Non-Linear Analysis of Laminated Composite Plates under General Out-Of-Plane Loading

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ABTRACT

The theoretical analysis depends on the Classical Laminated Plate Theory (CLPT) that is based on the Von-Káráman Theory and Kirchhov Hypothesis in the deflection analysis during elastic limit as well as the Hooke's laws of calculation the stresses. New function for boundary condition is used to solve the forth degree of differential equations which depends on variety sources of advanced engineering mathematics. The behavior of composite laminated plates, symmetric and anti-symmetric of cross-ply angle, under out-of-plane loads (uniform distributed loads) with two different boundary conditions are investigated to obtain the central deflection for mid-plane by using the Ritz method. The computer programs is built using Mat lab(R2011a), to solve non-linearity effects on the central deflection values of rectangular cross-ply composite laminated plates, aspect ratio, stresses, orthotropic factor (E/G) and orientations of fiber. The non-linear analysis results of (4.74%) for SSSS of $0^0/90^0/90^o/0^0$cross-ply angle plate, (8.77%) for CCCC of $0^0/90^0$cross-ply angle plate and (10.83%) for CCCC of $0^0/90^0/90^o/0^0$cross-ply angle plate showed a good agreement with **Reddy, 1997** results**.**

Comparing between the analytical linear, non-linear and experimental results gave a big difference between linear and non-linear results, while, non-linear showed close results with experimental results.

Key words: non-linear analysis, large deflection, composite laminated plate, classical laminated plate theory and Ritz method**.**

التحليل اللاخطي في الصفائح المركبه لحمل الانحناءالعام

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الخلاصه

التحليل النظري يعتمد على نظرية الصفيحه المركبه الكلاسيكيه التي تستند على نظرية فون كارمن وفرضيات كيرشوف ضمن حدود المرونه وكذلك قوانين هوك في حساب الاجهادات.استخدمت دوال جديده على المحاور المتعامده لحل المعادلات التفاضليه من الدرجه الرابعه حسب الشروط الحديه بالاعتماد على مصادر متنوعه في الرياضيات الهندسيه المتقدمه.التحقق في سلوك الصفائح الطبقيه المركبه,المتناظره وغير المتناظره,تحت حمل الانحناء العام(الموزع بشكل منتظم)لعشرة شروط حديه للحصول على الازاحه المركزيه في المستوي الوسطي باستخدام طريقة ريتز تم استخدام برامج الحاسوب,ماتلاب, لتنفيذ حل الاثار اللاخطيه على قيم الازاحه المركزيه في اللالواح الطبيقيه المركبه المتعامده ,نسبة الطول الى العرض , الاجهادات ,عامل التغاير

المتعامد واتجاهات الالياف.نتائج التحليل اللاخطي الحاليه تختلف عن نتائج البحوث السابقه كما يلي(4.7%) للمثبت بشكل بسيط من الجهات الاربع اللوح ذو الطبقات الاربع المتعامده(صفردرجه/تسعين درجه/تسعين درجه/صفر درجه) , (8.77%) للمثبت بقوه من الجهات الاربع للوح ذو الطبقتين المتعامدتين(صفر درجه/تسعين درجه) و(10.83%) للمثبت بقوه من الجهات الاربع للوح ذو الطبقات الاربع المتعامده (صفردرجه/تسعين درجه/تسعين درجه/صفر درجه).

تم مقارنة نتائج التحليل الخطي واللاخطي والعملي ووجدنا الفارق الكبير بين نتائج التحليل الخطي من جانب وبين نتائج التحليل اللاخطي والعملي من جانب اخر والذي يدعم الاعتماد على نتائج التحليل اللاخطي في التصميم لانها الاقرب الى الواقع.

الكلمات الرئيسيه: التحليل اللاخطي, الازاحه الكبيره, اللالواح الطبقيه المركبه, نظرية اللالواح الطبقيه المركبه وطريقة ريتز.

1. INTRODUCTION

During the last decades, needs for composite materials which contain two or three types of materials mixed together homogenously have appeared.

Composite materials have many advantages such as high strength with low weight compared with traditional engineering materials; furthermore, their properties can be controlled during mixing of their components to meet the suitable design requirements**.** When a flat plate subjected to out-of-plane loads (uniform distributed loads), the real shape of displacement of this plate is nonlinear shape, **Reddy,1997.**

Huai, and Hui, 1990, based on the Von-Káráman theory of plates and they used double Fourier series method to solve the nonlinear bending problems of simply supported symmetric laminated cross-ply rectangular plates under combined action of pressure and in-plane load. The solution which investigated and satisfies the governing equations and boundary conditions is obtained. **Singh, et al., 1991,** investigated the large deflection bending analysis of anti-symmetric rectangular cross-ply plate based on Von-Káráman plate theory is investigated, with one-term approximation for the in-plane and transverse displacement, under sinusoidal loading. The presence of bending-stretching coupling in such plates resulted in an additional square nonlinear term which made the solution load direction dependent, unlike isotropic, orthotropic, symmetric, square anti-symmetric cross-ply and symmetric and anti-symmetric angle-ply plates. **Savithri, and Varadan, 1993,** worked on the non-linear bending analysis of simply supported symmetrically laminated orthotropic plates subjected to uniformly distributed load, using an accurate displacement based higher-order theory is presented. The non-linear governing equations were solved by the Galerkin procedure with the Newton- Raphson method. Numerical given here, based on analytical investigation, will be useful for comparison in future**. Tanriöver, and Senocak, 2004,** discussed a large defection analysis of laminated composite plates. Galerkin method along with Newton-Raphson method was applied to large deflection analysis of laminated composite plates with various edge conditions. The Von-Káráman plate theory was utilized and the governing differential equations were solved by choosing suitable polynomials as trial functions to approximate the plate displacement functions. The solution is compared to that of dynamic relaxation and finite elements. A very close agreement had been observed with these approximating methods. In the solution process, analytical computation has been done wherever it is possible, analytical-numerical type approach has been made for all problems. **Nayyar, 2006,** examined static and vibration analysis of laminated plates were conducted conventional and hierarchical finite element formulation based on First-order Shear Deformation Theory (FSDT). The efficiency and accuracy of the developed formulation is also established in comparison with approximate solutions based on Ritz-method which are also developed for the cases under study. A detailed parametric study has been conducted on various types of laminated plates, in order to investigate the effects of boundary conditions, laminate configuration, aspect ratio values and elastic modulus to shear modulus (E/G) ratio. **Shfrin, et al, 2008,** studied a semi-analytic approach for the geometrically non-linear analysis of rectangular laminated plates with general boundary conditions and out-of-plane loads had been developed. The solution of non-linear partial differential equations of Von-Káráman plate theory has been reduced to an iterative (sequential) solution of a set of non-linear ordinary differential equations using multi-term extend Kantorovich method. Various combinations of boundary and loading conditions that are beyond the applicability of other semi-analytical methods have been considered the convergence, accuracy, and applicability of the proposed approach have been demonstrated through the quantitative study of various cases of large deflection non-linear response of laminated plates. The semi-analytical method proposed in this paper for the large deflection analysis of laminated plates subjected to out-of-plane loading can be further extended for non-linear analysis of plates with in-homogenous or mixed boundary conditions. **Kim, et al.**, **2008,** performed the non- linear structural analysis of higher- aspect- ratio structures. For the high-aspect-ratio structures, it is important to understand geometric nonlinearity due to large deflections. To consider geometric non- linearity, finite element analysis based on large deflection beam theory were introduced. Comparing experimental data and the present nonlinear analysis results, the current results were proved to be very accurate for the static and dynamic behaviors for both isotropic and anisotropic beams. **Saffari, and Mansouri, 2011,** solved non-linear algebraic equations by an iterative method, the non-linear equations being linearized by evaluating the non-linear terms with the known solution from the preceding iteration. The Newton-Raphson method, which is based on the Taylor series expansion and uses the tangent stiffness matrix, had been extensively used to solve non-linear problems. A new Newton-Raphson algorithm was developed for analyses involving non-linear behavior. **Nishawala, 2011,** studied a thin plate or beam; if the deformation is on the order of the thickness and remain elastic, linear theory may not produce accurate results as it does not predict the in-plane movement of the member. Therefore, a geometrical nonlinearity, large deformation theory is required to account for the inconsistencies. The equation of motion for plates with free and clamped edges was derived using model analysis in conjunction with the expansion theorem. Theoretical results were compared with a finite element simulation for plates.

Concluding Remarks:

In present work the considered cases are as follows:-

(a) Wide coverage to cases of static load (uniform distributed load) which is made of materials E-fiber glass and Polyester, multi-layers of cross-ply angle and thickness (4mm) was observed.

(b) Using a new orthogonal shape functions and Ritz method to cover our boundary conditions which are not used in previous papers.

(c) Making a new device for bending test.

2. ANALYTICAL SOLUTION (CLASSICAL LAMINATED PLATE THEORY)

The classical laminated plate theory is an extension of the classical plate theory to composite laminates. In the classical laminated plate theory (CLPT), it is assumed that the Kirchhoff hypothesis holds:-

1. Straight lines perpendicular to the mid surface (i.e. transverse normally) before deformation remain straight after deformation.
2. The transverse normal do not experience elongation (i.e. they are inextensible)
3. The transverse normal rotates such that they remain perpendicular to the mid surface after deformation.

