Enhancing the Performance of Piezoelectric Energy Harvesters Using Permanent Magnets

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ABSTRACT

A cantilevered piezoelectric beam with a tip mass at its free end is a common energy harvester configuration. This paper introduces a new principle of designing such a harvester which increases the generated voltage without changing the natural frequency of the harvester: The attraction force between two permanent magnets is used to add stiffness to the system. This magnetic stiffening counters the effect of the tip mass on the natural frequency. Three setups incorporating piezoelectric bimorph cantilevers of the same type in different mechanical configurations are compared theoretically and experimentally to investigate the feasibility of this principle. Theoretical and experimental results show that magnetically stiffened harvesters have important advantages over conventional setups. They generate more voltage and they can be tuned across a wide range of excitation frequencies.

Key words: piezoelectric bimorph, frequency tuning, generated voltage, magnetic stiffinig.

تحسين اداء حاصدات الطاقة الكهروضغطية باستخدام مغانط ثابتة

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الخلاصة

ان الهيكل الاكثر شيوعا لعمل حاصدة طاقة كهروضغطية هوعتبة من ماده كهروظغطية مثبتة من جانب واحد و تحمل كتلة على الجانب الحر. في هذا البحث تم تقديم مفاهيم جديدة لانتاج هكذا حاصدات حيث يتم زيادة الفولطية المتولدة بدون تغيير التردد الطبيعي الذي عنده تعمل الحاصدة بكفاءة. ويتم ذلك من خلال استخدم قوة الجذب بين مغناطيسيين لاضافة متانة الى الحاصدة وبذلك يلغى تاثير الكتلة الموضوعه على الجانب الحر على التردد الطبيعي للحاصدة. تم تصنيع و فحص ثلاثة نماذج مختلفة من الحاصدات الكهروضغطية. تم مقارنة اداء هذه الحاصدات نظرياً و عملياً و تبين ان الحاصدات التي تعمل بالمفاهيم الجديدة تستطيع ان تولد فولطية اعلى و كذلك تلك الحاصدات ممكن توليفها لتعمل بكفاءة لمدى من الترددات المسلطة.

الكلمات الرئيسية: عتبة كهروضغطية, توليف التردد, الفولتية المتولدة, التاثير المغناطيسي

1. INTRODUCTION

Energy harvesting or scavenging are two terms commonly describing process for obtaining useful electrical power from the available energy in the environment. There are three main techniques for energy harvesting: vibration harvesting, thermal harvesting and solar harvesting.
Piezoelectric material is one of three general vibration-to-electric energy conversion mechanisms, the other two are electrostatic and electromagnetic transduction **,Williams and Yates, 1996.** Literature of the last few years showed that piezoelectric transduction had received most attention in powering electronic circuits; numerous scientific journals and conferences are due to this subject. The main reason why piezoelectric transducers are preferred for mechanical

to electrical energy conversion is that their energy density is three times higher the density obtained from electrostatic or electromagnetic transduction **,Priya, 2007.**

Energy harvesting using piezoelectric materials is a promising technique, but there are a number of obstacles that currently limit the amount of the generated voltage. One major limitation is due to the necessary frequency matching: The maximum voltage is generated when the natural frequency of the harvester matches the excitation frequency. Manufacturing tolerances, excitation frequency changes and changes of electric load make frequency matching difficult. **Al-Ashtari, 2012a** had developed an analytical model for piezoelectric bimorphs and concluded that manufacturing tolerances lead to a variation of the natural frequency of up to 5%, which cause a considerable drop in power, this drop can be 95% of the generated voltage. Therefore, tuneable harvesters are essential for this technique to be commercially viable.

One technique for harvester frequency tuning is to exploit the magnetic forces between permanent magnets. Literature shows that the attractive magnetic force can be used for enhancing the operation of piezoelectric harvesters. Depending on the magnets separation distance, the alignment, and the orientation, these magnets can effect the harvester stiffness. Literature shows that the magents effect can be modelled as an additional nonconventional spring, which allows to analyse the operation of the system using linear equations, for example, **Challa, et al., 2008. and 2011** fixed two small cylindrical magnets at the free end of a cantilever, one on the top and one on the bottom, and vertically aligned two magnets above and under the first two magnets. They used magnetic repulsion for the lower side and magnetic attraction for the upper side and tuned the harvester by changing the separation distances between these magnets. **Zhu, et al., 2010** used the attraction force between two axially aligned permanent magnets to change the resonance frequency of a cantilever beam in an electromagnetic generator. The opposing faces of the relatively large magnets are curved to maintain a constant separation distance between the two magnets during operation. **Al-Ashtari, 2012b,** introduced a tuning technique using attractive magnetic force acting in longitudinal direction of the cantilever. They presented a comprehensive derivation for modelling the effect of magnetic force as that of a nonconventional spring whose stiffness depends on the nonlinear magnetic force.

Many research projects work on increasing the output power of energy harvesters but most of them focused on developing new or optimised power flow concepts based on modifying the electrical harvesting circuit such as **Ottman, et al., 2002 ,Badel, et al., 2006 ,Dicken, et al., 2009** and **,Ramadass and Chandrakasan 2010.** In contrast, a view number of researches had been conducted to investigate the increasing the generated power from manipulating the mechanical characterstics of the harvester such as the mass or stiffness.

Many physical models had been introduced for predicating the voltage generated across a resistive load connected to piezoelectric harvesters. There are two classes of models; distinguished by the way that physical parameters are handled: Models with distributed parameters and models with lumped parameters.

