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Slab-beam Interaction in One-way Floor Systems

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ABSTRACT

This study focuses on the slab-beam interaction in one-way systems. In the context of this study, slab-beam interaction means how beam deflection can affect moment distribution in one-way slabs. This interaction is usually neglected in the traditional approximate analysis that is adopted in engineering practice and design codes. Slab positive moments have been considered as indicators on the accuracy of approximate methods, as they overestimate negative moments while underestimating positive moments.

After proposing of effecting parameters in slab-beam interaction including of panel length and width, beam dimensions, and slab thickness, Buckingham's π theorem has been adopted to transform the dimensional-model into a non-dimensional qualitative one. Different case studies with finite element models have been adopted to generate points on the proposed qualitative non-dimensional model. Finally, linear regression analyses have been adopted to develop the corresponding quantitative models.

Case studies and corresponding regression analysis indicate that non-dimensional parameters adopted in the model are related linearly with a correlation coefficient in the range of 0.97 and that an error up to 250% may be noted due to neglecting the slab-beam interaction. Therefore, a condition related to the relative stiffness of supporting beams should be added to the current conditions for the approximated methods to be more accurate and more compatible with those adopted in the analysis of two-way systems.

Key Words: One-way slabs, slab-beam interaction, finite element analysis, regression analysis.

تداخل السقف-العتب في أنظمة الأرضيات باتجاه واحد

صلاح رحيمة عبيد
مدرس
كلية الهندسة-جامعة بغداد

الخلاصة

تركز هذه الدراسة على التداخل بين السقوف والأعتاب في أنظمة التسقيف باتجاه واحد. عرف التداخل في سياق هذه الدراسة بانه مقدار تأثير انحراف الأعتاب على توزيع العزوم في السقوف باتجاه واحد. هذا التداخل عادة ما يهمل في طرق التحليل التقريبية المتبعة في التطبيقات الهندسية والمدونات التصميمية. عزم السقف الموجب أخذ كمؤشر على دقة الطرق التقريبية وذلك لان هذه الطرق التقريبية تبالغ في تقدير العزوم السالبة وتقلل تقدير العزوم الموجبة.

بعد اقتراح العوامل المؤثرة في التداخل بين السقوف والأعتاب والمتضمنة طول وعرض السقف، أبعاد العتب، وسمك السقف، استخدمت نظرية الباي لبيكنهام في تحويل النموذج البعدي لأخر غير بعدي. تمت دراسة عدة حالات باستخدام نماذج العناصر المحددة لتوليد عدة نقاط على النموذج غير البعدي الكيفي المقترح. أخيرا استخدم تحليل الانحسار الخطي لأعداد النموذج غير البعدي الكمي. الحالات المدروسة مع تحليلات الانحسار بينت أن المعاملات غير البعدية المقترح مرتبطة بمعاملات ارتباط

بحدود 0.97 وبينت كذلك بان خطا بنسبة 250% قد يلحظ في الطرق التقريبية نتيجة إهمال التداخل بين السقف والأعتاب. وعلية شرطا" معرفا" للصلابة النسبية للأعتاب يجب إن يضاف للشروط الحالية للطرق التقريبية لجعلها أكثر دقة وأكثر انسجاما" مع تلك التي تستخدم في تحليل السقوف باتجاهين.

كلمات الرئيسية: السقوف باتجاه واحد، تداخل السقف-العتب، تحليل العناصر المحددة، تحليل الانحدار.

1 INTRODUCTION

Slab-beam-girder flooring system usually adopted in reinforced concrete buildings with its load path is presented in **Fig1(a)**, **Nilson, 2011**. According to **McGuire, 1959**, this system is commonly used with column spacing from 6m to 12m. Panel length to width ratio usually excesses 1.5 according to **MacGregor** and **Wight, 2005**.

Floor beams usually have a span up to 6m, **Callender, 1982**, with a depth about twice the width, and usually located at mid-points, at the third points, or at the quarter points of the girders, **McGuire, 1959**. For lighter loads, intermediate and deep girders may be eliminated and one-way slab to be supported by wide, shallow beams located along column lines as indicated in **Fig1(b)**, **McGuire, 1959**.

According to **ACI 318, 2008**, slab-beam-girder flooring system should cast monolithically resulting in a highly indeterminate system with deflected shape indicated in **Fig. 2**.

Many approximated methods have been offered to determine shear forces and bending moments in the slab including ACI coefficients methods, **ACI 318, 2008**, semi-analytical methods proposed by **Wang** and **Salmon, 1985**, and moment distribution method proposed by **Cross** and **Morgan, 1949**. In all these methods, beam deflection is neglected relative to slab deflection and actual deflected shape of **Fig. 2** is approximated with that of **Fig. 3**.

