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Investigation of the Static and Dynamic Performance of Leaf Springs under Different Shapes, Leaf Numbers, and Beam Widths

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ABSTRACT

This study is related to a new geometrical design of automotive leaf springs that involves substituting the common trapezoidal base and circular shape with a circular base plate and parabolic shape. The main aim is to explore how the shape of the base and the profile of curvature affect the performance of the leaf springs, both at rest and in dynamic conditions under realistic loading conditions. There were four geometrical combinations that were experimented with: trapezoidal base with circular curvature, circular base with circular curvature, trapezoidal base with parabolic curvature, and circular base with parabolic curvature. The influence of the leaf number and width distribution on the mechanical performance of each arrangement under the constant-width and variable-width scenarios was examined. ANSYS Workbench 20 and Mechanical APDL were used to conduct a finite element analysis to approximate the maximum primary stress, overall deformation, and natural frequency. Comparing the trapezoidal base with the circular foundation, it has been found that the circular foundation is better in terms of load distribution as well as minimization of stress concentration. Specifically, the circular base configuration reduced the maximum static bending stress by more than 36% when compared to the conventional design. A parabolic shape was used to further enhance dynamic performance, boosting natural frequency by up to 300% in certain combinations. The use of a parabolic curve improved dynamic performance significantly, increasing natural frequency by more than 300% in some combinations.

Keywords: Finite element method, Leaf spring, Leaf spring shape, Leaf spring curvature, Shape enhancement.

1. INTRODUCTION

Among machine and structural parts, springs are special because they are designed to undergo significant elastic deformation under stress, enabling them to store mechanical energy that is readily recovered **(Kader et al., 2021; Kumar and Kalam, 2016)**. A spring is an elastic body that bends in response to external forces before regaining its original shape

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when the load is released; the amount of recovery depends on the applied stress. This characteristic enables springs to be compressed, stretched, or twisted, while also acting as force and displacement transducers, energy-storage components, and shock and vibration absorbers in devices such as weighing scales **(Patunkar and Dolas, 2011)**.

Leaf springs are vital for trucks and other heavy-duty vehicles as they provide great load support and capacity **(Ben Sghaier et al., 2018; Besekar et al., 2023; Suresh et al., 2019)**. They are inexpensive to produce and maintain because of their simple design, which ensures dependability and longevity with no effort. Leaf springs, with their outstanding off-road performance, stability, and control, are ideal for rugged terrain and big loads **(Mallesh et al., 2021; Saini et al., 2013).**

Additionally, the modular design facilitates maintenance, as individual springs can be replaced without completely rebuilding the suspension system. Leaf springs are still highly valued for their practicality and efficiency in vehicles that require longevity, load carrying capacity, and ease of maintenance, even with more sophisticated suspension technology **(Besekar et al., 2023; Dighe, 2016).**

In recent research, there is a focus on altering the base shape and the curvature to enhance the fatigue life and stiffness-weight ratio of springs. Studies on various geometries (parabolic, exponential, and variable-width profiles) in the period between 2019 and 2024 found a circular or parabolic curve in bending stress, and significant enhancements in natural frequency and base shape **(Sakthivel et al., 2024; Varma et al., 2021)**. Such results suggest that a change could provide better performance, both in terms of static and dynamic performance, than conventional designs **(Ma et al., 2021; Rajendran and Vijayarangan, 2001)**.

Leaf springs are commonly used to take up shock loads in rail system suspension, big vehicles, and light car suspension systems. It also passes on driving torque, brake torque, and lateral loads, besides shock absorption **(Aggarwal et al., 2024; Jadhav et al., 2024; Tariq, 2020)**. Leaf springs have an advantage over helical springs in that they may be directed in a particular direction as they deflect, providing an energy-absorbing system as well as a structural element. The findings indicate that the most preferable material to use as a leaf spring is the one with the highest strength and the least longitudinal modulus of elasticity **(Aggarwal et al., 2024; Mallesh et al., 2021; Rahman and Kowser, 2010)**.

In recent years, scientists have focused on the enhancement of the performance of leaf springs by using composite materials, correcting curvatures, and optimizing geometries. As an illustration, **(Saini et al., 2013)** developed and exercised composite leaf springs in light automobiles, which led to a big reduction in weight without compromising on the right degree of strength. The E-glass/epoxy composite springs were found to realize significant stress reduction in both the static and fatigue tests as compared to the traditional steel designs. **(Rajendran and Vijayarangan, 2001)** found that parabolic steel springs and composite leaf springs were lighter yet retained rigidity. **(Winter et al., 2022)** observed that the study of shape optimization of leaf springs made of glass-fiber composites in 2022 was optimized using Bayesian methods, leading to large bulk and cost savings across various constraints. More recently, **(Wang et al., 2024)** investigated thermal-mechanical fatigue of sliding composite leaf springs and discovered that the structural optimization, relating to base shape and material selection, can raise life at least fourfold. These results indicate that base shape, curvature, material choice, and leaf architecture play a significant role in the leaf spring static, fatigue, and dynamic behavior. Although there has been a lot of research on composite materials and curvature manipulation in leaf springs, the impact of the base

geometry and curvature profile on the dynamic and static performance has not been well studied. In particular, the relationship between the base plate shape, the type of curvature, and the number of leaves has not been properly studied using numerical modeling.
Earlier research by **(Abdulkareem, 2018)** investigated the impact of curvature and base geometry on leaf spring behavior, but the combined impacts of leaf number variation and width distribution across diverse geometric configurations remain poorly understood. Consequently, the present study examines the behavior (both static and dynamic) of automotive leaf springs in four different geometric combinations formed by the combination of two base plate shapes (trapezoidal and circular) with two curvature shapes (circular and parabolic). The effects of the number of leaves and the breadth distribution are evaluated by the use of finite element analysis under the condition of variable leaf width and variable total base width. The aim is to determine a configuration that provides the optimal stress reduction, stiffness enhancement, and dynamic performance. Using exponential curvature instead of classical circular curvature increases the dynamic safety factor under entirely reversed load by around 300%. Prior research **(Abdulkareem, 2018)** claimed that the circular curvature guarantees the minimal deflection, and the trapezoidal design of leaf springs, which has been popular, has been known to provide consistent distribution of bending stress throughout its length. According to a previous study, the leaf spring base plate should have a trapezoidal shape, a consistent thickness, and be divided into a particular number of strips of varied lengths. The present study seeks to find a leaf spring model that would be more optimal in balancing dynamic behavior, stiffness, and strength. Based on the results, the proposed circular base-parabolic curvature scheme is more vibrationally active and stress concentrated than the previous ones. Such studies show that geometry ingenuity could be used to enhance leaf spring design in the modern automotive suspension

2. GEOMETRICAL MODELING AND THEORETICAL FORMULATION

Considering the following presumptions **(Abdulkareem, 2018)**. The trapezoidal shape guarantees a consistent bending stress along the spring length.
1. In the plane of its curvature, a beam deforms.
2. After bending, plane sections stay plane.
3. The homogenous beam material adheres to Hook's law.
4. No lingering pressures exist.
5. No interleaf friction is taken into account.

$$\sigma = 3WL/(2\,nbt^2) \quad (1)$$

$$\delta\_max = 3WL^3/(8\,Enbt^3) \quad (2)$$

where n: number of leaves, t: thickness of plate, W: applied load, b: width of each strip, L: base plate length, E: modulus of elasticity **(Hearn, 1997)**. No matter what kind of curvature a straight trapezoidal leaf spring has, equation (1) is used to determine its maximum bending stress. The greatest deflection of a leaf spring with a circular curvature, regardless of shape, is described by Eq. (2)**(Edition and Bauld, 1986)**.
Thus, under both variable and constant width conditions, the current work investigates four geometric configurations: trapezoidal base with circular curvature, circular base with circular curvature, trapezoidal base with parabolic curvature, and circular base with parabolic curvature. The analysis employs finite element analysis (ANSYS WORKBENCH Ver. 20) to determine the maximum principal stress, total deformation, and natural frequency.