Models with distributed parameters are based on Euler-Bernoulli beam theory. These models evaluate the physical equations along the whole length of the beam. They generally give more accurate results than lumped parameters models, but involve complicated mathematics and long mathematical expressions. Such models have for example been used by **,Lu, et al., 2004 ,Chen, et al., 2006 ,Lin, et al., 2007** and **Erturk and Inman, 2008.** Discretization of a model with distributed parameters leads to a lumped parameters model. Such models can be considered a less accurate approximation of the distributed parameters, but they are accurate enough for many applications. They also provided an explicit understanding of the operation of piezoelectric harvesters and can be handled with circuit theory by applying electro-mechanical analogies. This motivated **Erturk, and Inman, 2008,** to use their model with distributed parameters for deriving a correction factor for the lumped parameters model by **du Toit, et al., 2005** in order to improve its accuracy. Many researchers had used lumped parameters for modelling piezoelectric harvesters, for example **Roundy, et al., 2003 ,Sodano, et al., 2004 ,du Toit, et al., 2005 ,Shu, and Lien, 2006 ,Richter, et al., 2006 ,Twiefel, et al., 2007** and **Richter, 2010.**

In this contribution, it will be shown that the configuration introduced by **Al-Ashtari, 2013.** cannot only be used for tuning the frequency of energy harvesters over a wide range but that it can also significantly increase the harvested electrical power. The concept for power increasing introduced in this paper is based on manipulating mass and stiffness, i.e. mechanical quantities of the harvester. A new harvester configuration in which a tip mass is combined with magnetic stiffening has been developed. This allows increasing the power of energy harvesters without changing their efficient operation frequency. Compared to a simple cantilever beam this structure has two important advantages: It shows a considerable increase of the generated voltage and its natural frequency can be tuned over a wide range of frequencies.

Also in this paper, a model of lumped parameters for piezoelectric energy harvesters with and without magnetic stiffening is introduced. This model is described by simple mathematical expressions and gives fairly accurate results. This allows further development and optimization of piezoelectric harvesters.

Three setups incorporating piezoelectric bimorph cantilevers of the same type are compared due to theoretical and experimental results: One bimorph is unmodified,called reference setup; and the other two bimorphs have tip masses of different size, reducing their natural frequencies. These bimorphs are additionally stiffened to compensate the drop of the natural frequency caused by the tip masses. All bimorphs were excited at constant base velocity amplitude and tested for different resistive loads and excitation frequency.

Theoretical and experimental results show that magnetically stiffened harvesters have important advantages over conventional setups: They generate more voltage due to increasing the strain in the piezoelectric transducer and they can be tuned across a wide range of excitation frequencies. The high power output, tunability and good efficiency make magnetically stiffened harvesters a very promising option for future energy harvesting applications.

2. PIEZOELECTRIC HARVESTER MODELLING

A typical piezoelectric energy harvester is a cantilever beam consisting of a shim layer and one or two layers of piezoelectric ceramic. Often a tip mass M_t is attached to the free end of the cantilever to reduce the natural frequency and increase the deflection of the beam. The cantilever is attached to a vibrating host structure, generating an alternating voltage output $u(t)$ for powering an electric load R_l as shown in **Fig. 1**. For the following investigation, the electric load R_l is assumed to be purely resistive. $v_b(t)$ Is the velocity of the base excitation and $v_t(t)$ is the velocity of the cantilever tip. The relative velocity $v(t)$ describes the beam deflection and is expressed as

$$v(t) = v_t(t) - v_b(t) \tag{1}$$

The piezoelectric harvester is an electromechanical device with both mechanical and electrical characteristics. For system analysis and optimization it is convenient to introduce a single domain representation of the electromechanical system applying electromechanical analogies. **Fig. 2** shows the lumped parameters electrical equivalent system similar to the one used by **Richter, 2010.** Comparing to the model by **Roundy, 2003** this model allowed non-zero base velocities.

The mechanical parameters of the model are the equivalent mass M, the equivalent mechanical damping B, and the equivalent mechanical stiffness K. The parameters describing the electrical properties are the capacitance C_p, the electric load R_l and the generated voltage $u(t)$. $i(t)$ Is the current flowing through the load and α is the transfer factor between the mechanical and the electrical domain. For the investigations documented in this paper, the analytical model introduced by **Al-Ashtari, 2012a** had been used for calculating the aforementioned quantities from geometry and material parameters.
A physical model of the energy harvester is used to calculate the characteristics of the energy harvester such as input and output power, and vibration amplitude. This model can also be used to design and optimise energy harvesters. The derivation of the model starts with the governing equation of the piezoelectric harvester **,Al-Ashtari, 2013.**

$$M\dot{v}_t(t) + Bv(t) + K\int v(t)dt = -\alpha u(t) \tag{2}$$

After subtraction of $M\dot{v}_b(t)$ on both sides of Eq. (2), it leads to

$$M\dot{v}(t) + Bv(t) + K\int v(t)dt = -M\dot{v}_b(t) - \alpha u(t). \tag{3}$$

For the electrical system shown in **Fig. 2** the following equations are found:

$$C_p\dot{u}(t) - \alpha v(t) = -i(t) \tag{4}$$

$$u(t) = R_l i(t) \tag{5}$$

Laplace transformation of Eqs. (3), (4) and (5) at zero initial conditions results in:

$$\left(Ms + B + \frac{K}{s}\right)V(s) = -MsV_b(s) - \alpha U(s) \tag{6}$$

$$C_p sU(s) - \alpha V(s) = -I(s) \tag{7}$$

$$U(s) = R_l I(s) \tag{8}$$

where $V_b(s)$, $V(s)$, $U(s)$ and $I(s)$ are the Laplace transforms of base and relative velocity, generated voltage and output current, respectively. Based on Eqs. (6), (7), and (8) the following equation can be determined

$$\frac{U(s)}{V_b(s)} = -\frac{\alpha R_l M s^2}{MC_pR_l s^3 + \left(M + BC_pR_l\right)s^2 + \left(B + KC_pR_l + \alpha^2 R_l\right)s + K} \ . \tag{9}$$

