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Variable Structure Control Design for a Magnetic Levitation System

Shibly Ahmed Al-Samarraie
Assistant Professor
Control & Systems Eng. Dept.
University of Technology
E-mail: 60132@uotechnology.edu.iq

ABSTRACT

In this paper the variable structure control theory is utilized to derive a discontinuous controller to the magnetic levitation system. The magnetic levitation system model is considered uncertain, which subjected to the uncertainty in system parameters, also it is open-loop unstable and strongly nonlinear. The proposed variable structure control to magnetic levitation system is proved, and the area of attraction is determined. Additionally, the chattering, which induced due to the discontinuity in control law, is attenuated by using a non-smooth approximate. With this approximation the resulted controller is a continuous variable structure controller with a determined steady state error according to the selected control parameters. Finally the ability and the effectiveness of the proposed continuous variable structure controller to the magnetic levitation system are verified via numerical simulations. When state initiated inside the area of attraction, the results show that the ball position can be directed to follow various desired positions, with steady state error not exceeding$0.1mm$.
Keywords: Magnetic levitation, Variable structure control, Area of attraction, chattering attenuation.

تصميم مسيطر ذو هيكل متغير لنظام الرفع المغناطيسي

الاستاذ المساعد الدكتور شبلي أحمد السامرائي
قسم هندسة السيطرة والنظم
الجامعة التكنولوجية

الخلاصة

في هذا البحث تم استخدام نظرية التحكم في الهيكل المتغير لاشتقاق وحدة تحكم متقطعة في نظام الرفع المغناطيسي. يعتبر نموذج نظام الرافع المغناطيسي غير مؤكد ، والذي يخضع لعدم اليقين في معاملات النظام ، كما أنه غير مستقر ولاخطي. تم إثبات التحكم في الهيكل المتغير المقترح لنظام الرفع المغناطيسي ، وتم تحديد منطقة الجذب. بالإضافة إلى ذلك ، يتم تخفيف الثرثرة ، التي يسببها بسبب عدم الإستمرارية في قانون السيطرة، باستخدام تقريب غير سلس. مع هذا التقريب فإن وحدة التحكم الناتجة هي وحدة تحكم هيكل متغير مستمر مع وجود خطأ ثابت في الحالة المستقرة وفقًا لمعاملات التحكم المحددة. وأخيراً يتم التحقق من قدرة وفاعلية جهاز التحكم المتغير في الهيكل المتغير المقترح على نظام الرفع المغناطيسي من خلال عمليات محاكاة عددية. تظهر النتائج أنه يمكن توجيه موضع الكرة لمتابعة مختلف المواقف المطلوبة ، ومع شروط إبتدائية مختلفة ، والتي تبدأ في منطقة الجذب ، مع عدم تجاوز الخطأ $0.1mm$ عند الإستقرار.

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*Corresponding author
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الكلمات الرئيسية: الرفع المغناطيسي ، التحكم في الهيكل المتغير ، منطقة الجذب ، تخفيف التذبذب.

1. INTRODUCTION

Magnetic Levitation Systems (MLS) becoming popular in many applications. They can be used for precise positioning such as frictionless bearings, magnetic levitation trains, wind tunnels, and conveyor systems. And that because in magnetic levitation systems there is no mechanical contact, friction, or noise as presented by **Chih-Min, et al. 2014**. However, **Matthew, et al., 2006** show that the MLS suffers from many problem, witch complicate the design task like the nonlinear and open loop instability nature, the uncertainty in system model and the external disturbance. These challenges has led to a significant need for developing control technologies for magnetic levitation control systems.

Recently, a variety of control approaches have been used to design nonlinear control to the MLS. When a third order MLS model was considered an additional problem is added which represented by the mismatched problem. The feedback linearization was used, as a first step, by many authors in order to transform the MLS model to a linear model like **Samir, and Chia-Hsiang, 1997, Rudi, et al., 2013, Jinquan, et al., 2015, Jerzy, and Paweł, 2008, Zi-Jiang and Masayuki, 2001, Divyesh, et al., 2016,** and **Zi-Jiang, et al., 2011**. Accordingly, the linear control techniques can be used effectively to design a stabilizing or tracking controller. However to apply the feedback linearization method and then design a linear state feedback control, the system model must be either free from external disturbance or it is assumed constant. As a result, the control performance will be affected especially when the system model is uncertain and when the external disturbance is variable as was shown by **Khalil, 2002**. To solve this problem, a robust controller based on Lyapunov redesign approach was applied to MLS by **Jinquan, et al., 2015** to eliminate perturbation. However this type of controller induces chattering in system response, also it is required that the external disturbance is at least Lipschitz. A Backstepping using nonlinear damping approach for the MLS model was used by **Zi-Jiang, et al., 2011**, where the input to state stability property to the uncertainty term is required. Unfortunately, the Backstepping method cannot be used with the presence of non-vanishing external disturbance. A disturbance observer based control design methodology was introduced by **Divyesh, et al., 2016**, where a cascaded sliding mode control which use sliding mode controller disturbance-observer-based for the electrical loop and for the electromechanical loop. The stability was proved for the overall system under the proposed scheme and the results compared with an LQR plus PI controller in simulation and in experimental validation. The cascade features of Backstepping design with a simple disturbance observer was utilized by **Zi-Jiang, et al., 2011** to design a robust output feedback controller for the MLS systems in the presence of uncertainties.