Experience with current numerical analysis by finite element method indicates that aforementioned assumption may be in a serious error especially for slabs supported on flexible beams. Therefore, a condition of the relative stiffness of the supporting beam should be adopted for more accurate results. This condition would be similar to that adopted by ACI code in direct design method for two-way slabs.

This paper aims to show the effect of slab-beam interaction on moments in one-way slabs. Finite element method has been adopted for analysis of different case studies with and without beam interactions.

2 BUILDING OF THE MODEL AND THE DIMENSIONAL ANALYSIS

2.1 Basic Relation to the Model

As discussed above, this study aims to show how slab-beam interaction affects slab moments in a one-way system. Parameters that are important in this interaction have been summarized in Eq. 1 below.

$$f\left(\frac{M_I}{M_E},\frac{L_1}{L_2},L_1,L_2,b,h,t\right)=0 \quad (1)$$

With referring to **Fig. 4**, above parameters are defined in below:

M_I is slab moment including slab-beam interaction,

M_E is slab moment excluding slab-beam interaction,

L_1 is the beam span and the panel length,

L_2 is the spacing between beams and panel width,

b, and h are beam width and depth respectively,

t is the slab thickness.

Unfortunately, there is no systematic method to determine which parameters are significant in a specific problem, **Langhaar, 1951**. Therefore, parameters of Eq. 1 above are proposed based on a physical reasoning where the parameters b, h, and L_1 are included to reflect beam stiffness in the model, while the parameters t, and L_2 are included to simulate slab stiffness. Assuming that the slab and beams to be casted in a monolithic process with same concrete, the concrete properties are dropped from the model parameters.

2.2 Number of Independent Dimensionless Groups

A non-dimensional model form that based on Buckingham's theorem is useful in reducing problem parameters and in ensuring that case studies are significantly apart to be adopted in regression analysis, **Langhaar, 1951**.
The dimensional matrix of the model is presented in **Table 1**. According to **Langhaar, 1951**, the number of dimensionless products in a complete set is equal to the total number of parameters minus the rank of their dimensional matrix. Matlab code of **Table 2** indicated in the dimensional matrix of **Table 1** has a rank of one. Therefore, the number of dimensionless product for the proposed model would be:

$$No.\,of\ Non-dimensional\ paramters = 5 - 1 = 4 \tag{2}$$

2.3 Dimensionless Groups

Equating length dimension for both sides of Eq. 1 above according to the law of dimensional homogeneity, the following equation is the result:

$$0 = (L_1)^{\alpha_1}(\mathrm{L}_2)^{\alpha_2}(b)^{\alpha_3}(h)^{\alpha_4}(t)^{\alpha_5} \tag{3}$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 0$$

Solve for slab thickness exponent, α_5

$$\alpha_5 = -\alpha_1 - \alpha_2 - \alpha_3 - \alpha_4$$

The four non-dimensional groups would be as indicated in Eq. 4 below and the proposed model in its non-dimensional form would be as indicated in Eq. 5 below.

$$0 = \left(\frac{\mathrm{L}_1}{t}\right)^{\alpha_1}\left(\frac{\mathrm{L}_2}{t}\right)^{\alpha_2}\left(\frac{b}{t}\right)^{\alpha_3}\left(\frac{h}{t}\right)^{\alpha_4} \tag{4}$$

$$\frac{M_I}{M_E} = g\left(\left(\frac{L_1}{L_2}\right)\left(\frac{\mathrm{L}_1}{t}\right)^{\alpha_1}\left(\frac{\mathrm{L}_2}{t}\right)^{\alpha_2}\left(\frac{b}{t}\right)^{\alpha_3}\left(\frac{h}{t}\right)^{\alpha_4}\right) \tag{5}$$

3 FINITE ELEMENT MODEL

3.1 Basic Formulation

Shell element has been adopted to simulate slabs while space frame element has been adopted for beam simulation. Typical degrees of freedom for each node of the shell element has been indicated in **Fig. 5** below. In spite of neglecting of geometric nonlinearities in finite element modeling of this study, membrane action has been adopted to make DOF for shell similar to those of space frame element and in turn to simplify the assemblage process, **Rockey, et al., 1975**. Regarding bending action, Mindlin theory that includes shear deformation has been adopted. This theory is based on a kinematic assumption of normals to the mid-surface before deformation remain straight but not necessarily normal to the mid-surface after deformation, **Hinton** and **Owen, 1984**. Unfortunately, finite element simulation of Mindlin plate may overestimate energy due to shear deformations for thin plates. This numerical difficulty has been solved through adopted of a reduced integration scheme during formulation of stiffness matrix, **Huang, 1989**.

Regarding the supporting beams, they have been simulated using a space frame element. The Linear displacement field is used to derive terms for axial stiffnesses while Hermite cubical shape function is adopted for flexure stiffnesses ,**Chandrupatla** and **Belegundu, 1996**.

