





## Deterministic and Probabilistic Approaches for Estimating Oil Reserve in Carbonate Reservoir: A Case Study for the Buzrgan Oil Field, Southern Iraq

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### ABSTRACT

The accuracy in oil reserve estimation is very important, as decisions should be made based on this value. The present paper will deal with estimating oil reserves by using deterministic and probabilistic approaches. The case study is the Mishrif formation of the Buzrgan oil field that located in Missan Governorate, southern Iraq. The deterministic approach is represented by the material balance equation, while the probabilistic approach is represented by Monte Carlo simulation. The data used in the material balance equation are production, PVT, and pressure data, along with needing to calculate bulk volume and porosity, while the Monte Carlo simulation needs volumetric equation parameters, which are porosity, fluid saturation, net pay thickness, and reservoir bulk volume. The linearized material balance equation, the Havlena – Odah method, is used. Monte Carlo simulation is applied as a volumetric equation, but each input parameter is modeled to get the best value. The result of the oil reserve by the material balance equation was 4.368 billion stock tank barrel while by Monte Carlo simulation was classified by Hefner and Thompson's classification as *P90* proven reserves is 1.03279 billion stock tank barrel, proved and probable oil reserves *P50* is 1.98761 billion stock tank barrel, and proved, probable, and possibly reserve *P10* is 4.3694 billion stock tank barrel. The material balance equation cannot be used before the production stage, while the Monte Carlo simulation can be applied before that.

**Keywords:** Material balance, Monte carlo, Volumetric method, Mishrif formation, Probabilistic approach.

### 1. INTRODUCTION

The determination of hydrocarbon reserves is a core function in the oil and gas industry. The hydrocarbon reserve is defined as the oil and gas quantities with some of the associated materials that are expected to be economically recoverable from a reservoir at special time

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Peer review under the responsibility of University of Baghdad.

<https://doi.org/10.31026/j.eng.2026.04.02>



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Article received: 25/09/2025

Article revised: 15/03/2026

Article accepted: 27/03/2026

Article published: 01/04/2026



(Ampomah et al., 2016; Al-Dujaili, 2023). Its estimation with a high degree of certainty represents the greatest challenges for petroleum engineers (Thanh, 2002). The decisions for hydrocarbon reserve possession and divestiture depend on many parameters such as company plan, risks, financial state of the company, and payment considerations (Simpson et al., 1995). The hydrocarbon reserve estimation can be classified into two main approaches: deterministic and probabilistic. Many methods are categorized as deterministic, likewise: decline curve analysis, material balance, volumetric, analog, and reservoir simulation (Kosova et al., 2015), as shown in Fig. 1 (Cronquist, 2001). The deterministic approach provides a unique value with no uncertainty, while the probabilistic approach supplies a number of potential values with levels of uncertainty (Al-Dujaili et al., 2024). The Monte Carlo simulation is the main common method to construct probabilistic hydrocarbon estimation (Al-Khalifa et al., 2007).

One of the important methods from the deterministic approach is the material balance equation (MBE) for calculating hydrocarbon reserves. It is initially presented by (Schilthuis, 1936) as achieving mass or volume balance where the produced volume that defined as a reservoir withdrawal, is equal to fluid expansion, water encroachment, fluid injection, and rock water compressibility according to reservoir pressure decline (Dake, 1978). (Havlena and Odeh, 1963) made the development of the general material balance equation by plotting some variables against others based on the reservoir driving mechanism. This method is called the graphical method, and it mainly depends on the sequence of plotting points and the final resulting plot (Yee, 2011).

Monte Carlo simulation is the probabilistic approach that employs the ranges of input reservoir parameters to make the probability distribution and estimate the distribution ranges of the target parameter in a random way (Kim et al., 2014). Hefner and Thompson defined the hydrocarbon reserves level in terms of percent. The percentages are defined as  $P90$ ,  $P50$ , and  $P10$ , where these values refer to the probability of existing actual reserves greater than 90%, 50%, and 10%, respectively.  $P90$  refers to reserves with a 90% confidence of recovery, whereas  $P50$  represents proved and probable reserves, and  $P10$  represents proved, probable, and possibly recoverable reserves (Hefner and Thompson, 1993; Idrobo et al., 2001).

The aim of the present work is the estimation of oil reserve in the Mishrif formation of the Buzrgan oil field by achieving the following objectives according to information availability:

1. Using the material balance equation as a deterministic approach to estimate oil reserves.
2. Employment of Monte Carlo simulation to estimate oil reserve by using the volumetric equation as a probabilistic approach.
3. Make a comparison between two adopted approaches.

## 2. GEOLOGICAL SETTING

Buzrgan oil field is one of the southern Iraqi oil fields that is located specifically in the Missan governorate near the Iranian border. It is located 350 km southeast of Baghdad and 175 km north of Basra (Al-Ismaily, 2017; Khashman et al., 2025A), as appearing in Fig. 2. The field has an area of about 280 km<sup>2</sup>, and the Mishrif reservoir is the main producing zone in this field (Hussain, 2012; Jubair and Hadi, 2021a; Ahmed et al., 2024). The field was originally recognized in 1970, and its development period began in 1976. The Buzrgan oil field is part of the unstable Mesopotamian basin (Naji et al., 2023). Mishrif reservoir is mostly made up of dolomite and limestone with intersecting shale. A carbonate platform gradient may be detected in the depositional environment of the Mishrif reservoir (Al-Ali et



al., 2019). Based on many previous formation evaluation processes, the Mishrif formation had been divided into six units: MA, MB11, MB12, MB21, MC1, and MC2 (Alhuseini and Hamd-Allah, 2019). The Tertiary and Cretaceous phases form the stratigraphic column that was drilled by the Buzrgan oil field wells. The structures from younger to older indicate the tertiary period, represented by: Upper Fars (Injana), Lower Fars (Fatha), Asmari, Jaddala, and Aliji, while the Cretaceous is formed of a multitude of reservoirs: Shiranish, Hartha, Saadi, Tanuma, Khasib, Mishrif, Rumaila, Ahmadi, Maudud, and Nahr Umr, as depicted in Fig. 3 (Sissakian et al., 2017; Aldarraji and Almayahi, 2019).

