} \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \quad (15\text{-a})$$

$$\begin{Bmatrix} M_x^b \\ M_y^b \\ M_{xy}^b \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + \begin{bmatrix} D_{11}^s & D_{12}^s & D_{16}^s \\ D_{12}^s & D_{22}^s & D_{26}^s \\ D_{16}^s & D_{26}^s & D_{66}^s \end{bmatrix} \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \quad (15\text{-b})$$

$$\begin{Bmatrix} M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix} B_{11}^s & B_{12}^s & B_{16}^s \\ B_{12}^s & B_{22}^s & B_{26}^s \\ B_{16}^s & B_{26}^s & B_{66}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11}^s & D_{12}^s & D_{16}^s \\ D_{12}^s & D_{22}^s & D_{26}^s \\ D_{16}^s & D_{26}^s & D_{66}^s \end{bmatrix} \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + \begin{bmatrix} H_{11}^s & H_{12}^s & H_{16}^s \\ H_{12}^s & H_{22}^s & H_{26}^s \\ H_{16}^s & H_{26}^s & H_{66}^s \end{bmatrix} \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \quad (15\text{-c})$$

$$\begin{Bmatrix} Q_{yz}^a \\ Q_{xz}^a \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^a \\ \gamma_{xz}^a \end{Bmatrix} + \begin{bmatrix} A_{44}^a & A_{45}^a \\ A_{45}^a & A_{55}^a \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} \quad (15\text{-d})$$

$$\begin{Bmatrix} Q_{yz}^s \\ Q_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^a & A_{45}^a \\ A_{45}^a & A_{55}^a \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^a \\ \gamma_{xz}^a \end{Bmatrix} + \begin{bmatrix} A_{44}^s & A_{45}^s \\ A_{45}^s & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} \quad (15\text{-e})$$

Where:

$$\left(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{Q}_{ij}\,(1, z, z^2, f(z), z\,f(z), [f(z)]^2)\, dz \quad (15\text{-h})$$

Equations of motion are solved using Navier's series, which satisfy simply supported boundary conditions are given as **(Reddy, 2003):**

$$u = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn} \cos \alpha x \sin \beta y \quad (16\text{-a})$$

$$v = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn} \sin \alpha x \cos \beta y \quad (16\text{-b})$$

$$w_b = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{bmn} \sin \alpha x \sin \beta y \quad (16\text{-c})$$

$$w_s = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{smn} \sin \alpha x \sin \beta y \quad (16\text{-d})$$

$$w_a = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{amn} \sin \alpha x \sin \beta y \quad (16\text{-e})$$

where: $\alpha = \frac{m\pi}{a}$, $\beta = \frac{n\pi}{b}$ and (U_{mn} V_{mn} W_{bmn} W_{smn} W_{amn}) are arbitrary constants.

Substituting the above equations in equations of motion, the following dynamic equations are found in matrix form:

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & 0 \\ s_{12} & s_{22} & s_{23} & s_{24} & 0 \\ s_{13} & s_{23} & s_{33} & s_{34} & 0 \\ s_{14} & s_{24} & s_{34} & s_{44} & s_{45} \\ 0 & 0 & 0 & s_{45} & s_{55} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \\ W_{amn} \end{Bmatrix} + \begin{bmatrix} m_{11} & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} & m_{35} \\ 0 & 0 & m_{34} & m_{44} & m_{45} \\ 0 & 0 & m_{11} & m_{11} & m_{55} \end{bmatrix} \begin{Bmatrix} \ddot{U}_{mn} \\ \ddot{V}_{mn} \\ \ddot{W}_{bmn} \\ \ddot{W}_{smn} \\ \ddot{W}_{amn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q(t) \\ q(t) \\ q(t) \end{Bmatrix} \quad (17)$$