As the Hermite displacement filed is exact for beams loaded at nodes only, and to maintain compatibility with the supported slabs, a mesh size in the range 0.25m has been adopted to discretize the slabs and the supporting beams. According to **Cook**, **1995**, this mesh size is adequate to simulate the behavior of the problem.

When slab-beam interaction to be included, boundary conditions are simulated as indicated in **Fig. 6** while they have simulated as indicated **Fig. 7** when slab-beam interaction to be excluded with maintaining beam torsional effects.

3.2 Effect of Offset between Slab and Beams Center Line

As indicated in **Fig. 8** below, traditional finite element models usually connect neutral plane for the slab to the neutral axes of the supporting beams and implicitly neglect the actual offset between them. Therefore, before adopting of a traditional finite element model in the assessment of approximate methods for analysis of one-way slab system, the effect of neglected offset should be checked at first.

According to **Cook**, **1995**, offset between the slab and the supporting beam can be simulated either through adopting of a physical rigid link to connect between the node on the slab and the corresponding node on the beam, see **Fig. 9a** or through adopting of three-dimensional simulation indicated in **Fig. 9b**. When one adopts the three-dimensional modeling of **Fig. 9b**, he should consider the difference between clear span and center-to-center span for the slabs. As this aspect is out of our scope and needs a separate study, therefore, the model of the rigid link has been adopted here to show that offset between slab neutral plane and beam neutral axis affects slab moments in average by an amount indicated in Eq. 6.

$$\frac{M_{with\ offset}}{M_{without\ offset}} \approx 0.9 \qquad (6)$$

Results of Eq. 6 above can be interpreted if one notes that adopting of offset increases beams torsional stiffness, and in turn, it reduces positive moments and increases negative moments in the slab. With this results and interpretations, one can conclude that neglecting of offset between the slab and the supporting beams leads to a conservative estimation of the positive moments.

4 CASE STUDIES AND REGRESSION ANALYSES

4.1 One-way Floor Systems with Two Spans

Based on analysis parameters and finite element model discussed above, case studies indicated in **Table 3** have been considered.

As actual deformations presented in **Fig. 2** indicate that approximate analysis methods underestimate positive moments while overestimating the negative moments, therefore this study considers positive moments as an indicator on accuracy and adequacy of the approximated

methods.
Regression model indicated in Eq. 7 has been adopted to correlate slab moment with interaction, M_I, to slab moment without interaction, M_E. As the beam thickness, h, and the slab thickness, t, are related to beam stiffness and slab stiffness respectively, therefore they are included with cubical power.

$$M_I = \left(k_0 + k_1\left(\frac{L_1}{L_2}\right) + k_2\left(\frac{L_1}{t}\right) + k_3\left(\frac{L_2}{t}\right) + k_4\left(\frac{b}{t}\right) + k_5\left(\frac{h}{t}\right)^3\right)M_E \qquad (7)$$

In terms of variables of **Table 3**, Eq. 7 has been re-written as indicated in Eq. 8 below.

$$y = (k_0 + k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 + k_5x_5^3) \qquad (8)$$

To avoid nonlinear multiple regresses and deal with linear regression analysis where one can investigative the partial contribution of each parameter, Eq. 8 above has been linearized in term of $x_5^* = \left(\frac{h}{t}\right)^3$ as indicated in below:

$$y = (k_0 + k_1x_1 + k_2x_2 + k_3x_3 + k_4x_4 + k_5x_5^*) \qquad (9)$$

Using least square analysis in SPSS environment, the coefficients k_i have been determined and presented **Table 4**. SPSS regression analysis indicates that the parameter $x_3 = \frac{L_2}{t}$ has insignificant effect and has been excluded from the model. Therefore, in its final form, relation between positive slab moment with and without beam interaction has been presented in Eq. 10 below. From **Fig. 10** below, one concludes that the M_I/M_E determined from FE analysis and those determined from the regression analysis are highly correlated that Eq. 10 can be used to estimate of M_I from the corresponding value of M_E which can be determined from approximated the method for analysis of one-way slabs.

$$M_I = \left(-1.146 - 1.249\left(\frac{L_1}{L_2}\right) + 0.075\left(\frac{L_1}{t}\right) + 0.651\left(\frac{b}{t}\right) - 0.002\left(\frac{h}{t}\right)^3\right)M_E \qquad (10)$$

4.2 One-way Floor Systems with Three Spans

As for slabs with two-spans, parameters and results for case studies of slabs with three-spans have been presented in **Table 5**.
Correlation coefficients and corresponding relation between slab moment with beam interaction, M_I, and the corresponding moment with neglecting of beam interaction, M_E, have been presented in **Table 6** and Eq. 11 respectively.
Accuracy of proposed linear model to estimate M_I from corresponding M_E has been illustrated in **Fig. 11** below that indicates a high correlation, with R^2 value of 0.9794, between results of proposed linear model and corresponding results of the finite element analysis.