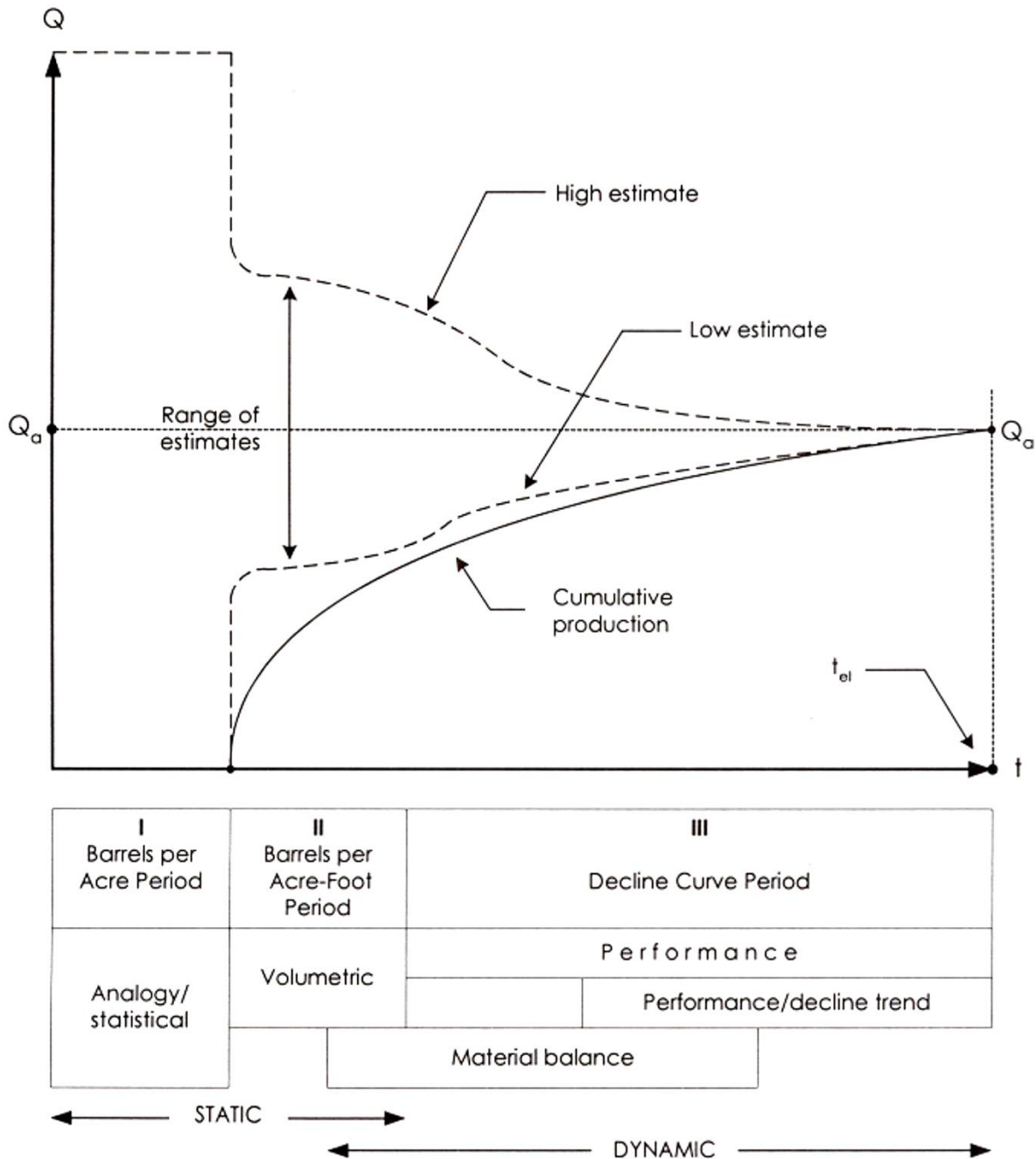
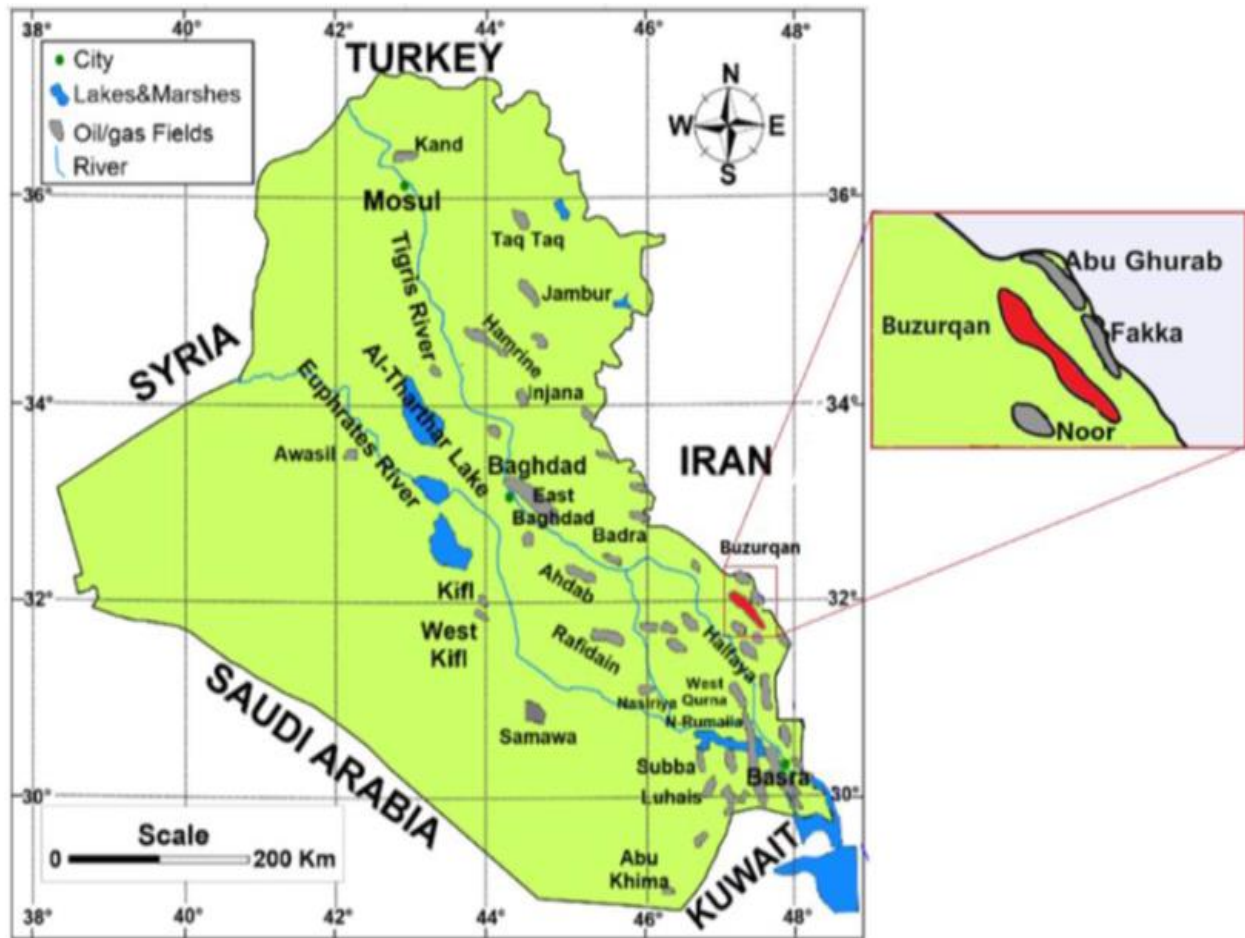


Figure 1. Chronological list of deterministic methods for reserve estimation (Cronquist, 2001)