The first two assumptions imply that the transverse displacement is independent of the transverse (thickness) coordinate and the transverse normal strain ε_{ZZ} is zero. The third assumption results in zero transverse shear strain, γ_{xz}=0, γ_{yz} =0. **Reddy, 1997.**

2.1 Displacements: Reddy, 1997

$$u(x,y,z) = u_\circ(x,y) - z\frac{\partial w_\circ}{\partial x} \quad (1.a)$$

$$v(x,y,z) = v_\circ(x,y) - z\frac{\partial w_\circ}{\partial y} \quad (1.b)$$

$$(x,y,z) = w_\circ(x,y) \quad (1.c)$$

2.2 Strains:

The Von-Káráman strains and the associated plate theory is named the Von-Káráman plate theory: **Reddy, 1997.**

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \quad (2.a)$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \quad (2.b)$$

$$\Gamma_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y}\right) \quad (2.c)$$

$$\gamma_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \quad (2.d)$$

$$\gamma_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \quad (2.e)$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} \quad (2.f)$$

In view of assumption of the classical laminated plate theory, Eq. (2) becomes:

$$\varepsilon_{xx} = \frac{\partial u_\circ}{\partial x} + \frac{1}{2}\left(\frac{\partial w_\circ}{\partial x}\right)^2 - z\frac{\partial^2 w_\circ}{\partial x^2} \quad (3.a)$$

$$\varepsilon_{yy} = \frac{\partial v_\circ}{\partial y} + \frac{1}{2}\left(\frac{\partial w_\circ}{\partial y}\right)^2 - z\frac{\partial^2 w_\circ}{\partial y^2} \quad (3.b)$$

$$\Gamma_{xy} = \frac{1}{2}\left(\frac{\partial u_\circ}{\partial y} + \frac{\partial v_\circ}{\partial x} + \frac{\partial w_\circ}{\partial x}.\frac{\partial w_\circ}{\partial y}\right) - z\frac{\partial^2 w_\circ}{\partial x\,\partial y} \quad (3.c)$$

$$\gamma_{xz} = \frac{1}{2}\left(-\frac{\partial w_\circ}{\partial x} + \frac{\partial w_\circ}{\partial x}\right) = 0 \qquad (3.d)$$

$$\gamma_{yz} = \frac{1}{2}\left(-\frac{\partial w_\circ}{\partial y} + \frac{\partial w_\circ}{\partial y}\right) = 0 \qquad (3.e)$$

$$\varepsilon_{zz} = 0 \qquad (3.f)$$

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix}\frac{\partial u^\circ}{\partial x} + \frac{1}{2}\left(\frac{\partial w_\circ}{\partial x}\right)^2\\ \frac{\partial v_\circ}{\partial y} + \frac{1}{2}\left(\frac{\partial w_\circ}{\partial y}\right)^2\\ \frac{\partial u_\circ}{\partial y} + \frac{\partial v_\circ}{\partial x} + \frac{\partial w_\circ}{\partial x}\cdot\frac{\partial w_\circ}{\partial y}\end{bmatrix} - z\begin{bmatrix}\frac{\partial^2 w_\circ}{\partial x^2}\\ \frac{\partial^\circ w_\circ}{\partial y^2}\\ 2\frac{\partial^2 w_\circ}{\partial x\,\partial y}\end{bmatrix} \qquad (4)$$

2.3 Plane Stress –Reduced Constitutive Relations

A state of generalized plane stress with respect to the XY-plane is defined to be one in which:-

$$\sigma_{xx}=\sigma_{xx}(x,y),\sigma_{yy}=\sigma_{yy}(x,y),\ \sigma_{zz}=0,\varepsilon_{zz}=0 \qquad (5)$$

The strain-stress relations of an orthotropic body in plane stress state can be written as:

$$[\sigma_{ij}]=[Q_{ij}][\varepsilon_{ij}] \qquad (6)$$

Most laminates are typically thin and experience a plane state of stress. For a lamina in the XY-plane, the transverse stress components are σ_{zz} , τ_{xz} and τ_{yz} . Although these stress components are small in comparison to σ_{xx} , σ_{yy} andτ_{xy}, they can induce failures because fiber-reinforced composite laminates are weak in the transverse direction (because the strength providing fibers are in the XY-plane). For this reason, the transverse shear stress is not neglected in shear deformation theories. However, in most equivalent-single layer theories the transverse normal stress σ_{zz} is neglected. Then the constitutive equations must be modified to this account for this fact. **Reddy, 1997.**

The condition σ_{zz}=zero results in following static constitutive equations for k^{th} layer that is characterized as orthotropic lamina uniform distributed loads **Reddy,1997.**

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{bmatrix}^{(k)} = [Q_{ij}]^{(k)}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{bmatrix}^{(k)} \qquad (7)$$

Where i,j=1,2,6 ,k the number of layers and $[Q_{ij}]^{(k)}$ are the plane stress-reduced stiffness founded by **Reddy, 1997.**

$$Q_{11}=\frac{E_1}{1-\nu 12\nu 21},\ Q_{12}=\frac{\nu 12E_2}{1-\nu 12\nu 21}=\frac{\nu 21E_2}{1-\nu 12\nu 21},$$
$$Q_{22}=\frac{E_2}{1-\nu 12\nu 21},Q_{66}=G_{12} \qquad (8)$$

2.4 Equations of Motion

As noted earlier, the transverse strains (γ_{xz}, γ_{yz}, ε_{zz}) are identically zero in the Classical Plate Theory. Consequently, the transverse shear stresses (τ_{xz} , τ_{yz}) are zero for a laminate made of orthotropic layers if they are computed from the constitutive relations. The transverse normal stress σ_{zz} is not zero by the constitutive relation because of the Poisson effect. However, all three stress components do not enter the formulation because the virtual strain energy of these stresses is zero due to the fact that kinematically consistent virtual strains must be zero [see Eq.(9)]:

$$\delta\varepsilon_{xz} = \delta\varepsilon_{yz} = \delta\varepsilon_{zz} = 0 \qquad (9)$$

Whether the transverse stresses are accounted for or not in a theory, they are present in reality to keep the plane in equilibrium. In addition, these stress components may be specified on the boundary. Thus, the transverse stresses do not enter the virtual strain energy expression, but they must be accounted for in the boundary condition and equilibrium of forces.

Here, the governing equations are derived using the principle of virtual displacement. In the derivations, we account for static effects.

The static version of the energy is:-

$$\delta W = \int_0^{L_x}\int_0^{L_y}(U + V)dxdy = 0 \qquad (10)$$

Where the virtual strain energy δU (volume integral of δU_0), the virtual potential virtual δV and the virtual work done by applied forces are given by:-

$$U = \int_0^{L_x}\int_0^{L_y}\int_{-\frac{h}{2}}^{\frac{h}{2}}[\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \tau_{xy}\gamma_{xy})dxdydz$$

$$U = \int_0^{L_x}\int_0^{L_y}\{\int_{-\frac{h}{2}}^{\frac{h}{2}}[\sigma_{xx}(\varepsilon_{xx}{}^{(0)} + z\varepsilon_{xx}{}^{(1)}) + \sigma_{yy}\left(\varepsilon_{yy}{}^{(0)} + z\varepsilon_{yy}{}^{(1)}\right) + \tau_{xy}\left(\gamma_{xy}{}^{(0)} + z\gamma_{xy}{}^{(1)}\right))\}dxdydz$$

(11)

$$V = -\int_0^{L_x}\int_0^{L_y}[q_b(x,y)w\left(x,y,\frac{h}{2}\right) + q_t(x,y)w\left(x,y,\frac{h}{2}\right)]dxdy \qquad (12)$$

Where q_b is the distributed force at the bottom ($z=\frac{h}{2}$) of the laminate, q_t is the distributed force at the top ($z=-\frac{h}{2}$).

$$\begin{bmatrix} N_{XX} \\ N_{YY} \\ N_{XY} \end{bmatrix} = \int_{-h/2}^{h/2}\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} dz \;\; ; \;\; \begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{bmatrix} = \int_{-h/2}^{h/2}\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{bmatrix} zdz \qquad (13)$$

The quantities (N_{xx} , N_{yy} , N_{xy}) are called the in-plane force resultants, and (M_{xx} , M_{yy} , M_{xy}) are called transverse force resultants All stress resultants are measured per unit length (e.g., N_i and Q_i in N/m and M_i in N.m/m).

[A], [B] and [D] are the common laminate stiffness matrices of membrane stiffness, bending–membrane coupling stiffness and bending stiffness. For arbitrary laminates, these matrices are defined as **Reddy, 1997.**

$$[A] = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}; \ [B] = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}; \ [D] = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}$$

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}(1, z, z^2)dz \qquad (14)$$

$$i, j = 1, 2, 6.$$

2.5 Ritz Method (Energy Method)

Components of the compatible infinitesimal virtual displacements (u, v, w) must be piecewise continuous functions of x, y and z in the interior domain of the body. In addition, they should satisfy the geometrically boundary condition of the elastic system and be capable of representing all possible displacement patterns. If these admissible displacement functions are chosen properly, very good accuracy can be attained.

According to this theorem, as discussed above, of all displacements that satisfy the boundary conditions, those making the total potential energy of the structure a minimum are the sought deflections pertinent to the stable equilibrium conditions.

