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Stress Concentration factor Analysis of Helical Gear Drives with Asymmetric Teeth Profiles

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ABSTRACT

This study investigates the influence of asymmetric involute teeth profiles for helical gears on the bending stress. Theoretically, bending stress has been estimated in spur involute gears which have symmetric teeth profile by based on the Lewis, 1892 equation. Later, this equation is developed by, Abdullah, 2012. to determine the effect of an asymmetric tooth profile for the spur gear on the bending stress. And then these equations are applied with stress concentration factor once for symmetric and once other for asymmetric teeth profile. In this paper, the bending stresses for various types of helical gear with various types of asymmetric teeth profile are calculated numerically for defined the stress concentration factor. The numerical solution based on the finite element method technique which that done by using the software simulation SolidWorks 2016. The results of this study indicate that the helical gear drive with asymmetric teeth profile having 'loaded side pressure angle' of (14.5˚) and 'unloaded side pressure angle' of (35˚) is better than a helical gear with standard teeth profile having pressure angle of (14.5˚) from the regarding of tooth bending strength. Also, notes that the great enhancement in the results of maximum tooth bending stress for modified involute of tooth profile compared with the standard teeth profile. In addition to, predict the equation of stress concentration factor which is a function of both unloaded side pressure angle and helix angle and then it used with Abdullah equation for to determine the nominal stresses in the root fillet.

Keywords: Asymmetric Helical Gear Tooth**,** Involute Profile**,** Stress Analysis, Bending Stress, Stress Concentration Factor, Solidworks2016 Simulation**.**

تحليل عامل تمركز الاجهادات للتروس الحلزونية ذات الاسنان الغير متماثلة الجوانب

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الخلاصة

هذه الدراسة تحقق في تأثير اسنان التروس الحلزونية الغير متماثلة الجوانب على اجهاد الانحناء. نظريا، تم تقدير انحناء الإجهاد في التروس ذات حافة الجبل التي لديها اسنان متناظرة الجوانب على أساس معادلة لويس (1892). في وقت لاحق، تم تطوير هذه المعادلة من قبل الباحث عبد الله (2012) لتحديد تأثير اسنان التروس الغير متناظرة على إجهاد الانحناء. ومن ثم تم تطبيق هذه المعادلات مع عامل تركيز الإجهاد مرة للاسنان المتناظرة ومرة أخرى للاسنان الغير متناظرة في هذه البحث، يتم حساب الضغوط الانحناء العظمى لأنواع مختلفة من التروس الحلزونية مع أنواع مختلفة من الأسنان غير المتماثلة عدديا للتنبؤ بعامل تركيز الإجهاد على أساس تقنية

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أسلوب العناصر المحدودة باستخدام برنامج المحاكاة سوليدووركس 2016 و مقارنتها مع المسننات المتناظرة الاكثر شيوعاً. النتيجة تشير الى ان المسننات المائلة التي لها اسنان بزوايا ضغط (14.5°) للجانب المحمل و (35°) للجانب غير محمل من السن هي الافضل من المسننات المائلة القياسية مع تخفيظ في قيم اجهادات الانحناء.
الكلمات الرئيسية: التروس الحلزونية الغير متماثلة, تحليل اجهاد الحناية, عامل تمركز الاجهادات , محاكاة سولودورك 2016.

1 INTRODUCTION

Gears are widely used in various industrial mechanisms and devices to transmit power and motion positively (without slip) between parallel, intersecting or non-intersecting nonparallel shafts, with or without a change in the direction of rotation and with or without a change in speed of rotation at any desired ratio. Conventional helical gears are designed for transformation of rotation between a parallel axis in relatively smooth, silent operation large and higher operating speed this that's lead to increase in load carrying capacity and this is cause it currently being using in the most industrial application as a power transmitting gear. But Sufficiency in bending load carrying capacity is led to the failures due to bending stress are at critical section of a gear tooth, **Litvin, et al., 2007**.

The stress concentration factor plays a great role in gear design where is represented the ratio between the numerical or experimental value of maximum tooth bending stress to the analytical value of nominal tooth bending stress and by it can control the magnitude of stress due to the geometrical shape of teeth, **Alemu, 2007.** In this study, gear involute teeth profiles have been modified by a change in the form of the geometrical shape of teeth in order to obtain reduction-bending stress. This modifier is represented by the asymmetric involute tooth which has load side of pressure angle (14.5°) and unloaded side of pressure angle (35°). Also, the key geometrical parameters that be constant of cases studied such as module 7 mm, tooth face width 60 mm and the same number of teeth on the pinion and gear $z_1 = z_2 = 14\ teeth$ and (speed ratio $\mu = 1$). Also, In this work has been studied five categories the first is reference standard case having symmetric teeth profiles with loaded and unloaded pressure angles ($14.5^{\circ}/\ 14.5^{\circ}$) and the other is non-standard cases having asymmetric teeth profiles with loaded side pressure angles 14.5° for unloaded side and various pressure angles variation from 20° to 35° for unloaded side spur gears and helical gears with several helix angles varied from 7.5° to 45° in each steps of 7.5° on the performance of these gears as shown in **Table 1.** all the details of these cases studied. This study included theoretical analysis and numerical analysis. All the results of bending stress by analytical or numerical method are depended on applied the same external normal load which equal to 2400 N . In addition to , Aluminum alloy AA6061 has been used in this study, with modulus of elasticity , E = 68100 N/mm^2 .

2 ANALYTICAL ANALYSIS METHOD

2.1 Lewis Equation

The bending stress is considered the primary parameters in the analysis of helical gear. The fracture failure will occur when the maximum bending stress at the root fillet of the tooth is greater than or equal to the yield strength of the gear tooth material. A formula was developed by Wilfred Lewis, **Niels L. Pedersen 2010.** for users to predict the bending stress in gears. This formula equation is used to estimate the bending stress in spur gear teeth and is by far the easiest method in order to get reasonable results for bending stress in helical gear. The assumptions which were simulated the bending stress acting on the gear was by bending of a cantilever beam as shown in **Fig 1.** And the Lewis equation is shown below:

$$\sigma_b = \frac{F_t}{b.m_0.Y} \tag{1}$$

Where : F_t is the tangential load induces bending stress which that tends to fracture.

b : is the face width of the gear.
m_0: is the module of the gear.
Y: is represented the Lewis form factor which play a major role for calculating the bending strength of gears and has a equation as shown:

$$Y = \frac{1}{6m_0}\left(\frac{t^2}{h}\right) \quad (2)$$

The value of (Y) is obtained from standard tables.

