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Effectiveness of Meso-Scale Approach in Modeling of Plain Concrete Beam

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ABSTRACT

The main aim of this research paper is investigating the effectiveness and validity of Meso-Scale Approach (MSA) as a modern technique for the modeling of plain concrete beams. Simply supported plain concrete beam was subjected to two-point loading to detect the response in flexural. Experimentally, a concrete mix was designed and prepared to produce three similar standard concrete prisms for flexural testing. The coarse aggregate used in this mix was crushed aggregate. Numerical Finite Element Analysis (FEA) was conducted on the same concrete beam using the meso-scale modeling. The numerical model was constructed to be a bi-phasic material consisting of cement mortar and coarse aggregate. The interface between the two consisting materials was assumed fully bonded interface. In the ABAQUS program, the Extended Finite Element Method (XFEM) was employed for the treatment of the discontinuity problems, which is accompanied by cracking during the fracture process of plain concrete. The behavior and response of the beam in both meso-scale numerical analysis and experimental test were found in a good agreement. Another check was added by comparing the results using thin-beam theory assuming the concrete as a homogenous linear-elastic material. The result of this comparison showed that the meso-scale model analysis lies between theoretical and experimental models.

Key Words: concrete fracture mechanics, meso-scale modeling, Extended Finite Element Method, flexural analysis.

فعالية النمذجة المتوسطة المدى للعتبات الخرسانية غير المسلحة

الهدف من هذا البحث هو التحقق من فعالية وصلاحية منهج ذي مقياس متوسط المدى (Meso-Scale Approach) كتقنية حديثة لنمذجة الاعتاب الخرسانية .تم تحميل عتب خرسانية في حالة اسناد بسيطة عند نقطتين للكشف عن استجابية قوى الانحناء.تجريبيا ، تم تصميم وتحضير خلطةخرسانةلإنتاج ثلاثة أعتاب خرسانية قياسية متشابهة لاختبار الانحناء .واستعمل الركام الخشن المكسر في هذه الخلطة. اجري التحليل العددي بأستعمال طريقة العناصر المحددة (FEA) على نفس العتب الخرسانية بأستعمال النمذجة المتوسطة المدى .تم بناء النموذج العددي للخرسانة على شكل مادة ثنائية الطور تتألف من مونة السمنت والركام الخشن .التماس بين الركام الخشن ومونة السمنت افترض في حالة تماسك كامل مع بعضهما البعض. في برنامج الـ(ABAQUS) استعملت طريقة العناصر المحددة الموسعة (XFEM) لعلاج مشاكل اللاستمرارية ، والمصاحبة التشقق أثناء مرورالخرسانة مرحلة التكسير. تم الحصول على سلوك واستجابة للعتب الخرسانية في اتفاق جيد في كل من التحليل العددي على نطاق متوسط المدى والاختبار التجريبي. وقد تم إضافة فحص آخر للنتائج بمقارنتها مع النتائج النظرية للاعتاب النحيفة على افتراض ان الخرسانة مادة متجانسة ومرنة ذات تصرف خطي. أظهرت نتائج هذه المقارنات أن نتائج تحليل نموذج المقياس المتوسط المدى تتوسط بين النتائج النظرية والتجريبية.

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1. INTRODUCTION

Concrete is the most common material used for the structural engineering work. It mainly consists of cement mortar and aggregate. Most of the concrete studies are in the macro scales for the simplicity in analyses and tests. The structural behavior of concrete is a result of the behavior of its components. The mechanical behavior of the materials is a result from the behavior of the smallest particle in the material, which might be in the atomic scale and gradually to the bigger particles at the macro scale of the material. Between the atomic and the macro scales, there are many analysis scales such as the micro and the meso-scales. For the better and accurate understanding of the mechanical behavior of the materials, these small effects of the small particles should be studied, that give the macro results seen by the naked eye.