**Figure 2.** Buzrguan oil field location on Iraq fields map (Jubair and Hadi, 2021b).

### 3. MATERIALS AND METHODS

As shown previously, the area studied is a Buzrguan oil field, and the target reservoir is the Mishrif, so all the described data in the following is related to this reservoir. The used materials data in deterministic and probabilistic approaches to accomplish the existing work classified to: (1) Reservoir data that comprised on the contour map of MB21 top unit of Mishrif reservoir as shown the **Fig. 4**. (2) Location of adopted wells and its vertical depth to bottom of Mishrif reservoir as appearing in **Table 1**. (3) Production data which comprised cumulative oil and water production with measured reservoir pressure listed in **Table 2**, where these data are showing decline in pressure with increasing of production from 1976 to 1980 after that the pressure re-increasing according to production shut in during Iraqi – Iranian war until the wells open to flow again in 1992.(4) The pressure-volume-temperature (PVT) data utilized are obtained from flash and differential tests at 110°C; they match some of the empirical correlations and are extrapolated to determine all parameters with respect to pressure ranges, and the best data are selected for use in modeling. The main PVT data are listed in **Table 3**. And finally, (5) The petrophysical properties, porosity, water saturation, net to gross thickness ratio, reservoir volume, as will be summarized and listed in a probabilistic approach.



	Age	Formation	Lithology	Note	
Tertiary	Pliocene	Bakhtiary	Marly clay + Gypsum + Sandy clay		
	Upper Miocene	Upper Fars	Gravel Sand + Siltstone + Anhydrite + Shale		
	Middle Miocene	Lower Fars – MB5		Shale + Anhydrite + Limestone + Dolomite	
		Lower Fars – MB4		Shale + Anhydrite + Salt + Limestone	
		Lower Fars – MB3		Anhydrite + Salt + Shale	
		Lower Fars – MB2		Salt	
		Lower Fars – MB1		Anhydrite + Shale + Dolomite	
	Burdigalian	Jeribe/Euphrates	Limestone + Dolomite		
	Aquitainian to Oligocene	Upper Kirkuk		Sand + Siltstone + Shale + Dolomite	Asmari Formation
		M-L Kirkuk		Sand + Siltstone + Shale	
Eocene to Paleocene	Jaddala		Limestone + Marl + Chert + Shale		
	Aaliji		Marl + Limestone		
Cretaceous	Lower Maastrichtian	Shiranish	Chalky Limestone		
		Hartha	Chalky Limestone		
	Campanian to Santonian	Sadi	Limestone + Marl		
	Coniacian to Turonian	Tanuma	Shale + Marl + Limestone		
		Khasib	Limestone + Shale	Unconformity	
	Lower Turonian to Upper Cenomanian	Mishrif	Limestone		
		Rumaila	Limestone		
		Ahmedi	Marl + Limestone		
Lower Cenomanian	Mauddud				
	Nahr Umr				

Figure 3. Sett stratigraphic column of Buzrgan oil field (Khashman et al., 2025b).

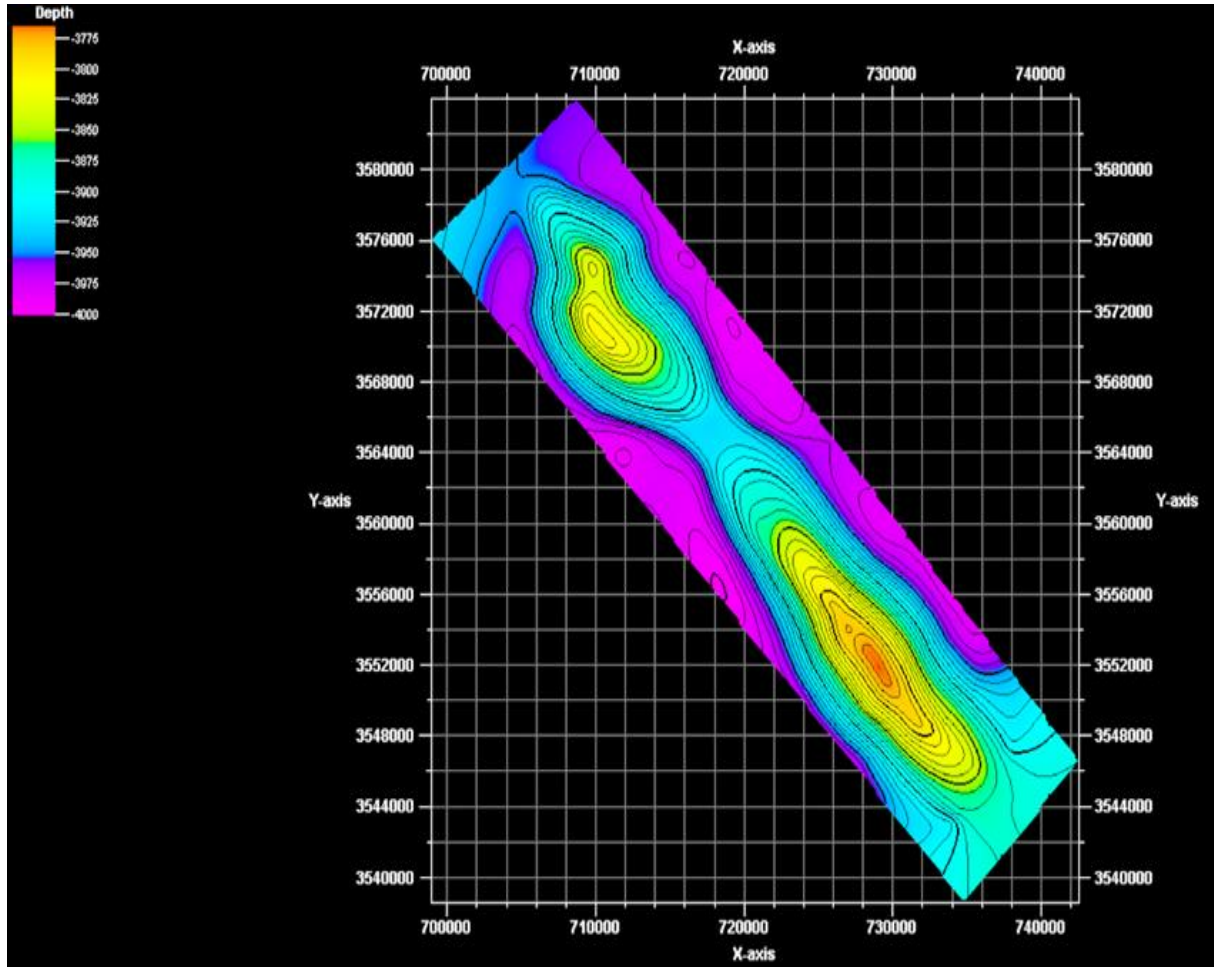


Figure 4. Contour map of the top of the MB21 unit (Naji et al., 2023).

Table 1. Wells location and measured depth to the Mishrif reservoir bottom.