5. TRANSIENT SOLUTION

Plate transient displacement calculated by principal mode method with orthogonality condition of modes **(Khdeir and Reddy, 1989)**:

$$(\omega_{mn}^2 - \omega_{sr}^2) \int_0^a \cdot \int_0^b \{[I_1 U_{mn}]U_{sr} + [I_1 V_{mn}]V_{sr} + [I_1 W_{amn} + I_1 W_{bmn} + I_1 W_{smn} + I_3 \ (\alpha^2 + \beta^2) W_{bmn}]W_{bsr} + [I_1 W_{amn} + I_1 W_{bmn} + I_1 W_{smn} + I_6 \ (\alpha^2 + \beta^2) W_{smn}]W_{ssr} + [I_1 W_{amn} + I_1 W_{bmn} + I_1 W_{smn}]W_{asr}\}\, dx\, dy = 0 \quad (18)$$

Generalized forces by orthogonality condition are:

$$f_{mn}(t) = \frac{\int_0^a \cdot \int_0^b (q * W_{mn})\, dx\, dy}{N_{mn}} \quad (19)$$

Where:

$$N_{mn} = \int_0^a \cdot \int_0^b \{I_1[U_{mn}^2 + V_{mn}^2 + 2(W_{amn}W_{bmn} + W_{amn}W_{smn} + W_{bmn}W_{smn}) + (W_{amn}^2 + W_{bmn}^2 + W_{smn}^2)] + I_3[(\alpha^2 + \beta^2)W_{bmn}^2] + I_6\left[I_3[(\alpha^2 + \beta^2)W_{smn}^2]\right]\}\, dx\, dy\; u_{i(x,y)} = \sum_{m=n=1}^{\infty} U_{imn(x,y)}\, T_{imn(t)} \quad (20)$$

Where (U_{imn}) are the plate mode for i = 1 to 5, while the unknown time function is $T_{mn(t)}$.

$$\ddot{T}_{mn} + \omega_{mn}^2 T_{mn} = f_{mn} \quad (21)$$

For zero initial conditions, the solution **(Gutierrez and Reddy, 2017)**:

$$T_{mn(t)} = \frac{1}{\omega_{mn}} \int_0^t f_{mn(\tau)} \sin \omega_{mn(k)}(t - \tau)\, d\tau \quad (22)$$

Response of plate under a load $q_{(x,y,t)} = q_0 f_{3(x,y)} F_{(t)}$, (m=n=1), can be presented as:

$$\begin{Bmatrix} u \\ v \\ w_b \\ w_s \\ w_a \end{Bmatrix} = \sum_{k=1}^{5} \frac{q_0}{J_{mn(k)} \omega_{mn(k)}} \begin{Bmatrix} U_{mn(k)} \\ V_{mn(k)} \\ f_3 \\ W_{smn(k)} \\ Wamn_{\,mn(k)} \end{Bmatrix} \int_0^t F_{(\tau)} \sin \omega_{mn(k)}(t - \tau)\, d\tau \quad (23)$$

For the sin pulse and step pulse, F (t) is **(Khdeir and Reddy, 1989)**:

$$F_{(t)} = \begin{Bmatrix} \sin\left(\pi t / t_1\right) & 0 \le t \le t_1 \\ 0 & t > t_1 \end{Bmatrix}$$

$$F_{(t)} = \begin{Bmatrix} 1 & 0 \le t \le t_1 \\ 0 & t > t_1 \end{Bmatrix}$$

6. RESULTS AND DISCUSSION

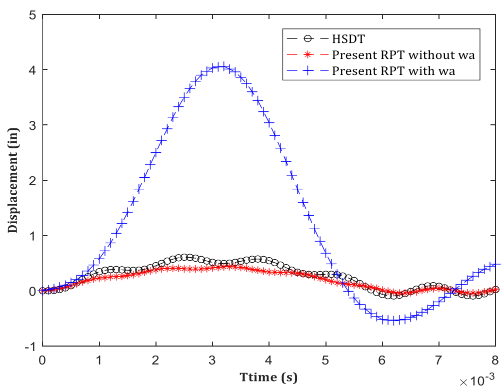
The transverse response of cross-ply laminated plates is analyzed by solving equations of motion using MATLAB R2014a programming for material properties used for all figures shown in **Table 1** (1(in) = 2.54 cm):

