Mathematical Modeling of Compaction Curve Using Normal Distribution Functions

Abdul Kareem Esmat Zainal
Assistant Professor
Engineering college- Baghdad University
Email:kareem_esmat@yahoo.com

ABSTRACT

Compaction curves are widely used in civil engineering especially for road constructions, embankments, etc. Obtaining the precise amount of Optimum Moisture Content (OMC) that gives the Maximum Dry Unit weight γ_{dmax}. is very important, where the desired soil strength can be achieved in addition to economic aspects.

In this paper, three peak functions were used to obtain the OMC and γ_{dmax}. through curve fitting for the values obtained from Standard Proctor Test. Another surface fitting was also used to model the Ohio's compaction curves that represent the very large variation of compacted soil types.

The results showed very good correlation between the values obtained from some published sample tests and the values obtained from curve fitting for both cases of the single curve and multiple Ohio's curves.

The easiness of obtaining OMC and γ_{dmax}. From the results of curve fitting encourage users to utilize this procedure, in addition to its accuracy.

Keywords: Compaction Curve, Optimum Moisture Content, Maximum dry density, Procter Test, Peak functions.

التمثيل الرياضي لمنحنيات الرص باستخدام دوال التوزيع الطبيعي

عبد الكريم عصمت زينل
استاذ مساعد
كلية الهندسة – جامعة بغداد

الخلاصة

تستعمل منحنيات الرص بشكل كبير في تخصص الهندسة المدنية وخاصة في عمليات انشاء الطرق، التعليات الترابية، الخ. وتعتبر عملية ايجاد محتوى الرطوبة الامثل ووحدة الوزن الجافة القصوى بشكل دقيق من الاهمية بمكان منحيث تحمل التربة والاعتبارات الاقتصادية.

في هذا البحث تم استخدام ثلاثة دوال من النوع الذي يمثل التوزيع الطبيعي للحصول على تمثيل رياضي باستخدام المنحنيات المناسبة وتطبيقها على البيانات المستحصلة من تجربة بروكتر Proctor Test. كما تم استخدام نموذج السطح المناسب Surface fitting لمجموعة منحنيات Ohio Curves التي تمثل مدى كبير من انواع الترب الطبيعية وتمثيلها على شكل معادلة رياضية تمثل سطح Surface.

اظهرت نتائج التمثل الرياضي وجود تطابق كبير جدا بين النتائج المستحصلة من المعادلات المقترحة بعد مقارنتها بالنتائج المذكورة في المصادر لجميع الحالات التي تمت دراستها.

تعتبر مسألة سهولة الحصول على نتائج دقيقة من خلال تطبيق المعادلات مشجعة جدا كطريقة استعمال بديلة عن الطرق الاخرى (اليدوية احيانا) اضافة الى دقة النتائج المستحصلة من خلال التطبيق.

1. INTRODUCTION

Compaction is the application of mechanical energy to a soil to rearrange the particles and reduce the void ratio. The principal reason for compacting soil is to reduce subsequent settlement under working loads; Compaction increases the shear strength of the soil and reduces the voids ratio making it more difficult for water to flow through soil. This is important if the soil is being used to retain water such as would be required for an earth dam.

The compaction is affected by the water content of the soil, the type of soil being compacted, and the amount of compactive energy used, **Das, 2014.**

Soil compaction is widely used in geoengineering and is important for the construction of roads, dams, landfills, airfields, foundations, hydraulic barriers, and ground improvements. Compaction is applied to the soil, with the purpose of finding optimum water content to maximize its dry density, and therefore, to decrease soil's compressibility, increase its shearing strength, and in some cases, to reduce its permeability.

A typical compaction curve presents different densification stages when the soil is compacted with the same apparent energy input but different water contents. The water content at the peak of the curve is called optimum moisture content (OMC) and represents the water content in which dry density is maximized for a given compaction energy.

Precise calculations to obtain the OMC are reflected on:

i) The cost of water and its transportation and distribution (e.g. road construction).

ii) The strength of the soil due to obtaining the maximum dry unit weight.

Even small amounts of differences may increase the cost of compaction due to the large amount of earthwork.

2. LITERATURE REVIEW

Kurucuk, et al., 2007, and **Kurucuk, et al., 2008,** work was on implementing theoretical prediction of the compaction curve for sand using unsaturated soil mechanics principles.

It highlights the fact that shape of the compaction curve can be predicted using unsaturated soil mechanics principles. The main insight gained was that the changes in matric suction are not important for the evolution of the compaction states, but the influence of matric suction on the material compressibility with respect to net stress is the governing factor determining the compaction density. Therefore, it can be reasoned that the inverted parabolic shape of the compaction curves is a direct function of the variation of the material compressibility with a degree of saturation.

Altun, 2008, mentioned the importance of obtaining precise values for, in the field, compaction control is commonly carried out by sand-cone and nuclear gauge tests. Whether conducted in the field or in the laboratory, these tests are intended to determine optimum water content and dry unit weight parameters, information required for design specifications.

The parameters of field soil densification obtained by various testing methods performed in the same region are compared: unit weight, water content, and densification percentage are measured by nuclear density and sand cone tests. The variations in the outcomes of nuclear density and sand cone tests, namely unit weight, water content, and densification percent, are recorded.