2.1 The Geometrical Representation of Leaf Spring

The basic concept behind leaf spring design is to enhance the second moment of area along the length of the spring by using a plate with a length-dependent leaf width, which results in constant stress levels. To create stress concentration zones, the well-known shapes are trapezoidal plates that are cut into a specific number of leaves with knife-edge ends, as shown in **Fig. 1 (Abdulkareem, 2018).**

The representation is divided into two sections. The first one substitutes rectangular leaves of the same width but shorter length (equivalent length Leq) for the knife-edge plates in both shapes, i.e., trapezoidal and circular plates. At the same time, the second section seeks to introduce various curvatures for every leaf shape. This work suggested using a circular plate shape rather than a trapezoidal one as the base to build up the leaf spring in order to increase its load-carrying capacity. In addition to the conventional circular curvature, two additional curvatures were introduced: the parabolic curvature and the exponential curvature **(Abdulkareem, 2018)**.

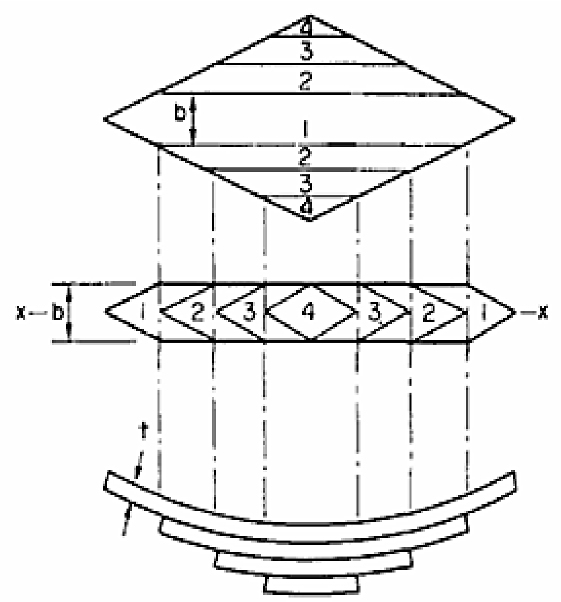


Figure 1. Semielliptical carriage spring showing initial pre-forming **(Abdulkareem, 2018)**

2.2 Calculating Equivalent Length

For both trapezoidal and circular base plate shapes, the basic principle for evaluating equivalent leaf lengths is area similarity for edge ends and rectangular leaves. To calculate the equivalent length (L eq), which represents the arc length of the leaf spring.

For a trapezoidal base plate:

$$L_{eq} = \frac{4L}{b}\int_{\frac{b}{2}(i-1)}^{\frac{b}{2}i}(0.5-\frac{y}{B})\,dy \tag{3}$$

For circular base plate;

$$L_{eq} = \frac{4}{b}\int_{\frac{b}{2}(i-1)}^{\frac{b}{2}i}\left(\sqrt{R^2-y^2}\right)dy \tag{4}$$

The dimensions of the laminated square leaf springs that correspond to the trapezoidal and circular knife-edge springs, respectively, are shown in Eqs. (3) and (4).

2.3 Leaf Spring Curvature

In this work, two types of curvatures were considered, circular and parabola to study the effect of curvature shape on leaf spring performance. Since the equivalent length is represented in the horizontal Figure of the leaf spring, while the leaf spring is going to have a curvature (regardless of its shape), this makes the horizontal length (x) shorter than the equivalent length. So, we need to calculate this distance for each curvature (circular curvature and parabolic curvature) **(Abdulkareem, 2018)**.

The different types of leaf spring curvatures (circular and parabolic) are shown in **Fig. 2** below. Analytical Eqs. (1) and (2) above use L (horizontal length) as the same as the arc length (L eq), in this study, the horizontal distance will be calculated for more accuracy as shown below in Eqs. (5) and (6). The horizontal span (L) in straight beam theory is used to produce the analytical stress and deflection equations **(Hearn, 1997)**, whereas the arc length (Leq) is used in geometrical modeling. The conversion between the two is done statistically and is dependent on curvature.

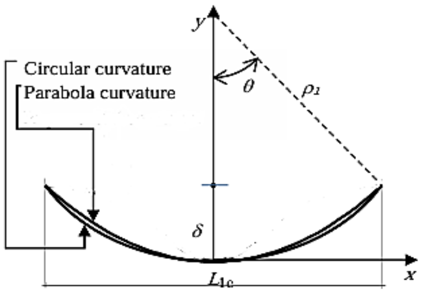


Figure 2. Different leaf spring curvature representations **(Abdulkareem, 2018)**

2.3.1 Circular Curvature

To determine the horizontal length (x) for circular curvature;

$$\rho^2 = x^2 + y^2 \; but \; y = \rho - \delta$$

$$\therefore \rho^2 = x^2 + (\rho - \delta)^2 \rightarrow \rho = \frac{x^2 + \delta^2}{2\delta}$$

$$\theta = tan^{-1}\frac{x}{\rho - \delta} = tan^{-1}\frac{x}{\frac{x^2 + \delta^2}{2\delta} - \delta} =) \; tan^{-1}\frac{2\delta x}{x^2 - \delta^2}$$

$$\frac{L_{eq}}{2} = \rho\,\theta = \frac{x^2 + \delta^2}{2\delta} * \frac{\pi}{180} * \; tan^{-1}\frac{2\delta x}{x^2 - \delta^2}$$

$$\therefore \frac{L_{eq}}{2} = \frac{\pi(x^2 + \delta^2)}{2 * 180 * \delta} * tan^{-1}\frac{2\delta x}{x^2 - \delta^2} \tag{5}$$

2.3.2 Parabola Curvature

$$y = ax^2 \; where \; at \; x_{max}, \; y = \delta \rightarrow \delta = a{x^2}_{max}$$

$$\frac{L_{eq}}{2} = \int ds = \int \sqrt{(dx^2 + dy^2)}$$

$$y = ax^2 \rightarrow dy = 2axdx$$

$$\frac{L_{eq}}{2} = \int_0^{x_{max}} \sqrt{(1 + 4a^2x^2)}dx$$

$$\frac{L_{eq}}{2} = \int_0^{x_{max}} \left\{\frac{sinh^{-1}(2ax)}{4a} + \frac{1}{2}x\sqrt{4a^2x^2 + 1}\right\}$$

$$\therefore \frac{L_{eq}}{2} = \int_0^{x_{max}} \{\frac{sinh^{-1}\left(2a\sqrt{\frac{\delta}{a}}\right)}{4a} + \frac{1}{2}\sqrt{\frac{\delta}{a}}\sqrt{4a^2x^2 + 1}\} \quad (6)$$

Both Eqs. (5) and (6) can be solved numerically to calculate the horizontal distance (x), the radius of curvature (ρ) and the constant value (a), keeping in mind that the parabola curvature is variable at each point.