2.2 JGMA Equation

Japanese Gear Manufacturer Association (JGMA**), Manak** and **Zafar, 2007**, inserted a special factor called helix angle factor and is denoted by (Y_β). This factor is calculated the effect of helix angle on tooth bending stress for helical gear which that have standard teeth profile. it is done by calculate the 'stress inside the tooth profile of a helical gear and put its as preparatory value in order to find the tooth-root stress with study the effect each of helix angle and overlap ratio on in these cases that have the oblique direction of the lines of mesh contact which was defined as follows:

If $\epsilon_\beta > 1$ and $\beta \le 30^\circ$, then

$$Y_\beta = 1 - \frac{\beta}{120} \quad (3)$$

If $\epsilon_\beta > 1$ and $\beta > 30^\circ$, then

$$Y_\beta = 0.75$$

If $\epsilon_\beta \le 1$ and $\beta \le 30^\circ$, then

$$Y_\beta = 1 - \epsilon_\beta . \frac{\beta}{120} \quad (4)$$

If $\epsilon_\beta \le 1$ and $\beta > 30^\circ$, then

$$Y_\beta = 1 - 0.25\,\epsilon_\beta \quad (5)$$

Where
ϵ_β : is axial overlap ratio.
The modified Lewis equation is shown below:

$$\sigma_b = \frac{F_t . Y_\beta}{b . m_0 . Y} \quad (6)$$

2.3 Equation M.Q, Abdullah.

Abdullah, 2012, presented an analytical method based on a trial graphical method to find the solution of bending stress for symmetric and asymmetric involute gear teeth for different gear design parameters. The same assumptions that considered from Lewis for asymmetric tooth have been adopted. Therefore; based on **Fig 2.** the tensile stress that is generated at the root of the symmetric and asymmetric tooth profile due to bend can be calculated using the following expression:

$$\sigma'_{bending} = \frac{M' . y'}{I'} \quad (7)$$

$$M' = F_t . h' - F_r . e \quad (8)$$

$$F_r = F_t \tan(\beta_l) \quad (9)$$

$$y' = \frac{t'}{2} \quad (10)$$

$$I' = \frac{b.t'^3}{12} \quad (11)$$

$$e = \frac{(t_u - t_l)}{2} \quad (12)$$

$$t' = (t_l + t_u) \quad (13)$$

M' : is the bending moment at the weakest section of asymmetric tooth.
I' : is the moment of inertia of the weakest section.
t' : is the total thickness of the weakest section.
t_l & t_u : are the weakest section thicknesses of the loaded and unloaded tooth sides respectively, with a radial load position (R) and with tooth design parameters of loaded side (ϕl & rfl) and for unloaded side (ϕu & rfu) respectively, with taking into account that the resulted value of (t) from this equation must be divided by (2).
h' is the load height from tooth critical section with a radial load position.
β_l : is the loading angle of loaded tooth side.

$$\sigma'_{bending} = \frac{F_t}{b.m_0.Y'} \quad (14)$$

$$Y' = \frac{1}{3.m_0}\left[\frac{(t_l + t_u)^2}{2h_l - (\tan(\beta_l)).(t_u - t_l)}\right] \quad (15)$$

Eq.(14) and Eq.(15) represent the developed Lewis bending stress represent and the form factor for asymmetric gear respectively.

In this study, Lewis bending stress equation, JGMA bending stress equation are used to calculate the bending stresses for symmetric involute tooth profile of spur and helical gears. Also, the Lewis stress modified by Abdullah equation (15) will use to calculate the bending stress for asymmetric involute tooth profile for spur gears. The Abdullah equation is modified to find the bending stress for asymmetric involute tooth profile for helical gears at a specified number of teeth that used in present paper. The modify based upon stress concentration factor results for different cases as shown in **Table 2.** that shows the geometrical properties of the gears tooth shape which that used in this work.

3 NUMERICAL ANALYSIS METHOD

In this study, the numerical method is used to analyze the stresses distributed in gear tooth profile. The bending strength of helical teeth is much more complex to determine because the contact line is placed at an angle on the 'tooth-working surface'. This problem may be solved by using the numerical analysis. FEA is a numerical method interpolate an approximate solution to boundary value problem. This technique that was done by the software SOLIDWORKS 2016 simulation. As for the generation process of helical gear have been created by using the software SOLIDWORKS 2016.

3.1 Helical Gear Modeling

A helical gear which that used in this work is modeled by using the software SOLIDWORKS 2016 program. The parametric equations of the involute curve are used to represent the involute tooth profile. The length of the involute curve for any portion can be determined by using the usual parametric equation of the involute curve which follows down:

$$x = R_b(\cos(\varphi) + \varphi * \sin(\varphi)) \quad (16)$$

$$y = R_b(\sin(\varphi) - \varphi * \cos(\varphi)) \quad (17)$$

Where " φ " is interval over which circles are drawn and the tooth profile is equation driven. To drawing the gears tooth profile using a program was built into SOLIDWORKS 2016. This technique depends on manage equations that will have been used to management involutomentry relationships and gear dimensions. The helical models of the based on **Table 1.** contains the specification was made in SOLIDWORKS 2016 modeling package.

3.2 Numerical Analysis Of Bending Stress

The 3D finite element method analysis is used to calculate the maximum bending stresses by using SOLIDWORKS 2016 simulation package. In this analysis has been chosen the static stress with linear material properties. In addition to that, the model three teeth of gear have been prepared for this study in order to the analysis of tooth bending stress. A mesh convergence study is an important tool that should be used in any finite element model to determine a solid mesh type with element type and number of elements and also to determine when the mesh density is sufficient enough to provide accurate results. Tetrahedral elements have been used to model solid objects and their element number is different from each model for helix angle. This of the type of tetrahedral elements supports three translational degrees of freedom available in higher-order formulation. In order to create a denser mesh around the vulnerable areas of the model should find zones of the maximum bending stress. Because of the maximum tooth bending stress will occur at the root fillet region of the tooth, these are the areas where the extreme stresses. So, the mesh size of an element of absolute type has been reduced around those zones and leave the coarse mesh for the other areas. And then to generate 3D-mesh for this model, the current mesh consists of element of size that equal (2.06393 mm), total number of elements that equal (89994) and to active the mesh control, the mesh size of surface refinement on the tooth root zones that equal (1.16103mm) have been selected depending on the convergence test, as shown in **Fig 3.**

In this work, the external normal force applied at the tip of the tooth and which is equal 2400 N. Also, the models were constrained with the boundary condition, where The surface on the two sides of the three teeth of the gear rim is considered as a fixed constraint. And apply surface pin constraints to cylindrical surfaces for a model of the three teeth for gear as fixed pin constraint. The 3D finite element method which analysis of gear teeth are applied to all other models with the same loading and boundary conditions.