Horpibulsuk, et al., 2009, showed compaction curves from 16 coarse and 9 fine-grained soils, which cover all soil types classified by the Unified Soil Classification System are analyzed to develop the Modified Ohio's curves. For all soils, the relationships between water content and degree of saturation on both the dry and the wet sides of optimum are represented by power functions. Their compaction curves under standard Proctor energy follow the Ohio's curves. The optimum degree of saturation, *ODS*, of coarse-grained soils is lower than that of fine-grained soils.

Prakash, et al., 2015, studied the water content variations along the compaction curve, where the degree of saturation of compacted soils also varies. In their experimental work, the study of the variation of degree saturation along the compaction curve for soils with widely varying clay mineralogical composition subjected to both Indian Standard light and heavy compaction efforts. It is observed that the variation of the degree of saturation with molding water content adopted for both light and heavy compaction tests is linear up to OMC. The degree of saturation of kaolinitic soils has been observed to be less than that of montmorillonitic soils on the dry side of optimum; whereas on the wet side of the optimum, the degree of saturation of kaolinitic soils at any molding water content can be more than that of montmorillonitic soils. In addition, the degree of saturation of compacted soils at optimum condition has been observed to be a function of the soil clay mineralogy.

Shrivastava, et al., 2016, mentioned that a need is felt to obtain the required compaction parameters from the basic soil test which are used for the classification of soil namely Atterberg's limits, gradation, specific gravity etc.

Basic soil parameters were collected from literature and Artificial Neural Network (ANN) techniques have been employed on the data collected, as ANN can better model the relation between compaction parameters and basic soil properties than statistical modeling.

They demonstrate application of five different ANN algorithms like LM (Levenberg-Marquardt), GDM (Gradient descent with momentum weight and bias learning function), SCG (Scaled Conjugate Gradient), and CFB (Circulating Fluidized bed) to predict standard compaction characteristics of varieties of soils with a large range variation in their basic soil properties. Multiple variable non-linear regression analysis was also carried out, in which establishment of an empirical relationship for prediction of compaction characteristics of Modified compaction.

Li, 2013, research was to improve the Compaction Forecasting Expert Database (CFED) by linking moisture-density-compaction energy relationships with shear strength and stiffness properties to predict and evaluate the compaction performance of geo-materials.

3. PROPOSED EQUATIONS

The compaction curve used to obtain the optimum moisture content (OMC) and Maximum dry density (or unit weight) after Proctor method ASTM D698. It is widely used in experimental tests in laboratories.

The shape of the compaction curve is usually a bell-shaped and the functions that may describe this kind of shape are peak functions or in a more precise words (Single Peak functions) through the compaction curve sometimes has multi-peaks and may be described by a multi-peak functions, but this work is devoted only to the single peak compaction curve.

3.1 First Approach

It is required to describe the compaction curve by a mathematical equation, which may be useful to obtain the (OMC) and the γdry_{max} easily and in a more precise way. Three functions are presented in this work and verified against some published compaction curves.

Three different compaction curves were taken as examples. **Fig. 1, 2, and 3** show these compaction curves as presented by **Das,** and **Sobhan, 2014**, **Budhu, 2011**, and **Fredlund, 2004**, respectively. The compaction curves were digitized and the values of the dry density or the dry unit weight against water content were obtained as shown in **Table 1**.

Three functions were proposed as single peak functions that can be implemented as a mathematical model to describe the compaction curves, which are:

1- GaussAmp Function, Eq. (1)

$$y = y_o + A \times e^{-\frac{(x-x_c)^2}{2w^2}} \tag{1}$$

The OMC and γ_{dmax} can be easily obtained where:
OMC = x_c, γ_{dmax} = y_o + A. These values are the constants that can be obtained from any Curve fitting computer program, (e.g. MatLab, LabFit, Origin, etc.) to mention but a few.

2- Log Gaussian function, Eq. (2)

$$y = A \times e^{-\frac{[Ln(x)-B]^2}{C}} + D \tag{2}$$

where OMC = e^B, γ_{dmax} = A + D

3- Inverse Poly Function, Eq. (3)

$$y = y_o + \frac{A}{1+A_1\left(2\frac{x-x_c}{w}\right)^2+A_2\left(2\frac{x-x_c}{w}\right)^4+A_3\left(2\frac{x-x_c}{w}\right)^6} \tag{3}$$

where OMC = x_c, γ_{dmax} = y_0 + A

Through these functions, the compaction curve may be described mathematically and the (OMC) and $\gamma dry_{max.}$ may be obtained more correctly and accurately.
Plate 1 shows the results of the application of Eq. (1) for the three compaction curves with the result parameters that represents the values of the (OMC) and $\gamma dry_{max.}$, in addition to the correlation factor R^2.
The fitted curve for each compaction curve is also shown with the original data also displayed. The calculated values of (OMC) and $\gamma dry_{max.}$ is shown against the values obtained from the references to compare with.
Plate 2 and **3** show the results of the application of Eq. (2) and Eq. (3) respectively and also shows the curve fitting parameters for each of the compaction curves already mentioned.