Table 1. Material properties **(Ali and Majeed, 2021)**

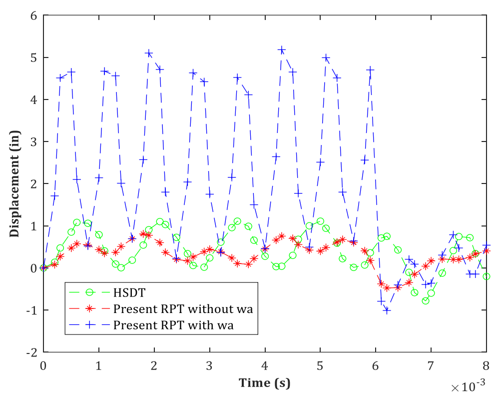
E_1	172.369 GPa
G12 = G13	3.448 GPa
ν_{12}	0.25
E_2	6.895 GPa
G_{23}	1.379 GPa
ρ	1603.03 kg/m^3
a/h	5
q_o	68.9476 MPa

6.1 Comparison Study

Many plate theories are used to investigate the response of a cross simply supported plate, refined plate theory with five variables, refined plate theory with four variables, and higher order shear deformation theory. The transient displacement results of laminated plates are compared with higher-order shear deformation theory **(Ali and Majeed, 2021)** of a cross-ply (0°/90°/ 0°) square plate (a = b). The response of laminated plates under transient loading (sine pulse and step pulse) obtained from four refined theories is closer than that obtained from five refined theories when compared to those given from high-order theory, as shown in **Fig. 2** and **Table 2.** Therefore, the present study focused on the four-variable refined plate theory. Central deflection and non-dimensional normal stress σ_1 under both step and sine pulses are shown in **Figs. 3 and 4,** respectively**.** Which results obtained from the four variables theory have the same behavior as those given by higher order theory, but are under-predicted with small time shifting.



a- Sine pulse

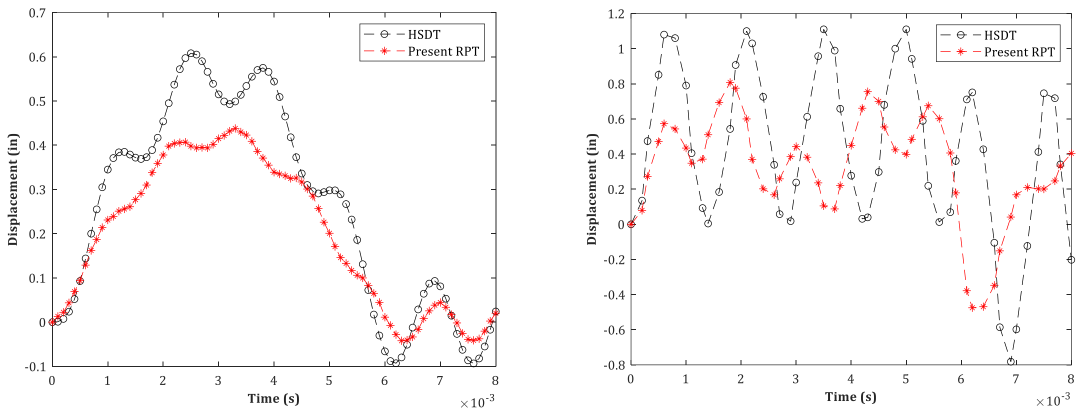


b- Step Pulse

Figure 2. Center displacement as a function of time, for different plate theories and pulses. (a) sine; (b) step; (HSDT; 4 variable RPT; 5 variable RPT).