The behavior of the concrete was studied in the meso-scale, which is a scale that falls between the macro and micro scales **Murayama, 2001**. In general, the meso-scale modeling can be categorized into two types: continuum models and lattice models. In the continuum model, concrete is considered as a composite material consisting of coarse aggregate particles, cement mortar, and the interface between them, while in the lattice model, concrete is considered as a discrete model consisting of lattice elements, where elements are sub-divided according to their geometry and location of either mortar, aggregate, or interface element**, Nitka, 2015**. The discrete method requires enormous effort for the construction of the meso-scale model of concrete. In this paper, concrete considered as a continuum composite material consists of coarse aggregate particles, cement mortar, and the interface between them. This type of modeling was adopted by **Huang, Chen, and Sun, 2014**, **Eftekhari, et al., 2014**, and **Rubin, et al**., **2015.** For simplicity, the interface between aggregate particles and cement mortar was considered as a fully bonded state i.e. the aggregate is fully bonded with the cement mortar and there is no fracture is expected between them.

The meso-scale modeling has two approaches, the image based method, and the parametrization modeling method. In the image-based method, modeling is done by taking a two-dimensional image of the concrete model that is assembled to get a three-dimensional numerical model**, Bentz, et al., 1994**. This approach is more expensive and time-consuming than the parametrization approach, where the concrete is randomly modeled with a suitable finite element mesh**, Wang, et al., 2016**. In the parametrization method, aggregate particles can be modeled in the stochastic system i.e. the shape, orientation, size, and location of the particles are randomly controlled in the cement mortar space, which is more compatible for the meso-scale modeling. In this paper, the parametrization approach is for the meso-scale modeling of concrete.

Like other brittle materials when concrete is subjected to external stresses near its strength limits, it will reach the fracture point. During the loading period, cracks are introduced in concrete and gradually propagate. There are many methods for the analysis of the concrete fracture mechanics. For example, the smeared crack method introduced by **Rashid, 1968** in which the tensile stress of the finite elements is limited to a specific value depending on the tensile strength of the concrete. When this tensile strength is reached, a stress relaxation in the element will be introduced and a crack is considered and a strain-softening is obtained. However, this method has a drawback, that is the results are affected by the element size and refining of the mesh**, Menin, et al., 2009**. Another common method is the re-meshing method**, Swenson, and Ingraffea, 1988**. In this method, a re-meshing is produced near the crack tip for every crack propagation step. This technique is uneconomic from the time-consuming point of view.

Mathematically the crack propagation and fracture analysis problems might be solved. In the last few decades, a numerical method was developed. Such method is the Partition of Unity (PU), which is first produced by **Melenk and Babuska, 1996**. In this method, a set of functions are defined on a certain domain, and the enrichment method, which is developed by **Gifford and Hilton, 1978**, depends on the enriching region. The displacement approximation in this method is considered to be the summation of the finite elements standard solution u_{std}, and the enrichment solution u_{enr}, as can be seen in Eq. (1) below:

$$u = u_{std} + u_{enr} \tag{1}$$

Belytschko, 1988 proposed an enrichment method that is specialized in the enriching of localized regions. The Extended Finite Element Method (XFEM) developed by **Belytschko and Black, 1999** is an enrichment method that is used for discontinuities problems such as crack propagation in concrete. It employs the PU technique for the numerical solution.

In this paper, plain concrete beam subjected to flexural stresses was analyzed numerically in the meso-scale model. Flexural stresses were produced by a two-point loading system. The XFEM was utilized for the discontinuity problem produced by the existing of the crack interface in the concrete's domain. ABAQUS program was used for the numerical modeling and analyzing. To examine the validity of numerical modeling, the results of the numerical model was compared with the experimental results of the same plain concrete beam subjected to the same loading. In addition, they were compared with the theoretical results of thin beam theory application assuming the concrete as a linear elastic material.

2. THE EXTENDED FINITE ELEMENT METHOD (XFEM)

The Extended Finite Element Method (XFEM), is a numerical technique used to solve the discontinuities problems that occur in brittle materials such as concrete**, Belytschko and Black, 1999**. These discontinuities might be a result of crack interface produced from the fracture of the material in a certain region of the domain, or might be a material interface that produced in a composite material located at the interface zone between the two materials, (aggregate particles and cement mortar)**, Khoei, 2015**.