2.6 Boundary Conditions

In general, the definition of the boundary conditions are procedure to fix edges the elements, plates, as free(F), simply supported(S), clamped(C), built-in or hinged. The boundary conditions that are (clamped-clamped-clamped-clamped, for four edges case, CCCC) or mixing two or more than of these types (clamped-free-simply supported-free, CFSF). The way of fixing edges depend on the application of a structure, plate, dimensions of a structure, the load type and the load amount**. Robbert M. Jones, 1999.**

(a)Free-edges (F):

$M_{x|x=a}=M_{y|y=b}=0$; $V_{x|x=a}=V_{y|y=b}=0$

(b)Simply supported edges (S):

$w|_{x=a,y=b}=0$; $M_{x|x=a}=M_{y|y=b}=0$

(c)Clamped edges(C):

$w|_{x=a,y=b}=0$; $(\frac{\partial w}{\partial x})|(x=a) = (\frac{\partial w}{\partial x})|(y=b)=0.$

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2.7 Displacement Function

The behavior of a structure, plate, is different from fixing way to other. In present work, the orthogonal arbitrary displacement functions are used to find the exact solution of these cases as following:-

1. CCCC boundary condition,Szilard, 2004.

$u(x,y)=(1/4)\, A_{mn}\, (1-\cos(\alpha x))(1-\cos(\beta))$, $v(x,y)=(1/4)\, B_{mn}\, (1-\cos(\alpha x))\, (1-\cos(\beta))$,
$w(x,y)=(1/4) C_{mn}\, (1-\cos(\alpha x))\, (1-\cos(\beta))$

2. SSSS boundary condition, Szilard,2004.

$u(x,y)= A_{mn} \sin(\alpha x) \sin(\beta y)$, $v(x,y)= B_{mn} \sin(\alpha x) \sin(\beta y)$, $w(x,y)= C_{mn} \sin(\alpha x) \sin(\beta y)$

3. CCCF boundary condition[Present work]

$u(x,y)= (1/2)\, A_{mn}\, (1-\cos(\alpha x))(-1)^{\left(\frac{2n-1}{2}\right)} \left(e^{\beta y}-1\right)$,$v(x,y)= (1/2)\, B_{mn}\, (1-\cos(\alpha x))\, (-1)^{\left(\frac{2n-1}{2}\right)} \left(e^{\beta y}-1\right)$

$w(x,y)= (1/2)\, C_{mn}\, (1-\cos(\alpha x))(-1)^{\left(\frac{2n-1}{2}\right)} \left(e^{\beta y}-1\right)$

4. SSSF boundary condition (Present work):

$u(x,y)= A_{mn} \sin(\alpha x)(-1)^{\left(\frac{2n-1}{2}\right)} e^{\beta y}$, $v(x,y)= B_{mn} \sin(\alpha x)(-1)^{\left(\frac{2n-1}{2}\right)} e^{\beta y}$, $w(x,y)= C_{mn} \sin(\alpha x)(-1)^{\left(\frac{2n-1}{2}\right)} e^{\beta y}$

2.8 Analytical Solution

The solution of SSSS for E-fiber, Polyester (volume fraction 0.3) for cross-ply angle plate(0^0, $0^0/90^0$, $0^0/90^0/0^0$, $0^0/90^0/90^0/0^0$, $0^0/90^0/0^0/90^0$)and dimensions(0.2*0.2*.004m,0.2*0.1,.004m and 0.2*0.05*.004m).It depends on the last sections as follows:

The load q(x,y) can be expanded as Fourier series as:

$$p(x, y)= \sum_{i,j=1}^{m,n} \frac{16p_0}{\pi^2(2m-1)(2n-1)} \sin\left(\frac{(2i-1)\pi x}{a}\right) \sin\left(\frac{(2j-1)\pi y}{b}\right) \quad (15)$$

i, j=1, 2, 3…………, and m, n =3(mode shape).

In the case of uniform distributed load over the surface of plate: $q_{mn} = \frac{16q_\circ}{\pi^2 mn}$

The potential energy related with the uniformly distributed load q(x,y) is:

$$V = \frac{16q}{\pi^2 mn} \int_0^a \int_0^b \left\{ \begin{array}{c} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sin\frac{m\pi_x}{a} \sin\frac{n\pi_y}{b} * \\ \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} C_{mn} \sin\frac{m\pi_x}{a} \sin\frac{n\pi_y}{b} \end{array} \right\} dxdy$$

Strain energy: - $U = \frac{1}{2} \int_0^a \int_0^b \left(\sigma_{xx}\,\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \tau_{xy}\gamma_{xy}\right) d_x d_y$

For pure bending: -

$$U = \frac{1}{2}\int_0^a \int_0^b \left\{\left[B_{11}\frac{\partial u}{\partial x} + \left(\frac{1}{2}\frac{\partial w}{\partial x}\right)^2 + B_{12}\frac{\partial u}{\partial y} + \left(\frac{1}{2}\frac{\partial w}{\partial y}\right)^2 - D_{11}\frac{\partial^2 w}{\partial x^2} - D_{12}\frac{\partial^2 w}{\partial y^2}\right]\left[\frac{\partial u}{\partial x} + \left(\frac{1}{2}\frac{\partial w}{\partial x}\right)^2\right] + \left[B_{12}\frac{\partial u}{\partial x} + \left(\frac{1}{2}\frac{\partial w}{\partial x}\right)^2 + B_{22}\frac{\partial u}{\partial y} + \left(\frac{1}{2}\frac{\partial w}{\partial y}\right)^2 - D_{12}\frac{\partial^2 w}{\partial x^2} - D_{22}\frac{\partial^2 w}{\partial y^2}\right]\left[\frac{\partial u}{\partial y} + \left(\frac{1}{2}\frac{\partial w}{\partial y}\right)^2\right] + \left[B_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x\,\partial y}\right) - 2D_{66}\frac{\partial^2 w}{\partial x\,\partial y}\right]\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x\,\partial y}\right]\right\} dxdy = \frac{1}{2}\int_0^a \int_0^b \left\{B_{11}\left(\frac{\partial u}{\partial x}\right)^2 + B_{11}\frac{\partial u}{\partial x}\left(\frac{\partial w}{\partial x}\right)^2 \frac{B_{11}}{4}\left(\frac{\partial w}{\partial x}\right)^4 + B_{12}\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \frac{B_{12}}{2}\frac{\partial u}{\partial x}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{B_{12}}{2}\frac{\partial v}{\partial y}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{B_{12}}{4}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial w}{\partial y}\right)^2 - D_{11}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2} - \frac{D_{11}}{2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial x}\right)^2 - D_{12}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial y^2} - \frac{D_{11}}{2}\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial x}\right)^2 + B_{12}\frac{\partial u}{\partial x}.\frac{\partial v}{\partial y} + \frac{B_{12}}{2}\frac{\partial u}{\partial x}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{B_{12}}{2}\frac{\partial u}{\partial y}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{B_{11}}{4}\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial w}{\partial y}\right)^2 + B_{22}\left(\frac{\partial v}{\partial y}\right)^2 + B_{22}\frac{\partial v}{\partial y}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{B_{22}}{4}\left(\frac{\partial w}{\partial y}\right)^4 - D_{12}\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial x^2} - \frac{D_{12}}{2}\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial w}{\partial y}\right)^2 - D_{22}\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial y^2} - \frac{D_{22}}{2}\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial y}\right)^2 + B_{66}\left(\frac{\partial u}{\partial y}\right)^2 + B_{66}\left(\frac{\partial v}{\partial x}\right)^2 + 2B_{66}\frac{\partial u}{\partial y}.\frac{\partial v}{\partial x} + 2(B_{66} - D_{66})\frac{\partial u}{\partial y}\frac{\partial^2 w}{\partial x\,\partial y} + 2(B_{66} - D_{66})\frac{\partial v}{\partial x}\frac{\partial^2 w}{\partial x\,\partial y} + (B_{66} - D_{66})\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right\} dxdy$$

The displacement functions of SSSS Boundary condition is:

$$u(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\sin\frac{m\pi_x}{a}, v(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\sin\frac{m\pi_x}{a}\sin\frac{n\pi_y}{b}$$
$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sin\frac{m\pi_x}{a}\sin\frac{n\pi_y}{b} \qquad (16)$$

By using separation of variables technique as:

$$X_m(x) = S_{mn}\sin\alpha x\ , \alpha = \frac{m\pi}{a}, Y_n(y) = Z_{mn}\sin\beta x\ , \beta = \frac{n\pi}{b} \qquad (17)$$

Becomes:

$u(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}X_m(x)Y_n(y)$,v(x,y)= $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}X_m(x)Y_n(y)$
w(x,y)= $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}X_m(x), Y(y)$

The function Xm(x) and Yn(y) are chosen so as to satisfy the boundary conditions.

Then the strain energy equation for SSSS becomes:-

$$U = \left\{\frac{1}{2}\int_0^a \int_0^b \alpha^3 B_{11}(\cos\alpha x\sin\beta y)^2 A^2 m + \alpha^3 B_{11}(\cos\alpha x\sin\beta y)^3 A_m C^2 m + \frac{\alpha^4 B_{11}}{4}(\cos\alpha x\sin\beta y)^4 C^4 mn + \alpha\beta B_{12}(\cos\alpha x\sin\beta y)(\sin\alpha x\cos\beta y)A_m B_m + \frac{\alpha\beta^2 B_{12}}{2}(\cos\alpha x\sin\beta y)(\sin\alpha x\cos\beta y)^2 A_m C^2 m + \right.$$