After calculating the master leaf curvature radius, we can apply the following equation:

$$\rho_{i+1} = \rho_i + t \quad (7)$$

Eq. (7) describes the graduated leaves curvature radius by adding the master leaf's radius of curvature to the thickness of the base plate. However, their distribution angle is:

$$\theta_1 = tan^{-1}\frac{x}{\rho_1 - \delta} \quad (8)$$

Eq. (8) describes how to calculate the master leaf angle, or we can simply use;

$$\frac{L_{eq\,i}}{2} = \rho_i\,\theta_i \quad (9)$$

3. MATERIALS AND METHODS

This study employed finite element techniques to analyze the stress distribution, deflection characteristics, and the natural frequency of a multi-leaf spring. Under the effect of changing the total width (B), as the leaf spring width (b) is constant (=70mm) **(Ehab, 2021; Kumar and Kalam, 2016; Srikanth and Tarun, 2020; Tadesse and Fatoba, 2022)**, and vice versa, as the number of the leaf springs increases from (2 to 10) with the same charge using ANSYS 20 workbench and mechanical APDL. Static analysis was performed on the chosen dimensions and properties shown in **Table 1.**

Table 1. Dimensions and mechanical properties for different leaf spring geometries **(Abdulkareem, 2018).**

Parameter	Value
Length L (mm)	1000
Thickness t (mm)	10
Initial deflection δ (mm)	200
Modulus of elasticity E (GPa)	200
Passion ratio μ	0.3
σ_e(GPa)	86
σ_y(GPa)	250
Length L (mm)	1000

The leaf springs were designed to be made of structural spring steel, a material typically utilized in heavy vehicle suspension systems. The material was modeled as a linear elastic isotropic material with 200 GPa Young's modulus, 0.3 Poisson's ratio, and a density of 7840 kg/m^3.

3.1 Calculations Results

3.1.1 Variable Width (b)

Taking into consideration the dimensions in **Table 1**, for trapezoidal and circular base plates, each plate has circular and polynomial curvatures, by applying the Eqs. (3) to (9) to calculate the equivalent length (L_{eq}), the horizontal distance (x), the radius of curvature (ρ), and the angle of curvature (θ). Taking the number of leaf spring plates to be variable from (2 to 10). In this case, the width varies gradually from (500 mm when n=2) to (100 mm when n=10). The dimensions for all four configurations are listed in **Tables 2 and 3.**

Table 2. Dimensions of the trapezoidal and circular base plates with circular curvature when the width (b) is variable.

Trapezoidal base-circular curvature		
Parameter	**Value**	
N	2	
$L_{eq}/2$ (mm)	375	125
ρ (mm)	310.9756	320.9756
θ (degree)	69.09235	22.31317
Circular base-circular curvature		
Parameter	**Value**	
N	2	
$L_{eq}/2$ (mm)	478.5	307
ρ (mm)	545.3	555.3
θ (degree)	51.21636	31.67622

Table 3. Dimensions of the trapezoidal and circular base plates with parabolic curvature when the width (b) is variable.

Trapezoidal base-parabolic curvature		
Parameter	**Value**	
N	2	
A	0.002197	
$L_{eq}/2$ (mm)	375	125
x (mm)	301.717131	119.7
Circular base-parabolic curvature		
Parameter	**Value**	
N	2	
A	0.001126	
$L_{eq}/2$ (mm)	478.5	478.5
x (mm)	412.51	412.51

3.1.2 Variable Total Base Width (B)

Taking into consideration the same dimensions in **Table 1,** for trapezoidal and circular base plates, each plate has circular and polynomial curvatures, by applying the Eqs. (3) to (9) to calculate the equivalent length (L_{eq}), the horizontal distance (x), the half radius of curvature (ρ), and the angle of curvature (θ). Taking the number of leaf spring plates to be variable from (2 to 10). In this case, the total width (B) varies gradually from 140 mm when n=2 to 700 mm when n=10. The dimensions for all four configurations are listed in **Tables 4 and 5.**

Table 4. Dimensions of the trapezoidal and circular base plates with circular curvature when the total base width (B) is variable

Trapezoidal base-circular curvature		
Parameter	**Value**	
N	2	
$L_{eq}/2$ (mm)	375	125
ρ (mm)	310.9756	320.9756
θ (degree)	69.09235	22.31317
Circular base-circular curvature		
Parameter	**Value**	
N	2	
$L_{eq}/2$ (mm)	449.6	497.15
ρ (mm)	587.2615	597.2615
θ (degree)	48.74311	47.692

Table 5. Dimensions of the trapezoidal and circular base plates with parabolic curvature when the total base width (B) is variable

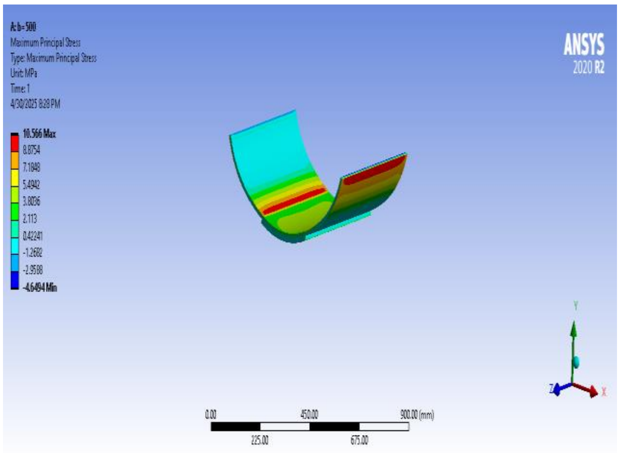
Trapezoidal base-parabolic curvature		
parameter	**Value**	
n	2	
a	0.002197	
$L_{eq}/2$ (mm)	375	125
x (mm)	301.717131	119.7
Circular base-parabolic curvature		
Parameter	**Value**	
n	2	
a	0.001008	
$L_{eq}/2$ (mm)	499.6	497.15
x (mm)	445.45	443.6123