The sinusoidal transfer function is

$$\frac{U(j\omega)}{V_b(j\omega)} = \frac{\alpha R_l M \omega^2}{\left[K - \left(M + BC_pR_l\right)\omega^2\right] + j\left[\left(B + KC_pR_l + \alpha^2 R_l\right)\omega - MC_pR_l\omega^3\right]}\,. \tag{10}$$

In terms of natural frequency ω_n and damping ratio ζ it can be written as

$$\frac{U(j\omega)}{V_b(j\omega)} = \frac{\alpha R_l}{\left[\frac{\omega_n^2}{\omega^2} - 2\zeta C_p R_l \omega_n - 1\right] + j\left[2\zeta\frac{\omega_n}{\omega} + \frac{C_pR_l}{\omega}\left(\omega_n^2 - \omega^2 + \frac{\alpha^2}{MC_p}\right)\right]} \tag{11}$$

where

$$\omega_n = \sqrt{\frac{K}{M}} \tag{12}$$

And

$$\frac{B}{M} = 2\zeta\omega_n\,. \tag{13}$$

According to the equivalent electrical circuit in **Fig. 2** , the ratio $U(s)/V_b(s)$ represents an impedance, called the electromechanical impedance of the piezoelectric harvester in the following. If the base excitation velocity is given by

$$v_b(t) = V_b \sin\omega t \tag{14}$$

where V_b is the base velocity amplitude, the generated voltage will be

$$u(t) = U\sin\left(\omega t + \varphi_{u-v_b}\right) \tag{15}$$

with the voltage amplitude

$$U = \frac{\alpha R_l V_b}{\sqrt{\left[\frac{\omega_n^2}{\omega^2} - 2\zeta C_p R_l \omega_n - 1\right]^2 + \left[2\zeta\frac{\omega_n}{\omega} + \frac{C_pR_l}{\omega}\left(\omega_n^2 - \omega^2 + \frac{\alpha^2}{MC_p}\right)\right]^2}} \tag{16}$$

and the phase difference between voltage and base velocity

$$\varphi_{u-v_b} = -\tan^{-1}\left(\frac{2\zeta\frac{\omega_n}{\omega} + \frac{C_pR_l}{\omega}\left(\omega_n^2 - \omega^2 + \frac{\alpha^2}{MC_p}\right)}{\frac{\omega_n^2}{\omega^2} - 2\zeta C_p R_l \omega_n - 1}\right). \tag{17}$$

the amplitude of the relative velocity $\hat{v}$ can be calculated from Eqs. (6), (7), and (8), thus

$$\frac{V(s)}{V_b(s)} = -\frac{Ms^2\left(C_pR_ls+1\right)}{MC_pR_ls^3+\left(M+BC_pR_l\right)s^2+\left(B+KC_pR_l+\alpha^2R_l\right)s+K} \quad , \tag{18}$$

which, following Eqs. (12) and (13), can be written in terms of natural frequency and damping ratio as

$$\frac{V(j\omega)}{V_b(j\omega)} = \frac{1+jC_pR_l\omega}{\left[\frac{\omega_n^2}{\omega^2}-2\zeta C_pR_l\omega_n-1\right]+j\left[2\zeta\frac{\omega_n}{\omega}+\frac{C_pR_l}{\omega}\left(\omega_n^2-\omega^2+\frac{\alpha^2}{MC_p}\right)\right]}. \tag{19}$$

Thus, if the relative velocity is expressed as:

$$v(t) = V\sin\left(\omega t+\varphi_{v-v_b}\right), \tag{20}$$

its amplitude can be written as

$$V = \frac{V_b\sqrt{1+\left(C_pR_l\omega\right)^2}}{\sqrt{\left[\frac{\omega_n^2}{\omega^2}-2\zeta C_pR_l\omega_n-1\right]^2+\left[2\zeta\frac{\omega_n}{\omega}+\frac{C_pR_l}{\omega}\left(\omega_n^2-\omega^2+\frac{\alpha^2}{MC_p}\right)\right]^2}} \tag{21}$$

and the phase difference between the two velocities is described by

$$\varphi_{v-v_b} = \tan^{-1}\left(C_pR_l\omega\right)-\tan^{-1}\left[\frac{2\zeta\frac{\omega_n}{\omega}+\frac{C_pR_l}{\omega}\left(\omega_n^2-\omega^2+\frac{\alpha^2}{MC_p}\right)}{\frac{\omega_n^2}{\omega^2}-2\zeta C_pR_l\omega_n-1}\right]. \tag{22}$$

The amplitude of the beam deflection, which is a measure for the strain inside the bimorph, is expressed as

$$X = \frac{V}{\omega} \tag{23}$$

3. MAGNETIC STIFFENING TECHNIQUE

It can be deduced from simulating Eq.(16) that the output voltage can be increased by increasing the equivalent mass M or the natural frequency ω_n. A conventional method is to add a mass to the tip of the cantilever. This also reduces the resonance frequency of the harvester and can therefore not increase the harvested power effectively. This paragraph introduces a new configuration of a piezoelectric harvester, which increases the power of energy harvesters without changing their natural frequency. This is achieved by adding a tip mass and compensating the drop in natural frequency by an additional stiffness. The technique increases the harvested power for a given volume of piezoelectric material, keeping frequency and strain constant. If desired, the strain of the piezoelectric transducer can additionally be increased, leading to an even larger power increase.