$\frac{\alpha^2\beta B_{12}}{2}$ $(\sin\alpha x\cos\beta y)(\cos\alpha x\sin\beta y)^2 B_m C^2 m + \frac{\alpha^2\beta^2 B_{12}}{4}(\cos\alpha x\sin\beta y)^2(\sin\alpha x\cos\beta y)^2\ C^4 m +$
$\alpha^3 D_{11}(\sin\alpha x\cos\alpha x\sin^2\beta y)A_m C_m + \frac{\alpha^4 D_{11}}{2}(\sin\alpha x\cos^2\alpha x\sin^3\beta y)\ C^3 m +$
$-\alpha\beta^2 D_{12}(\sin\alpha x\cos\alpha x\sin^2\beta y)A_m C_m + \frac{\alpha^2\beta^2 D_{12}}{2}(\sin\alpha x\cos^2\alpha x\sin^3\beta y)\ C^3 m +$
$\alpha\beta B_{12}\ (\sin\alpha x\cos\alpha x\sin\beta y\cos\beta y)A_m B_m + \frac{\alpha\beta^2 B_{12}}{2}\ (\sin^2\alpha x\cos\alpha x\sin\beta y\cos^2\beta y)A_m C^2 m +$
$\frac{\alpha^2\beta B_{12}}{2}(\sin\alpha x\cos^2\alpha x\sin^2\beta x\cos\beta y)B_m C^2 m + \frac{\alpha^2\beta^2 B_{12}}{4}(\sin^2\alpha x\cos^2\alpha x\sin^2\beta y\cos\beta y)C^4 m +$
$\beta^2 B_{22}(\sin\alpha x\cos\beta y)^2 B^2 m + \beta^3 B_{22}(\sin\alpha x\cos\beta y)^3 B_m C^2 m + \frac{\beta^4 B_{22}}{4}\ (\sin\alpha x\cos\beta y)^4 C^4 m +$
$\alpha^2\beta D_{12}(\sin^2\alpha x\sin\beta y\cos\beta y)B_m C_m + \frac{\alpha^2\beta^2 D_{12}}{2}\ (\sin^3\alpha x\sin\beta y\cos^2\beta y)\ C^3 m +$
$\beta^3 D_{22}\ (\sin^2\alpha x\sin\beta y\cos\beta y)B_m C_m + \frac{\beta^4 D_{22}}{2}(\sin^3\alpha x\sin\beta y\cos^2\beta y)\ C^3 m +\ \beta^2 B_{66}(\sin\alpha x\cos\beta y)^2 A^2 m +$
$\alpha^2 B_{66}(\cos\alpha x\sin\beta y)^2 B^2 m + 2\alpha\beta B_{66}(\sin\alpha x\cos\alpha x\sin\beta y\cos\beta y)A_m B_m +$
$2\alpha\beta^2(B_{66} - D_{66})(\sin\alpha x\cos\alpha x\cos^2\beta y)A_m C_m\ + 2\alpha^2\beta(B_{66} - D_{66})(\cos^2\alpha x\sin\beta y\cos\beta y)B_m C_m +$
$\alpha^2\beta^2(B_{66} - D_{66})(\cos\alpha x\cos\beta y)^2 C^2 m\Big\}\ dxdy$

$U = \frac{1}{2}\int_0^a\int_0^b\{K_1 A^2 m + K_2 A_m C^2 m + K_3 C^4 m + K_4 A_m B_m + K_5 A_m C^2 m + K_6 B_m C^2 m + K_7 C^4 m + K_8 A_m C_m + K_9 C^3 m + K_{10} A_m C_m + K_{11} C^3 m + K_{12} A_m B_m + K_{13} A_m C^2 m + K_{14} B_m C^2 m + K_{15} C^4 m + K_{16} B^2 m + K_{17} B_m C^2 m + K_{18} C^4 m + K_{19} B_m C_m + K_{20} C^3 m + K_{21} B_m C_m + K_{22} C^3 m + K_{23} A^2 m + K_{24} B^2 m + K_{25} A_m B_m + K_{26} A_m C_m + K_{27} B_m C_m + K_{28} C^2 m\ \}d_x d_y$ (18)

K_i=constant as follow:

$K_1=\alpha^3 B_{11}(\cos\alpha x\sin\beta y)^2$, $K_2=\alpha^3 B_{11}(\cos\alpha x\sin\beta y)^3$, $K_3=\frac{\alpha^4 B_{11}}{4}\ (\cos\alpha x\sin\beta y)^4$,

$K_4= \alpha\beta B_{12}(\cos\alpha x\sin\beta y)(\sin\alpha x\cos\beta y)$, $K_5=\frac{\alpha\beta^2 B_{12}}{2}(\cos\alpha x\sin\beta y)(\sin\alpha x\cos\beta y)^2$,

$K_6=\frac{\alpha^2\beta B_{12}}{2}\ (\sin\alpha x\cos\beta y)(\cos\alpha x\sin\beta y)^2$, $K_7=\frac{\alpha^2\beta^2 B_{12}}{4}(\cos\alpha x\sin\beta y)^2(\sin\alpha x\cos\beta y)^2$,

$K_8=\alpha^3 D_{11}(\sin\alpha x\cos\alpha x\sin^2\beta y)$

By using the Ritz method, the coefficients Amn , Bmn and Cmn are determined boundary conditions can be obtained from :-

From: $\frac{\partial U}{\partial A_{mn}} = \frac{\partial V}{\partial A_{mn}}$

$\int_0^a\int_0^b\{2K_1 A_m + K_2 C^2 m + K_4 B_m + K_5 C^2 m + K_8 C_m + K_{10} C_m + K_{12} B_m + K_{13} C^2 m + \ + 2K_{23} A_m + K_{25} B_m + K_{26} C_m\ \}d_x\ d_y = 0$

$\int_0^a\int_0^b\{R_{11} A_m + R_{12} B_m + R_{13} C_m + R_{14} C^2 m\}\ d_x d_y = 0$ (19)

Where:

$R_{11}=2(K_1+K_{14}), R_{12}=K_4+K_{12}+K_{25}, R_{13}=K_8+K_{10}+K_{26}, R_{14}=K_2+K_5+K_{13}$

from: $\frac{\partial U}{\partial B_{mn}} = \frac{\partial V}{\partial B_{mn}}$

$\int_0^a \int_0^b \{ K_4A_m+K_6C^2m + K_{12}A_m + K_{14}C^2m + 2K_{16} B_m+ K_{17}C^2m +K_{19}C_m + K_{21}C_m +2K_{24}B_m+ K_{25}A_m +K_{27}C_m\}d_x\, d_y=0$

$$\int_0^a \int_0^b \{ R_{21}A_m+R_{22}B_m+R_{23}C_m+R_{24}C^2m \}d_x\, d_y=0 \quad (20)$$

Where:

$R_{21}=K_4+K_{12}+K_{25}, R_{22}=2(K_{16}+K_{12}), R_{23}=K_{19}+K_{21}+K_{27}, R_{24}=K_6+K_{17}+K_{17}$

And from $\frac{\partial U}{\partial C_{mn}} = \frac{\partial V}{\partial C_{mn}}$

$$\int_0^a \int_0^b \{ 2K_2A_mC_m+4K_3C^3m +2K_5A_m C_m+2K_6B_mC_m +4K_7C^3m + K_8A_m+3K_9C^2m + K_{10}A_m+3K_{11}C^2m +2K_{13}A_m C_m +2 K_{14}B_mC_m +4 K_{15}C^3m +2K_{17}B_mC_m +4K_{18}C^3m +K_{19}B_m +3 K_{20}C^2m + K_{21}B_m +3 K_{22}C^2m +K_{26}A_m + K_{27}B_m +2 K_{28}C_m \}d_xd_y=\int_0^a \int_0^b \{ F_{31}\}d_xd_y \quad (21)$$

Where:

$R_{31}=K_8+K_{10}+K_{26}, R_{32}=K_{19}+K_{21}+K_{27}, R_{33}=2K_{28}, R_{34}=3(K_9+K_{11}+K_{20}+K_{22}), R_{35}=4(K_3+K_7+K_{15}+K_{18}), R_{36}=2(K_2+K_5+K_{13}), R_{37}=3(K_9+K_{11}+K_{20}+K_{22}), F_{31}=\frac{16q}{\pi^2 mn}C_{mn} \sin^2\frac{m\pi_x}{a} \sin^2\frac{n\pi_y}{b}$

The set of equations (19), (20) and (21) are solved by using program Mat lab (R2011a)

2.9 Verification of Case Studies

For verifying the solution of present work, we compare our results with those obtained by other researchers as shown in **Tables (1, 2 and 3)** which give good agreement with non-linear results of, **Reddy, 1997.**

3. EXPERIMENTAL WORK

Experimentally, the steps of manufacturing specimens (plates) from E-fiber and Polyester are as follows:-

1. Taking volume fraction (0.3) from E-fiber glass and Polyester.

2. Connecting fiber coils on the edges of mold.

3. Adding the Polyester on fiber coils and move to release bubbles.

4. Loading weights on the plates to comprise to thickness (4mm).

Finally, the plates are left a while to dry and undergo to three tests: tensile test to find mechanical properties of plates, tensile test to find the Poisson's ratio and the bending test,50 specimens are manu factored to find the displacement of cross-ply angle plates undergoes to uniform distributed load(0-8750N/m^2) for 10-different boundary conditions.

3.1 Tensile Test

Each laminate was oriented in longitudinal, transverse, (45^0) angle relative to designated (0^0) direction and sample for pure polyester to determine the engineering parametersE_1,E_2, G_{12}. Every samples are divided according to dimensions, as set by ASTM Number (D3039/D03039M) as shown in **Fig. 1**.And the sample shape for present study before and after the tensile test. The specimen tensile test is mounted vertically in a servo-hydraulic testing machine, and pulled hydraulically with stroke control with large steel grips, maximum capacity of tensile machine (50KN) and it occurred in Ministry of scientific search as shown:

3.2 Bending Test

The displacement of composite plate under uniform distributed load which can be measured by bending device. It measures results for any dimension of plates (length, width and thickness) as shown in **Fig. 3**.

4. RESULT AND DISCUSSION

4.1 Analytical Results

The purpose of the study is to find a theoretical analysis of one of the famous engineering applications, as well as to increase precision in the analysis to achieve results closer to reality. By using the Ritz method in the analysis of nonlinear deformation in composite plates with dimensions (0.2*0.2*0.004m), multi cross-ply angle layers, various boundary conditions and variable of uniform distributed load from (0-8750N/m^2) to obtain the central deflection of the plate. The non-linear behavior of plates or laminated plates begins after ($w > 0.3h$) and the consequent application of theories mathematically complex need to be solved by engineering software such as Mat lab (R2011a). The results obtained from linear analysis, it can be observed that central deflection increases with increasing load with (10-100% linearly steps) as well as from layer to others. The results obtained from non-linear analytical analysis with different boundary conditions and three values of aspect ratios how that an increasing aspect ratio caused decreasing the central deflection with(18.5-27%) for aspect ratio(a/b)=2, and(40-53%) for aspect ratio(a/b)=4 with relative to aspect ratio(a/b)=1.

4.2 Experimental Results

The results obtained from non-linear experimental analysis of two different boundary conditions and three values of aspect ratio, it can be observed that same aspect ratios equal to one, two and four.

From the results shown in Tables (1,2 and 3), it can observed that the boundary conditions always effect on the central deflections while changing the lamination from symmetric to un-symmetric may increase the central deflection for aspect ratio as shown in following discussion.