3.2 Second Approach

Ohio's compaction curves describe a variety of test results for most of the soils (about 10,000 compaction tests on different types of soils), **Joslin, 1959**, as cited by **State of Ohio Department of Transportation, 2010**, the results of these tests are shown in **Fig. 4.**

Horpibulsuk, et al., 2013, also showed the compaction curves of the fine-grained soils, and the lateritic soils and crushed rocks. They mentioned that all the fine-grained soils (under standard Proctor energy) follow the Ohio's compaction curves, **Joslin, 1959**. All test data were collected from the Bureau of Rural Road 6, the Department of Rural Roads, and Thailand. The results are shown in **Fig. 5** for the fine-grained soils only just as an example.

In this approach, a 3-D mathematical surface model is found to describe these curves that make it easy to determine the values of the wet density **Fig. 4** or dry unit weight, **Fig. 5** by giving the desired water content and the required curve (which is described by a number), the numbers follow a sequence as (A=1, B=2, C=3, … etc. for **Fig. 4**), and (number 1 for the most upper curve in **Fig. 5**, the curve number increases as we go down to lower curves respectively).

The curves for each figure were digitized, then three of the most suitable surface equations were applied that relates the wet density to the curve number and the water content **Fig 4** and the dry unit weight to the curve number and the water content **Fig. 5.** They were found to be of the form:

1- Poly surface fit of 2nd degree

$$z = a0 + a1x + a2y + a3x^2 + a4xy + a5y^2 \quad (4)$$

2- Mathematical equation

$$y = \frac{(A + x1)}{(B + Cx2)} + DLn(x2) \quad (5)$$

3- Rational2D

$$z = \frac{z0 + A01x + B01y + B02y^2 + B03y^3}{1 + A1x + A2x^2 + A3x^3 + B1y + B2y^2} \quad (6)$$

where a0, a1, a2, a3, a4, a5, A, B, C, D, z0, A01, B01, B02, B03, A1, A2, A3, B1, and B2 are constants

3.2.1 Ohio's compaction curve

Data of curve in figure 4 were subjected to the three curve fitting equations and the results were as shown:

1- For the poly surface fit of 2nd-degree equation, the results were:

$$\rho_{wet} = 140.63 - 2.573C_{no.} + 1.4371w - 0.10978{C_{no.}}^2 + 0.18819C_{no.}w - 0.09278w^2 \quad (7)$$

where ρ_{wet} *=wet density lb/ft^3 ,$C_{no.}$=Curve number, and w=water content percent*
With correlation coefficient R^2=0.9727. As shown in figure 6a.

2- For the mathematical equation, the results were:

$$\rho_{wet} = \frac{(33.56 - C_{no.})}{(0.2935 + 0.005719w)} + 26 \times Ln(w) \quad (8)$$

With correlation coefficient R^2=0.956, as shown in figure 6b

3- For the Rational2D equation, the results were:

$$\rho_{wet} = \frac{124.58358 + 0.90112C_{no.} - 8.68w + 3.2w^2 - 0.0102w^3}{1 + 0.0693C_{no.} - 0.00557C_{no.}^2 + 3.56221\times10^{-4}C_{no.}^3 - 0.11233w + 0.02248w^2} \quad (9)$$

With correlation coefficient R^2 = 0.98438, as shown in figure 6c.

3.2.2 Compaction Curves after, **Horpibulsuk, et al., 2013**

Data of curve in figure 5 were subjected to the three curve fitting equations and the results were as shown:

1- For the poly surface fit of 2nd-degree equation, the results were:

$$\rho_{wet} = 21.67 - 0.48947C_{no.} + 0.11626w - 0.00995{C_{no.}}^2 + 0.02555C_{no.}w - 0.01388w^2 \quad (10)$$

With correlation coefficient R^2=0.977. As shown in figure 7a.

2- For the mathematical equation, the results were:

$$\rho_{wet} = \frac{(44.33 - C_{no.})}{(2.223 + 0.03283w)} + 2.022 \times Ln(w) \quad (11)$$

with correlation coefficient R^2=0.969, as shown in figure 7b

3- For the Rational2D equation, the results were:

$$\rho_{wet} = \frac{19.52351 + 1.51715C_{no.} - 4.20977w + 0.63134w^2 - 0.00589w^3}{1 + 0.1041C_{no.} - 0.00218C_{no.}^2 + 2.95037\times10^{-4}C_{no.}^3 - 0.22571w + 0.02852w^2} \quad (12)$$

With correlation coefficient R^2 = 0.9912, as shown in figure 6c.

For all the proposed equations, when the derivative of each equation with respect to the water content (w) is equated to zero ($\partial\rho/\partial w$=0) or ($\partial\gamma/\partial w$=0) gives the equation of a line that represents the connections between all OMC values in the family of the compaction curves.

4. CONCLUSIONS

Results demonstrated to show that reliable parameters can be obtained by applying for the non-linear curve fitting programs. This deduction is based on the high correlation factor (R^2) shown against each compaction curve data.

The easiness of determining the values of (OMC) and $\gamma dry_{max.}$ from fitting parameters makes this method faster and more accurate compared to using graph paper for example.

Computer programs that apply non-linear curve fitting are widely used, this paper focuses and sheds the light on the easiness of obtaining the compaction curve parameters accurately.

In addition, the figures show that even a family of compaction curves can be simulated through mathematical equations. These equations can describe the fitting surface and the value of γdry_{max} can be obtained by supplying the required curve number and the moisture content.