The power increasing technique is based on the magnetic tuning method introduced by **Al-Ashtari, 2012b.** If the tip mass of a setup as shown in **Fig. 1** is replaced by a magnet and a

second magnet is attached to the vibrating structure as shown in **Fig. 3**, the resonance frequency of the harvester can be adjusted by changing the distance d between the magnets. If this technique is used to increase the resonance frequency of the harvester to match the resonance frequency of the original harvester without tip mass or magnets, the resulting magnetically stiffened harvester uses the same piezoelectric element and has the same resonance frequency, but delivers much more power compared to the original harvester without tip mass.
The equivalent electrical model of the proposed harvester setup is the same as shown in **Fig. 2**, with the value of the motional capacitance decreased to $1/(K+K_M)$ due to the additional “magnetic” stiffness K_M which can be calculated as

$$K_M = \left(\frac{15}{14l} + \frac{1}{d}\right) F_M \,, \tag{24}$$

where l is the length of the vibrating beam, d is the distance between the magnets, and F_M is the magnetic force which is a nonlinear function of magnet properties and separation distance. These parameters are constant during harvester operation. Details on the harvester model, the derivation of above formula, and the nonlinear calculation of the magnetic force can be found in **Al-Ashtari, 2012b.**

4. EXPERIMENTAL SETUP

Three different harvester setups are investigated experimentally, all using the same type of piezoelectric bimorph, Piezo Bending Actuators 427.0085.11Z” from Johnson Matthey. The specifications of the birmophs are given in **Table 1**. Their vibrating length is about 40 mm in the experiments.
As tip masses and for the magnetic stiffening, two types of neodymium magnets from HKCM Engineering were used, Q08.5x02x01.5Ni-48H with a mass of 0.19 g, and Q10x04.5x04.5Ni-N52 with a mass of 1.51 g.
The characteristics of the harvester setups are summarized in **Table 2**. While “reference setup” refers to the original cantilever beam without any mass or magnetic stiffening, “stiffened setup 1” refers to the cantilever beam with smaller magnet as a tip mass and magnetic stiffening. finally, stiffened setup 2” refers to the cantilever beam with larger magnet as a tip mass and magnetic stiffening.
The mechanical quality factor Q_m of the piezoelectric harvester for each setup can be identified by measuring the frequency sweep of the electrical admittance. A typical frequency sweep of a piezoelectric harvester is shown in **Fig. 4**.
From **Fig. 4**, the mechanical quality factor Q_m can be calculated as **,Zickgraf, 1996.**

$$Q_m = \frac{f_r}{f_2 - f_1} \,. \tag{25}$$

The frequency sweep of the electrical admittance of each setup was obtained using impedance analyzer type HP 4192A.
Fig. 5 schematically shows the experimental setup used in this paper. The harvesters are excited by an electro-dynamic shaker. The base velocity is monitored using a laser vibrometer and the amplitude of the shaker voltage is manually adjusted to achieve the desired amplitude of the base velocity. The harvester base frame is rigid compared to the bimorph structure, so that the measured velocity at any location on the base frame is the same.

The experimental setups with tip masses are similar to the ones depicted in **Figs. 1** and **3**, respectively. For technical reasons, the larger magnet has not been glued to the face of the bimorph but on top of it, aligned with the face. This reduces the free vibrating length of the bimorph assumed in the calculations by the width of the magnet. **Fig. 6** shows this setup.
In the experiments, the harvesters are excited by an electro-dynamic shaker. The base velocity is monitored using a laser vibrometer and the amplitude of the shaker voltage is manually adjusted to achieve steady base velocity amplitude. The harvester base frame is designed to be rigid compared to the bimorph structure, so that the measured velocity at any location on the base frame is the same. For measuring the beam deflection, a differential laser vibrometer is used with one beam pointed to the tip mass and the other pointed to the base frame.
It is well known that the power generated by a piezoelectric energy harvester is highly load-dependent. The output terminals of the bimorphs are therefore connected to a resistor decade to investigate the influence of the load. The load is varied between 100 Ω and 10 MΩ. The base velocity is kept constant for all loads and the steady-state amplitudes V of the beam velocity and U of the generated voltage across the load are measured.

5. RESULTS AND DISCUSSION

Figure 7 compares the theoretically and experimentally determined generated voltage of the different setups with a common resonance frequency of 250 Hz: Reference setup, stiffened setup 1 and stiffened setup 2. It is obvious that the magnetically stiffened harvesters generate much more voltage than the reference setup especially at high load resistance. Stiffened setup 2 with a larger tip mass and accordingly larger additional stiffness generates more voltage than stiffened setup 1.
The deflection of the bimorph tip is proportional to the strain inside the bimorph and as such is relevant for determining the lifetime and maximum allowable excitation amplitude. **Fig. 8** shows the corresponding deflection of the three setups. As expected, a larger tip mass leads to a larger deflection of the bimorph. The maximum deflection of stiffened setup 2 is more than 2.1 times the maximum deflection of the reference setup, and thus the generated voltage increased by the same factor.
Fig. 9 compares also the theoretically und experimentally determined generated voltage of the different setups with a common resonance frequency of 250 Hz: Reference setup, stiffened setup 1 and stiffened setup 2. These harvesters were tested at open circuit condition. Also, the magnetically stiffened harvesters generate much more voltage than the reference setup. The deflection of each setup is shown in **Fig. 10**.
All figures given above shows that the introduced model gives fairly accurate results. The difference between the simulated and the experimental results ranges from 1% to 3%. One possible reason for getting such deviations is using the lumped-parameter model which lacks the effect of the mode shape i.e. the description of the strain distribution along the beam. Also, assuming that the equivalent damping and the equivalent stiffness are linear can cause inaccurate results, especially if the excitation amplitude is large.