4.3 Linear, Non-linear Analytical and Experimental Results

For the linear, non-linear analytical and experimental results the plates have a=b=0.2m, h=4mm,E_1 =22.049Gpa, E_2=4.163Gpa,G_{12}=1.365Gpa, ν_{12}=0.334, ν_{21}=0.063.The results for various techniques, experimental, linear and non-linear analytical results are shown in **Figs. (4.a, b and c).**

4.4. Influence Orthotropic Factor (E/G) on Deflection:

In the elasticity law's, the deflection related with modulus of elasticity, from the results we proved that analytical and experimental, as shown in **Figs. 5.a, b and c**, the deflection increased when modulus of elasticity decreased, therefore the reinforcement for any composite sections depended on the increased modulus of elasticity etc, Fiber, Steel coils or any stiffened material was in direction of maximum load. Two of types material, from, **Reddy, 1997,** with different mechanical properties to examine its impact on the results of central deflection for same loads and dimensions as shown in **Figs. 5 a, b and c.**

(a) Material-, Reddy, 1997:
a=b=0.2m,h=4mm,E_1=12.605Gpa,E_2=12.628Gpa,G_{12}=2.154Gpa, ν_{12}=0.2395, ν_{21}=0.239.

(b)Material-2 (present material):-a = b = 0.2m, h = 4mm, E_1 = 22.049Gpa, E_2=4.163Gpa ,G12=1.365Gpa, ν_{12}=0.334,ν_{21}=0063.

(c) Material-3, Reddy N.J, 1997:
a=b=0.2m, h=4mm, E_1=275.8Gpa, E_2=6.895Gpa, G_{12}=0.6E_2 ν_{12}=0.25, ν_{21}=6.25E-3.

4.5 Deflection with Boundary Conditions

In this section, the behavior of cross ply angle plate that exposed to bending distributed load of 2-boundary conditions is examined. The maximum value of deflection strongly connected with the boundary condition for the plates a = b = 0. 2 m, h = 4 mm, E_1 = 22.049 Gpa, E_2 = 4.163 Gpa, G_{12}=1.365Gpa, ν_{12}=0.334,ν_{21}=0.063 as shown in Fig.(3.a and b).

4.6 Influence of Aspect Ratio on the Deflection

Based on the experimental and theoretical results the effect of aspect ratio non-linearity limit($w > 0.3h$) and large deflection limit($w \geqq h$) for the ten cases of the boundary conditions are discussed and shown in **Figs. 7.a, b, c and d**.When the value of b is b ,b/2 or b/4 sameness the value of R is R ,R/2 or R/4 respectively. The difference of deflection was nonlinearly when changes the value of aspect ratio(R). So, its nonlinearly after changing the boundary conditions. This inquiring in all cases which are using in present work increasing or decreasing for plates, a=b=.2m,h=4mm,E1=22.049Gpa,E2=4.163Gpa,G12=1.365Gpa,ν_{12}=0.33,ν_{21}

4.7 Stress Analysis

To verify present work results, stresses values in X-and Y-axes (respectively σ_{xx} and σ_{yy}) are compared with those obtained by other researchers, **Reddy, 1997** are shown in **Fig. 8.a and b** for $0^o/90^o$ cross-ply angle plate which shows a good agreement.

5. CONCLUSION

The present analytical investigation is carried out to study non-linear analysis of large deflection of rectangular composite plate undergoes to uniform distributed loads for 10-boundary conditions. Using Classical Laminated Plate Theory and Ritz method were used to solve the forth degree of differential equation and used many shape functions which are changing with the change of boundary condition. A new shape function which depends on the behavior of plate subjects' uniform distributed load and boundary condition are used. Additionally, experimental program is developed to makes the composite plates from E-Fiber glass and Polyester of volume fraction (0.3). The following conclusions can be made:-

(a) Mechanical properties for E-Fiber glass and Polyester with volume fraction (0.3) were obtained. In addition, composite plates were manufactured and subjected to uniform distributed load to find the amount of large deflection.

(b) The elasticity modulus of composite plate (Fiber-Polyester) increased with increasing the Fiberglass coils. Conversely, if underestimated the proportion yarns to less than the value of the modulus of elasticity.

(c) The deflection depends on thickness, width, length of plate, number of layers and orientation of plate.

(d) Comparing between the analytical linear, non-linear and experimental results gave a big difference between linear and non-linear results, while, non-linear showed close results with experimental results.

REFERENCES

- Jones R. M., 1999, *Mechanics of Composite Materials*, Taylor & Francis, 2nd edition.

- Reddy J. N., 1997, *Mechanics of Laminated Composite Plates and Shells Theory and Analysis*, CRC PRESS, 2nd ed.

- Szilard R., 2004, *Theories and Applications of Plates Analysis-Classical Numerical and Engineering Methods*, John Wiley& Sons, Inc.

- Ugrural A .C., 1999, Stress *in Plates and Shells*, nbMcGraw-Hill, printed in Singapore.

- Huai L. R. and Hui H. L.,1990, *Non-linear Bending Simply Supported Symmetric Laminated Cross-ply Rectangular plates* ,Applied Mathematics and Mechanics, English 2ed.Vol.11, 9Sep.

- Singh G., Rao Gz.V., and Iyengar N.G.R., 1991, Some O*bservations on the Large Deflection Bending of Rectangular Anti-symmetric Cross-Ply Plates*, Composite Structures 18p.p.77-91.

- Savithri S., Varadan T. K., 1993, *Large Deflection Analysis of Laminated Composite Plates*, International Journal of Non-Linear Mechanics, Vol. 28, No.1 p.p.(1-12), Printed in Great Britain.

- Tanriöver H., and Senocak E., 2004, *Large Deflection Analysis of Un-symmetrically Laminated Composite Plates: Analytical-Numerical Type Approach*, International Journal of Non-Linear Mechanics 39 p.p.1385-1392.

- Nayyar K., 2006, *Static and Model Analysis of Laminated Composite Plates Using Heretical Finite Method*, Concordia University, Canada.

- Shfrin I., Rabinovitch O. and Eisenberger M., 2008, *A Semi-analytical Approach for Non-linear Large Deflection Analysis of Laminated Rectangular Plates Under General Out-Of-Plane Loading*, International Journal of Non-Linear Mechanics 43 p.p. 328-340.

- Kim K. S., Yoo S. J., Lim I. G. and Lee I., 2008, *Nonlinear Structural Analysis of High-Aspect-Ratio Structures Using Large Deflection Beam Theory*, KASA International journal,Vol.2, p.p. 41-47.

- Saffari H., and Mansouri I., 2011, *Non-linear Analysis of Structures Using Two-Point Method*, International Journal of Non-Linear Mechanics 46, p.p.834-840.

- Nishawala V. V., 2011, A *Study of Large Deflection of Beam and Plates*, New Brunswick, New Jersey.

NOMENCLATURE

a =length of plate, X-axis, meter.

A_{ij}=coefficients of stretching stiffness matrix of composite plate.

b=width of plate, Y-axis meter.

B_{ij}=coefficients of bending- stretching coupling matrix of composite plate.

C =clamped edge.

d =derivative of x, y or z, dimensionless.

D_{ij} =coefficients of bending stiffness matrix of composite plate.

$\hat{e}_x$, $\hat{e}_y$, $\hat{e}_z$=unit vectors in X-,Y- and Z-axes, dimmensionless.

E_1 (E_x or E_0) =modulus of Elasticity in X- xis, GPa.

E_2 (E_y or E_{90})=modulus of Elasticity in Y- Axis, GPa.

E_{12} (E_{xy} or E_{45})=modulus of Elasticity in XY-axis, GPa.

F=free edge.

G_{12}=shear modulus in XY-axis, GPa.

H=thicknesses of plate, meter.

$h_1, h_2, \ldots, h_n, h_k, h_{k+1}$=thicknesses of layers, meter.

Lx, Ly=length of plate in X-, Y- axes, meter.

m, n=the number of modes in X-, Y-axes, dimensionless.

M_{xx}, M_{yy},M_{xy}=moment in X-,Y-,XY-axis per length, N.m/m.

N_{xx}, N_{yy}, N_{xy}=force in X-,Y-,XY-axis per length, N/m.

q(x, y)=function of uniform distributed load, pressure, N/m^2.

Q_{xx}, Q_{yy}, Q_{xy}=shear force in X-,Y-,XY- axis per length, N/m.

R =aspect ratio (a/b), dimensionless.

$u_\circ$,$v_\circ$,$w_\circ$=mid-plane displacement in X-,Y-,Z- axis, meter.

u(x, y),v(x, y),w(x, y)=function of displacement in X-, Y-,Z-axis.

U=strain energy ,Joule.

V=potential energy, Joule.

W=work done, Joule.

$\propto$= m*π/a , m^{-1}.

β= n*π/b, m^{-1}.

π= constant Ratio (22/7), dimensionless. ν = poisson's ratio, dim. less.

σ_{xx}, σ_{yy}, τ_{xy}=stress in X-,Y-, XY- axis , MPa.

ε_{xx}= strain in X- axis , dimensionless.

ε_{yy}= strain in Y- axis, dimensionless.

γ_{xy} =shear strain in XY- axis, dimensionless.

ε_x^{NL}=non-linear strain in X-axis (dim.less).

ε_{21}= transverse strain when the lamination angle0^0, dimensionless

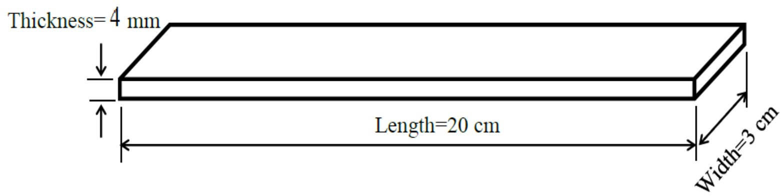

Figure 1. Shape and dimension of tensile test samples.