Many other authors used the reduced MLS model where the coil current assumed as the control input. The uncertainty and the external disturbance in this model, as showed by **Matthew, et al., 2006**, satisfy matching condition. Unfortunately, another problem arises which it is a unidirectional force input. Consequently, due to satisfying matching condition the control design based on disturbance observer was used in several publications. For example **Matthew, et al., 2006**, utilized a learning-based disturbance estimator to asymptotically regulate the target mass to a desired set point position. Also, the learning-based disturbance estimator was used by **Fang, Y., et al., 2003** in the presence of a nonlinear, bounded, periodic disturbance.

Variable Structure Control (VSC) recognized as an efficient tool to design of robust controllers for nonlinear, complex and high order linear plants and time-delay systems with parameter perturbations and external disturbances. Variable structure control systems are characterized by a suit of feedback control algorithms and a decision rule termed as the switching function. The measurable state variables are decision rule input while the output is the particular feedback controller (linear or nonlinear) that should be used at that instant in time. In other words, a variable structure system consists of a set of linear or in general nonlinear subsystems with a proper switching function logic. **Elbrous**, **2009**, indicated that these systems can be named also as multi-structure systems. Historically, sliding modes were discovered as a special mode in VSS. Furthermore, in a sliding mode, these include the insensitivity to certain (so-called matched) model uncertainties and external disturbances as well as robustness to parasitic dynamics, are the main system properties in sliding mode. The development of these novel ideas began in the Soviet Union in the late 1950s as indicated by **Shtessel, et al., 2014**.

For the third order MLS model, the sliding mode control (SMC) theory was applied successfully by many authors based on different approaches. These approaches summarily include the following: classical SMC used by **Mahdi, 2004**, terminal SMC by **Boonsatit and Pukdeboon, 2016**, integral SMC by **Zhen, et al., 2009**, dynamic SMC by **Al-Muthairi , and Zribi, 2004**, PI-SMC by **Vithal S. Bandal, and Pratik, 2009,** and the higher order sliding mode control by **Azar, and Quanmin, 2015**. In all of these works the feedback linearization was used as a first step. Moreover, **Utkin, et al., 2009** showed that the chattering problem cannot be avoided unless by using some approximation for the discontinuous term. This step will degrade the SMC performance.

In the reduced MLS model, Lipschitz condition on the disturbance term is removed, also the matching condition is satisfied. **Jing-Chung, 2002** designed the H^{∞} SMC and the PID to the MLS. The objective of the H^{∞} and the proportional–integral–derivative (PID) controller is the disturbance attenuation control while for the SMC is the disturbance estimation and compensation control. Experimental results showed that the performance of the H^{∞} controller is superior to that of the SMC, while the SMC performance is superior to that of the PID controller. Generally, for a system that satisfies matching condition, the performance of the SMC, where the disturbance is rejected, is superior to any other control techniques like the H^{∞} controller, where the disturbance is attenuated only. Other SMC design to the MLS which based on different approaches are; classical SMC used by **Cho, et al**., **1993**, the classical SMC with the integral term by **Chiang, et al., 2006**, and disturbance estimator based SMC by **Yu-Sheng, and Jian-Shiang, 1995** and **Yao, et al., 2015**.

In this work, a variable structure control is designed to the MLS. The MLS is modeled as second order system with parameters uncertainty and in presence of external disturbance. The proposed robust VSC depends on simple decision rules with considering the saturation in control input. To validate the performance of the proposed VSC, different numerical simulation results for tracking various desired ball position are included.

The organization of this paper is as follows; the mathematical model for the magnetic levitation system is presented in section two. In section three, the proposed variable structure controller is presented, after that the chattering problem is solved in section four via approximating the control law. Finally the simulation results and discussion are found in sections five and six, respectively.