The XFEM is based on the PU method. In spite the XFEM uses a localized enrichment function, an enrichment of nodes is developed nearby the discontinuity. The enrichment is done mathematically by the use of the enrichment functions. The basic equation of the XFEM solution is shown in Eq. (2), **Khoei, 2015**:

$$u(x) = \sum_{i=1}^{N} N_i(x)\bar{u}_i + \sum_{i=1}^{N} \bar{N}_i(x)\left(\sum_{j=1}^{M} p_j(x)\,\bar{a}_{ij}\right) \tag{2}$$

where;

$u(x)$: displacement of the domain.

N: the total number of nodes of the standard finite element domain.

$N_i(x)$: shape function of the standard nodes.

$\bar{u}_i$: degree of freedom of the standard nodes.

$\bar{N}_i(x)$: shape function of the enriched nodes.

$p_j(x)$: enrichment function.

$\bar{a}_{ij}$: degree of freedom of the enriched nodes.

M: number of enrichment nodes.

There are two major types of discontinuities in the concrete structure. The weak discontinuity and strong discontinuity, **Khoei, 2015.** The weak discontinuity is described by the interface between aggregate particles and cement mortar, while the strong discontinuity is described by the crack interface in the domain.

3. ENRICHMENT FUNCTIONS

The enrichment functions are the basis of the XFEM approximation. There are many types of enrichment functions, that are included in the solution according to their functionality. Such enrichment function is the Heaviside enrichment function *H(x)*, **Moe¨ s N., 1999**. This function is used for the strong discontinuities problems as shown below.

Let a domain Ω be considered with crack interface that split the domain into a positive region Ω^+ and a negative region Ω^- by the interface Γ as shown in **Fig. 1**. The Heaviside enrichment function for this domain can be illustrated in Eq. (3):

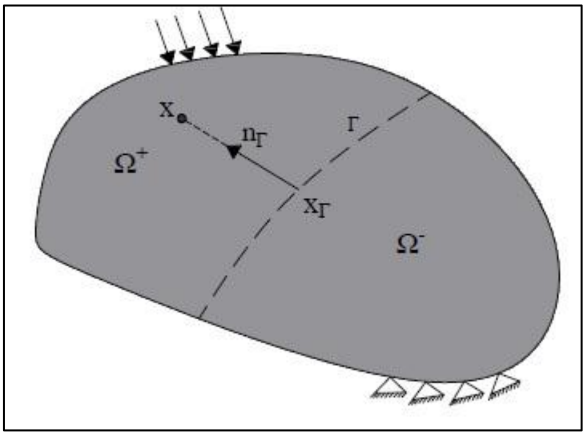

Figure 1. Body domain crossed by crack discontinuity.

$$H(x) = \begin{cases} 1 & x \in \Omega^+ \\ 0 & x \in \Omega^- \end{cases} \quad (3)$$

The most important matter in the analysis of the fracture mechanics is the representation of the crack geometry in the domain. The XFEM facilities this representation by the use of the Level Set Method (LSM), which is a function used for the tracking of the crack and the crack tip geometry**, Sethian, 1999**. The basic idea of the LSM is to define a level set function *φ(x)* that the discontinuity is at zero level set function. The LSM can be defined in Eq. (4) shown below:

$$\varphi(x) = \| x - x_\Gamma \| \, sign(n_\Gamma(x - x_\Gamma)) \quad (4)$$

where x_Γ is the projection of the point x on the interface, and n_Γ is the vector from the point x to the interface. The symbol || || denotes the distance between x_Γ and x, the term sign represents a function that takes three values (-1), (0), or (1). Eq. (4) shown above is also called the signed distance.

There is another technique for enrichment that is used extensively in the XFEM, which is called the ramp function. This function is calculated from the absolute value of the level set method (LSM) as shown in Eq. (5) shown below:

$$|\varphi(x)| = \begin{cases} -\varphi(x) & if\ \varphi(x) < 0 \\ +\varphi(x) & if\ \varphi(x) \geq 0 \end{cases} \quad (5)$$

4. LOADING SYSTEM AND MODEL DIMENSIONS

Two point loading system for a plain concrete beam was produced for the numerical and experimental analysis. The dimensions, boundary conditions, and loading system are illustrated in **Fig. 2**.
The dimension of the beam model was accomplished due to the British Standard for the hardened concrete prisms test **BS 1881: Part 109, 1983**. The concrete beam was notched at the mid-span length of the beam on the bottom face for crack initiation. In the experimental test, an electric strain gauge (of

a resistance 120 ohm and an effective length of 50 mm) was attached at the mid-span of the bottom face of the middle width of the beam for the strain measurement due to the applied load.