$$M_I = \left(1.142 + 0.361\left(\frac{L_1}{L_2}\right) + 0.007\left(\frac{L_1}{t}\right) - 0.031\left(\frac{b}{t}\right) - 0.002\left(\frac{h}{t}\right)^3\right)M_E \qquad (11)$$

5 CONCLUSIONS

- Proposed regression models indicated that an error up to 250% can occur due to the neglect of slab-beam interaction in one-way systems.
- From a practical point of view, proposed regression models can be used to modify bending moments of one-way slabs estimated from approximated methods to reflect the effect of slab-beam interaction.
- On the other hand, the proposed regression models can be adopted to define a new limitation on the applicability of approximated methods for analysis of one-way systems. To be similar to the corresponding limitation in two-way systems, this new limitation should be written in terms of the relative stiffness of the supported beams.

6 RECOMMENDATIONS FOR THE FUTURE WORKS

More elaborate models for interaction between slab and supporting beams in one-way systems can be achieved through:

- Using three-dimensional finite element analysis to show how slab-beam inaction can be affected by the difference between center-to-center span and the clear span.
- Using a finite element model with material nonlinearities to show how slab-beam interaction is affected by cracking of concrete and yielding of reinforcement. The moment distribution is generally significant in slabs that usually have high ductility levels.
- Using a finite element model with geometric nonlinearities to show how slab-beam interaction may be affected by membrane forces in the slabs and the supporting beams.

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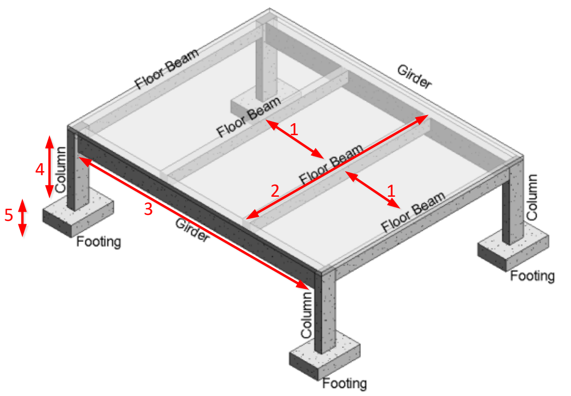
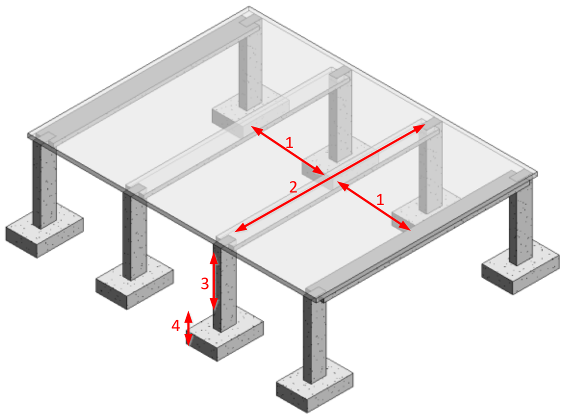
8 NOMENCLATURE

L_1 is the beam span and the panel length, m.
L_2 is the spacing between beams and the panel width, m.
M_E is slab moment excluding slab-beam interaction, $kN.m\ per\ m$.
M_I is slab moment including slab-beam interaction, $kN.m\ per\ m$.
k_i are coefficients for the regression model, dimensionless.
h is the beam depth, m.
b is the beam width,
t is the slab thickness,
α_i are the exponents for non-dimensional model,



(a) With traditional floor beams and girders.



(b) With wide, shallow beams.

Figure 1. One-way flooring system.

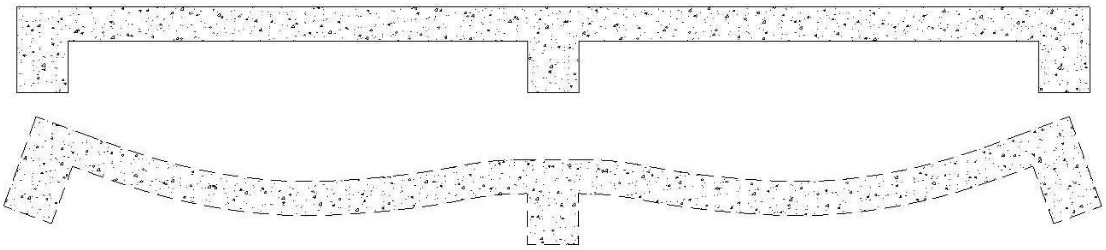


Figure 2. Exaggerated deflected shape for two spans one-way slab system.