4 RESULTS AND DISCUSSIONS.

4.1 Results FEA For Tooth Bending Stress

The aim of numerical solution which represents by FEA, to estimate the maximum tooth bending stresses that will distribute inside the tooth profile of gears**.** And also, that the 'bending strength' of helical gear teeth is much more complex to calculate because the contact line occurs at an angle on the tooth-working surface, **Klebanov, et al., 2007.** This problem may be solved by using FEM. In this study, to estimate the tooth bending stress by through The maximum tensile stresses that will occur in the root of the helical tooth. In **Fig 4.** and **Fig 5** the bending stresses distribution inside gear tooth profile for spur and helical gear with helix angle equal to 45° are shown.

These maximum tensile stresses occur due to the tangential force acts at the tipping point of the tooth. **Table 3.** shows the results of FEM for all cases that used in this work. The relationship between the maximum bending stresses with helix angles will show in **Fig 6.** This figure will be noted There is a reduction in the tooth bending stresses whereas the stresses for asymmetric tooth profile helical gears at all the helix angle will be decreased with increased the helix angle. Also, The **Fig 7.** shows the relationship between the enhancement percentage with helix angle. This enhancement percentage is compared with the standard of reference case**.** It's clear that the enhancement percentage in asymmetric involute of tooth gear with the reference standard case is increased with increasing the helix angle for each type of asymmetric gears. conceptually, enhancement in tooth bending stress is due to increase in the tooth weakest section thickness caused by using asymmetric tooth profile. And the maximum enhancement percentage is asymmetric helical gear has unloaded sided pressure angle (35°) and loaded sided pressure angle (14.5°) with helix (45°).

4.2 Results Of Analytical Solution

The analytical results for nominal tooth bending stresses have been calculated by using Lewis equation and JGMA equation for each type of gears such as standard gear 'symmetric tooth profile' and modified gears 'asymmetric tooth profile' for spur and helical gears with several different helix angles. On the other hand, Eq.(14) is used to calculated tooth bending stresses for asymmetric spur gears only that will achieve after calculating the tooth loading angle, tooth weakest thickness, and load height and thereby, Lewis form factor was got. According to the nominal tooth bending stress that has been calculated from analytical results and with FEA results which represent the maximum tooth bending stress that will determine the value of stress concentration factor for symmetric and asymmetric teeth for spur and helical gears with several different helix angles. The formula that used to evaluate the value of stress concentration factor is:

$$K_t = \frac{\sigma_{\mathrm{bending}_{\mathrm{max}}}}{\sigma_{\mathrm{bending}_{\mathrm{nom}}}} \tag{18}$$

The comparison between the relationship of the nominal stress value which that was calculated by Lewis equation, JGMA equation, and Abdullah equation as showed in **Fig 8.** It was noted that the tooth bending stress was improved with increasing the helix angle for each type of gears.

4.3 Curve Fitting of Stress Concentration Factor

Macro geometry is represented by gear parameters such as unloaded pressure angle and helix angle, performs a very important role in gear design process. In general, unexpected change in the cross-section may act as the stress raiser leading to stress concentration and increases the amount of localized stress or can decrease the transmission error and smooth engaging and much more. In **Table 4.** that showed the comparison between the value of tooth stress concentration factor for spur gear with respect to the reference case study (No.1) that when the load acts at the tip of the tooth. According to the maximum bending stress value for spur and helical gears with several different helix angles which that used and with respect to the nominal stress of the reference case study (No.1) can be evaluated the stress concentration factor for helical gears as shown below.

It has been noticed that the stress concentration factor behaves nonlinearly with respect to the design unloaded pressure angle and helix angle. Where that the effect appears to be nonlinear of second degree so that a direct nonlinear relationship may describe the stress concentration as:

$$K_t(\emptyset_u, \beta) = a_0 + a_1\emptyset_u + a_2\emptyset_u^2 + a_3\beta + a_4\beta^2 \tag{19}$$

The difference (residual) between the true value of $K_t(\emptyset_u, \beta)$ and the polynomial approximation is:

$$c_j = K_t(\emptyset_u, \beta) - (a_0 + a_1\emptyset_u + a_2\emptyset_u^2 + a_3\beta + a_4\beta^2) \tag{20}$$

Hence, the sum of squared difference for n of data points is

$$S_r = \sum_{i=1}^{n} (c_j)^2 \tag{21}$$

From Eq.(20) and Eq.(21),

$$S_r = \sum_{i=1}^{n} (K_t(\emptyset_u, \beta) - (a_0 + a_1\emptyset_u + a_2\emptyset_u^2 + a_3\beta + a_4\beta^2))^2 \tag{22}$$

Should be reliable values of coefficients a1, a2, a3, a4, a5 must tune S_r to be as minimum as possible which is based on the principle of least square regression method using non-multi-polynomial regression. Eq. (22) will be partially derived five times to find the value of the coefficient.

$$\frac{\partial S_r}{\partial a_g} = \frac{\partial}{\partial a_g}\sum_{i=1}^{n}\left(K_t(\emptyset_u, \beta) - \left(a_0 + a_1\emptyset_u + a_2\emptyset_u^2 + a_3\beta + a_4\beta^2 \right)\right)^2 = 0 \qquad (23)$$