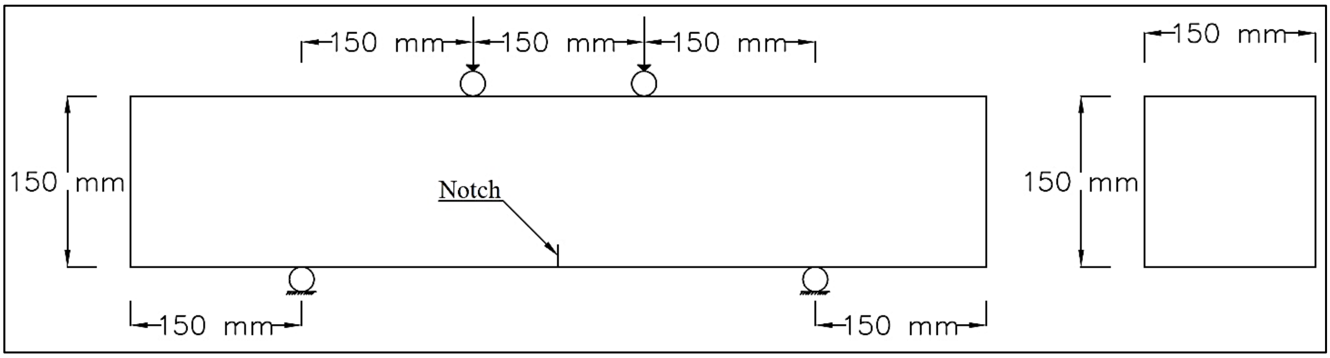

Figure 2. Loading system of the concrete beam model.

5. FINITE ELEMENT MODEL PROPERTIES AND CONSTRUCTION

The finite element model was constructed, meshed, and analyzed using the ABAQUS program with the help of EXCEL sheets for the necessary calculations. In this study, concrete was modeled as a bi-phasic material consisting of coarse aggregate particles and cement mortar. The total quantity of the coarse aggregate particles for the two-dimensional area of the beams was computed from the mix design of the concrete. This quantity was sub-divided according to the grading percentage of the coarse aggregate. The area percentage of each aggregate particle size is shown in **Table 1** below:

Table 1. Percentage of coarse aggregate particles for each size segment.

Particle Size Range, mm	Percentage of The Total Area, %
37.5-20	3.20
20-10	72.33
10-4.75	23.55
4.75-2.36	0.92

The quantities of concrete used for beam production are shown in **Table 2**.

Table 2.Concrete mix design quantities for the beam models.

Material	Quantity
Cement, kg/m^3	342
Mixing Water, kg/m^3	205
Coarse Aggregate, kg/m^3	976
Fine Aggregate, kg/m^3	775
Estimated air Voids, %	2

The coarse aggregate used for the numerical was modeled like as crushed aggregate, by assuming polygon shapes for the coarse aggregate particles. The air voids in the numerical model were assumed to be circular in shape with a diameter of 4 mm, **Wang, et al., 2015**.

The size of the coarse aggregate particles was selected randomly between the range limits of each aggregate size segment as shown in **Table 1**. The orientation and coordinate of each aggregate particle were also selected randomly in the two-dimensional model.

The properties of the cement mortar and the aggregate particles used for the numerical analysis are illustrated in **Table 3** shown below:

Table 3.Meso-scale material properties.