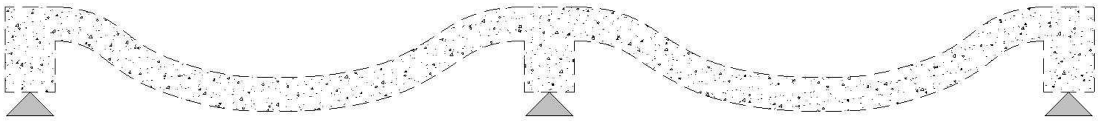


Figure 3. Approximated deflected shape adopted by traditional analysis methods of the one-way system.

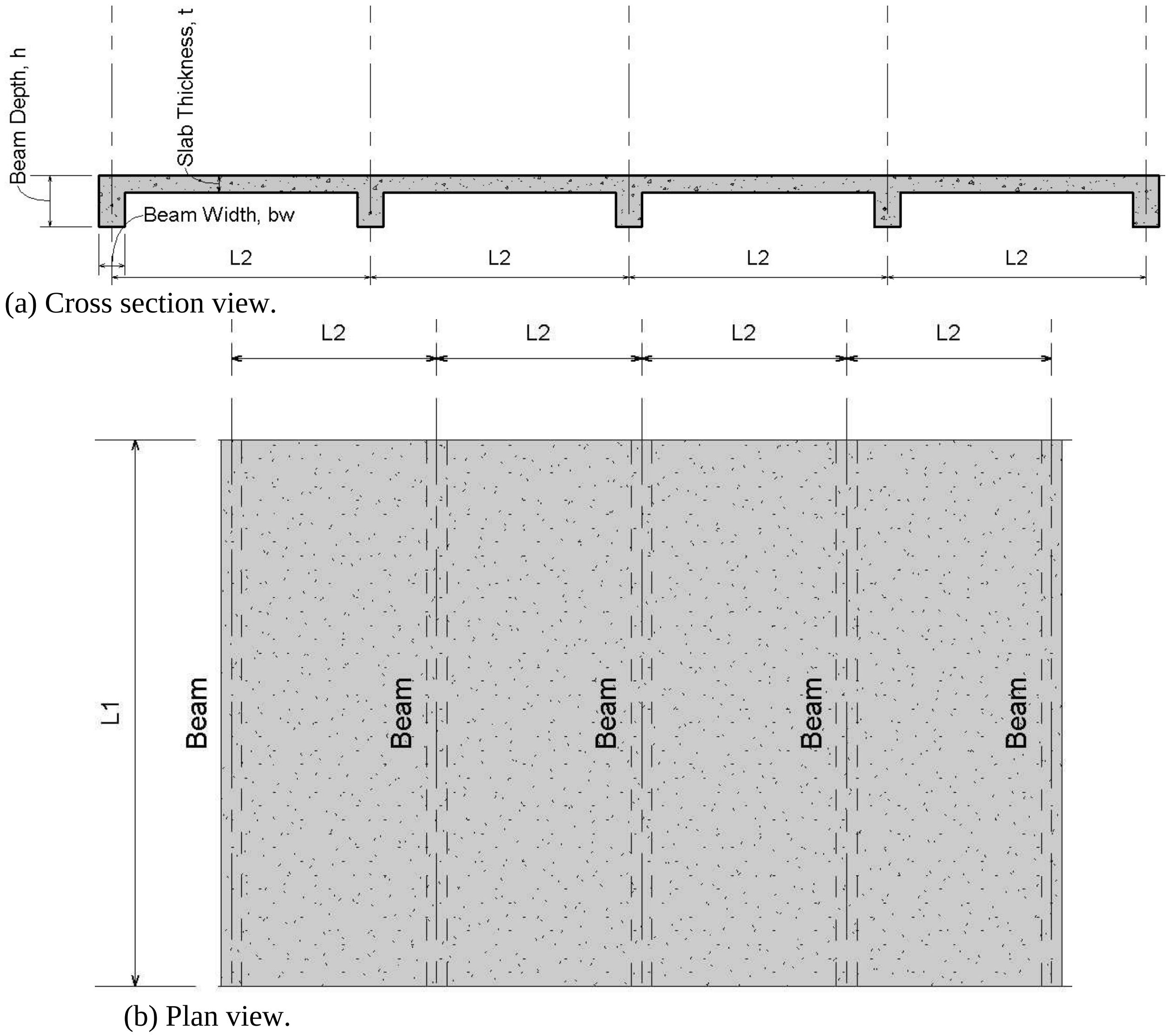


Figure 4. Parameters and notations adopted in case studies.