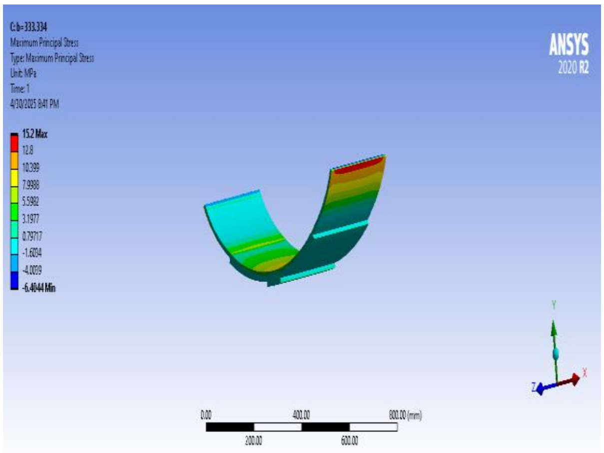
3.2 Finite Element Method and Analysis

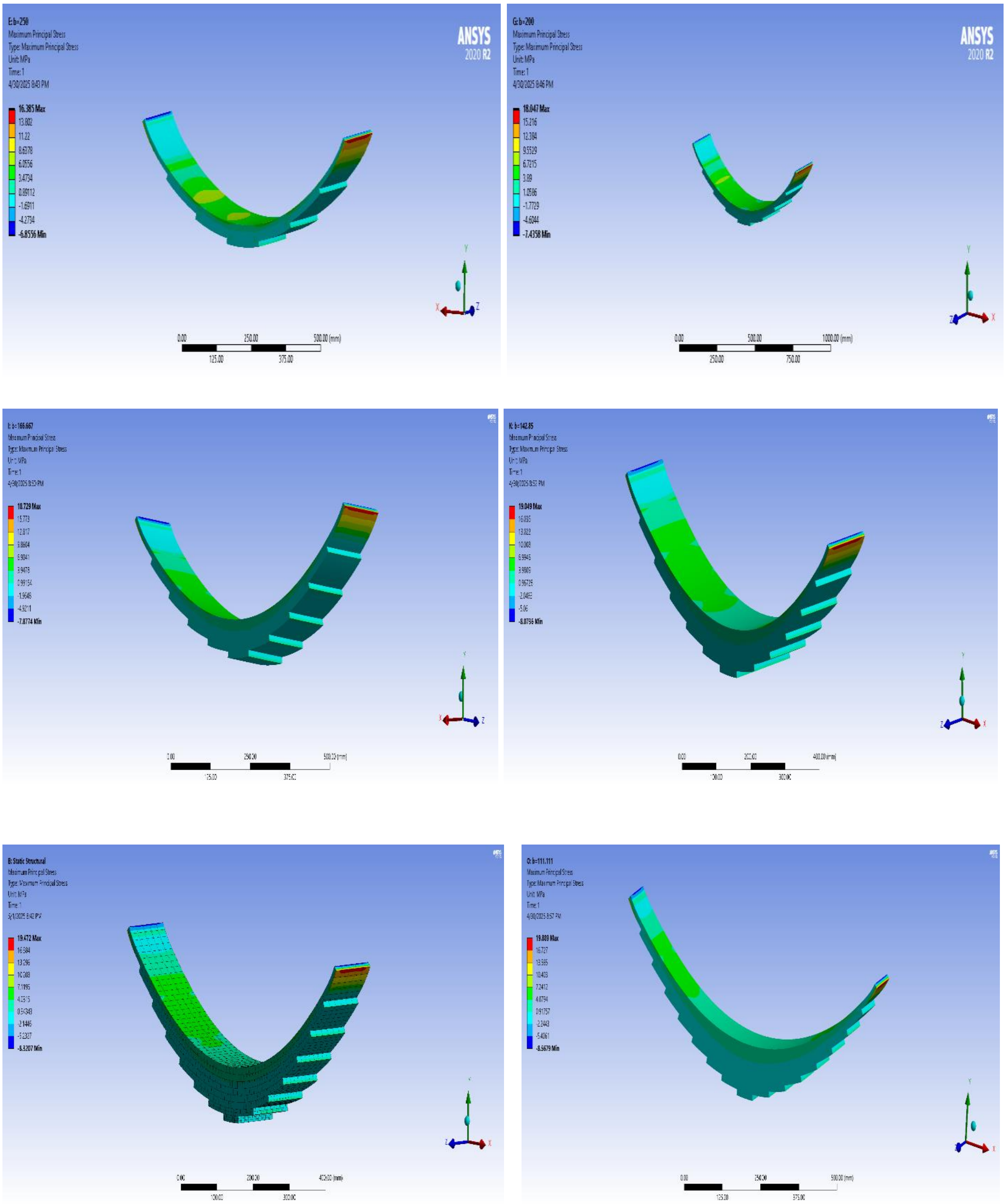
Affordable use, high accuracy results, and ease of use, in addition to the time saving with no limits of analytical models' complexities, the finite element method is an alternative engineering tool to simulate a wide range of disciplines and applications that interact with each other, which is so-called Multiphysics **(Kotha et al., 2024; Kumar and Kalam, 2016; Saaka et al., 2024; Suribabu et al., 2018)**. In applied mechanics, aerodynamics, stresses, vibrations, wear in bearings, and contacting elements are the most common applications that adopt FEM **(Hammza et al., 2022)**. The present work adopts the ANSYS APDL software version 20 and WORKBENCH to evaluate the induced maximum principal stress, deflection, and natural frequency in leaf springs. Starting from the geometrical representation of successive layers, till the full embodiment. Leaf spring models are discretized using two types of elements: solid brick elements with 20 nodes for normal stresses and contact elements to mesh the contacting surfaces between the mating layers, which are target 170 and surface 174. The boundary conditions are simply supported with a central upward load equivalent to the effective payload **(Varma et al., 2021)**. The material used for leaf springs is usually plain carbon steel having 0.90 to 1.0% carbon. For automobiles: 50Cr 1, 50 Cr 1 V 23, and 55 Si 2 Mn 90 all used in the hardened and tempered state. For railroad springs: C

55 (water - hardened), C 75 (oil-hardened), 40 Si 2 Mn 90 (water-hardened), and 55 Si 2 Mn 90 (oil-hardened) **(Takim, 2014; Venkatesan and Devaraj, 2012)**. Simulated cases are made of structural steel with a modulus of elasticity of 200 GN/m^2, and 0.3 Poisson's ratio, a yield strength of 250 MPa, and 7840 kg/m^3 **(Abdulkareem, 2018; Hmoad et al., 2020; Mohamed et al., 2020),** which are the same properties as the 55 Si Mn 90 used to manufacture the steel leaf springs. Simulated cases are made of structural linear elastic material with a modulus of elasticity of 200 GN/m^2, and 0.3 Poison's ratio and 7840 kg/m^3 **(Abdulkareem, 2018; Hmoad et al., 2020; Mohamed et al., 2020)**. Leaves are supposed to slide against each other with uniformly distributed contact stress. The studied cases involve trapezoidal and circular base plates, each with circular and polynomial curvatures, featuring a variable number and width of leaves. Classical beam theory was used to test the FEM results against analytical solutions. The accuracy of the model was confirmed by the greatest stress deviation, which was within 5–8%.

To make sure that the numerical results were independent of the mesh density, a mesh convergence analysis was carried out. The structural domain of the models was discretized using 20-node solid brick elements (SOLID186), and the interaction between neighboring leaves was simulated using surface-to-surface contact elements (TARGE170 and CONTA174). Depending on the arrangement, the simulations' average element size of 8–10 mm produced between 65,000 and 85,000 elements. To find the system's initial natural frequency, modal analysis was conducted after static structural analysis under the assumption of linear elastic behavior. Boundary conditions are equivalent to a simply supported semi-elliptic leaf spring, where the effective payload is represented by a center vertical load. The stress distribution (max. principal stress) when (b) is variable for a trapezoidal base plate with circular curvature as the number of leaf springs (n) increases from (2 to 10) is shown in **Fig. 3** below.