Material	Modulus of Elasticity, MPa	Poisson's Ratio	Max. Tensile Stress, MPa	Fracture Energy, N-mm/mm^2
Cement Mortar	25000	0.2	1.894	0.06
Aggregate	75000	0.2	-	-

The properties of the material shown in **Table 3** were a quote from **Wang et. al. 2015**. The maximum tensile stress shown in **Table 3**, is defined as the maximum stress located at the crack tip before the loading is exhibited. It was assumed to be equal to the maximum tensile splitting strength of the concrete, which was measured in the laboratory according to **ASTM C496-04, 2004**. The splitting strength of the concrete was (f_t =1.894) MPa for a compressive strength of(f_c' =27.5) MPa.
The numerical modeling of the plain concrete beam by the meso-scale method is illustrated in **Fig. 3** shown below;

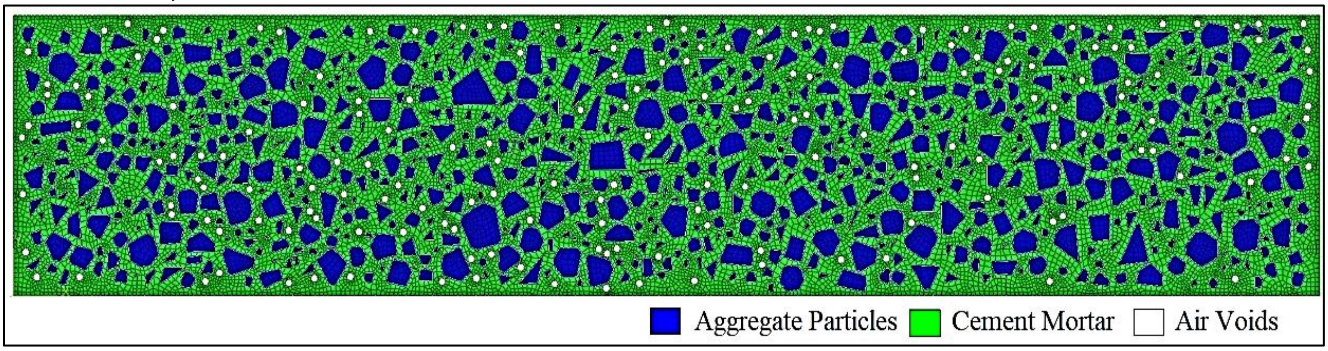

Figure 3. Meso-scale numerical model for the plain concrete beam.

6. RESULTS AND DISCUSSION

A plain concrete beam subjected to flexural was modeled in the meso-scale for the fracture analysis using the XFEM. In addition, an experimental test specimen was fabricated and tested to compare the results of the numerical analysis with the experimental test results. In the experimental test, three similar specimens were made with the same mix design proportions, and the average results were considered. Also, a splitting and compressive tests were made for tensile, and compressive strengths respectively. **Plate.1** below shows the loading machine during the testing period of the concrete beam.

Plate 1. Laboratory testing machine for the plain concrete beam.

The average applied failure load in the experimental test was 20.99 kN, while in the finite element model, the maximum applied load was 21.20 kN. The load versus strain curves in the experimental, numerical, and theoretical analyses for the concrete beam are shown in **Fig. 4**:

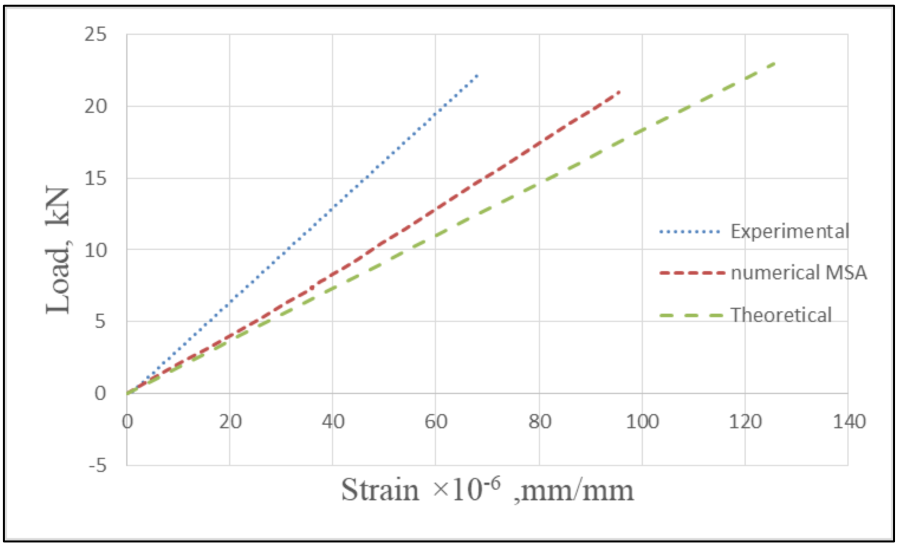

Figure 4. Load vs. strain for the experimental, numerical, and theoretical analysis.