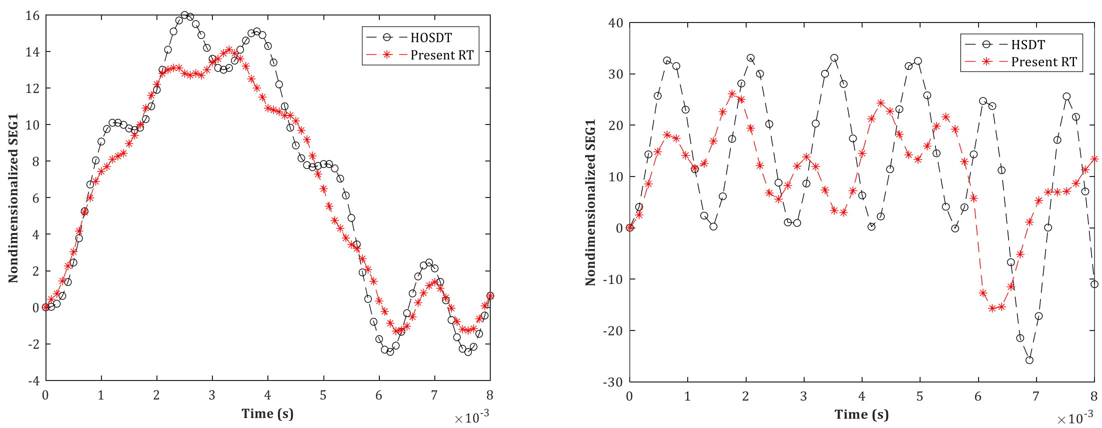
Table 2. Discrepancy between higher-order and refined plate theory

W in (time)				
Sin pulse				
HSDT	4RPT	5RPT	Discrepancy of 4RTP	Discrepancy of 5RTP
0.608(0.0025s)	0.407(0.0024s)	4.06(0.0032s)	33.059%	567.763%
W in (time)				
Step pulse				
HSDT	4RPT	5RPT	Discrepancy of 4RTP	Discrepancy of 5RTP
1.08(0.0006s)	0.573(0.0006s)	4.65(0.0005s)	46.944%	330.555%



a- Sine pulse b- Step Pulse

Figure 3. Center displacement as a function of time, for different pulses. (a) sine; (b) step; (4 variable RPT).

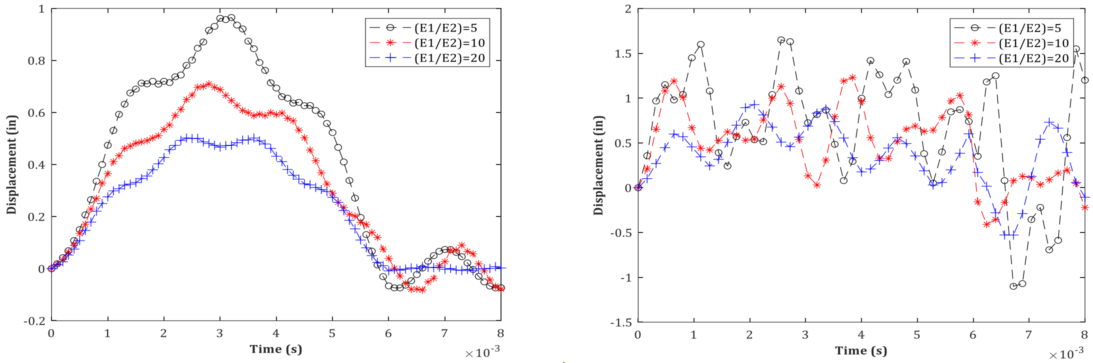


a- Stress1 under sin pulse b- Stress1 under step pulse

Figure 4. Nondimensional normal stress σ_1 as a function of time, for different pulses. (a) sine; (b) step; (4 variable RPT).