Name	X	Y	KB(M)	TD(M)
BU 1	711070.7	3571608	55	4074.8
BU 3	730708.2	3551217	30.7	4055.6
BU 4	725748.8	3556986	33.7	4085.6
BU 5	727116.4	3555338	22.4	4044.5
BU 6	728122.7	3552573	25.6	4038
BU 9	711832.9	3570610	42.4	4010
BU 10	726836.4	3552690	24.2	4059
BU 13	729292.6	3551248	26	4018
BU 14	727055.4	3553933	23.5	4038
BU 16	728207.7	3553852	24.5	4024.5
BU 19	725929.5	3555554	23.5	4038
BU 20	730599.1	3549730	28	4048

**Table 2.** Summarized production data of Mishrif reservoir conducted from twelve wells.

Time (date (d/m/y))	Reservoir Pressure (psia)	Cum Oil Produced (MMSTB)	Cum Water Produced (MMSTB)
01/01/1976	6244.27	0.062005968	0.00003144
01/01/1977	6084.28	9.012294924	0.082739627
01/01/1978	5984.64	23.8023616	0.182502161
01/01/1979	5608.89	37.39056363	0.359877629
01/01/1980	5481.97	46.3532097	0.40049659
01/01/1981	5572.93	46.3532097	0.40049659
01/01/1992	5714.64	46.3532097	0.40049659
01/01/1997	6063.9	46353209.7	0.40049659
01/01/1998	6030.87	49.47954435	0.40049659
01/01/1999	5856.78	59.4775642	0.40049659
01/01/2000	5797.9	60.32616923	0.40049659
01/01/2001	5772.88	65.65373499	0.40049659
01/01/2002	5620.85	72.64970799	0.40049659
01/01/2003	5577.24	78.91412499	0.40049659

**Table 3.** Main input of PVT data.

Property	Value
Solubility at $P_b$ ( $m^3/m^3$ )	104.12
API	22.7
Gas Gravity	0.8218
Water Salinity (PPM)	200000
$B_{ob}$ (RB/STB)	1.40985
$P_b$ (Psia)	2660
T ( $F^\circ$ )	233
$\mu_{ob}$ (cp)	0.716722

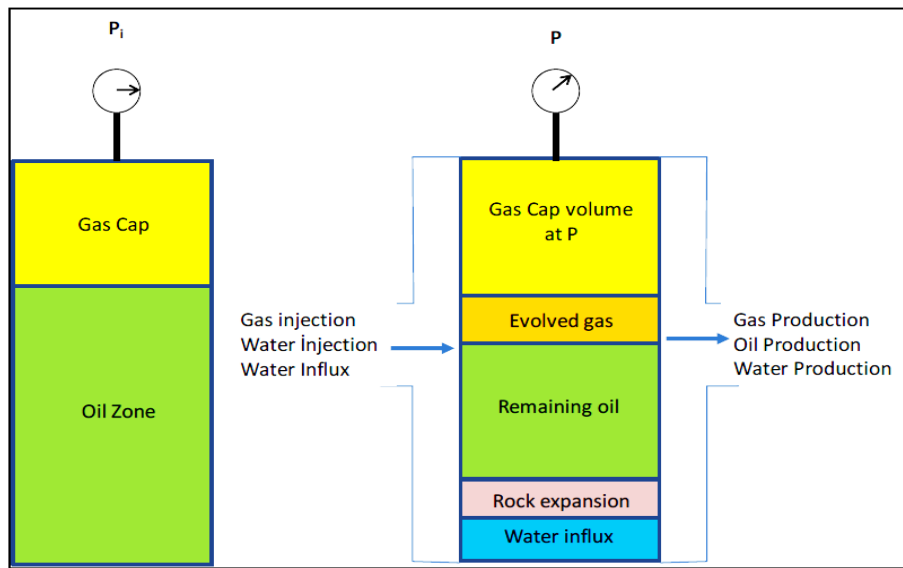
### 3.1 Deterministic Approach

For many years till now, the reservoir engineers have been relying on the material balance equation (*MBE*) as a basic deterministic tool for predicting and interpreting reservoir performance. *MBE* is used for the determination of initial oil in place ( $N$ ), initial gas in place ( $G$ ), water influx ( $We$ ), reservoir pressure, prediction of reservoir future performance, and recovery factor with forecasting of the type of drive mechanism. The hypothesis *MBE* was presented for the first time by **(Schilthuis, 1936)**. It implies that the volume of the reservoir remains constant, implying that the volumetric balance between the withdrawals underneath and the cumulative production must equal the expansion of fluids in the reservoir caused by pressure decrease. In other words, expansion must equal loss, and the net loss (production minus injection, minus water influx) must be represented by the expansion of in-place materials in its most basic form **(Turgay, 2014)**. The volumetric change that occurs during the production withdrawal is considered by assuming the reservoir as a tank, as depicted in **Fig. 5**, so the general form of the *MBE* expression is the following **(Dake, 1978)**:

$$N = \frac{N_p[B_o + (R_p - R_s)B_g] - (W_e - W_p B_w) - G_{ing}B_{ging} - W_{ing}B_w}{(B_o - B_{oi}) + (R_{si} - R_s)B_g + mB_{oi} \left[ \frac{B_g}{B_{gi}} - 1 \right] + B_{oi}(1 + m) \left[ \frac{S_{wi}C_w + C_f}{1 - S_{wi}} \right] \Delta P} \quad (1)$$

The definition of all parameters in the above equation is shown in the nomenclature part.

**(Havlena and Odeh, 1963 and 1964)** introduced a new approach for interpreting the *MBE* in terms of a straight-line equation in two published manuscripts, where the first manuscript showed the method of converting *MBE* to a straight-line equation, while the second manuscript applied the developed approach to a case study. The representation of *MBE* in terms of a straight-line equation is accomplished by curtailment of each group of parameters in Eq. (1), as one parameter, as follows **(Craft and Hawkins, 1991)**:



**Figure 5.** Tank model representation **(Turgay, 2014)**

$$F = N_p[B_o + (R_p - R_s)B_g] - (W_e - W_p B_w) - G_{ing}B_{ging} - W_{ing}B_w \quad (2)$$

Where *F* (bbl) is the underground withdrawal.

$$E_o = (B_o - B_{oi}) + (R_{si} - R_s)B_g \quad (3)$$

Where *E<sub>o</sub>* (bbl/STB) is the representation of reservoir fluids expansion.

$$E_g = B_{oi} \left[ \frac{B_g}{B_{gi}} - 1 \right] \quad (4)$$

Where *E<sub>g</sub>* (bbl/STB) is the gas cap expansion.

$$E_{f.w} = B_{oi}(1 + m) \left[ \frac{S_{wi}C_w + C_f}{1 - S_{wi}} \right] \Delta P \quad (5)$$

Where *E<sub>f.w</sub>* is the reduction of pore volume and initial water saturation expansion.



The straight-line method is done by plotting several parameter groups against each other on a Cartesian plot. The plotting group is determined based on the drive mechanism of reservoir production. The method of straight line can be used for the prediction of future reservoir performance by calculating some of the unknown parameters (Tarek, 2001).