6. CONCLUSION

In this paper, a new principle for increasing the power generated by piezoelectric energy harvesters without changing their resonance frequency has been introduced and investigated. The attraction force between two permanent magnets is used to add stiffness to the system to counter the effect of a tip mass on the resonance frequency. Similar configurations that use the attraction force between two permanent magnets to manipulate the effective stiffness of the harvester have been introduced before tuning the resonance frequency of energy harvesters **,Al-Ashtari, 2012b.**

A physical model for the piezoelectric harvester has been introduced. Different comparisons between theoretical and experimental results show the fair accuracy of the proposed model especially when a tip mass is attached to the harvester.
The magnetically stiffened harvester has important advantages over all other investigated setups with and without tip mass: It generates more voltage and it can be tuned across a wide range of excitation frequencies. These make magnetically stiffened harvesters a very promising option for future energy harvesting applications.

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NOMENCLATURE

B = equivalent mechanical damping of a piezoelectric device, Ns/m
C_p = equivalent capacitance of the piezoelectric material, F
F_M = attraction force between magnets, N
K = equivalent mechanical stiffness of a piezoelectric device, N/m
K_M = equivalent stiffness due magnetic attraction, N/m
M = total equivalent mass of a piezoelectric device, kg
M_t = tip mass attached to the free end of the vibrating beam, kg
Q_M = mechanical quality factor of the piezoelectric device
R_l = connected Resistive load, Ω
U = amplitude of the generated voltage, V
V = amplitude of the piezoelectric cantilever velocity, m/s
X = amplitude of the piezoelectric cantilever displacement, m
d = separation distance between magnets, m
f_r = resonance frequency of the piezoelectric device, Hz
$f_1$and f_2 = frequencies at which the maximum magnitude of the electrical admittance decreases by 3db, Hz
$i(t)$ = current through conncted load , A
l = length of the piezoelectric cantilever, m
t = time, s
$u(t)$ = generated AC voltage of the piezoelectric harvester, V
$v(t)$ = relative velocity equals to $v_t(t) - v_b(t)$, m/s
$v_b(t)$ = excitation velocity of the base of the piezoelectric device, m/s
$v_t(t)$ = tip velocity of the piezoelectric cantilever, m/s
α = conversion factor between the mechanical and electrical domains of a piezoelectric device, N/V
ζ = equivalent damping ratio of the piezoelectric device
φ_{u-v_b} = phase difference between the excitation velocity $v_b(t)$ and the generated voltage $u(t)$, rad
φ_{v-v_b} = phase difference between the excitation velocity $v_b(t)$ and the relative velocity $v(t)$, rad
ω = angular frequency of the excitation, rad/s
ω_n = natural frequency of piezoelectric cantilever, rad/s

Table 1. Bimorph specifications.

Parameter	Value
Total length of piezoelectric layers	45.00 ± 0.1 mm
Beam width	7.20 ± 0.1 mm
Total beam thickness	0.78 ± 0.03 mm
Shim layer thickness	0.28 ± 0.05 mm
Piezoelectric layer density	8000 kg/m^3
Shim layer density	1800 kg/m^3
Piezoelectric coupling factor	0.38
Piezoelectric compliance	15.8×10^{-12} m^2/N
Piezoelectric dielectric constant	61.95 nF/m
Beam mechanical quality factor	45
Shim layer modulus of elasticity	120×10^9 N/m^2

Table 2. Characteristics of harvester setups.

Harvester Type	**Tip Mass [g]**	**Mechanical Quality Factor [-]**	**Anti-Resonance Frequency [Hz]**
Reference Setup	-	45	250
Stiffened Setup 1	0.19	63	250
Stiffened Setup 2	1.51	95	250

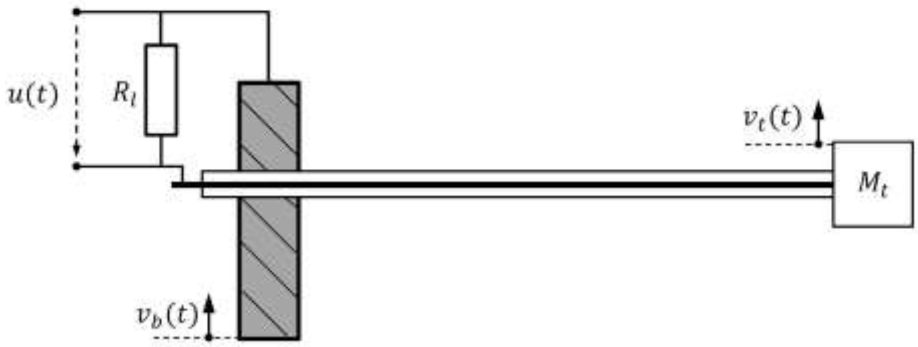

Figure 1. Typical energy harvesting system.