The three equations proposed for fitting single compaction curve and the three equations proposed for fitting a surface of a family of curves can be further investigated and any other mathematical equations can be implemented if more precise values can be obtained.

5. REFERENCES

- ALTUN, Selim, A. Burak G¨OKTEPE, Alper SEZER, 2008, “Investigation of Parameters of Compaction Testing”, Turkish J. Eng. Env. Sci., 32 (2008) , 201 – 209.
- Budhu, Muni, 2011, “Soil Mechanics and Foundations”, Third Edition, JOHN WILEY & SONS, INC.
- Das, Braja M., Khaled Sobhan, 2014, “Principles of Geotechnical Engineering”, Eighth Edition, SI, Cengage Learning.
- Fredlund, Murray, 2004, “SoilVision Theory Manual”, SoilVision Systems Ltd. Saskatoon, Saskatchewan, Canada.
- Horpibulsuk, Suksun, Apichat Suddeepong, Pokin Chamket, Avirut Chinkulkijniwat, 2013, “Compaction behavior of fine-grained soils, lateritic soils and crushed rocks”, The Japanese Geotechnical Society. Production and hosting by Elsevier B.V. Soils and Foundations Volume 53, Issue 1, February 2013, Pages 166–172.
- Horpibulsuk, Suksun, Wanchai Katkan, and Anek Naramitkornburee, 2009, “Modified Ohio’s Curves: A Rapid Estimation of Compaction Curves for Coarse- and Fine-Grained Soils” Geotechnical Testing Journal, Vol. 32, No. 1.
- Joslin, J.C., 1959, “Ohio’s typical water-density curves”, American Society for testing and materials. Special Technical Publication STP 239, 111–118.
- Kurucuk Nurses, Jayantha Kodikara, and Delwyn Fredlund, 2007, “Prediction of compaction curves”, 10th ANZ Conference on Geomechanics, 2: 115–119.
- Kurucuk, N., and J. Kodikara, D.G. Fredlund, 2008, “Theoretical modeling of the compaction curve”, Unsaturated Soils: Advances in Geo-Engineering – Toll et al. (eds), Taylor & Francis Group, London, ISBN 978-0-415-47692-8.
- Li, Cheng, 2013, “Using the Iowa K test and the gyratory compaction test to improve the Compaction Forecasting Expert Database”, A thesis submitted to the graduate faculty In partial fulfillment of the requirements for the degree of Master of Science, Iowa State University Ames, Iowa.
- Prakash, K., H. S. Prasanna, and A. Sridharan, 2015, “Variation of Degree of Saturation Along The Compaction Curve”, 50th Indian Geotechnical Conference 17th – 19th December 2015, Pune, Maharashtra, India.
- Shrivastava, K., P. K. Jain, Maulana Azad, 2016, “Prediction of Compaction Parameters using Regression and ANN Tools”, International Journal for Scientific Research & Development| Vol. 3, Issue 11, 2016 | ISSN (online): 2321-0613.
- State of Ohio Department of Transportation, 2010, Supplement 1015 Compaction Testing of Unbound Materials.

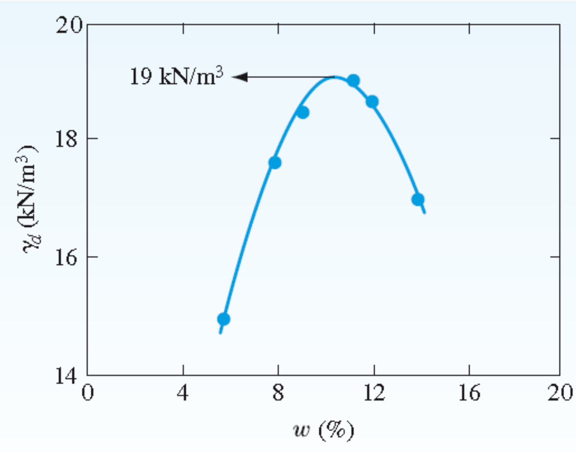


Figure 1. Compaction Curve after, **Das and Sobhan, 2014.**

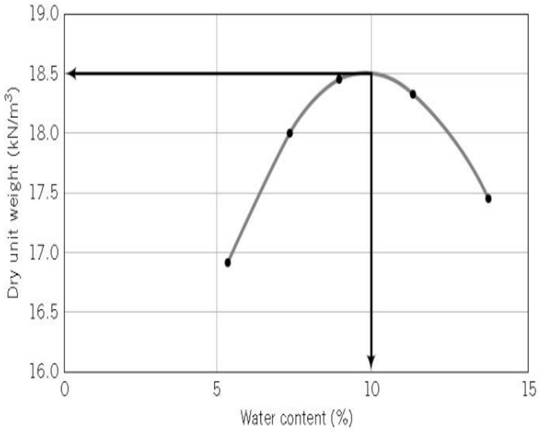


Figure 2. Compaction Curve after**, Budhu, 2011.**

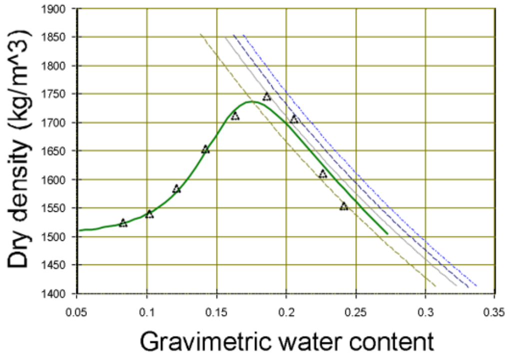


Figure 3. Compaction curve after, **Fredlund, 2004**.