Where subscript k ranges from 0 to 4 and identifies the coefficients. five simultaneous nonlinear equations are inferred from Eq. (23) that expressed in matrix form as:

$$[A]\bar{a} = \bar{c} \qquad (24)$$

Eq. (24) could be expanded to:

$$\begin{bmatrix} n & \sum_{i=1}^{n}\emptyset_u & \sum_{i=1}^{n}\emptyset_u^2 & \sum_{i=1}^{n}\beta & \sum_{i=1}^{n}\beta^2 \\ \sum_{i=1}^{n}\emptyset_u & \sum_{i=1}^{n}\emptyset_u^2 & \sum_{i=1}^{n}\emptyset_u^3 & \sum_{i=1}^{n}\emptyset_u\beta & \sum_{i=1}^{n}\emptyset_u\beta^2 \\ \sum_{i=1}^{n}\emptyset_u^2 & \sum_{i=1}^{n}\emptyset_u^3 & \sum_{i=1}^{n}\emptyset_u^4 & \sum_{i=1}^{n}\emptyset_u^2\beta & \sum_{i=1}^{n}\emptyset_u^2\beta^2 \\ \sum_{i=1}^{n}\beta & \sum_{i=1}^{n}\emptyset_u\beta & \sum_{i=1}^{n}\emptyset_u^2\beta & \sum_{i=1}^{n}\beta^2 & \sum_{i=1}^{n}\beta^3 \\ \sum_{i=1}^{n}\beta^2 & \sum_{i=1}^{n}\emptyset_u\beta^2 & \sum_{i=1}^{n}\emptyset_u^2\beta^2 & \sum_{i=1}^{n}\beta^3 & \sum_{i=1}^{n}\beta^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n}K_t \\ \sum_{i=1}^{n}\emptyset_u K_t \\ \sum_{i=1}^{n}\emptyset_u^2 K_t \\ \sum_{i=1}^{n}\beta K_t \\ \sum_{i=1}^{n}\beta^2 K_t \end{bmatrix} \qquad (25)$$

By using data from numerical solution and Eq.(25):

$$\begin{bmatrix} 14 & 385 & 11375 & 315 & 10237.5 \\ 385 & 11375 & 356125 & 8662.5 & 281531.25 \\ 11375 & 356125 & 11624375 & 255937.5 & 8317968.8 \\ 315 & 8662.5 & 255937.5 & 10237.5 & 372093.75 \\ 10237.5 & 281531.25 & 8317968.8 & 372093.75 & 14396484 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 15.628 \\ 419.633 \\ 12140.433 \\ 323.632 \\ 10129.022 \end{bmatrix} \qquad (26)$$

By solving the matrix, get the following equation:

$$K_t(\emptyset_u, \beta) = 2.35 - 0.073\,\emptyset_u + 0.00095\emptyset_u^2 - 0.004124\,\beta - 0.0000743\beta^2 \qquad (27)$$

$$\sigma_{bending_{nom}} = \frac{\sigma_{bending_{max}}}{K_t(\emptyset_u, \beta)} \qquad (28)$$

Fig 9. shows the relationship between the stress concentration factor with helix angle for the standard has pressure angle ($14.5^\circ/14.5^\circ$) for both sides and different types of asymmetric spur and helical gears have pressure angle (14.5°) loaded side and several different pressure from (20°) to (35°) for the unloaded side, when the load applied the tip of the tooth.

5 CONCLUSIONS

1- The tooth bending stress was improved with increasing the helix angle for the symmetric and asymmetric tooth of the gear.
2- Derive a new equation bounding tooth Stress Concentration Factor based the relationship between the unloaded pressure angles with helix angle for helical gear.
3- The optimum helix angle of 45° has the highest enhancement percentage of maximum tooth bending stress for each type of gears as when the applied load acts the tip of the tooth.
4- Helical gear which has asymmetric teeth profiles with unloaded side pressure angle (35°) and loaded side pressure angle (14.5°) is better than each type of asymmetric that used in this work and from the standard reference gear with pressure angle (14.5°) for both sides for a certain helix angle that has been from the point of view of tooth bending strength.

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NOMENCLATURES

b	tooth face width (mm)
E	modulus of elasticity (N/mm)
F	normal applied force at the tip of tooth (N)
h_a , h_d	addendum and dedendum heights (mm)

m_o	module (mm)
P_c	circular pitch (mm)
r_f	fillet radius (mm)
R_a	radius of addendum circle (mm)
R_b	radius of base circle (mm)
R_d	radius of dedendum circle (mm)
R_p	radius of pitch circle (mm)
Z	number of gear teeth
β_l	Tooth loading angle of the normal applied load (degree)
β_u	Tooth unloading angle of the normal applied load (degree)
β	helix angle (degree)
v	poisson's ratio
$\emptyset_l$, $\emptyset_u$	pressure angles for loaded and unloaded sides (degree)

Table 1. Cases studies.

Case No.	$\emptyset_u$	$\emptyset_l$	h_a	h_d	r_f
1	14.5°	14.5°	m_0	$1.157m_0$	$0.209m_0$
2	20°	14.5°	m_0	$1.25m_0$	$0.39m_0$
3	25°	14.5°	m_0	$1.25m_0$	$0.39m_0$
4	30°	14.5°	m_0	$1.25m_0$	$0.39m_0$
5	35°	14.5°	m_0	$1.25m_0$	$0.39m_0$

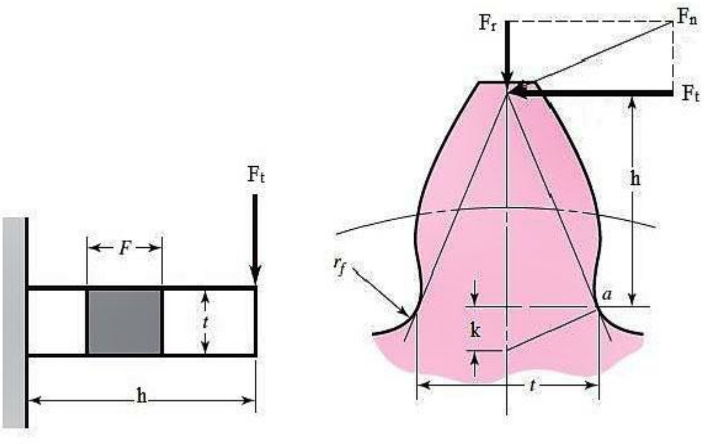


Figure 1. Beam strength of gear teeth.