Figure 2. Tensile test machine.

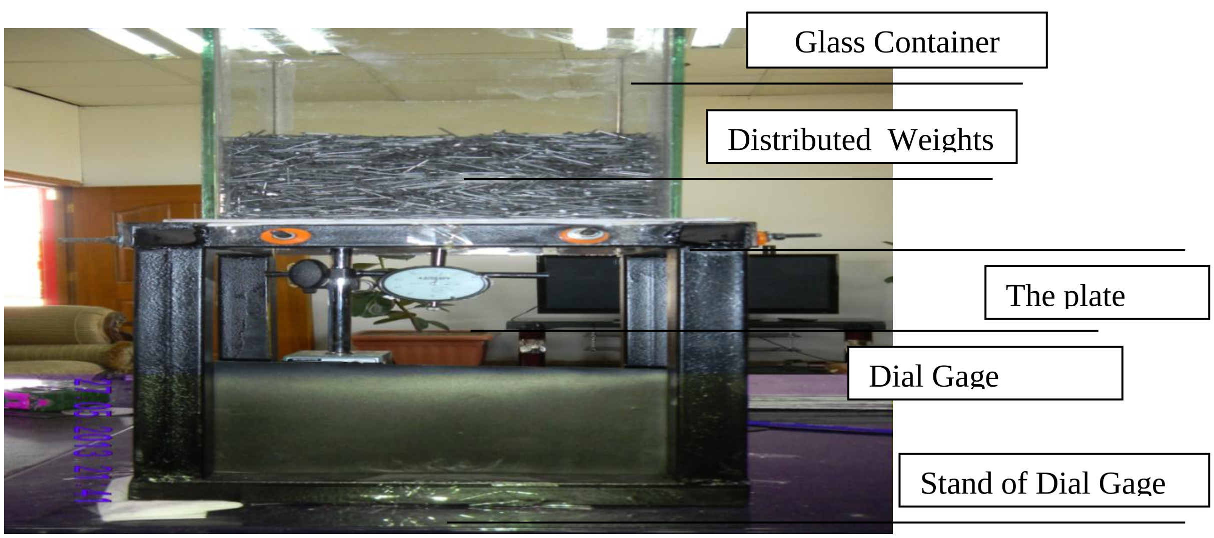

Figure 3. The mechanism of bending test**.**

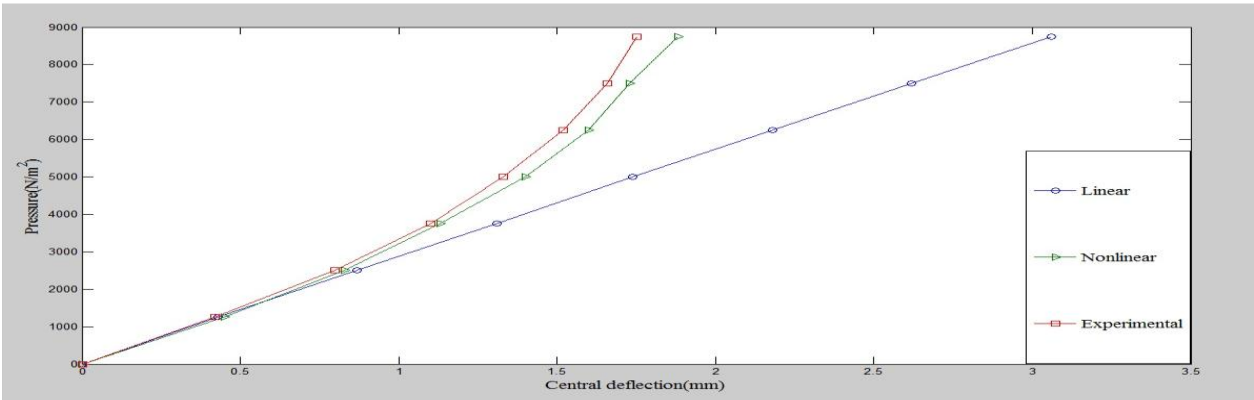

Figure 4.a. Comparison of central deflection for $0^0/90^0$ angle of CCCC B.Cs.

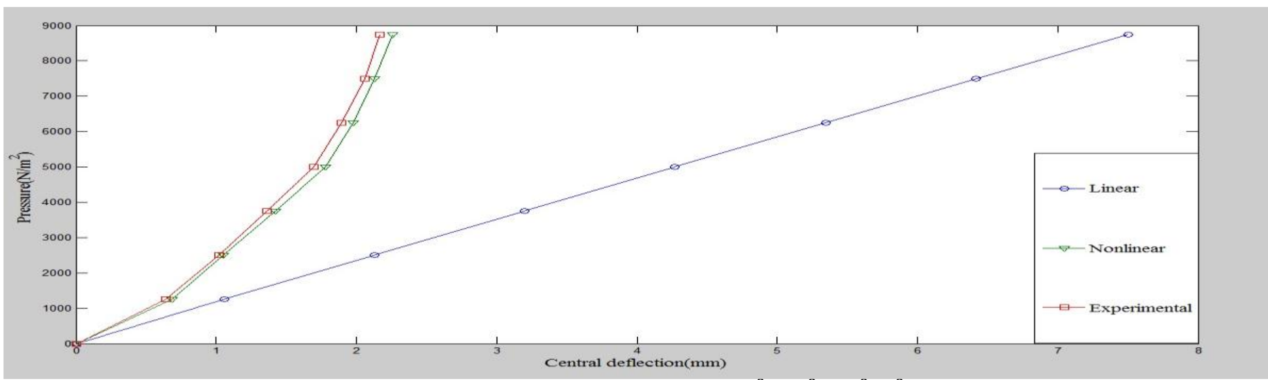

Figure 4.b. Comparison of central deflection for $0^0/90^0/90^0/0^0$ angle of CCCC B.C.

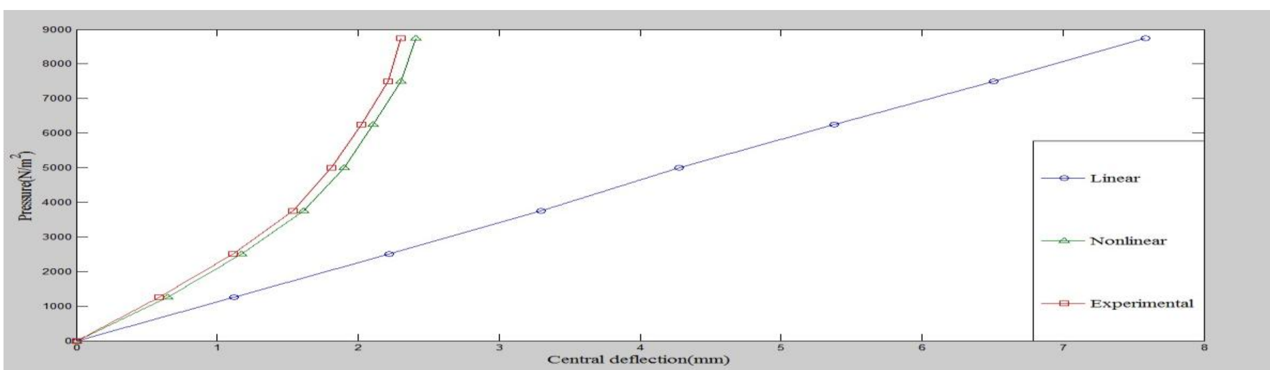

Figure 4.c. Comparison of central deflection for $0^0/90^0/0^0/90^0$ angle of CCCC B.Cs.

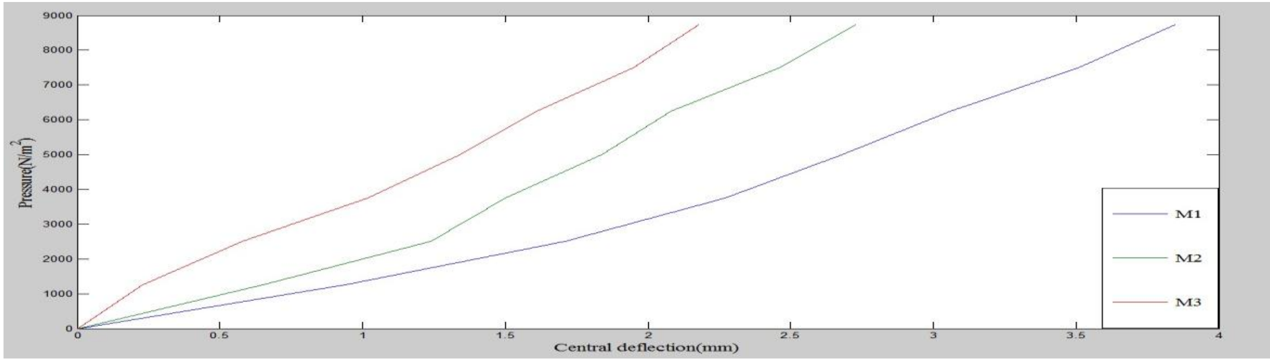

Figure 5.a. Comparison of central deflections of three different materials for $0^0/90^0/90^0/0^0$cross-ply angle plates of SSSS B.Cs.