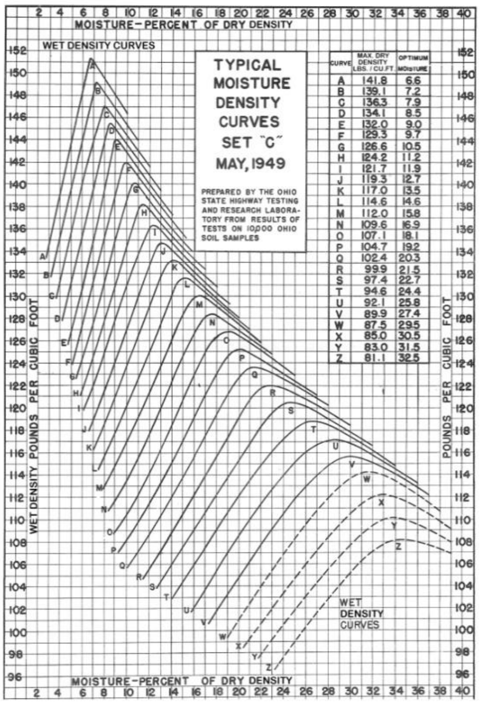


Figure 4. Ohio’s compaction curves wet density after, **Ohio’s state department of transportation, 2010**.

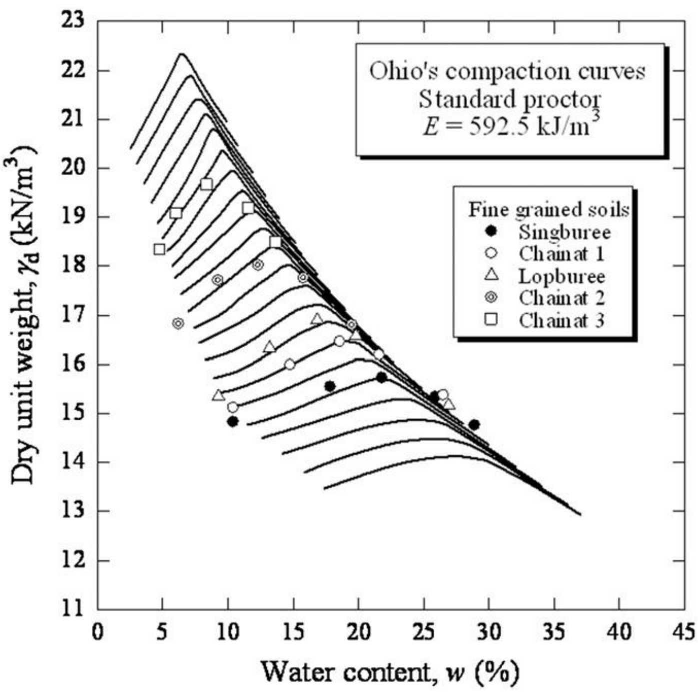
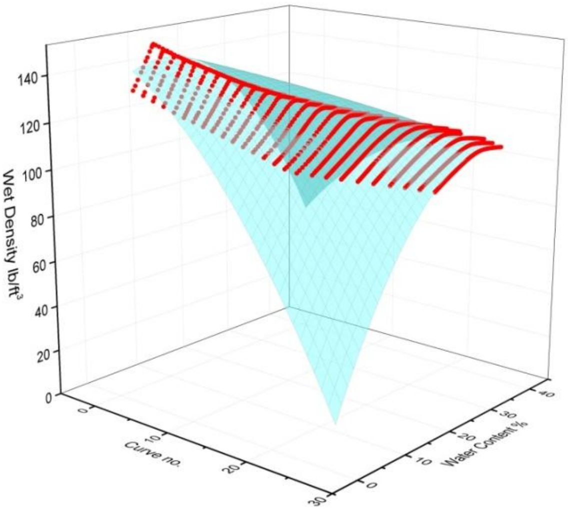
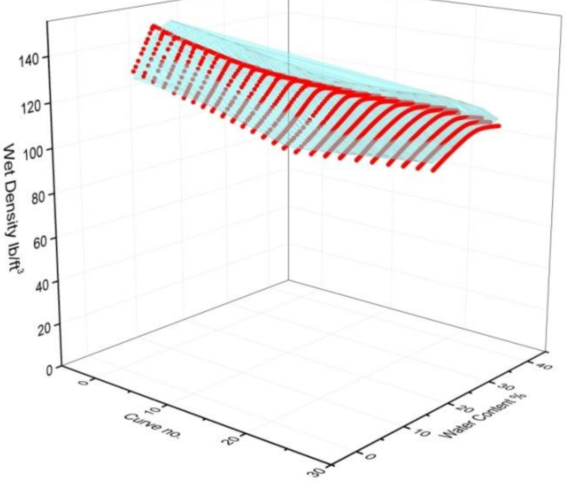