### 3.2 Probabilistic Approach

Monte Carlo simulation is an important probabilistic method for determining oil reserves by capturing uncertainties. It can be easily modeled and run extremely fast, where its outputs may be inspected both visually and quantitatively (Al-Khalifa et al., 2007). With the progress of technology and scientific research, many parameters are examined in the estimation of oil reserves, and that lead to high uncertainties (Ampomah et al., 2016), so the probabilistic approach is applying number of iterations on each participating parameter in oil reserves estimation. In theory, there is no really right approach for determining the number of iterations needed to generate trustworthy results from a probabilistic process. On the other hand, the necessity for many repeats means that sampling time may become an issue (Agnia et al., 2021). The mathematical formula that will be used in the probabilistic approach is the volumetric equation (Komlosi and Komlosi, 2009):

$$N = \frac{7758 V_B \frac{N}{G} \phi (1 - S_{wi})}{B_{oi}} \tag{6}$$

Simply, the oil in place ( $N$ ) is calculated based on single values of reservoir bulk volume ( $V_B$ ), average reservoir porosity ( $\phi$ ), oil saturation ( $S_o$ ) based on initial water saturation ( $S_{wi}$ ), net reservoir to gross thickness ( $N/G$ ), and initial oil formation volume factor ( $B_{oi}$ ). But when applying Monte Carlo simulation, the input data is modeled by using one of the distribution methods, such as normal, lognormal, uniform, and triangular, as illustrated in Fig. 6. The distribution method for each parameter in Eq. (6), with the statistical summary of adopted values, is shown in Table 4.

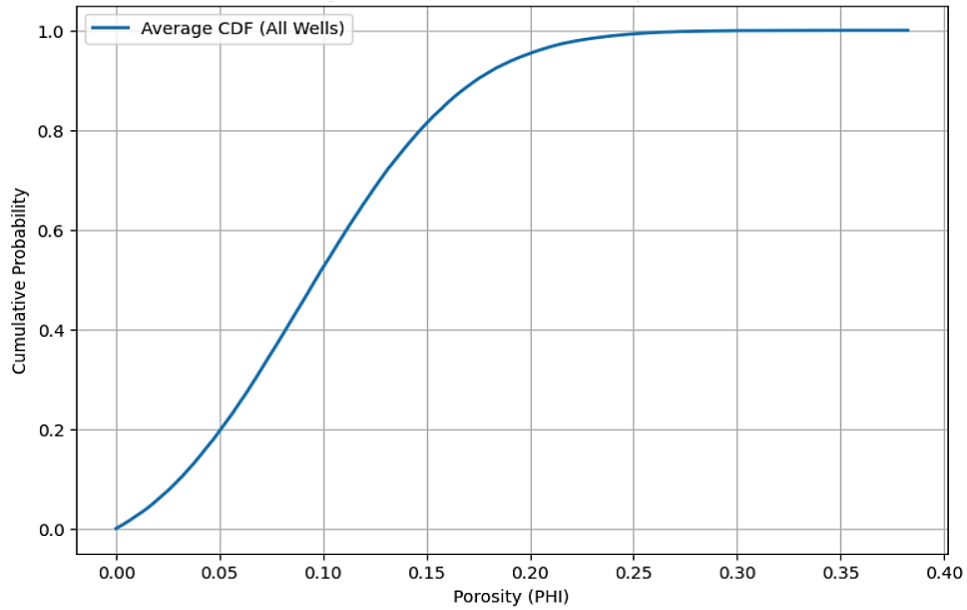
**Table 4.** Statistical summary and distribution methods of Monte Carlo simulation.

Parameter	Distribution method	MAX	Mode	MIN	Mean	STD
$V_B$ (MM ft <sup>3</sup> )	Fixed Value	293543				
$N/G$	Triangular	0.9884	0.539 2	0.0670	0.5301	0.1928
$\phi$	Normal	0.3825	0.17	0.0001	0.1006	0.0548
$S_w$	Triangular	0.9971	0.095 6	0.0006	0.5072	0.2256
$B_{oi}$ (bbl/STB)	Fixed Value	1.4				

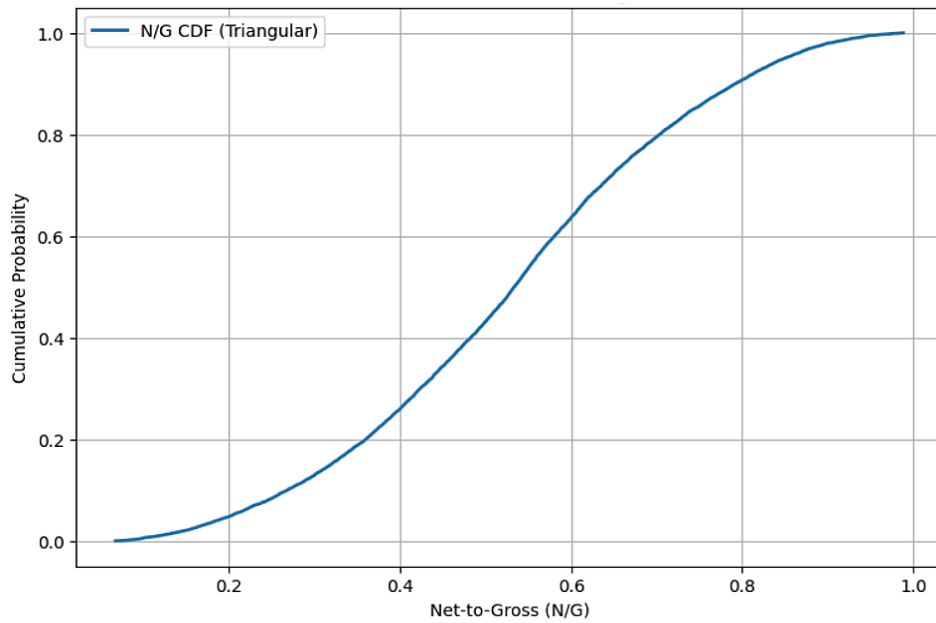
The cumulative distribution function ( $CDF$ ) had been plotted for  $\phi$ ,  $N/G$ , and  $S_w$  to show the probability of these parameters for certain values for each one. The selection of the distribution method for each parameter was based on some past literature, such as (Cosentino, 2001; Al-Khalifa et al., 2007; Mohammed et al., 2018; Agnia et al., 2021). The  $CDF$  of  $\phi$  made by the normal distribution method, as shown in Fig. 6, for  $N/G$  and  $S_w$  constructed by the triangular distribution method, as appeared in Figs. 7 and 8. The