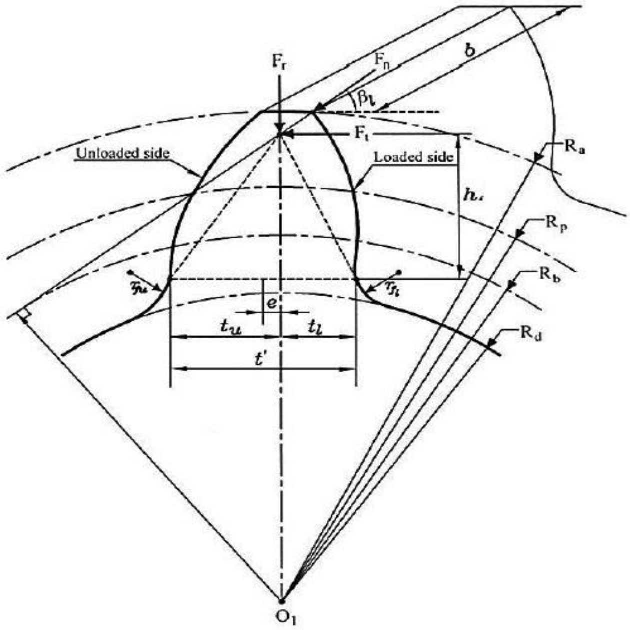
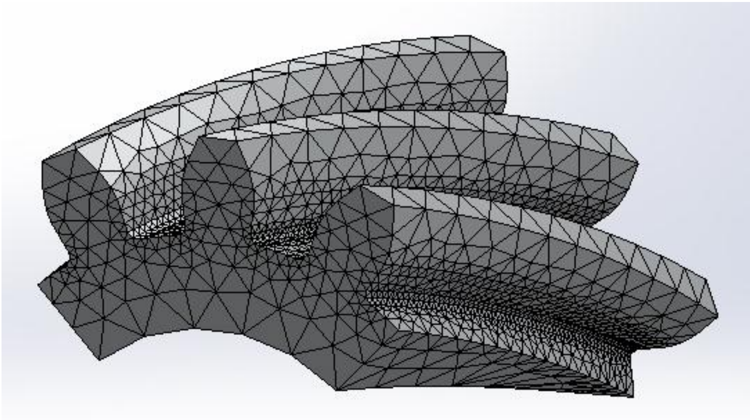


Figure 2. Estimation of weakest section for asymmetric tooth.

Table 2. The geometrical properties for spur gear tooth profile.

Case study	Symmetric Tooth Profile 14.5/14.5	Asymmetric Tooth Profile 20/14.5	Asymmetric Tooth Profile 25/14.5	Asymmetric Tooth Profile 30/14.5	Asymmetric Tooth Profile 35/14.5
β_l	29.194	29.194	29.194	29.194	29.194
β_u	29.194	32.378	35.868	39.829	44.194
t_l	4.989	5.0745	5.0745	5.0745	5.0745
t_u	4.989	5.466	6.120	7.7245	8.1111156
h_l	12.743	12.6105	12.6105	12.6105	12.6105
h_u	12.743	12.687	13.343	14.221	15.112
Y	0.186	–	–	–	–
Y'	–	0.2116	0.242	0.3285	0.3519



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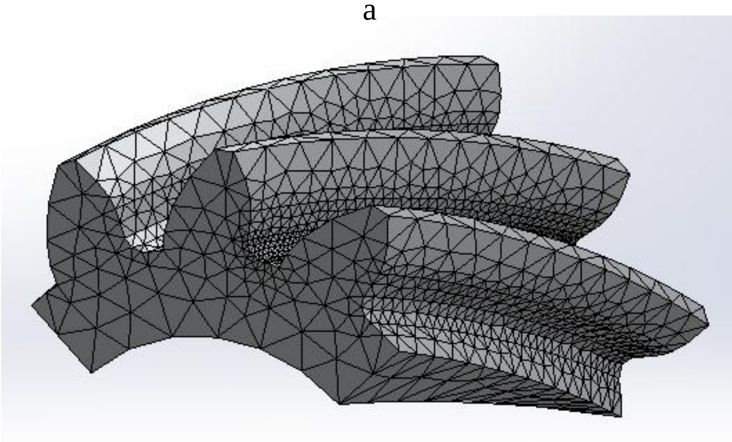
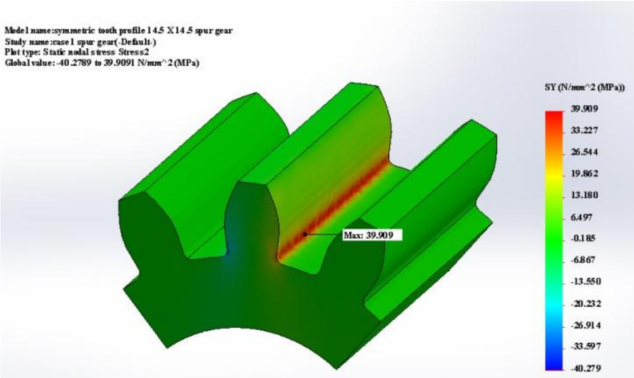


b

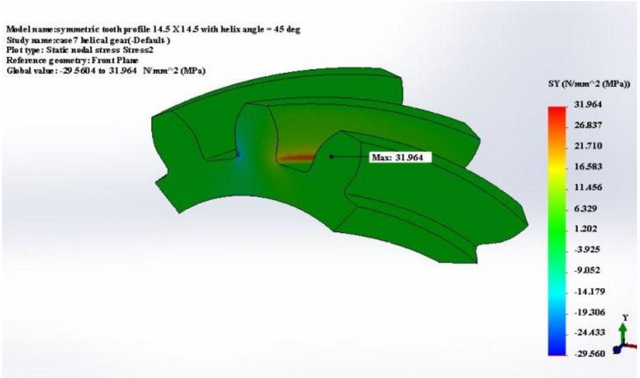
Figure 3. Three dimension model with mesh for helical gear (a) standard gear with pressure angle ($14.5^{\circ}/ 14.5^{\circ}$) for both sides with helix angle 45°. (b) asymmetric gear with pressure angle (35°) for unloaded side and (14.5°) for loaded side with helix angle 45°.

Table 3. The numerical results of maximum bending stress for helical gears, when the load acts at the tip of the tooth.