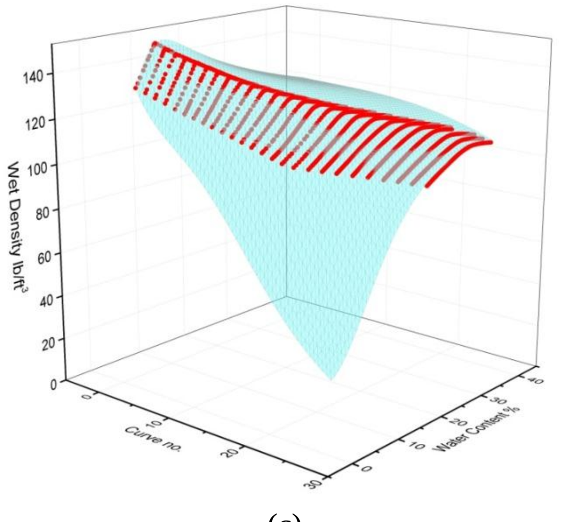
Figure 5. compaction curves dry unit weight for fine soils , tests conducted as Ohio’s compaction curves after, **Horpibulsuk, et al., 2009**.



(a)

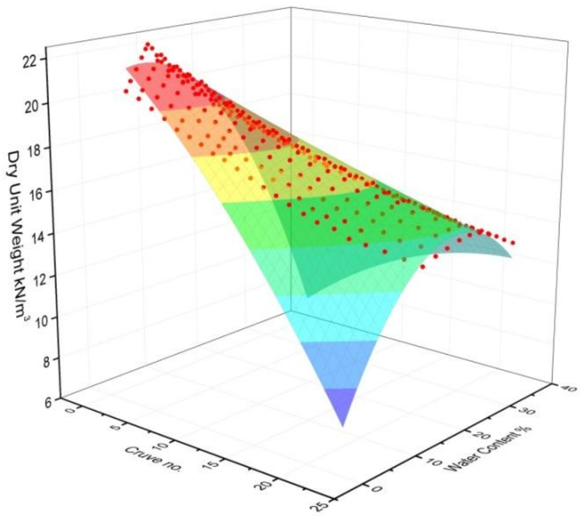


(b)

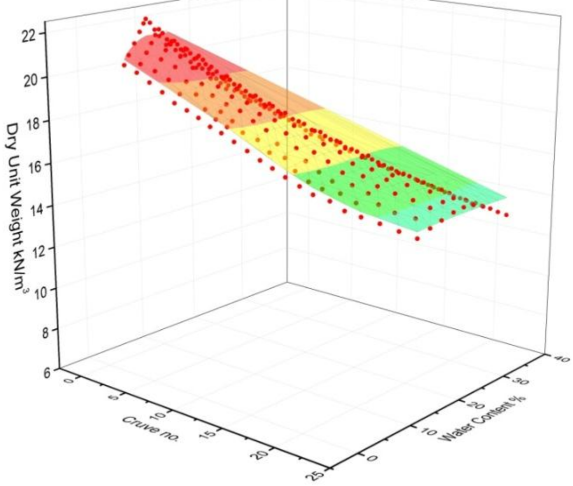


(c)

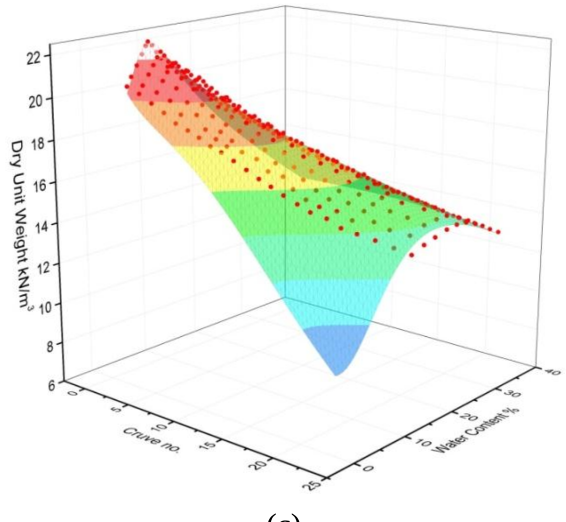
Figure 6. Ohio Compaction Curves a) Poly2dohiofinal ,
b) mathematical equation, c) Rational 2D.



(a)



(b)



(c)

Figure 7. Compaction Curves after, **Horpibulsuk, et al., 2009**. a) Poly2dohiofinal ,
b) mathematical equation, c) Rational 2D.

Table 1. Digitized data for the three curves.

Curve of Figure 1	Dry unit weight kN/m^3	Water Content %
	14.96	5.67
	17.65	7.82
	18.52	8.97
	19.02	11.09
	18.66	11.88
	17.01	13.86
Curve of Figure 2	Dry unit weight kN/m^3	Water Content %
	16.923716	5.28161
	18.006425	7.27273
	18.451093	8.87632
	18.334564	11.2771
	17.453734	13.7087
Curve of Figure 3	Dry unit weight kg/m^3	Water Content
	1523.35	0.08331
	1537.191	0.10178
	1581.517	0.12102
	1651.708	0.14213
	1710.299	0.16354
	1744.05	0.18625
	1704.517	0.2059
	1608.118	0.22643
	1551.755	0.24143

Plate 1. Applying function 1 to the three compaction curves.