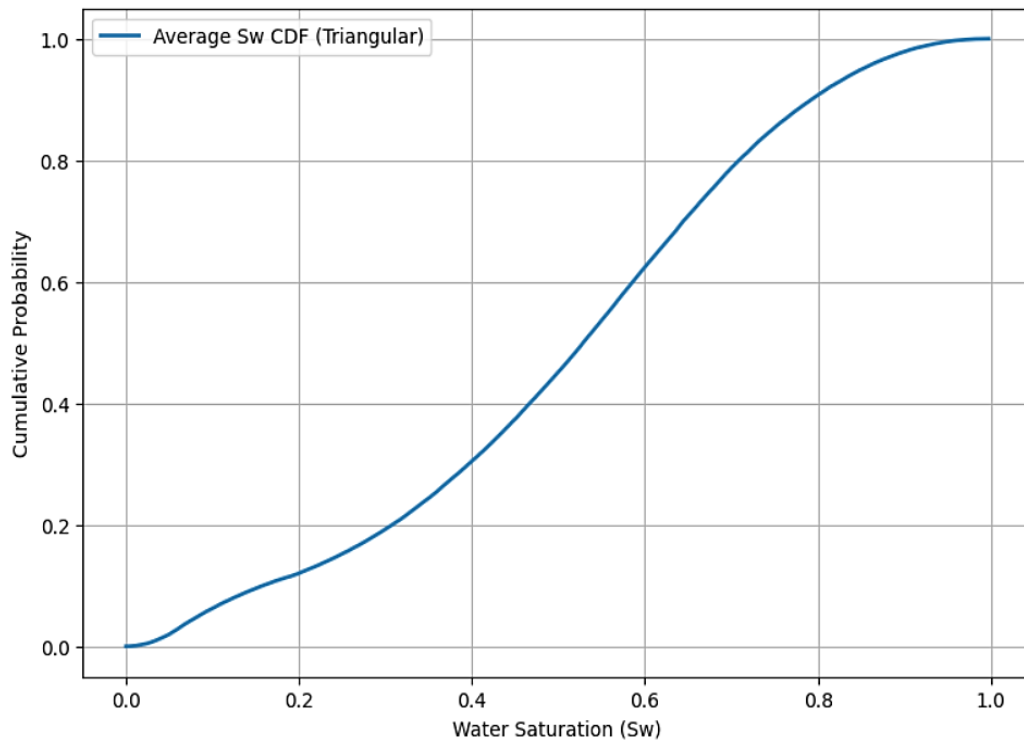
reservoir volume ( $V_B$ ) and initial oil formation volume factor ( $B_{oi}$ ) are used as single fixed values as shown above in **Table 4**.



**Figure 6.** CDF from the normal distribution of porosity collected from all wells.



**Figure 7.** CDF from the triangular distribution of  $N/G$  collected from all wells.



**Figure 8.** CDF from the triangular distribution of  $S_w$  collected from all wells.

#### 4. RESULTS AND DISCUSSION

The obtained results that will be discussed and compared are classified into results of *MBE* as a deterministic approach and Monte Carlo simulation as a probabilistic approach, as follows:

##### 4.1. Material Balance Equation (*MBE*)

The *MBE* employed by the Havlena–Odeh method to obtain the results in **Fig. 9** as a straight line. The axes of the resulting plot are production efforts normalized by expansion on X-axis and the friction of total expansion produced on the Y-axis. The listed data in **Tables 2 and 3** are utilized as input for the Havlena–Odeh method. As appearing, the initial oil in place ( $N$ ) of Mishrif formation in Buzrgan oil field is 4.368 billion stock tank barrels, this value is relatively acceptable in comparison with the obtained value from (**Naji et al., 2023**) and (**Alhousseini and Hamd-Allah, 2019**). The ideal state of the straight-line method all the points must be on the line, but indeed this is not achieved due to many errors in measurements of pressure or production data, as well as *PVT* data. The derived mechanism is determined based on straight line location in the resulting plot, where it shows a small deviation upward, pointing to the existence of multiple mechanisms. The **Fig. 10** illustrates the energy plot of Mishrif reservoir drive mechanisms in terms of driving index, where the blue area represents the production by fluid expansion, the green area is the production by pore volume compressibility, and the pink area shows the index of production by water drive. The performance of all three drive mechanisms is changing and not stable trends that are open – shut – open for production periods due to the Iraqi–Iranian war. The plot provides the balance in three production drive mechanisms along all periods, where in 1977, the fluid expansion, and pore volume compressibility as 0.41 and 0.58 respectively, were dominant



as drives indexes while after shut in of production, the water influx increased to 0.52 until open wells to production at 1992 the first two mechanisms started increasing at the expense of water influx.

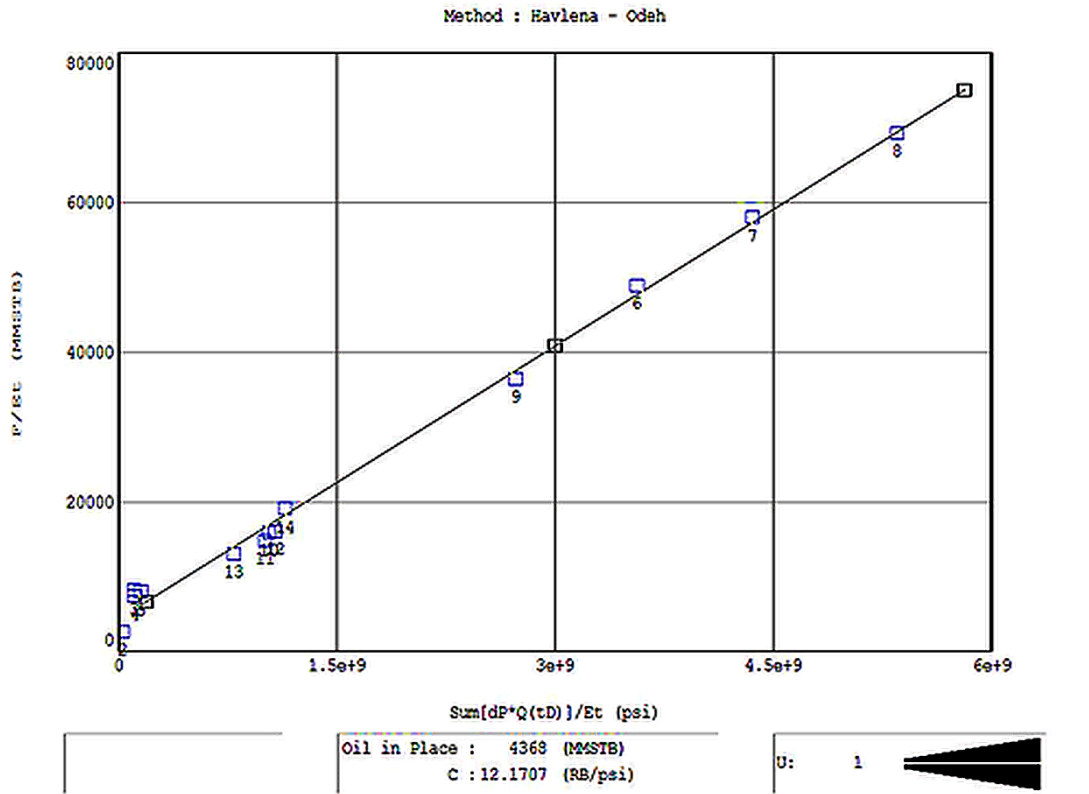


Figure 9. Result of Havlean- Odah straight line method.