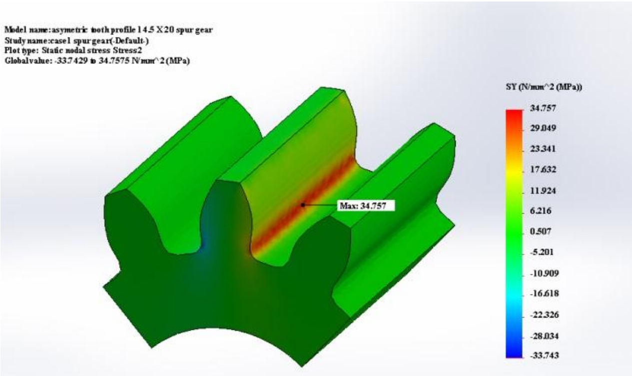
Helix angle β	$\sigma_{bending/\ numerical}$ N/mm^2				
	Symmetric gear	Asymmetric gear			
	$14.5^{\circ}/ 14.5^{\circ}$	$20^{\circ}/ 14.5^{\circ}$	$25^{\circ}/ 14.5^{\circ}$	$30^{\circ}/ 14.5^{\circ}$	$35^{\circ}/ 14.5^{\circ}$
0	39.909	34.757	30.154	27.719	26.122
7.5	38.710	33.481	28.878	26.443	24.846
15	37.635	32.3198	27.716	25.281	23.684
22.5	36.468	31.106	26.503	24.068	22.471
30	34.965	29.708	25.105	22.670	21.073
37.5	33.461	27.459	22.856	20.421	18.824
45	31.964	25.210	20.607	18.172	16.575



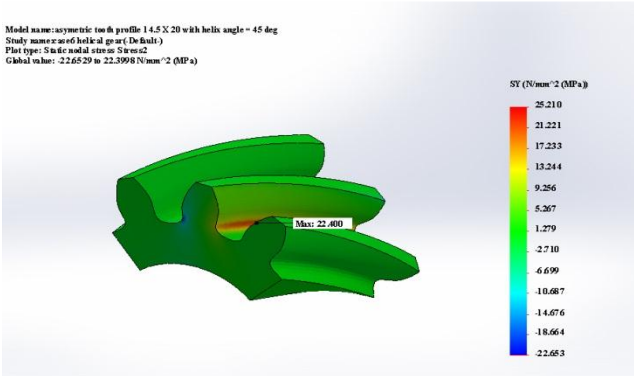
symmetric tooth profile with 14.5 / 14.5



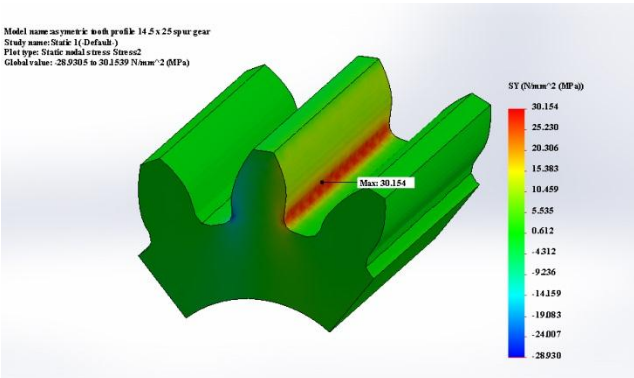
symmetric tooth profile with 14.5 / 14.5 and helix angle 45



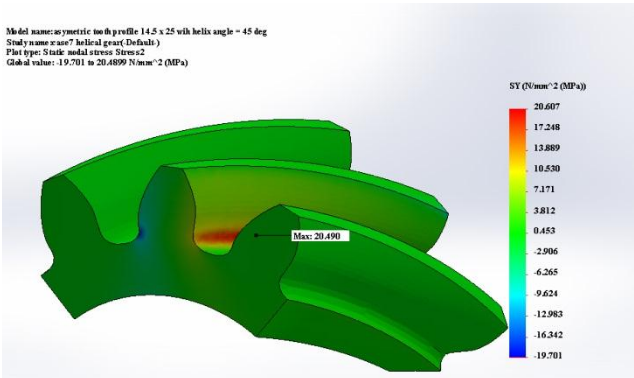
asymmetric tooth profile with 20 / 14.5



asymmetric tooth profile with 20/14.5 and helix angle 45

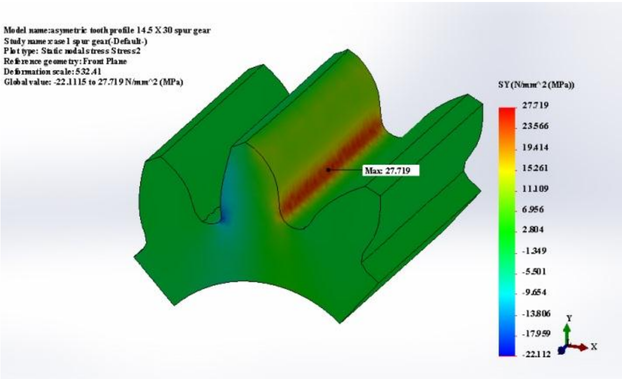


asymmetric tooth profile with 25/14.5

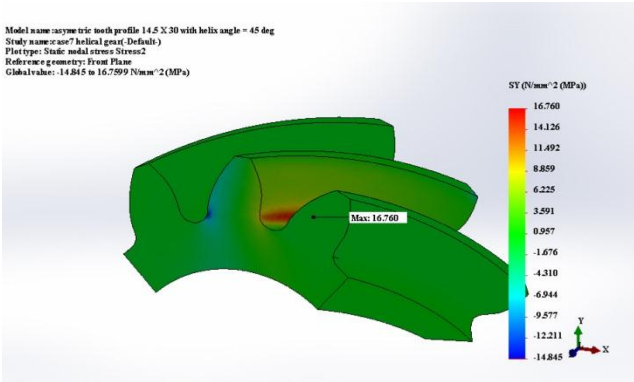


asymmetric tooth profile with 25/14.5 and helix angle 45

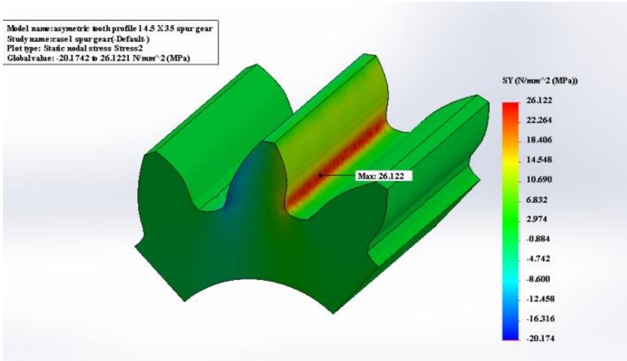
Figure 4. Three dimension bending stresses inside tooth of spur and helical gear when the load acts at the tip of tooth.