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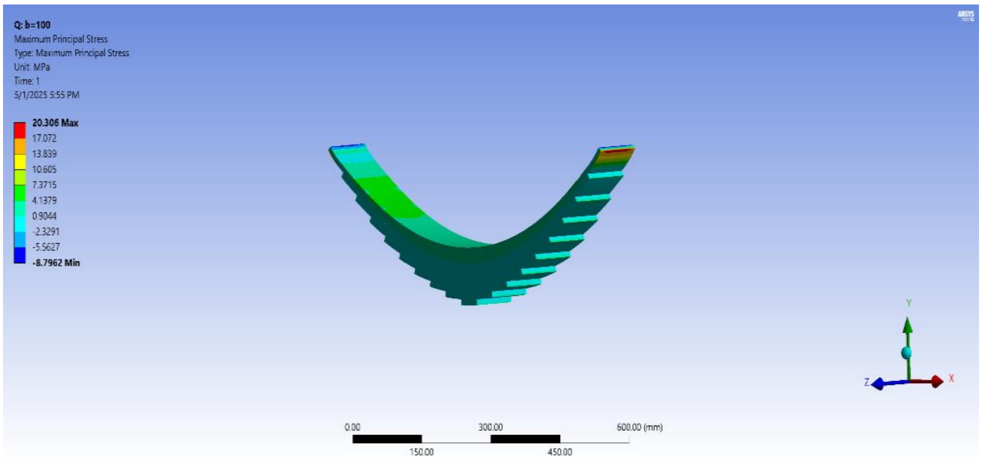

Figure 3. The stress distribution (max. principal stress) when the (width=b) is variable for a trapezoidal base plate with circular curvature as the number of leaf springs (n) increases from (2 to 10).

3.2.1 Finite Element Method Modeling and Analysis with Variable Width (b)

ANSYS WORKBENCH (ver. 20) is used exclusively to model and analyze trapezoidal and circular base plates with circular curvature, while mechanical APDL is used to model the trapezoidal and circular base plates with parabolic curvature. The dimensions in **Tables 2 and 3** are used to create models of the leaf spring. The investigation focused on the natural frequency, total deformation, and maximum principal stress.

3.2.2 Finite Element Method Modeling and Analysis with Variable Total Base Width (B)

Models of the leaf spring are created using the dimensions in **Tables 4 and 5**, and only ANSYS WORKBENCH (ver.20) for modeling and analyzing trapezoidal and circular base plates with circular curvature, and mechanical APDL for modeling and analyzing the models with parabolic curvature. The investigation focused on the natural frequency, total deformation, and maximum principal stress.

4. RESULTS AND DISCUSSION

The numerical results from finite element analysis are examined in light of previous analytical and numerical research on the geometry, load distribution, and vibration behavior of leaf springs. Under two loading situations, the effects of base shape, curvature profile, leaf number, and breadth variation are investigated: The width of individual leaves varies, while the total base width remains constant.

4.1 When the Width (b) Is Variable

4.1.1 The Maximum Principal Stress Results

When the individual leaf width varies with leaf number, distinct stress evolution trends appear throughout the investigated geometries. This behavior is primarily regulated by load-sharing efficiency and stress redistribution among stacked leaves, according to multiple findings in both classical and modern leaf spring research **(Patunkar and Dolas, 2011; Rajendran and Vijayarangan, 2001)**. The maximum primary stress of the trapezoidal base with circular curvature progressively increases as the number of leaves increases. Similar stress-building tendencies were seen in tapered bases by **(Raghavedra et al., 2012; Venkatesan and Devaraj, 2012)**, who attributed this behavior to nonuniform bending stiffness over the leaf length. Even though the additional leaves help with load sharing, the trapezoidal design maintains localized stress concentration around the narrower portions.

In contrast, as the number of leaves increases, the circular base with circular curvature loses tension. This tendency is consistent with findings by **(Al-Qureshi, 2001; Saini et al., 2013).** This indicates how smoother base geometries prevent sudden changes in stiffness, which lowers bending stress. Greater stress homogeneity and efficient force transfer are suggested by the steady drop in stress observed with an increase in leaf count.

The trapezoidal-parabolic shape has higher peak values and more unpredictable stress swings. Similar instabilities were discovered by **(Khan et al., 2018).** In tapered-parabolic systems, the geometric mismatch between base taper and curvature results in imbalanced stress channels and localized bending amplification.

The most consistent and dependable trend in stress reduction is demonstrated when the circular base and parabolic curve are combined. It has been shown that parabolic curvature more evenly distributes bending stress along the span by reducing peak bending moments at the center **(Agarwal et al., 2017; Shi et al., 2016)**. The present results confirm that the combination of a circular base and parabolic curvature improves stress smoothing beyond what is achievable with circular curvature alone. Quantitatively, at a representative leaf number (n = 6), the circular base reduces the maximum stress by approximately (36.275%) compared to the trapezoidal base under circular curvature. Furthermore, introducing parabolic curvature to the circular base yields an additional reduction of approximately. The trapezoidal–parabolic configuration exhibits the highest stress, exceeding the optimal configuration by approximately (59.57%). **Fig. 4** shows how the maximum principal stress is distributed when the width (b) is variable.

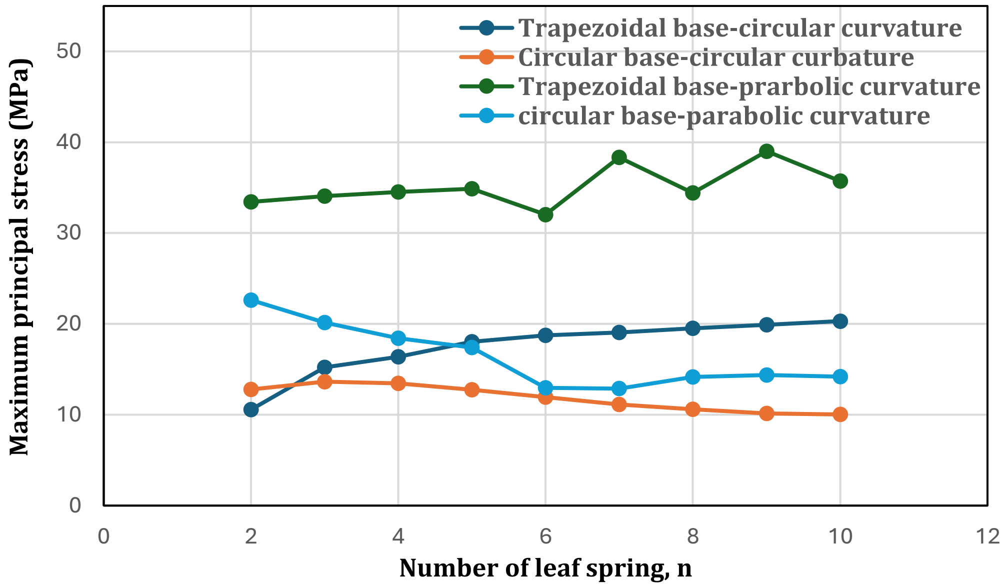

Figure 4. The maximum principal stress (MPa) when the width (b) is variable for all four cases

4.1.2 The Total Deformation Results

The deformation results under different width conditions demonstrate the combined effects of bending stiffness, leaf stacking, and geometric efficiency. More leaves reduce overall deformation for all designs due to increased overall stiffness, according to both analytical and FEM-based studies **(Sonawane et al., 2014; Takim, 2014)**.