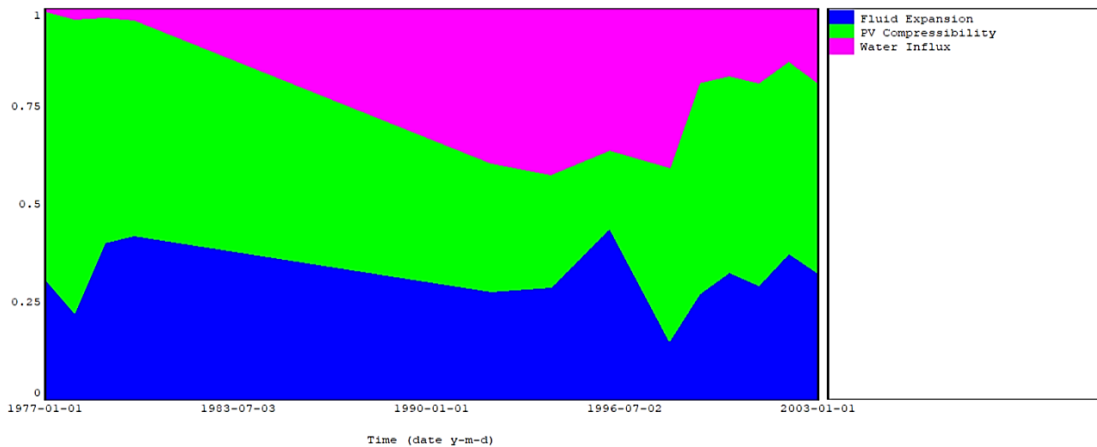


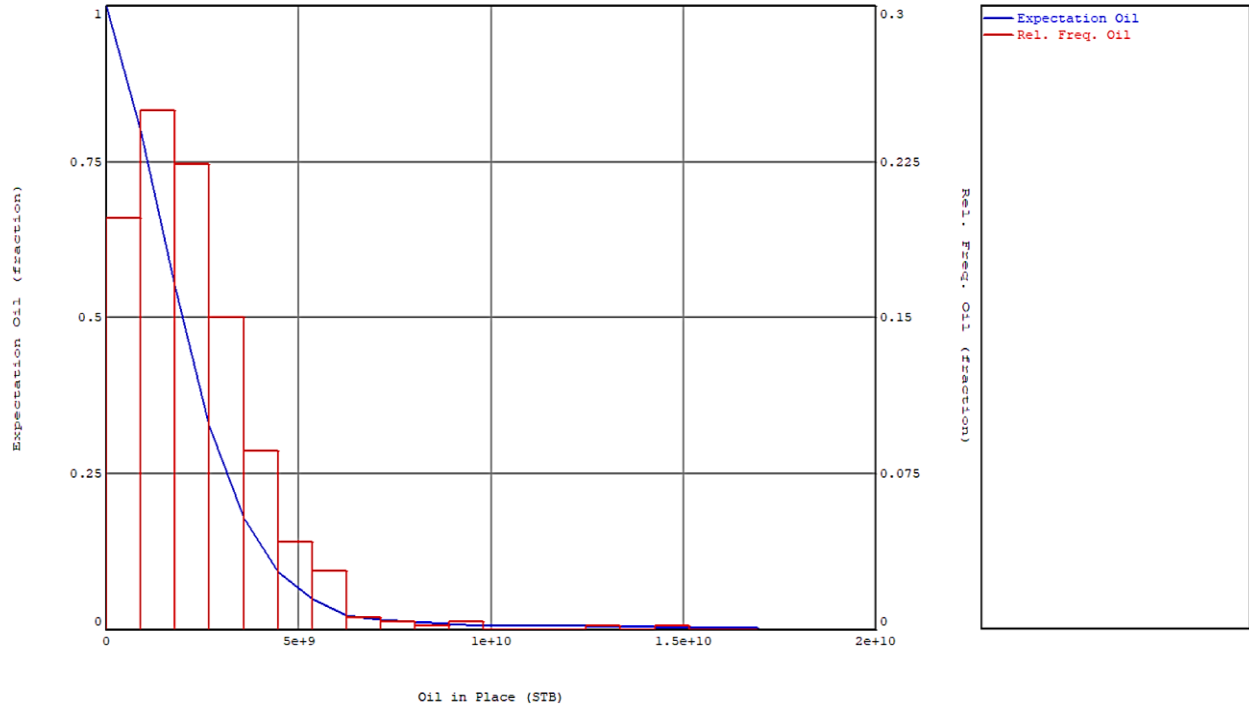
Figure 10. Energy plot of driving mechanisms dominant in Mishrif reservoir.

#### 4.2. Monte Carlo Simulation

As shown in previous part, in Monte Carlo simulation method each parameter in Eq. 6 is modeled to make a CDF distribution for  $\phi$ ,  $N/G$  and  $S_w$  and  $V_B$  and  $B_{oi}$  used as fixed values in Table 3. This diversity in distribution was based on reliable past studies for each parameter. The target of using distribution without fixed values is to reduce errors and obtain the best



value for the oil reserve. **Fig. 11** shows the Monte Carlo simulation results, where the blue curves refer to expected oil while the vertical red rectangle illustrates the relative frequency of oil reserves. The listed values in **Table 5** are derived from **Fig. 11**; it shows the oil reserves values in the Mishrif reservoir based on Hefner and Thompson’s classification. The *P90* proven reserves are 1.03279 MMSTB, the *P50* proved and probable oil reserves is 1.98761 MMSTB, the *P10* proved, probable, and possibly reserve is 4.3694 MMSTB, while the average oil reserve value is 2.31516 MMSTB. The standard deviation value 1.8906 is very small, and the number of iterations is 500.



**Figure 11.** Result of the Monte Carlo Simulation method.

**Table 5.** Summary of Monte Carlo simulation results.

Parameter	Oil in Place (MMM STB)
Average	2.31516
90 Percent Probability	1.03279
50 Percent Probability	1.98761
10 Percent Probability	4.3694

The resulting oil reserve is very sensitive to the adopted parameters in Eq. (6), so each parameter has an impact on these results. The sensitivity analysis of the oil reserve to  $\phi$ ,  $N/G$ , and  $S_w$ , accomplished using probabilistic Spearman correlation as applied in **Fig. 12**. The  $\phi$  is the highly dominant positive impact on oil reserve value is followed by  $N/G$  in the same direction, while the  $S_w$  had a negative impact on oil reserve. The  $V_B$  analytically has a positive effect on oil reserve, while  $B_{oi}$  has a negative effect.

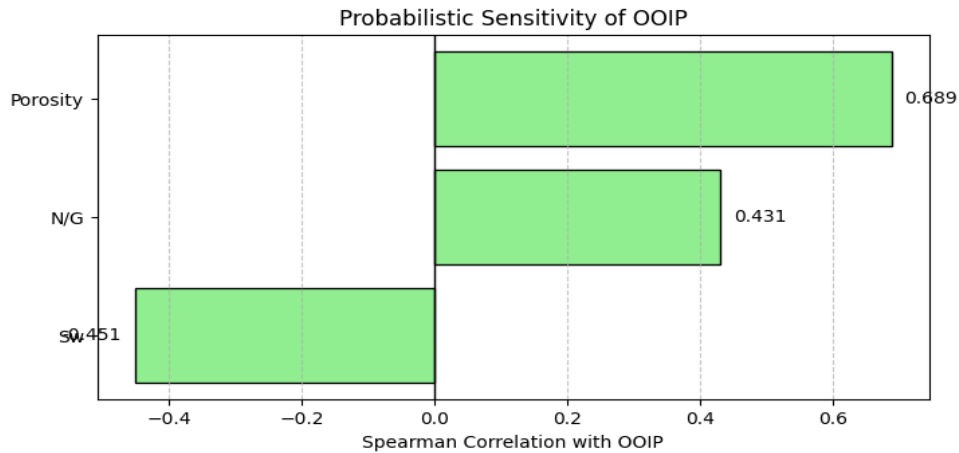


Figure 12. Oil reserve sensitivity analysis to  $\phi$ ,  $N/G$ , and  $S_w$ .