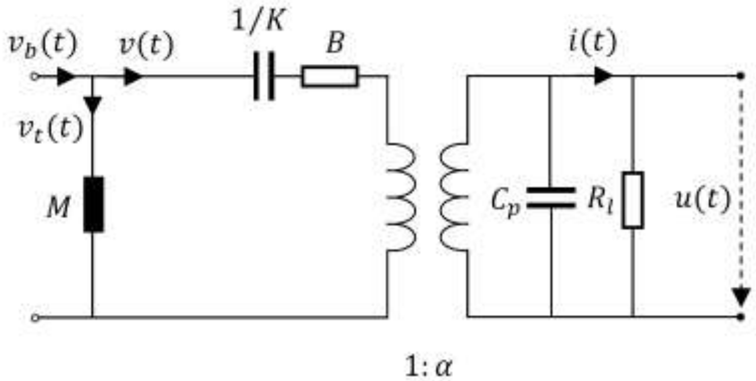

Figure 2. Equivalent electrical model of a base-excited piezoelectric energy harvester.

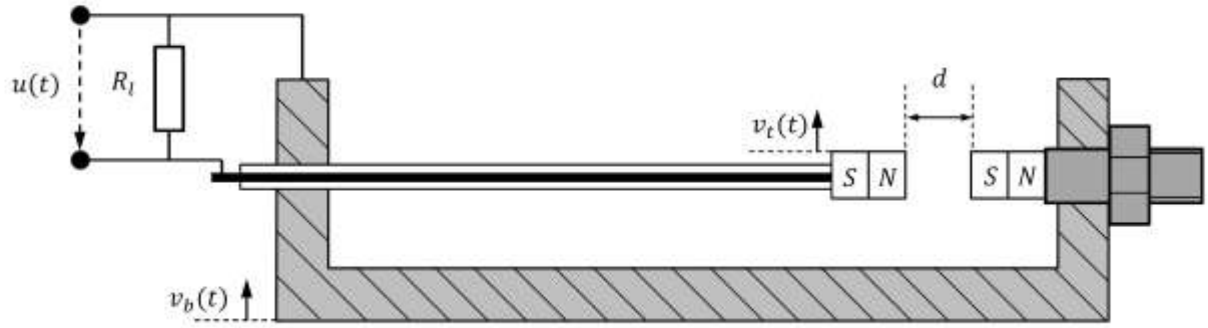

Figure 3. Principle setup of a magnetically stiffened harvester.

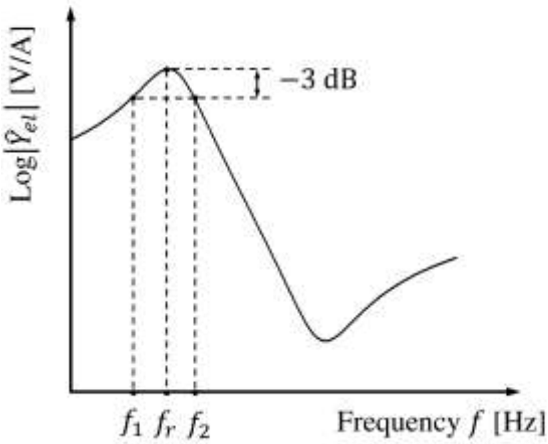

Figure 4. Typical frequency sweep of the electrical admittance of a piezoelectric harvester.

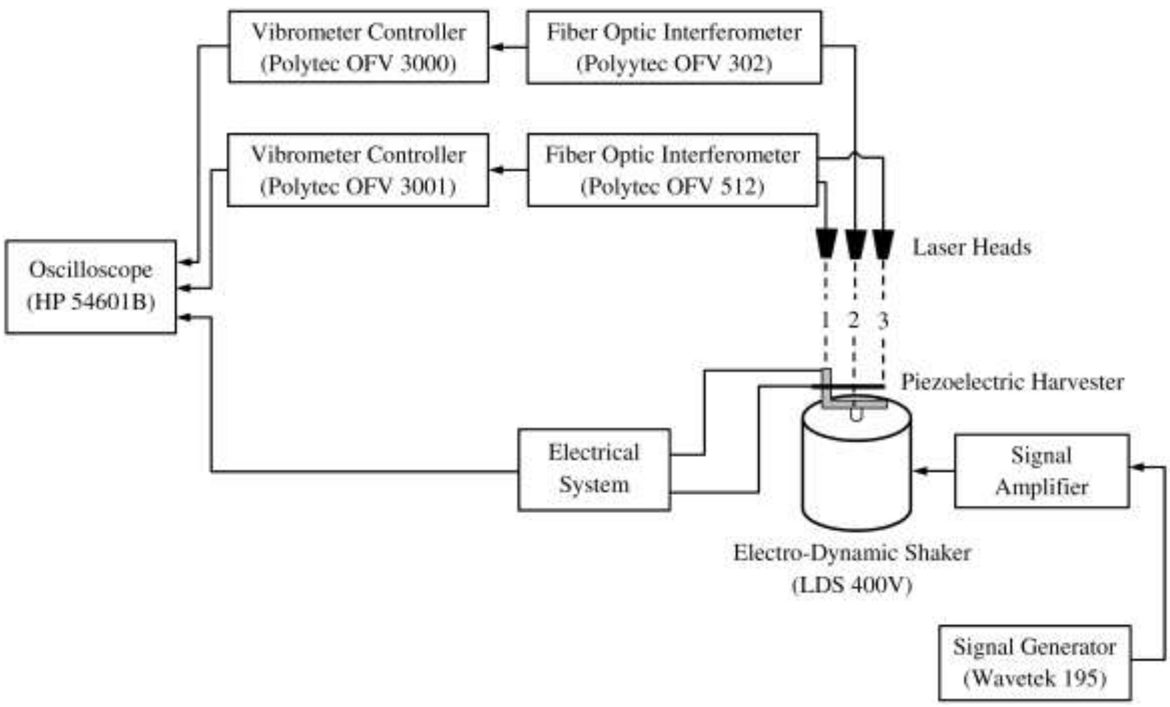

Figure 5. Schematic of the experimental setup.