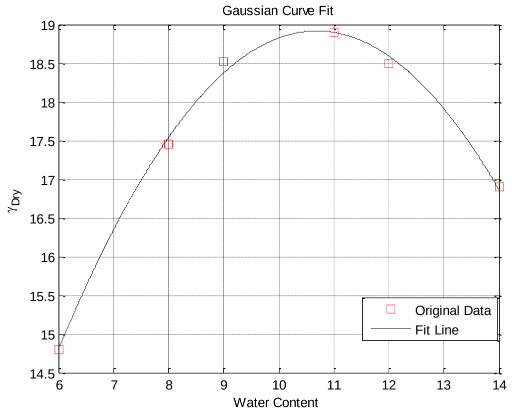
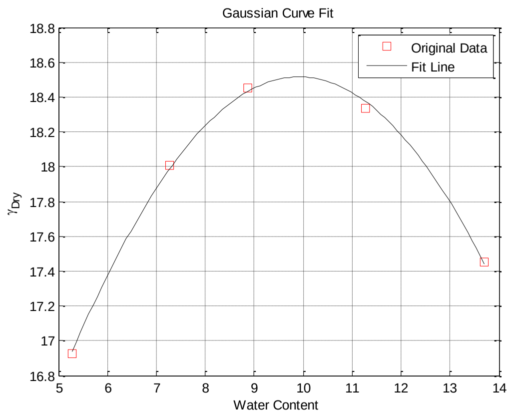
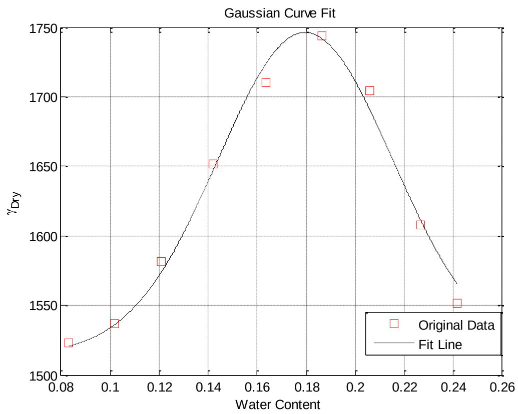
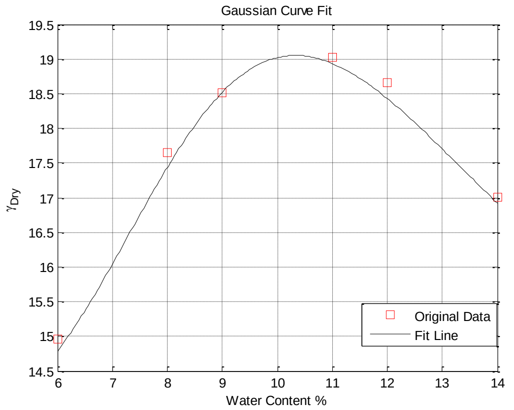
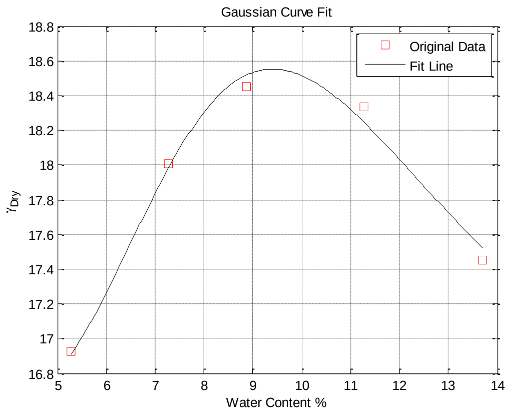
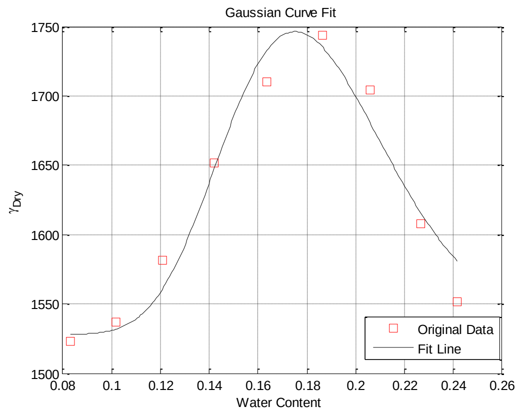
Graph	Values according to Reference			
	γ_{dmax}=	19kN/m^3	OMC=	10.4%
 Curve 1	Fitting Parameters			
	y_o	A	x_c	R^2
	–75.5	94.42	10.7	0.9965
	γ_{dmax}=	18.92 kN/m^3	OMC=	10.7%
 Curve 2	Values according to Reference			
	γ_{dmax}=	18.5 kN/m^3	OMC=	10%
	Fitting Parameters			
	y_o	A	x_c	R^2
	-2237.1	2255.6	9.904	0.9992
	γ_{dmax}=	18.5 kN/m^3	OMC=	9.9 %
 Curve 3	Values according to Reference			
	γ_{dmax}=	1744 kg/m^3	OMC=	0.186
	Fitting Parameters			
	y_o	A	x_c	R^2
	1514.6	231.5	0.1796	0.9180
	γ_{dmax}=	1746.1 kg/m^3	OMC=	0.1796

Plate 2. Applying function 2 to the three compaction curves.