### 3.3. Comparison Between Two Approaches

Based on the results provided in 4.1 and 4.2, the results of deterministic MBE (4.368 MMSTB) are closely with the P10 estimate from the Monte Carlo simulation approach. This indicates that the deterministic approach shows a high estimation of oil reserve, which must be compared with the value of the probabilistic method (P50 = 1.9875 MMSTB). This variation was expected according to differences in adopted data accuracy and availability. The estimated value of MBE is very close to the forecasted values in previous literature (Alhusseini and Hamd-Allah, 2022; Naji et al., 2023). But many aspects must be compared, and this will help in choosing the appropriate method before starting to work on it, as shown in the following Table 6.

Table 6. Comparison between the material balance and Monte Carlo simulation approaches.

Comparison Criteria	Deterministic Approach (MBE)	Probabilistic Approach (Monte Carlo)
Availability	More availability according to the simple data needed	Also available, but depending upon pressure measurement, production and PVT
Data	Porosity, fluid saturation and reservoir volume data collected from well logs analysis and calculations.	Production, PVT, pressure data, with bulk volume and porosity collected from different sources, field, laboratory and calculations.
Limitations	Cannot be used in a heterogeneous reservoir.	Cannot use before the production stage or without pressure measurement.

### 4. CONCLUSIONS

The present study shows the comparison between the deterministic approach represented by MBE and the probabilistic approach represented by Monte Carlo simulation. The following points are concluded:

1. This MBE approach showed a high estimation of oil reserve (4.368 MMSTB) in comparison with the value of the probabilistic method (P50 = 1.9875 MMSTB).
2. The MBE approach cannot be used for oil reserve estimation before production begins because it depends on PVT, production, and reservoir pressure measurement.



3. The production from the Mishrif reservoir is under three drive mechanisms: water influx, pore volume compressibility, and fluid expansion.
4. The Monte Carlo simulation approach can be used before wells open to flow because it depends on data that can be provided during the exploration stage.
5. The oil reserve is inversely sensitive to  $B_{oi}$  and  $S_w$  while positively sensitive to  $V_B$ ,  $\emptyset$ , and  $N/G$ .

### Nomenclature

Symbol	Description	Symbol	Description
$\Delta P$	Initial minus current pressure, psi.	F	The underground withdrawal, bbl.
$B_g$	Gas formation volume factor, ft <sup>3</sup> /scf.	$G_{ing}$	Cumulative injected gas, scf.
$B_{gi}$	Initial gas formation volume factor, bbl/scf.	m	Ratio of the Initial Gas Cap Volume to the Initial Oil Volume, bbl/bbl.
$B_{ging}$	Formation volume factor of injected gas, bbl/scf.	N	Initial oil in place, STB.
$B_o$	Oil formation volume factor, bbl/STB.	$N_p$	Cumulative oil production, STB.
$B_{oi}$	Initial oil formation volume factor, bbl/STB.	$R_p$	Produced gas oil ratio, scf/STB.
$B_w$	Water formation volume factor, bbl/STB.	$R_s$	Dissolved gas oil ratio, scf/STB.
$C_f$	Formation compressibility, psi <sup>-1</sup> .	$R_{si}$	Initial dissolved gas oil ratio, scf/STB.
$C_w$	Water compressibility, psi <sup>-1</sup> .	$S_{wi}$	Initial water saturation.
$E_{f,w}$	the reduction of pore volume and initial water saturation expansion.	$W_e$	Cumulative water influx, bbl.
$E_g$	The gas cap expansion, bbl/STB.	$W_{ing}$	Cumulative injected water, STB.
$E_o$	Reservoir fluids expansion, bbl/STB.	$W_p$	Cumulative produced water, STB.

### Credit Authorship Contribution Statement

Jassim M. Al Said Naji: Writing – review & editing, Writing – original draft, Validation, Software, Methodology. Mohammed A. Ahmed: Writing – review & editing, Software.

### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## الأساليب الحتمية والاحتمالية لتقدير احتياطي النفط في المكامن الكربونية: نموذج الدراسة لحقل بزركان النفطي, جنوب العراق

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### الخلاصة

الدقة في تقدير احتياطي النفط مهمة جدًا وفقًا للقرارات التي ستؤخذ بناءً على هذه القيمة. ستتناول هذه الورقة تقدير احتياطي النفط باستخدام مناهج احتمالية وحتمية. نموذج الدراسة هو تكوين مشرف في حقل بزركان النفطي الواقع في محافظة ميسان جنوب العراق. يتم تمثيل النهج الحتمي بمعادلة توازن المادة بينما يتم تمثيل النهج الاحتمالي بمحاكاة مونت كارلو. البيانات المستخدمة في تطبيق معادلة توازن المادة هي فحوصات الموائع المكمنية وبيانات الضغط مع الحاجة إلى الحجم الكلي والمسامية، بينما تحتاج محاكاة مونت كارلو إلى مدخلات المعادلة الحجمية وهي المسامية وتشبع السوائل وسمك الدفع الصافي وحجم المكن الكلي. يتم تطبيق معادلة توازن المادة كمعادلة خطية بطريقة هافلينة عودة. يتم تطبيق محاكاة مونت كارلو كمعادلة حجمية ولكن يتم نمذجة كل مدخل بصورة احصائية للحصول على أفضل قيمة. بلغت نتيجة احتياطي النفط باستخدام معادلة توازن المادة 4.368 مليار برميل من مخزون المكن، بينما صنفت محاكاة مونت كارلو وفقًا لتصنيف هيفنر وتومسون على أنها P90، حيث تبلغ الاحتياطيات المؤكدة 1.03279 مليار برميل من مخزون المكن، والاحتياطيات المؤكدة والمحتملة P50 تبلغ 1.98761 مليار برميل من مخزون المكن، والاحتياطيات المؤكدة والمحتملة P10 تبلغ 4.3694 مليار برميل من مخزون المكن. القيم الناتجة من كلا النهجين مُحكمة للغاية، وهذا يُشير إلى جودة الإجراءات والبيانات المُستخدمة. لا يُمكن استخدام معادلة توازن المادة قبل مرحلة الإنتاج، بينما يُمكن تطبيق محاكاة مونت كارلو قبل ذلك.

**الكلمات المفتاحية:** توازن المادة، مونت كارلو، الطريقة الحجمية، تكوين مشرف، النهج الاحتمالي