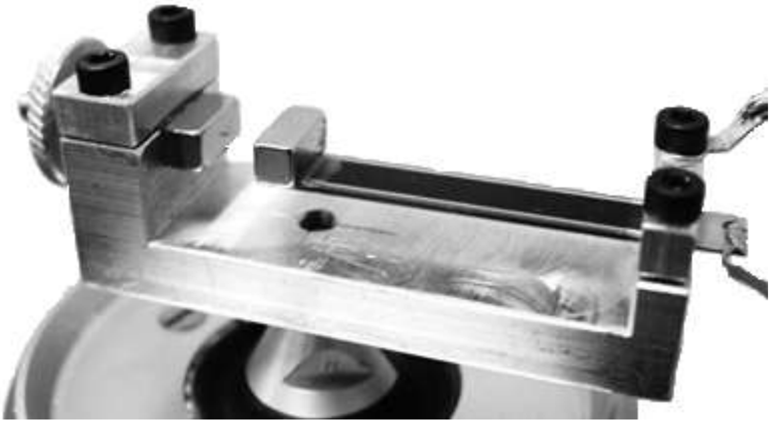

Figure 6. Magnetically stiffened harvester (Stiffened setup 2).

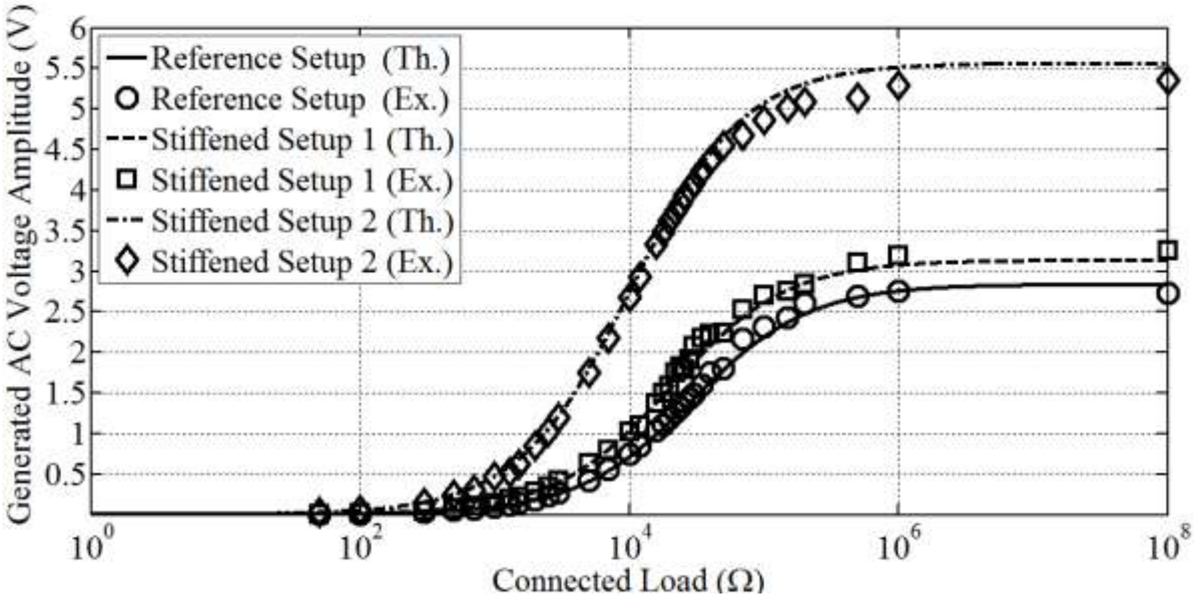

Figure 7. Variation of the generated voltage amplitude versus connected load.

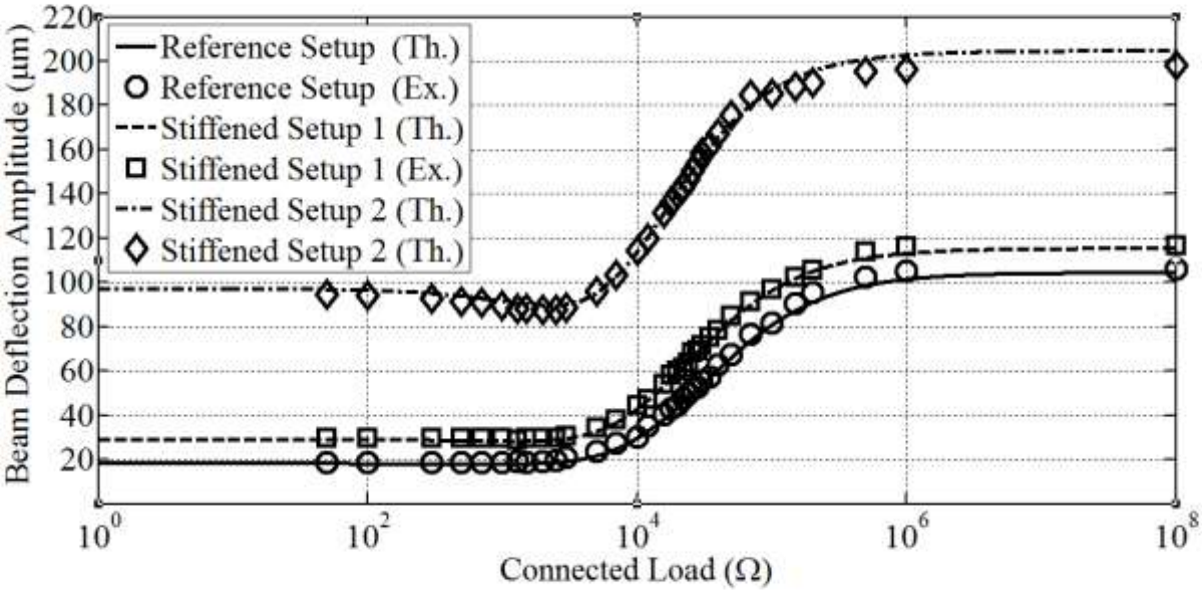

Figure 8. Variation of the beam deflection amplitude versus connected load.