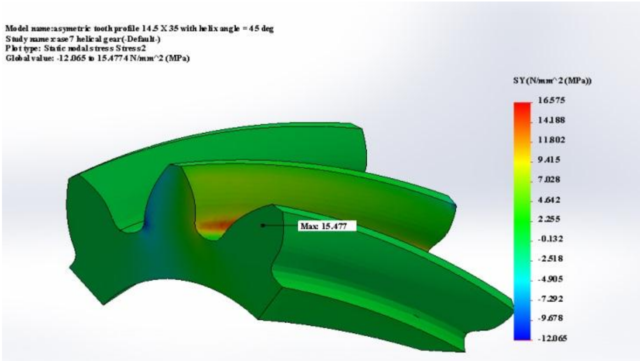
asymmetric tooth profile with 30/14.5



asymmetric tooth profile with 30/14.5 and helix angle 45



asymmetric tooth profile with 35 / 14.5



asymmetric tooth profile with 35 / 14.5 and helix angle 45

Figure 5. Three dimension bending stresses inside tooth of spur and helical gear when the load acts at the tip of tooth.

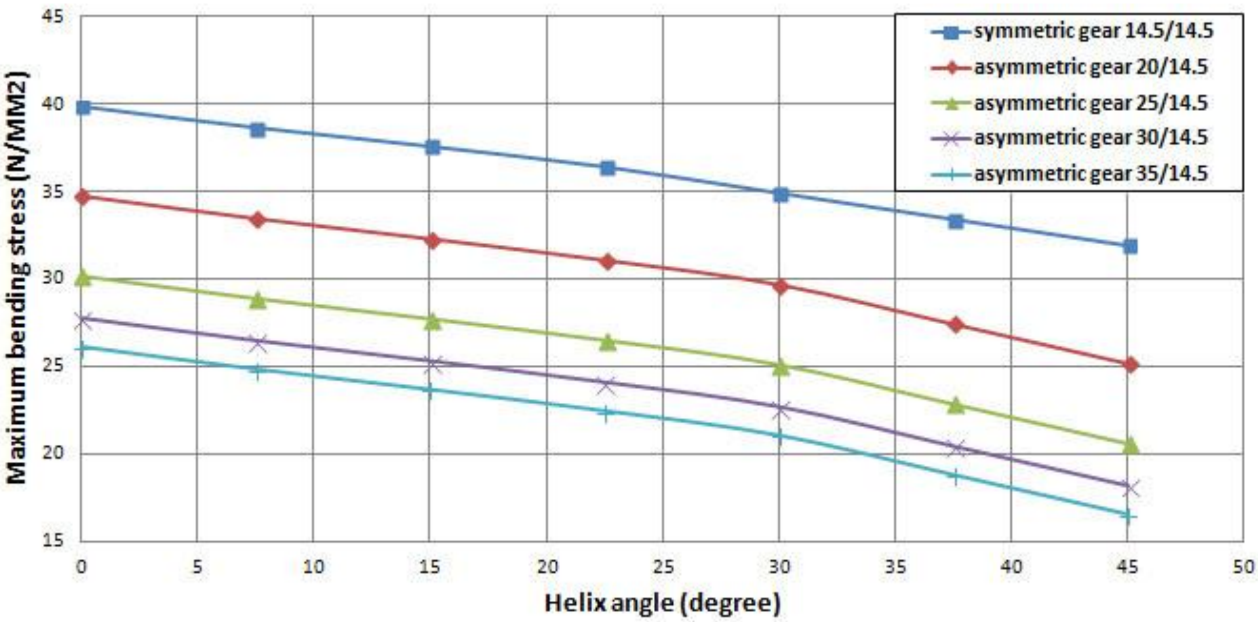


Figure 6. The relationship between the maximum bending stresses with helix angle.

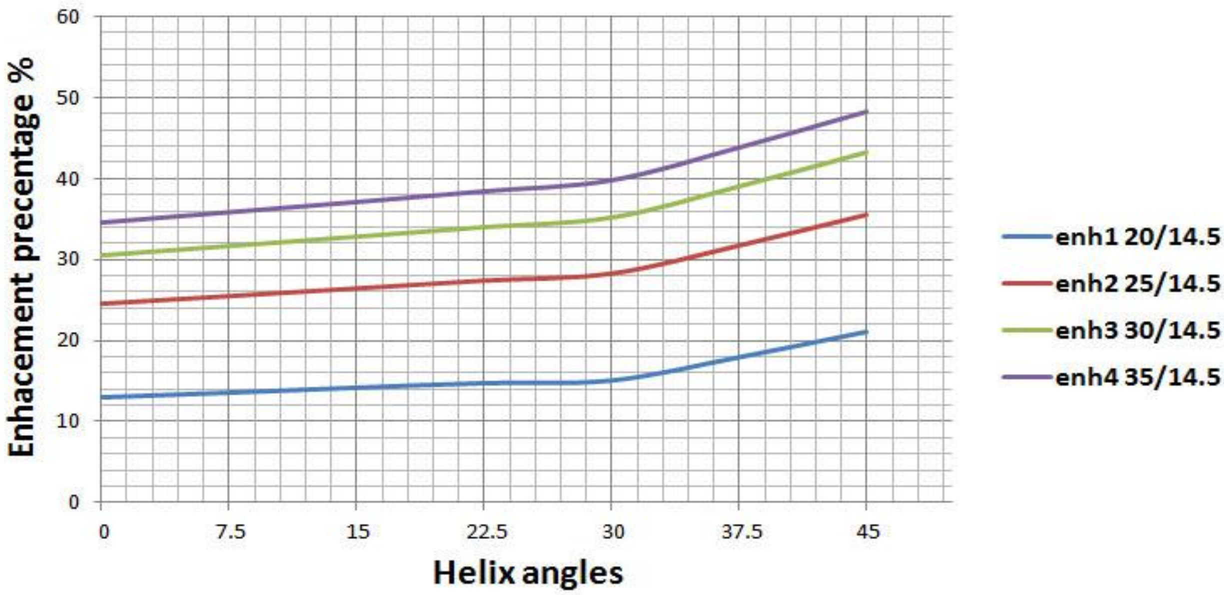


Figure 7. The relationship between the enhancement percentage with helix angle that compare with the reference case No1.