There is initially more distortion in the trapezoidal-circular configuration, but it gradually decreases. This nonmonotonic response has been linked to the sensitivity of tapered geometries to width variation, where stiffness advantages from more leaves partially offset reduced individual leaf width **(Singh and Brar, 2018)**. Circular base arrangements have significantly lower degrees of deformation. The circular design shows a smooth and consistent rise in stiffness, which is consistent with results reported by **(Larco et al., 2015)**, who highlighted the enhanced bending rigidity of uniform-base geometries. The highest deformation reduction in the circular–parabolic configuration shows that parabolic curvature increases stiffness efficiency by optimizing curvature-induced bending resistance **(Agarwal et al., 2017)**. Trapezoidal-parabolic springs sustain mid-range anomalies and higher deformation levels. Even at higher leaf counts, tapered bases demonstrated localized flexibility, according to **(Karditsas et al., 2014)**.

The system's increased equivalent bending stiffness controls the decrease in deformation. The parabolic curvature lessens peak bending effects along the span, while the circular base offers a more consistent stiffness distribution. In comparison to the trapezoidal base, the circular base quantitatively reduces total deformation at n = 6 by around (52.5%). In comparison to the circular–circular configuration, the circular–parabolic configuration exhibits an extra reduction of about (-48.77%), whereas the trapezoidal–parabolic scenario produces deformation values that are greater by about (63.8%). **Fig. 5** shows how total deformation is distributed when the width (b) is variable.

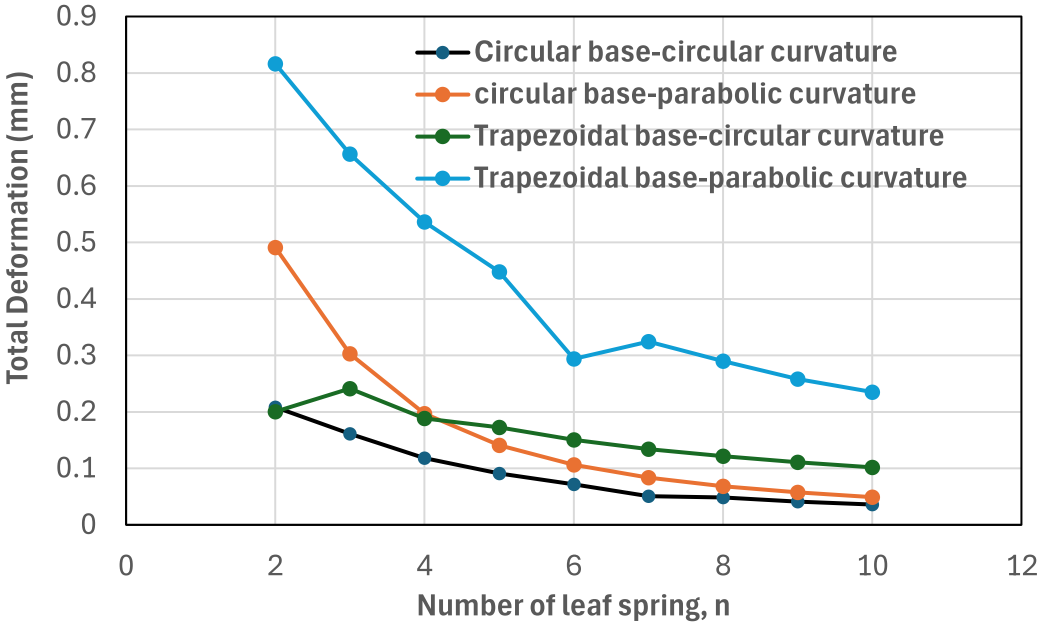

Figure 5. The total deformation (mm) when the width (b) is variable for all four cases

4.1.3 The Natural Frequency Results

The rise of natural frequency with the number of leaves will occur in all designs, suggesting an increase in dynamic stiffness as compared to system mass. The relationship has been actively confirmed in suspension research in vehicles; it is in accordance with vibration theory **(Polilov et al., 2019; Srikanth and Tarun, 2020)**. Circular base designs create much more frequent increases than do trapezoidal bases. The results of **(Ma et al., 2021)**, found out that uniform-based leaf springs with increased modal stiffness are in line with the observed frequency increase in the circular-circular set-up. Circular-parabolic design has the highest overall increase in frequency, which means that parabolic curvature enhances dynamic stiffness, which is a decrease in flexural compliance. When the number of leaves is moderate, trapezoidal-parabolic designs have small oscillations and a slower frequency rise. A similar modal sensitivity was shown in **(Winter et al., 2022; Zou et al., 2022)** and explained by the effects of interleaf interaction and the mode coupling due to geometry. The ratio of stiffness to mass in the system is directly proportional to the increase in the natural frequency. Although the parabolic curve adds no mass and only resistance to bending, the parabolic shape and the circular base give greater homogeneity to stiffness. The circular base increases the natural frequency by roughly 16.05% in comparison to the trapezoidal base. The most significant rise of 5.263 %, compared to the trapezoidal-circular baseline architecture, is recorded in the circular-parabolic architecture. **Fig. 6** demonstrates how the natural frequency changes as the width (b) varies.

The link between structural stiffness and mass accounts for the considerable increase in natural frequency found in parabolic curvature settings. To calculate a structure's natural frequency, take the square root of its stiffness-to-mass ratio ($f \propto \sqrt{k/m}$). The increase in frequency is mostly due to an increase in effective bending stiffness, while the overall mass of the leaf spring remains nearly unchanged across the geometries under consideration.

In addition to minimizing localized flexural compliance and enhancing the distribution of bending forces over the leaf length, parabolic curvature boosts the overall stiffness of the spring.

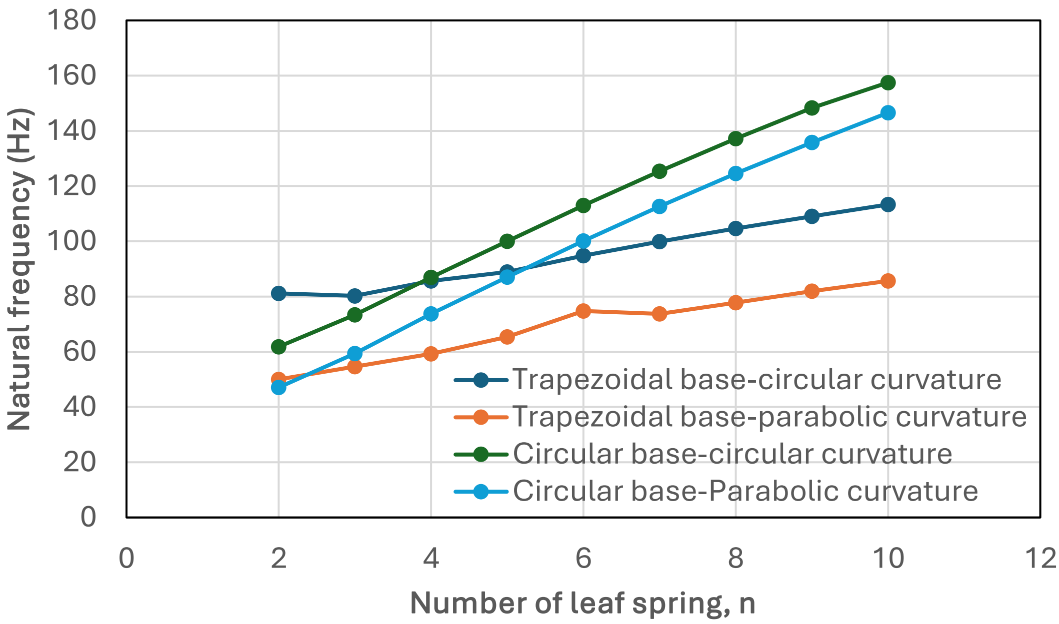

Figure 6. The natural frequency (Hz) when the width (b) is variable for all four cases