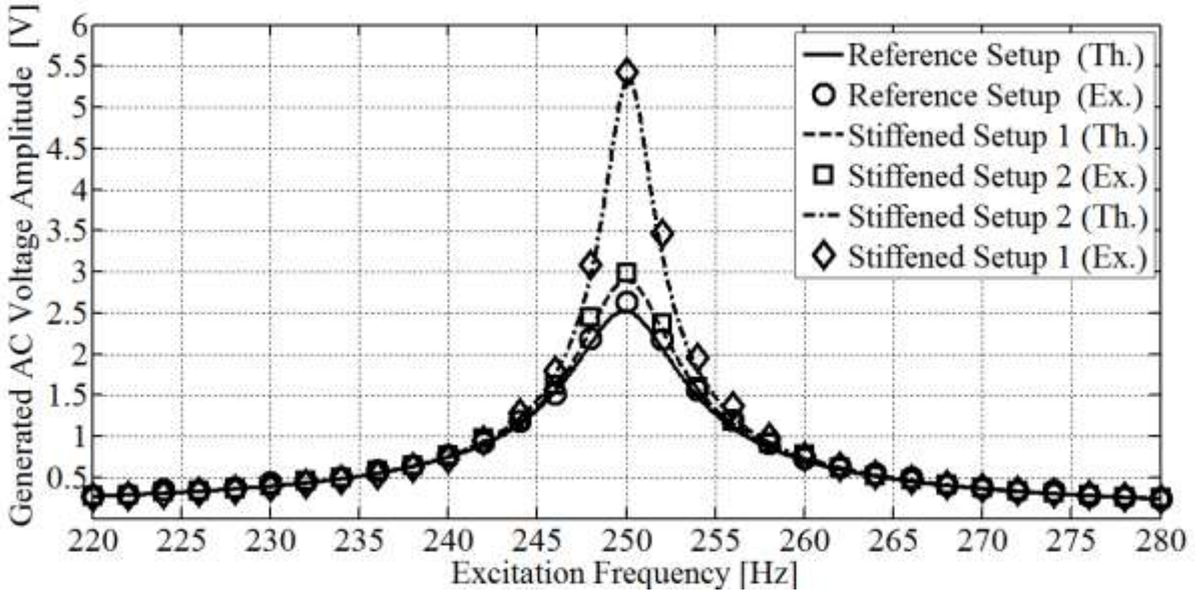

Figure 9. Variation of the generated voltage amplitude versus the excitation frequency (Excited by velocity of amplitude 3.5 mm/sat open-circuited condition.

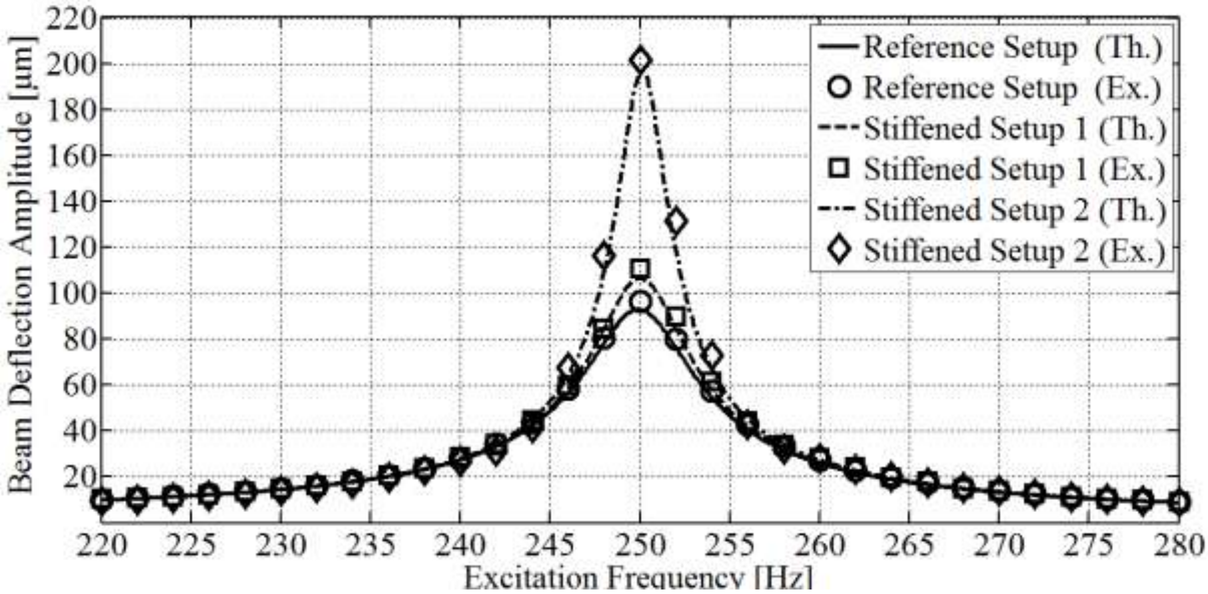

Figure 10. Variation of the generated voltage amplitude versus the excitation frequency (Excited by velocity of amplitude 3.5 mm/s at open-circuited condition.