Theoretical values were calculated according to the Bernoulli's thin beam theory which illustrated in Eq. (6):

$$\epsilon_c = \frac{\sigma}{E_c} = \frac{M.C}{E_c.I} \qquad (6)$$

Where;
ϵ_c: concretetensile strain in (mm/mm).
σ: concrete bending stress in (MPa).
Ec: modulus of elasticity of concrete in (MPa), which is equal to $4700\sqrt{f_c'}$ according to **ACI 318-14, 2014**, where f_c' is the compressive strength of the concrete in (MPa).
M: bending moment produced by the externally applied forces in (N-mm).
C: the distance between the neutral axis to the outer face of the beam in (mm).
I: the moment of inertia of the cross-section area in (mm^4).

As shown in **Fig. 4**, the strain magnitudes of the finite element meso-scale model have a good agreement with these of theoretical solution. While the experimental test exhibited a decrease in strain magnitudes in comparison with the theoretical and numerical results. This is may be attributed to the approximation in estimating of some of the concrete properties.

The bending stress distribution along the concrete beam predicted by numerical analysis is shown in **Fig. 5** below. As shown in this figure, the bending stress along the beam span has a coarse curve. The coarseness in the bending stress curves is a result of the non-homogeneity of the concrete material, which is produced from extremely difference in strength components (coarse aggregate particles and the cement mortar). Moreover, the bending stress curve vanishes at the mid-span of the beam, where the crack was initiated and propagated later.

The existing of the air voids in the numerical beam model produced a stress concentration zone surrounding these voids, consequently increasing the probability of crack development and propagation. **Fig. 6** shows the effect of the air voids existence on the distribution of the bending stress inside the concrete.

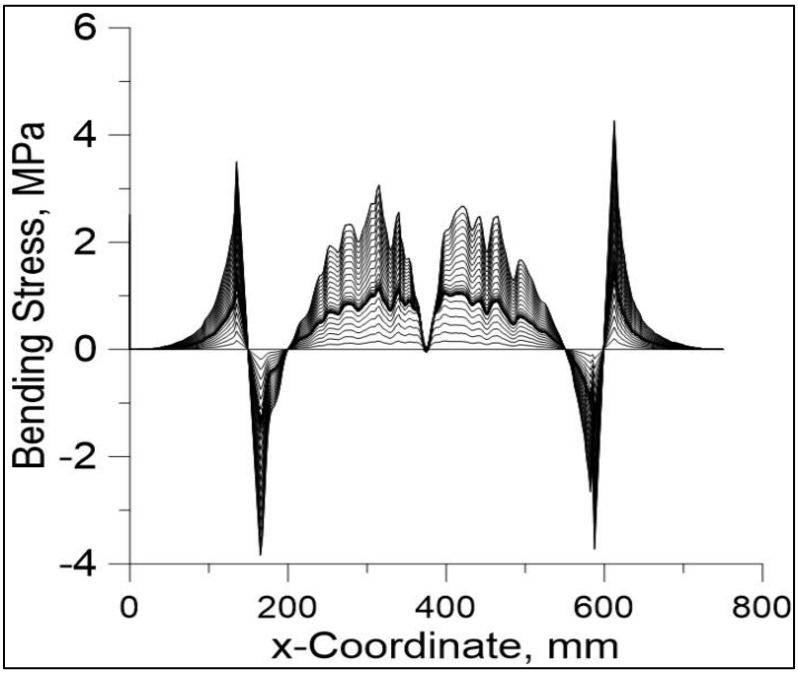

Figure 5. Meso-scale bending stress diagram along the concrete beam.