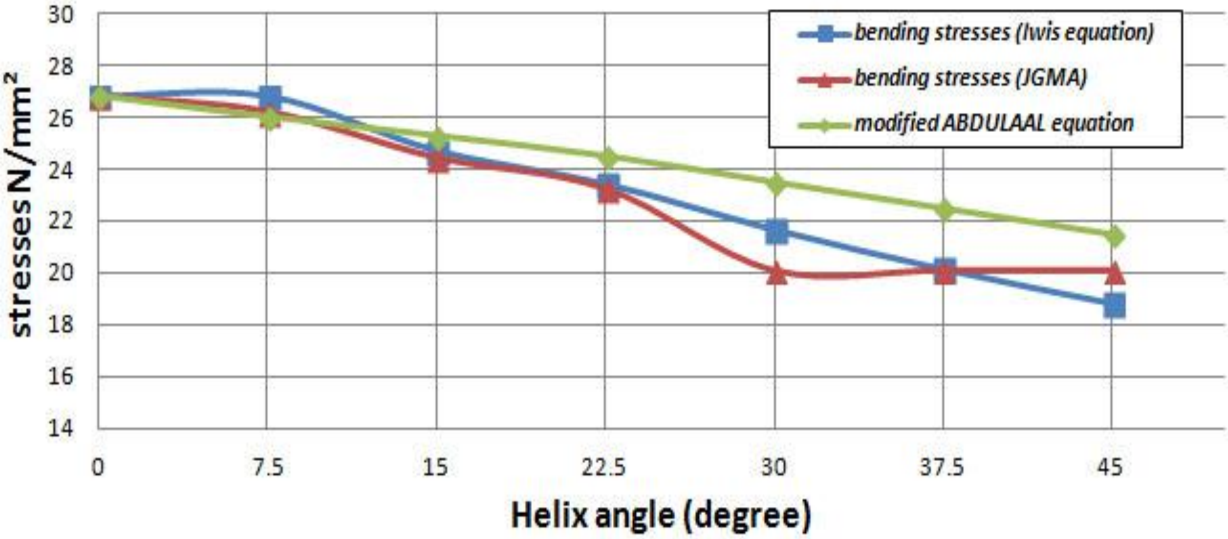


Figure 8. The relationship between the nominal bending stresses with helix angle.

Table 4. Results of relative tooth stress concentration factor with respect to the reference.

Helix angle	K_t relative				
	$14.5^{\circ}/14.5^{\circ}$	$20^{\circ}/14.5^{\circ}$	$25^{\circ}/14.5^{\circ}$	$30^{\circ}/14.5^{\circ}$	$35^{\circ}/14.5^{\circ}$
0	1.488531	1.296371	1.124688	1.0338667	0.9743016
7.5	1.44381	1.24878	1.0770952	0.9862743	0.92670919
15	1.4037148	1.20544	1.033755	0.942934	0.8833688
22.5	1.36018798	1.160195	0.9885122	0.8976912	0.838126142
30	1.304128902	1.108053	0.9363694	0.8455485	0.78598336
37.5	1.2480325	1.0241692	0.85248592	0.76166498	0.702099884
45	1.192197232	0.940286	0.768602439	0.67778151	0.618216404

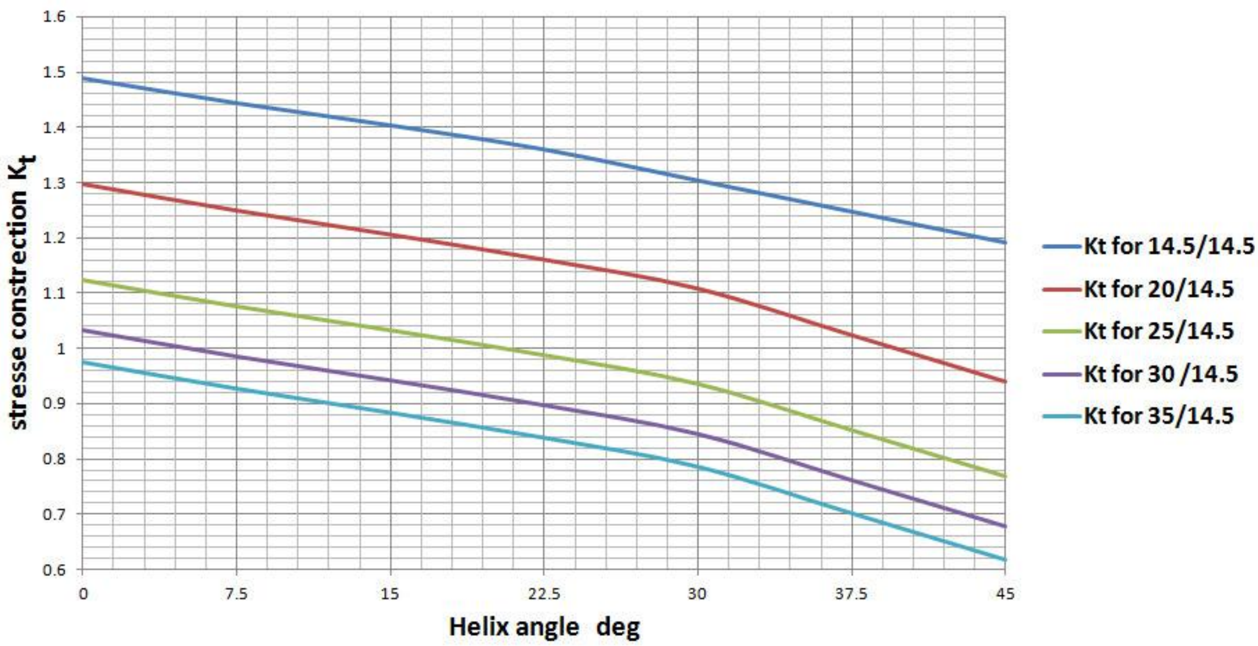


Figure 9. The relationship between the K_t with helix angle.