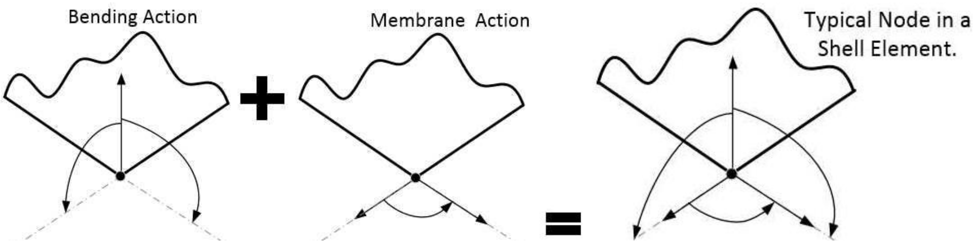
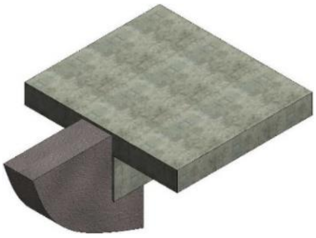
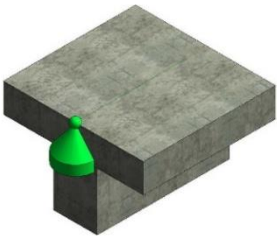


Figure 5. A typical node for quadrilateral shell element adopted to simulate foundations.



(a) Actual support.



(b) Ideal support.

Figure 6. Boundary conditions when slab-beam interaction is included.

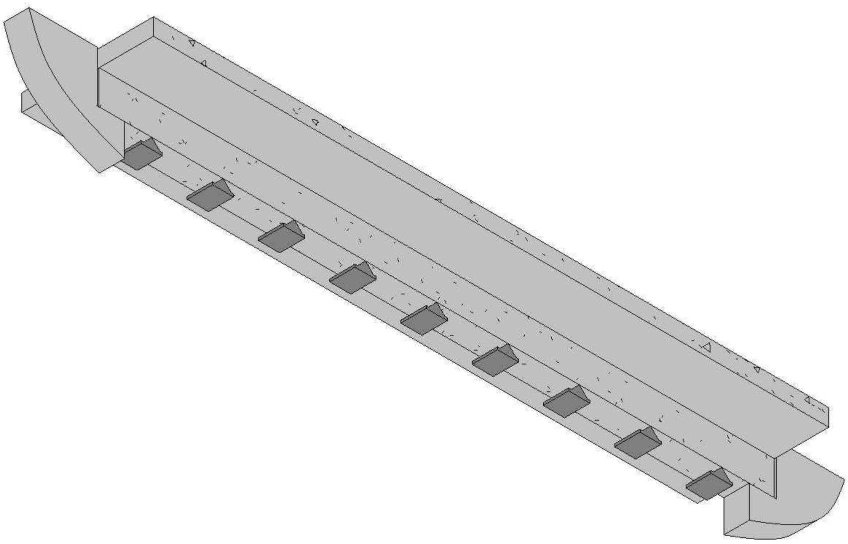
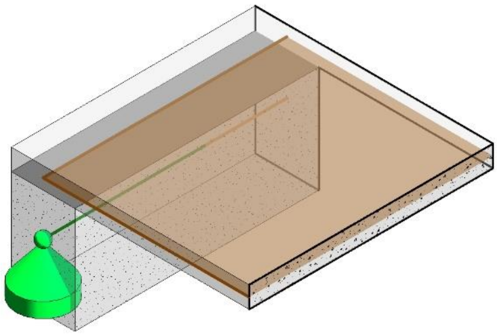
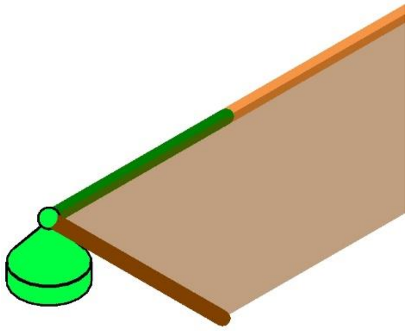


Figure 7. Boundary conditions when slab-beam interaction is excluded.

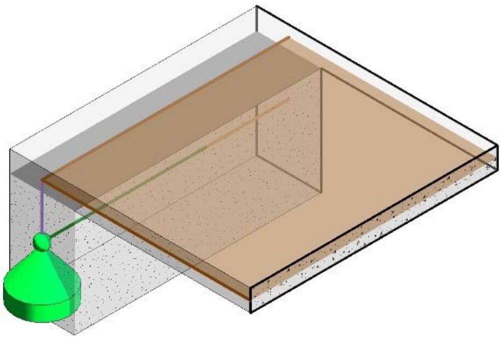


(a) Actual relation with offset.

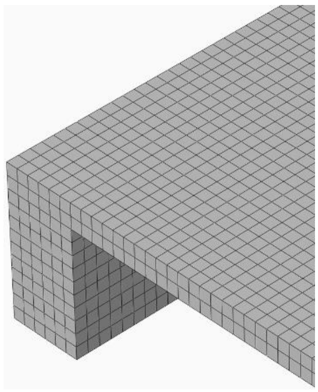


(b). Approximated relation adopted in traditional finite element models.