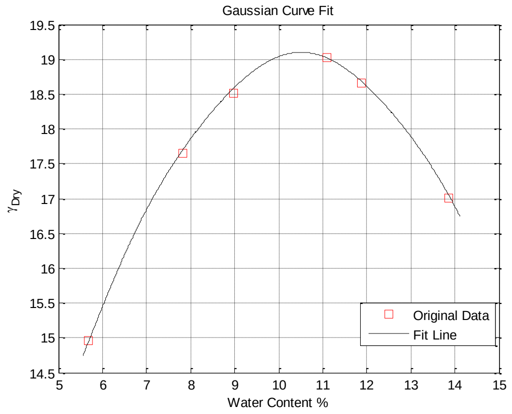
<table>
<tr><th rowspan="2">Graph</th><th colspan="5">Values according to Reference</th></tr>
<tr><td>γ_{dmax}=</td><td>19kN/m^3</td><td>OMC=</td><td colspan="2">10.4%</td></tr>
<tr><td rowspan="4">

Curve 1</td><td colspan="5">Fitting Parameters</td></tr>
<tr><td>A</td><td>B</td><td>C</td><td>D</td><td>R^2</td></tr>
<tr><td>5.27</td><td>2.335</td><td>–0.18</td><td>13.78</td><td>0.999</td></tr>
<tr><td>γ_{dmax}=</td><td>19.05 kN/m^3</td><td>OMC=</td><td colspan="2">10.33%</td></tr>
<tr><td rowspan="6">

Curve 2</td><td colspan="5">Values according to Reference</td></tr>
<tr><td>γ_{dmax}=</td><td>18.5 kN/m^3</td><td>OMC=</td><td colspan="2">10%</td></tr>
<tr><td colspan="5">Fitting Parameters</td></tr>
<tr><td>A</td><td>B</td><td>C</td><td>D</td><td>R^2</td></tr>
<tr><td>2.033</td><td>2.24</td><td>-0.2</td><td>16.52</td><td>0.989</td></tr>
<tr><td>γ_{dmax}=</td><td>18.55 kN/m^3</td><td>OMC=</td><td colspan="2">9.4 %</td></tr>
<tr><td rowspan="6">

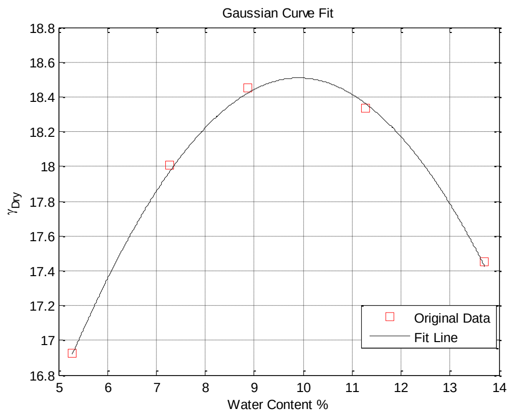
Curve 3</td><td colspan="5">Values according to Reference</td></tr>
<tr><td>γ_{dmax}=</td><td>1744 kg/m^3</td><td>OMC=</td><td colspan="2">0.186</td></tr>
<tr><td colspan="5">Fitting Parameters</td></tr>
<tr><td>A</td><td>B</td><td>C</td><td>D</td><td>R^2</td></tr>
<tr><td>218.4</td><td>–1.74</td><td>–0.073</td><td>1528</td><td>0.953</td></tr>
<tr><td>γ_{dmax}=</td><td>1746.4 kg/m^3</td><td>OMC=</td><td colspan="2">0.175</td></tr>
</table>

Plate 3. Applying function 3 to the three compaction curves.



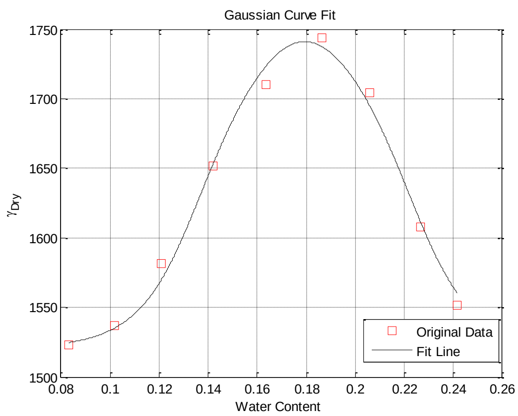
Curve 1

Values according to Reference			
γ_{dmax}=	19kN/m^3	OMC=	10.4%
Fitting Parameters			
y_o	A	x_c	R^2
11.2924	7.81	10.52	0.9995
γ_{dmax}=	19.1 kN/m^3	OMC=	10.52 %



Curve 2

Values according to Reference			
γ_{dmax}=	18.5 kN/m^3	OMC=	10%
Fitting Parameters			
y_o	A	x_c	R^2
14.728	3.78076	9.91758	0.9975
γ_{dmax}=	18.5 kN/m^3	OMC=	9.91758 %



Curve 3

Values according to Reference			
γ_{dmax}=	1744 kg/m^3	OMC=	0.186
Fitting Parameters			
y_o	A	x_c	R^2
1519.7	221.41	0.17945	0.99
γ_{dmax}=	1741.1 kg/m^3	OMC=	0.179