4.2 When the Total Base Width (B) is Variable

4.2.1 The Maximum Principal Stress Results

Increasing the number of leaves dramatically lowers the maximum primary stress in all geometries while the overall base width stays constant. Analytical simulations that demonstrate that a constant total width encourages more efficient stress transfer across stacked leaves are consistent with this outcome **(Patunkar and Dolas, 2011; Rajendran and Vijayarangan, 2001).** Circular base designs outperform trapezoidal bases in this scenario. The most uniform stress decay in the circular–circular design supports the findings of **(Saini et al., 2013)**, who showed that circular bases decrease stress concentration under constant-width constraints.

The circular–parabolic design achieves comparable stress reduction and better stability across leaf counts. Trapezoidal-parabolic designs continue to exhibit the greatest stress values, confirming earlier research that tapered bases and nonlinear curvature can raise bending stress due to stiffness mismatch **(Khan et al., 2018; Raghavedra et al., 2012).** The circular base quantitatively lowers the maximum stress at a representative leaf number (n = 6) by approximately (32.1%) as compared to the trapezoidal base under circular curvature. Additionally, adding parabolic curvature to the circular base yields an additional reduction, the trapezoidal-parabolic design, which is approximately (76.784%) more stressful than the optimal form, has the most stress. **Fig. 7** shows how the maximum principal stress is distributed when the total base width (B) is variable.

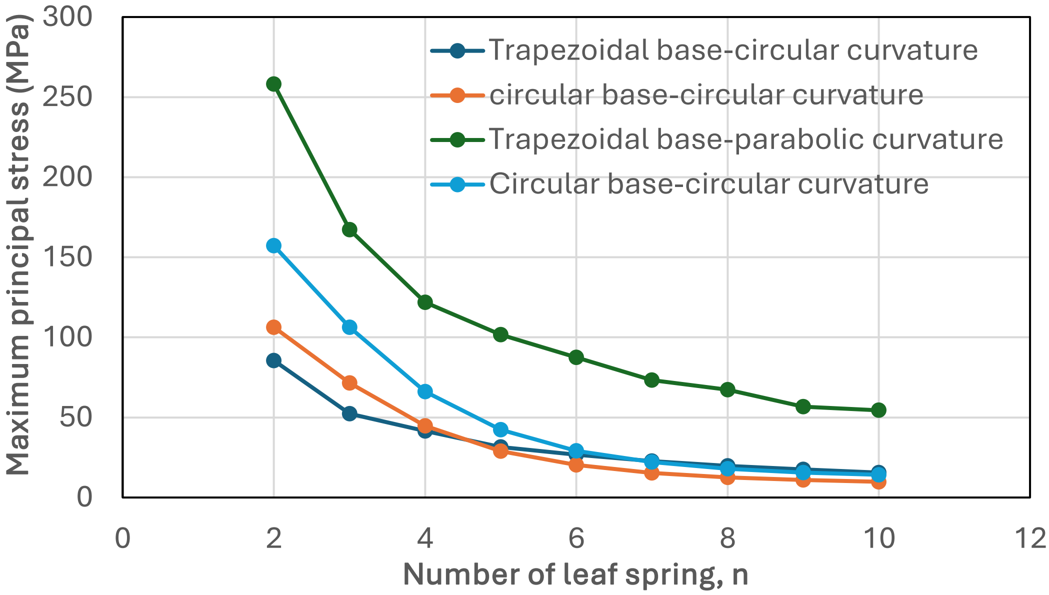

Figure 7. The max. principal stress (MPa) when the total base width (B) is variable for all four cases

4.2.2 The Total Deformation Results

As the number of leaves increases, all designs show a notable decrease in deformation under conditions of constant overall width. Again, circular base designs show excellent stiffness development with smooth and predictable deformation patterns. These results are in line with those of **(Agarwal et al., 2017; Larco et al., 2015)**, who emphasized the deformation efficiency of uniform-base designs. Trapezoidal-parabolic springs show higher initial deformation and minor increases with large leaf counts. Similar behavior has been associated with interleaf friction and local flexibility effects in tapering arrangements **(Karditsas et al., 2014; Winter et al., 2022)**. At n = 6, the circular base lowers overall deformation by (-127.1%) more than the trapezoidal base. The circular-parabolic configuration increases deformation values by about (19.447%) more than the circular-circular configuration, but the trapezoidal-parabolic configuration increases deformation values by around (85.25%). **Fig. 8** shows how the total deformation varies when the total base width (B) is variable.

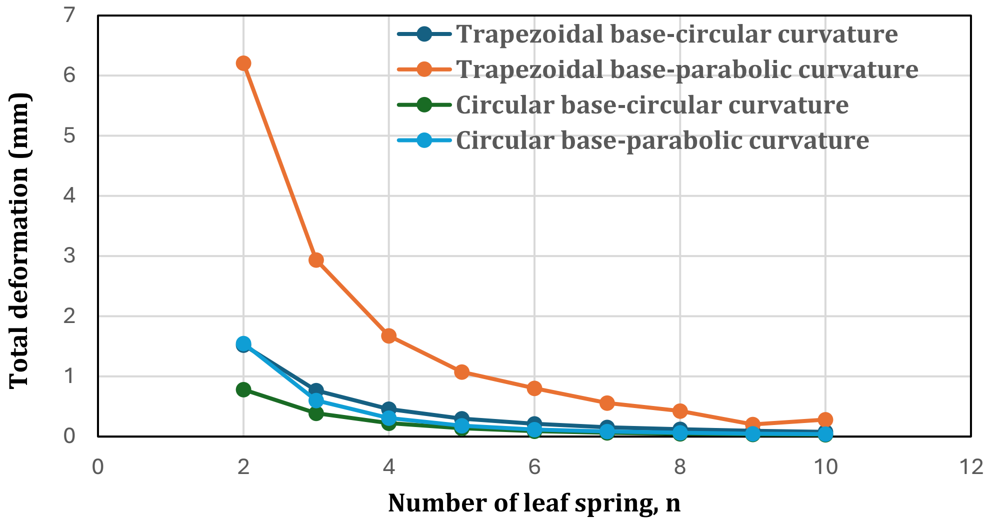

Figure 8. The total deformation (mm) when the total base width (B) is variable for all four cases