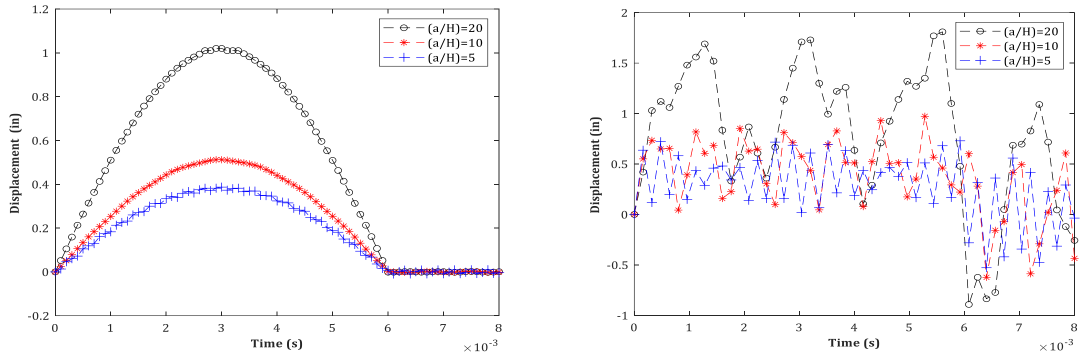
6.2 Design Parameters

The thickness ratio, modulus ratio, and number of layers are investigated as design parameters of cross-ply plates. It is observed that increasing the modulus ratio leads to a decrease in the response of the plate under sine and step pulse, as shown in **Fig. 5,** because stiffness increases. However**, Fig. 6** shows that central deflection increases when (a/H) decreases; also, the response of several ply (2, 4, and 6) layers illustrated in **Fig. 7**, having the same trend, displacement decreases. Increase and decrease of displacement due to an increase and decrease in stiffness for the same material properties.



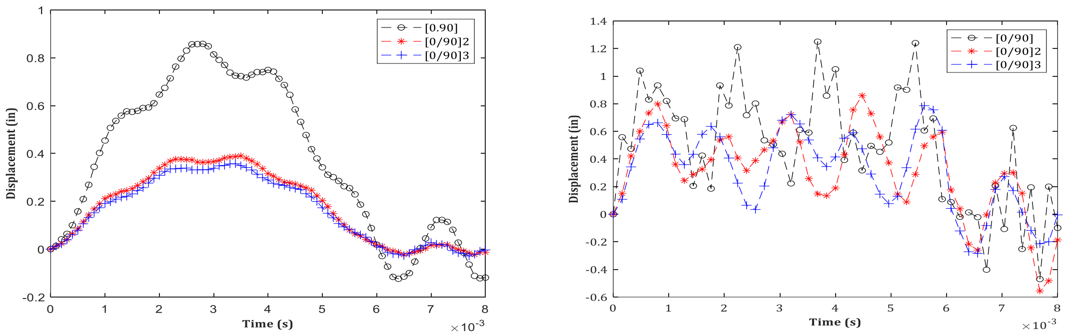
a- Sine pulse b- Step pulse

Figure 5. Center displacement as a function of time, for different pulses and (E1/E2). (a) sine; (b) step; (4 variable RPT).[0/90]2



a- Sine pulse b- Step pulse

Figure 6. Center displacement as a function of time, for different pulses and (a/H). (a) sine; (b) step; (4 variable RPT). [0/90]2



a- Sine pulse b- Step pulse

Figure 7. Variation of the center deflection as a function of time, for various pulses and no. of layers. (a) sine; (b) step; (4 var. RPT).

7. CONCLUSIONS

In the present work, different plate theories are investigated to obtain the dynamic response of a simply supported plate, from which the following points are concluded:

1. Dynamic response of laminated cross-ply plates using analytical solutions studied using different plate theories, higher-order theory, five variables refined plate theory, and four variables plate theory. To compare the obtained results under different loadings, the five variables of refined plate theory give results that are considerably different from the four variables of refined plate theory and higher-order theory.
2. The transient response of a plate using four-variable refined theory has similarity with classical plate theory, but it describes transverse shear stresses and shear strains closer to those obtained from higher-order theory.
3. Four-variable reined theory is easy to use when compared with higher-order theory.
4. Changing design parameters using the four-variable refined theory gives the same behavior compared with other theories

Some recommendations for future work are:
1. Transient response investigation of angel-ply plates using refined theory.
2. Static and dynamic analysis of laminated plates with different boundary conditions based on refined theory.
3. Transient response investigation of laminated plates using refined theory in thermal environment.