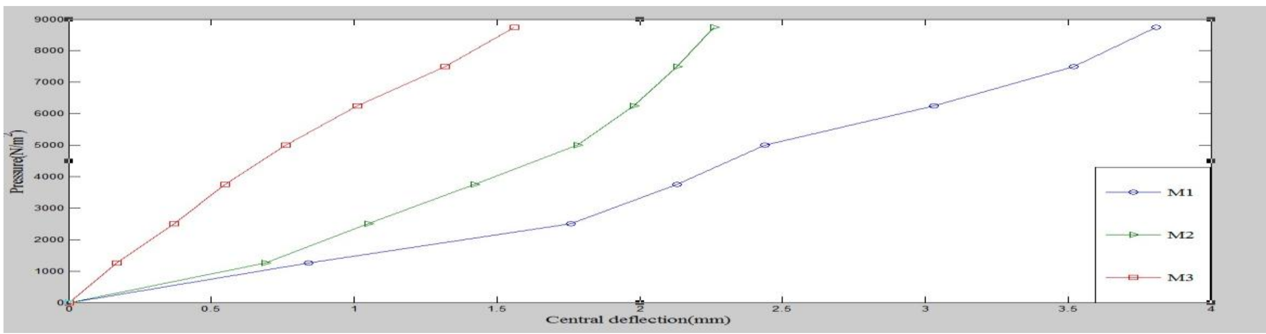

Figure 5.b. Comparison of central deflections of three different materials for $0^0/90^0/90^0/0^0$cross-ply angle plates of CCCC BCs.

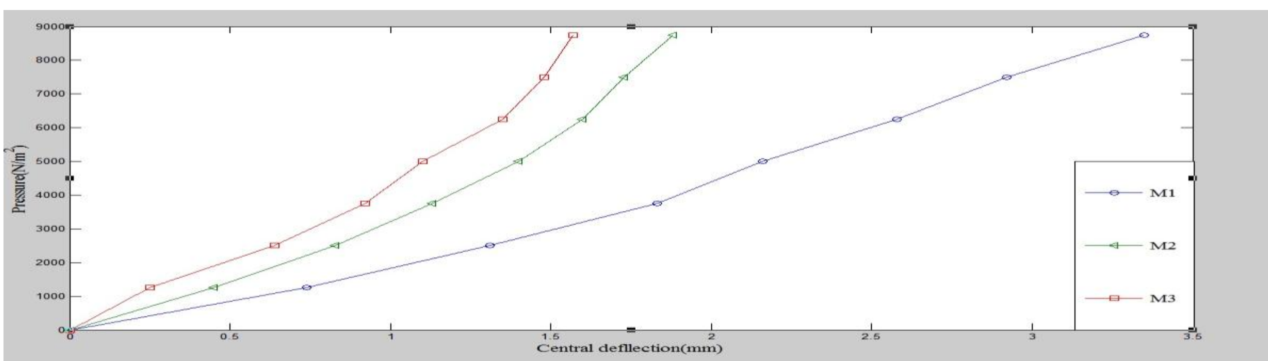

Figure 5.c. Comparison of central deflections of three different materials for $0^0/90^0$cross-ply angle plates of CCCC B.Cs.

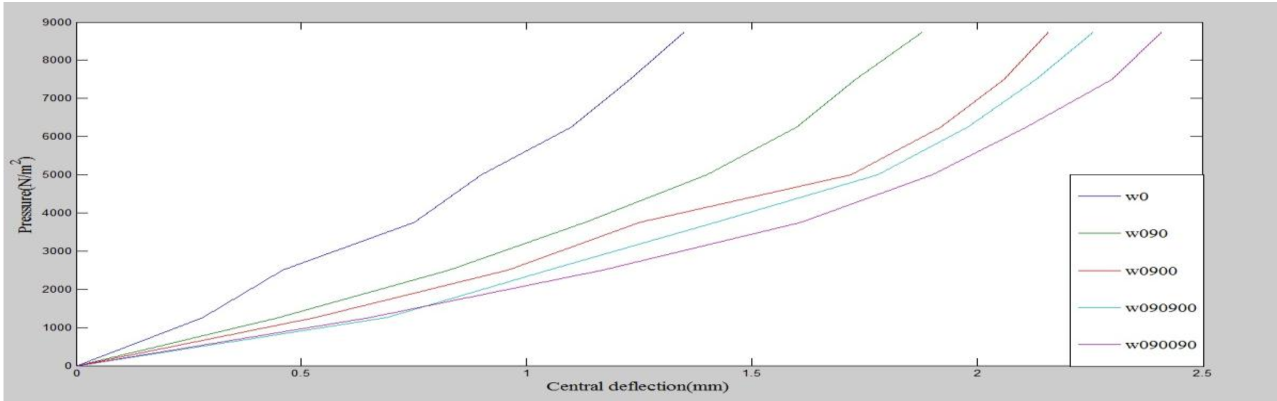

Figure 6.a. Analytical central deflection of 5-cross-ply angle plates of CCCC B.Cs.

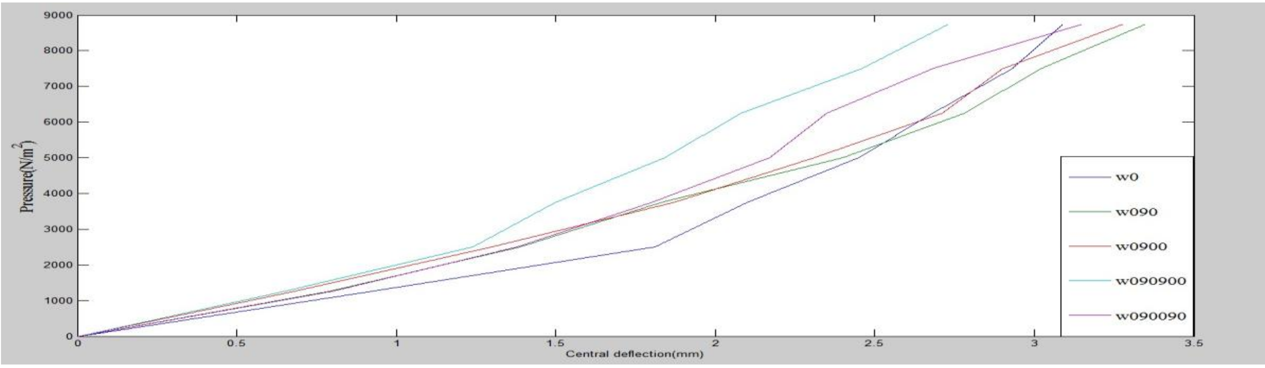

Figure 6.b. Analytical central deflection of 5-cross-ply angle plates SSSS B.Cs.

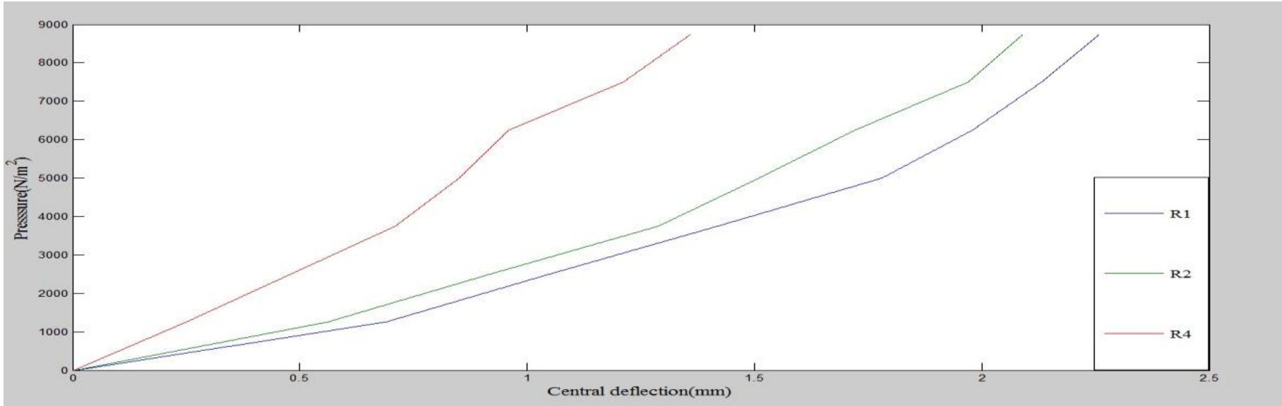

Figure 7.a Comparison the effect of aspect ratio on the central deflection of $0^0/90^0/90^0/0^0$ for CCCC B.Cs.

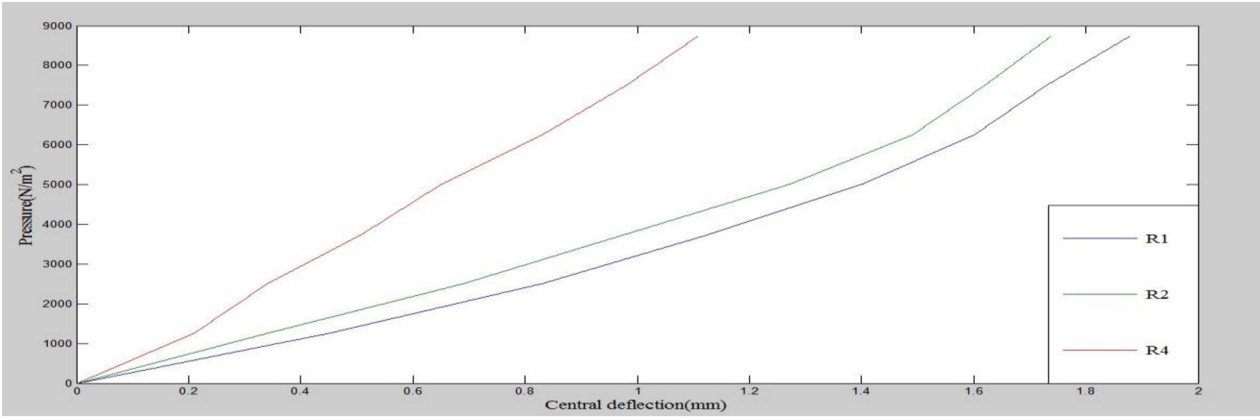

Figure 7.b. Comparison the effect of aspect ratio on the central deflection of $0^0/90^0$ for CCCC B.Cs.