4.2.3 The Natural Frequency Results

Natural frequency trends under constant base width conditions indicate the important influence of geometry on dynamic behavior. Circular base layouts have higher frequency values than trapezoidal designs, indicating greater dynamic stiffness. The circular–parabolic shape again displays the greatest frequency response, according to studies showing that parabolic curvature increases vibrational resistance **(Ma et al., 2021; Varma et al., 2021)**. A modest frequency drop with high leaf counts in some designs indicates modal interaction effects, which **(Zou et al., 2022)** has been shown in multilayer spring systems. Natural frequency rises in direct proportion to the system's stiffness-to-mass ratio. The circular base improves stiffness uniformity, while the parabolic curve increases bending resistance without adding significant bulk. The circular base does not increase the natural frequency by much (4.384%) as compared to the trapezoidal base. The circular-parabolic arrangement outperforms the trapezoidal-parabolic structure by up to 28.0.35%. **Fig. 9** depicts how natural frequency changes when the entire base width (B) is changed. The relationship between structural stiffness and mass helps explain the significant increase in natural frequency observed for parabolic curvature structures. A structure's natural frequency is proportional to its stiffness-to-mass ratio ($f = \sqrt{k/m}$). Because the overall mass of the leaf spring is generally consistent across the examined geometries, the increase in frequency is mostly related to an increase in effective bending stiffness. Parabolic curvature improves the distribution of bending moments over the leaf length while minimizing localized flexural compliance, increasing the overall stiffness of the spring system.

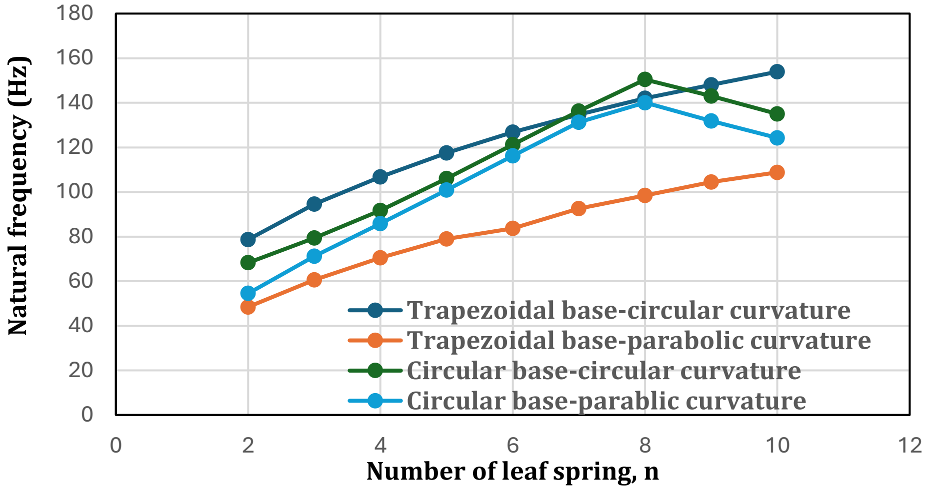

Figure 9. The natural frequency (Hz) when the total base width (B) is variable for all four cases

5. CONCLUSIONS

When stress, deformation, and natural frequency are all considered, the circular-parabolic arrangement performs best, despite the fact that the circular-circular structure has lower stress levels at times. Consequently, an optimal design is established by a combination of performance properties and not one parameter.

- The geometric arrangement of leaf springs has a significant effect on both the static and dynamic performance.
- Stress distribution is increased, and concentration is decreased over the length of the spring when a circular base is used instead of a trapezoidal base.
- The more leaves, the more structural stiffness and reduction in deformation owing to better distribution of loads across layers.

The parabolic curvature elevates the natural frequency of the system by distributing the bending forces evenly and elevating the ratio of stiffness to mass. To reduce stress, parabolic curvature of the base enhances deformation control and dynamic stability, compared to other arrangements. Geometrically optimizing base form and curvature can be done to provide the mechanical and vibrational properties of vehicle leaf springs.

NOMENCLATURE

Symbol	Description	Symbol	Description
σ	Bending stress (MPa).	n	Number of Leaves.
W	Load (N).	b	Strip Width (mm).
L	Base Plate Length (mm)	t	Thickness (mm).
E	Modulus of Elasticity.	L_{eq}	Equivalent Length (mm).
B	Total Base Width (mm).	ρ	Radius of Curvature (mm).
δ	Initial Deflection (mm).	Θ	Distribution angle (degree).
X	Horizontal distance (mm).	Y	Vertical axis
R	Circular base plate radius (mm).	A	Parabolic equation constant

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Credit Authors Contributions

Aveen Ahmed Abdulkareem: conceived the proposed idea, created the theory, verified the analytical procedures, and oversaw the findings of this study. Ibtehal Abdul Hussain Bani did the computations, analyzed the data, and authored the manuscript. All authors discussed the findings and contributed to the final text.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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دراسة قوة وخصائص الاهتزاز لنابض ورقي في ظل اختلاف (الشكل الاساسي المكون له وعدد صفائح مختلفة وعرض كل صفيحة)

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الخلاصة

تتناول هذه الدراسة تصميمًا هندسيًا مبتكرًا لنوابض السيارات الورقية، وذلك باستبدال القاعدة شبه المنحرفة التقليدية والانحناء الدائري بقاعدة دائرية ذات انحناء مكافئ. يهدف البحث بشكل أساسي إلى تقييم تأثير شكل القاعدة ونمط انحنائها على الأداء الساكن والديناميكي للنوابض الورقية في ظل ظروف تحميل واقعية. تمت دراسة أربع تركيبات هندسية مختلفة: قاعدة شبه منحرفة ذات انحناء دائري، وقاعدة دائرية ذات انحناء دائري، وقاعدة شبه منحرفة ذات انحناء مكافئ، وقاعدة دائرية ذات انحناء مكافئ. في كل تركيب، تم تقييم تأثير عدد الصفائح وتوزيع عرضها على الأداء الميكانيكي في حالتي العرض الثابت والمتغير. استُخدم تحليل العناصر المحدودة باستخدام برنامجي ANSYS Workbench 20 وMechanical APDL لتحديد أقصى إجهاد أولي، والتشوه الكلي، والتردد الطبيعي. أظهرت النتائج أن القاعدة الدائرية تُحسّن بشكل ملحوظ توزيع الحمل وتقلل من تركيز الإجهاد مقارنةً بالقاعدة شبه المنحرفة. على وجه التحديد، قلل تصميم القاعدة الدائرية من أقصى إجهاد انحناء ثابت بأكثر من 36% مقارنةً بالتصميم التقليدي. كما أدى اعتماد منحنى مكافئ إلى تحسين الأداء الديناميكي بشكل ملحوظ، حيث رفع التردد الطبيعي بنسبة تصل إلى 300% في بعض التكوينات. وقد تحسن الأداء الديناميكي بشكل كبير أيضًا باستخدام منحنى مكافئ، مما أدى في بعض التكوينات إلى زيادة التردد الطبيعي بنحو 300%.

الكلمات المفتاحية: طريقة العناصر المحدودة، نابض ورقي، شكل النابض الورقي، انحناء النابض الورقي، تحسين الشكل.