Figure 8. Offset between neutral plane for the slab and neutral axis for the supporting beam.



(a) Rigid link to overcome offset problem.



(b). Three-dimensional model to overcome offset problem.

Figure 9. Two common simulations to overcome the problem of offset between the slab and the supporting beams.

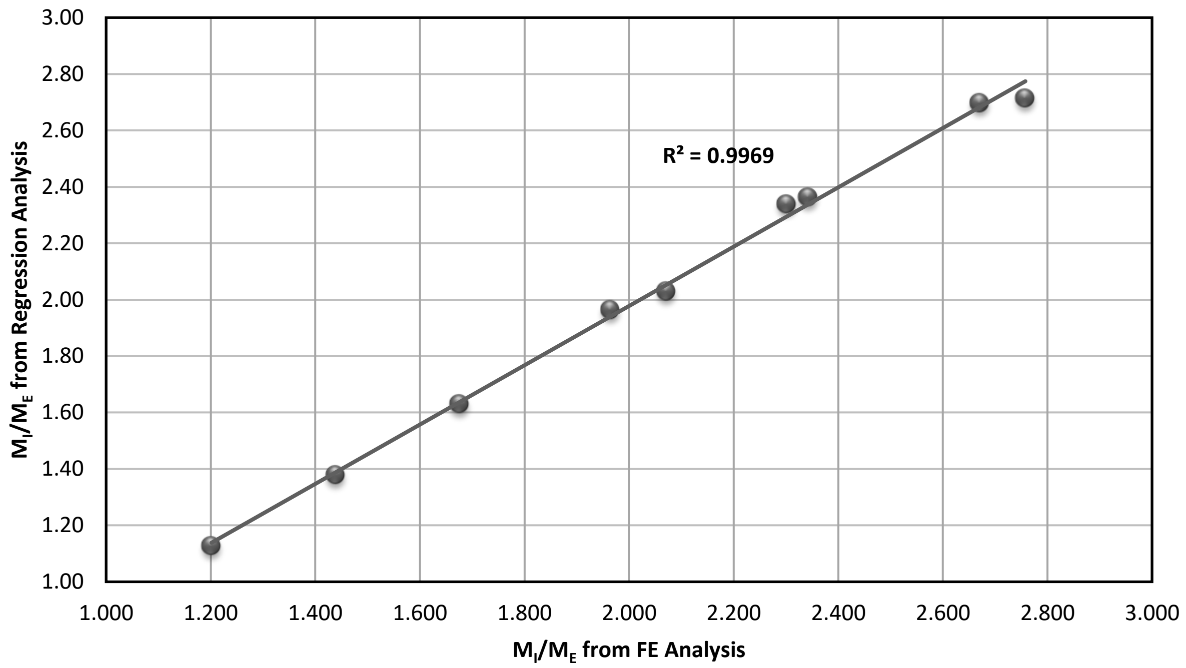


Figure 10. Correlation between $\mathbf{M_I/M_E}$ computed from FE analysis to those estimated from regression analysis for one-way systems with two spans.