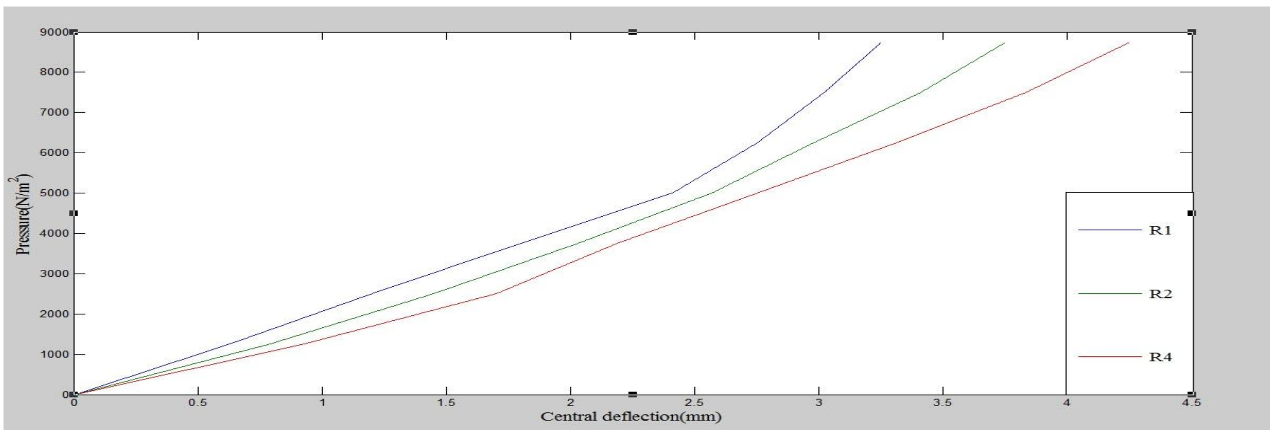

Figure 7.c. Comparison the effect of aspect ratio on the central deflection of $0^0/90^0/90^0/0^0$ for SSSS B.C.s

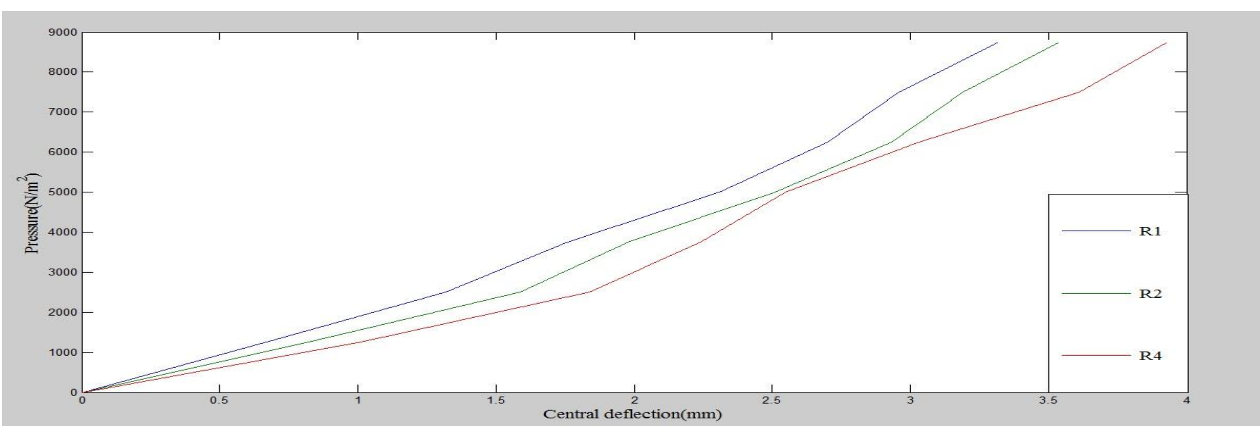

Figure 7.d. Comparison the effect of aspect ratio on the central deflection of $0^0/90^0$ for SSSS B.Cs.

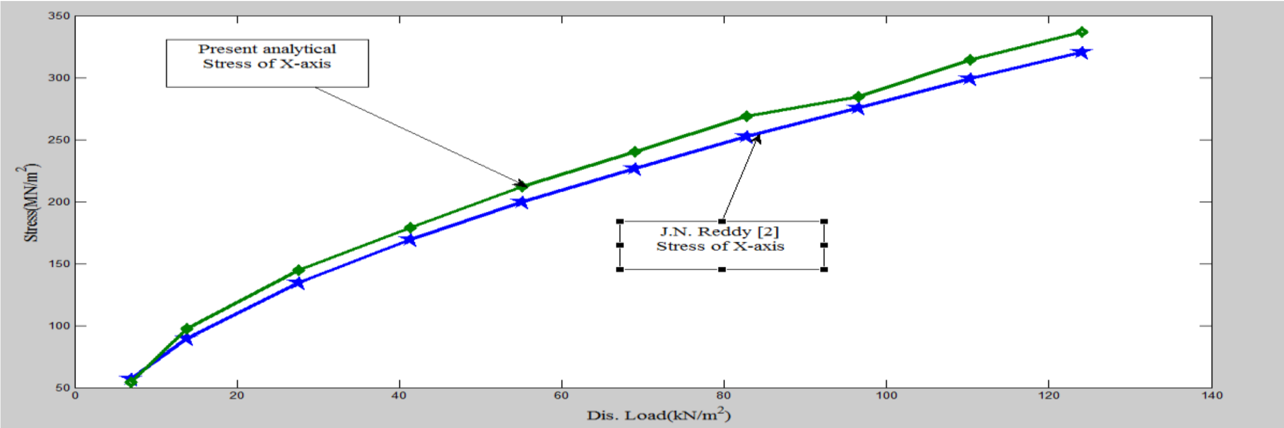

Figure 8.a. Stress analysis of X-axis between Reddy, 1977, and present analytical results for $0^o/90^o$ of CCCC B.Cs.

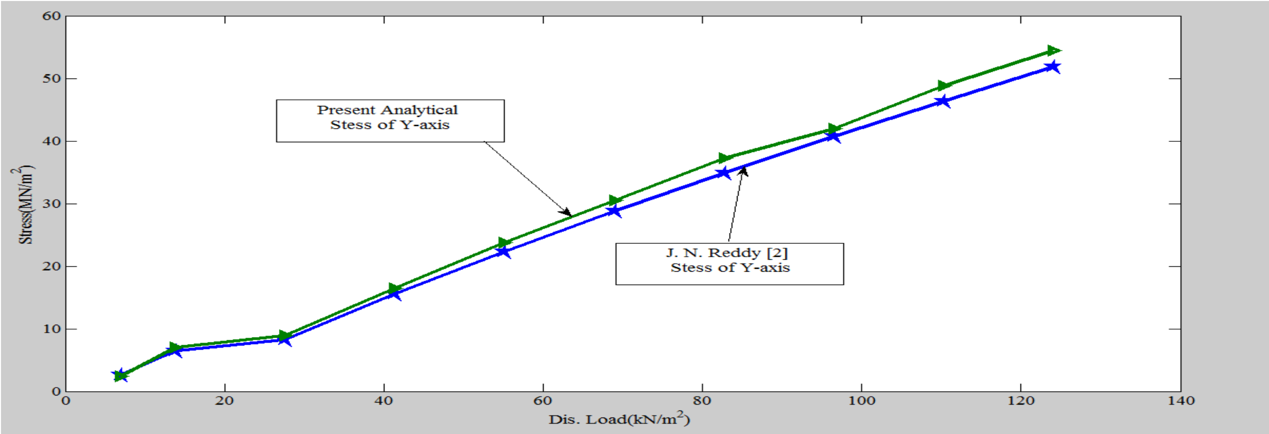

Figure 8.b. Stress analysis of Y-axis between Reddy, 1977, and present analytical results for $0^o/90^o$ of CCCC B.Cs.

Table 1. First verification of central deflection between Reddy, 1977, results and present analytical results of CCCC, $0^0/90^0/90^o/0^0$cross-ply angle plate.

a=b=0.3048m, h=2.4384mm, E1=12.605Gpa, E2=12.628Gpa, G12=2.154Gpa, ν_{12}=0.2395, ν_{21}=0.239.			
Pressure (N/m^2)	Reddy, 1997 results (mm)	Present analytical results (mm)	% Difference
1379	0.96	1.00	4
2758	1.65	1.8	8.33
4137	2.16	2.4	10.83
5516	2.55	2.8	8.93
6895	2.87	3	4.33
8274	3.15	3.4	7.35
9653	3.38	3.7	8.65
11032	3.61	4	9.75
12411	3.81	4.2	9.28
13790	4.00	4.40	9.09

Table 2. Second verification of central deflection between Reddy, 1997, results and present analytical results of SSSS, $0^0/90^0/90^o/0^0$cross-ply angle plate.

a=b=0.3048m, h=7.62mm, E1=275.8Gpa, E2=6.895Gpa, G12=0.6E2, ν_{12}=0.25, ν_{21}=6.25E-3.			
Pressure (N/m^2)	Reddy, 1997 results (mm)	Present analytical results (mm)	% Difference
6895	3.91	3.70	5.37
13790	5.70	6.20	8.77
27580	7.80	8.40	7.69
41370	9.17	9.70	5.78
55160	10.24	10.90	6.44
68950	11.14	11.80	5.92
82740	11.91	12.70	6.63
96530	12.6	13.00	3.17
110320	13.21	13.90	5.22
124110	13.80	14.50	5.07

Table 3. Third verification central deflection between Reddy, 1997,results and present analytical results CCCC,0°/90°cross-ply angle.

a=b=0.3048m, h=2.4384mm, E1=12.605Gpa, E2=12.628Gpa, G12=2.154Gpa, ν_{12}=0.2395, ν_{21}=0.239.			
Pressure (N/m^2)	Reddy, 1997, results (mm)	Present analytical results (mm)	% Difference
1379	1.96	1.98	1.02
2758	2.53	2.65	4.74
4137	2.97	3.07	3.36
5516	3.31	3.42	3.32
6895	3.58	3.70	3.35
8274	3.83	3.95	3.13
9653	4.04	4.18	3.46
11032	4.23	4.38	3.55
12411	4.41	4.52	2.50
13790	4.57	4.68	2.00