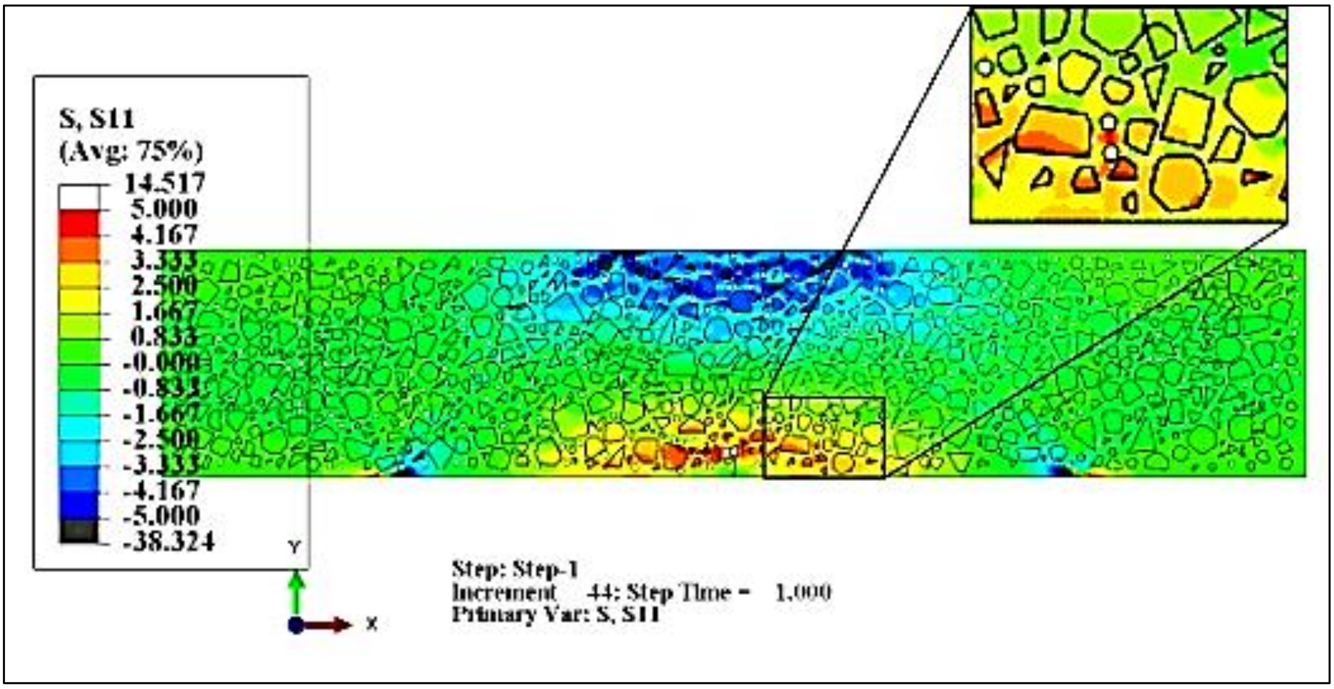

Figure 6. Meso-scale bending stress diagram along the concrete beam.

7. CONCLUSIONS

The two-dimensional plane stress numerical analysis was applied to plain concrete beam subjected to a two-point loading case for fracture response detection. The meso-scale model was employed for the finite element model construction of the concrete. The concrete was assumed to be consisting of aggregate and cement mortar. The interface between the two components was assumed to be fully bonded. XFEM was employed for the analysis of the crack propagation in the concrete beam. the numerical analysis was done using the ABAQUS program. Moreover, an experimental specimen was done for the comparison between the numerical and the experimental results. From the numerical, theoretical, and experimental results, the following conclusions were drawn:

1. The meso-scale model of the concrete was found as a good approach that takes the non-homogeneity of the concrete into account and shows the importance of the non-homogeneity on the macro behavior of the concrete.
2. The XFEM is a powerful tool used in the FE analysis of the crack evolution and propagation of concrete under flexural.
3. The coarse aggregate particles which are floating in cement mortar affects the total behavior of the concrete and might increase its strength in some solo case when the particle opposed the crack propagation path.
4. The existence of the air voids inside the concrete introduces a stress concentration in the vicinity of them result in a decrease, in overall concrete strength.
5. The results of the electric strain gage may effect by many factors such as the existing of the air voids around the area where the strain gage was attached.

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