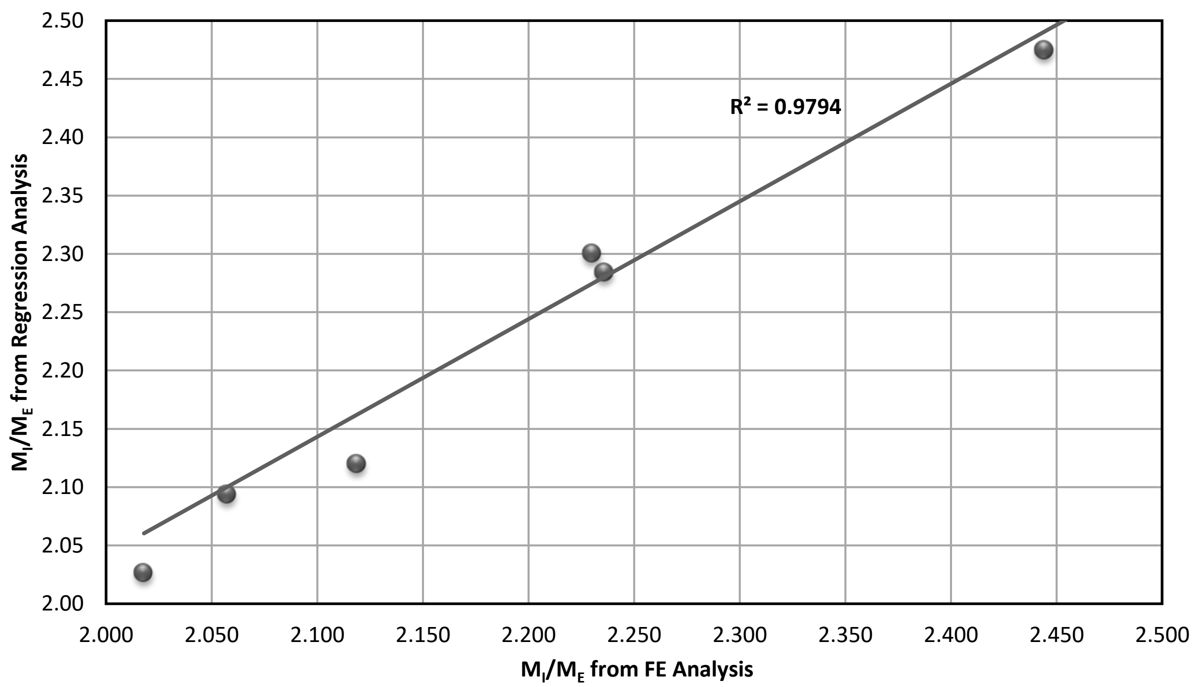


Figure 11. Correlation between M_I/M_E computed from FE analysis to those estimated from regression analysis for one-way systems with three spans.

Table 1. Dimensional matrix for the proposed model of Eq. 1.

	L_1	L_2	b	h	t
M	0	0	0	0	0
L	1	1	1	1	1
T	0	0	0	0	0

Table 2. Matlab code to determine the rank of the dimensional matrix for the proposed model.

```
clc
% The script file aims to determine the rank of dimensional matrix
% for the slab-beam interaction in one-way systems.
DM = [0 0 0 0 0
  1 1 1 1 1
  0 0 0 0 0]
k = rank(DM)
```

Table 3. Case studies for slab with two spans.

No	Slab Thickness, t, mm	L_1, m	L_2, m	b_w, mm	h, mm	$x_1 = \frac{L_1}{L_2}$	$x_2 = \frac{L_1}{t}$	$x_3 = \frac{L_2}{t}$	$x_4 = \frac{b}{t}$	$x_5 = \frac{h}{t}$	$x_5^* = \left(\frac{h}{t}\right)^3$	$y = \frac{M_I}{M_E}$
1	100	6	2	300	600	3.00	60.00	20.00	3.00	6.00	216.00	1.20
2	100	8	2	300	600	4.00	80.00	20.00	3.00	6.00	216.00	1.44
3	100	10	2	300	600	5.00	100.00	20.00	3.00	6.00	216.00	1.68
4	150	8	4	400	600	2.00	53.33	26.67	2.67	4.00	64.00	1.96
5	150	10	4	400	600	2.50	66.67	26.67	2.67	4.00	64.00	2.30
6	150	12	4	400	600	3.00	80.00	26.67	2.67	4.00	64.00	2.76
7	200	12	6	400	800	2.00	60.00	30.00	2.00	4.00	64.00	2.07
8	200	14	6	400	800	2.33	70.00	30.00	2.00	4.00	64.00	2.34
9	200	16	6	400	800	2.67	80.00	30.00	2.00	4.00	64.00	2.67

Table 4. Coefficients for multiple linear regressions for the proposed model of slabs with two spans.

k_0	-1.146
k_1	-1.249
k_2	.075
k_4	.651
k_5	-.002

Table 5. Case studies for slab with three spans.

No	**Slab Thickness, t, mm**	L_1, m	L_2, m	b_w, mm	h, mm	$x_1 = \frac{L_1}{L_2}$	$x_2 = \frac{L_1}{t}$	$x_3 = \frac{L_2}{t}$	$x_4 = \frac{b}{t}$	$x_5 = \frac{h}{t}$	$x_5^* = \left(\frac{h}{t}\right)^3$	$y = \frac{M_I}{M_E}$
1	100	6	2	300	600	3.00	60.00	20.00	3.00	6.00	216.00	2.119
2	100	8	2	300	600	4.00	80.00	20.00	3.00	6.00	216.00	2.456
3	100	10	2	300	600	5.00	100.00	20.00	3.00	6.00	216.00	3.106
4	150	8	4	400	600	2.00	53.33	26.67	2.67	4.00	64.00	2.018
5	150	10	4	400	600	2.50	66.67	26.67	2.67	4.00	64.00	2.230
6	150	12	4	400	600	3.00	80.00	26.67	2.67	4.00	64.00	2.541
7	200	12	6	400	800	2.00	60.00	30.00	2.00	4.00	64.00	2.057
8	200	14	6	400	800	2.33	70.00	30.00	2.00	4.00	64.00	2.236
9	200	16	6	400	800	2.67	80.00	30.00	2.00	4.00	64.00	2.444

Table 6. Coefficients for multiple linear regressions for the proposed model of slabs with three spans.

k_0	1.142
k_1	0.361
k_2	0.007
k_4	-0.031
k_5	-0.002