NOMENCLATURE

Symbol	Description	Symbol	Description
a	Plate dimension in x-direction (m)	T_{mn}	Time function
b	Plate dimension in y-direction (m)	t	time (s)
h	Plate thickness	x, y, z	Cartesian Coordinate system
f_{mn}	Applied load (N)	w_a , w_b, w_s	Displacement in extension, ending, and shear, respectively
A_{ij} , B_{ij} , D_{ij} , B^s_{ij} , D^s_{ij} , H^s_{ij}	Extension, bending, extension coupling (N/m)	u_s, v_s	Displacement in x and y direction ue to shear, respectively
E1, E2, E3	Elastic modulus components (GPa)	ε_x, ε_y, ε_z	Strain components (m/m)
G_{12}, G_{23}, G_{13}	Shear modulus components (GPa)	γ_{xz}, γ_{yz}	Transverse shear strain (m/m)
n	Total number of plate layers	v_{12} v_{21}	Poisson's ratio
N_x, N_y ,N_{xy}	In-plan force per unit length (N/m)	ω_{mn}	natural frequency (rad/s)
M^s_x , M^s_y , M^s_{xy}	Force per unit length due to shear moment (N/m)	σ_x σ_y σ_{xy} σ_{yz} σ_{xz}	Stress components (Gpa)
Q^a_{xz}, Q^a_{yz} Q^s_{yz},Q^s_{xz}	Transverse shear force (N)		

STEPS OF THE ALGORITHM

1. Input geometrical and mode specifications a, b, h, N, m, n
2. Input material properties, $E_1(T)$, $E_2(T)$, ρ, $\upsilon_{12}(T)$, υ_{13}, υ_{23}, $G_{12}(T)$, $G_{13}(T)$, $G_{23}(T)$
3. Calculate transformed stiffness $\overline{Q}^k_{ij}$

4. Calculate stiffness A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}
5. Calculate mass matrix & stiffness matrix
6. Calculate mode shapes and their frequencies
7. Input period, load amplitude, load function
8. Solution

Credit Authorship Contribution Statement

Ibtehal Abbas Sadiq: Writing – review & editing, Writing – original draft, Validation, Software, Methodology, data collection. Widad Ibraheem Majeed: review & editing, Validation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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دراسة الاستجابة العابرة للصفائح المتعددة الطبقات باستخدام النظرية المحسنة

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الخلاصة

تم الحصول على استجابة الصفائح الطبقية ذات الالياف المتعامدة والمعرضة الى حمل عابر باستخدام نظرية محسنة بخمسة وأربع متغيرات. تم اشتقاق معادلات الحركة والتي تعتمد على مبدأ الشغل الافتراضي وتم استخدام سلسلة نافير للصفائح الطبقية ذات المساند البسيطة. تم تقديم نتائج هذا العمل لعوامل مختلفة مثل عدد الطبقات ونسبة السمك ومعامل المرونة وبتأثير احمال ميكانيكية ذات النبضة الجيبية والمتدرجة، وتم مقارنة النتائج مع نظرية ذات الرتبة العالية. نظرية محسنة بخمسة متغيرات اعطت نتائج مختلفة عن النظرية المحسنة باربع متغيرات و مع نظرية ذات الرتبة العالية. نتائج النظرية المحسنة باربع متغيرات لها نفس التصرف لتل التي تم احصول عليها من النظرية ذات الرتبة العالية ولكن بقيم اقل مع ازاحة بسيطة للفترة الزمنية.

الكلمات المفتاحية: الصفائح الطبقية المركبة، تحليل الاهتزاز العابر، نظرية الصفائح المحسنة، نظرية القص ذات الرتبة العالية.