2. MATHEMATICA MODEL

The magnetic levitation system, which considered in this work consists of a steel ball affected by a magnetic force in the vertical motion only. The magnetic force is controlled by controlling the input current, which enables the ball to reach the desired position. The schematic diagram of the MLS is shown in **Fig. 1** and the nonlinear dynamic model of it is described by **Yao, et al., 2015**:

$$\ddot{x} = \mathrm{g} - \varphi(x)i^2 + d(t) \tag{1}$$

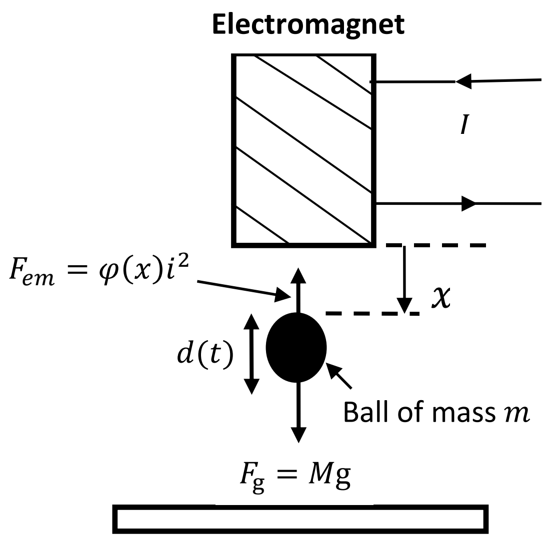

Figure 1. The schematic diagram of the MLS

Where

$$\varphi(x) = \frac{Q}{2M(X_\infty + x)^2} \tag{2}$$

x is the ball-position, g is the gravitational constant, i is the coil current, M is the steel ball mass, while Q and X_∞ are positive constants determined by the characteristics of the coil, magnetic core and steel ball. Finally the un-modeled external disturbances is represented by$d(t)$.

The magnetic levitation model can also be described in term of nominal and perturbation terms. This form is more appropriate in designing a discontinuous control, like VSC, in order to attenuate chattering, which represent the undesirable behavior that arises due to discontinuity. So the MLS model is rewritten as;

$$\ddot{x} = \mathrm{g}_o - \varphi_o(x)i^2 + \delta(t, x) \tag{3}$$

where $g_o = 9.81\ \frac{m}{s^2}$, $\varphi_o(x)$ is the nominal function of $\varphi(x)$ (see appendix A) and $\delta(t,x)$ the perturbation term is given by

$$\delta(t,x) = \Delta g + \Delta\varphi(x)i^2 + d(t) \tag{4}$$

Here Δg is the uncertainty in g, and $\Delta\varphi(x)$ is defined as

$$\Delta\varphi(x) = \varphi(x) - \varphi_o(x) \tag{5}$$

Assumption (1) : The bound on $\delta(t,x)$ can be taken as

$$|\delta(t,x)| \leq \alpha\varphi_o(x)i^2 + d^+ \tag{6}$$

where, $|\Delta\varphi(x)| \leq \alpha\varphi_o(x)$, $|\Delta g + d(t)| < d^+$, d^+ and α are positive constants. For the calculation of α see Appendix A.

Let the system states and control be defined as; $x_1 = x,\ x_2 = \dot{x}$ and define $v = i^2$ as a new control input where the current i is the actual control input signal to the electromagnet coil. According to these definitions the state space form to the magnetic levitation system can be written as;

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= g_o - \varphi_o(x)v + \delta(t,x) \end{aligned}\right\} \tag{7}$$

In the next section, based on the variable structure control theory, a non-smooth controller is proposed to the MLS using Eq. (7) as the mathematical model.

3. VARAIBLE STRUCTURE CONTROL

A variable structure control design to the magnetic elevation system is proposed in this section. At first, Eq. (7) is rewritten with respect to the error function $e(t)$ as follows; define

$$\left.\begin{aligned} e = e_1 &= x_1 - x_d \\ \dot{e} = e_2 &= x_2 - \dot{x}_d \end{aligned}\right\} \tag{8}$$

Where x_d and $\dot{x}_d$ are the desired ball-position and its time derivative respectively. Then,

$$\left.\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= g_o - \varphi_o(x)v + \delta(t,x) - \ddot{x}_d \end{aligned}\right\} \tag{9}$$

The sliding variable s is defined here as

$$s = e_2 + \lambda e_1 \tag{10}$$

Where $\lambda > 0$ is the switching line slope. Differentiating s with respect to time yields

$$\dot{s} = \dot{e}_2 + \lambda\dot{e}_1 = g_o - \varphi_o(x)v + \delta(t,x) - \ddot{x}_d + \lambda e_2 \tag{11}$$

For the sliding variable dynamics (Eq. (11)), we use the candidate non-smooth Lyapunov function defined as

$$V = |s| > 0 \ , \ \forall t \geq 0 \tag{12}$$

With the aid of non-smooth derivative which was given by **Frank, 1990**, the derivative of V is given by;

$$\left.\begin{aligned} \dot{V} &= \dot{s} * \mathrm{sign}(s) = \{\mathrm{g}_o - \varphi_o(x)v + \delta(t,x) - \ddot{x}_d + \lambda e_2\} * \mathrm{sign}(s) \\ &= \{\psi - \varphi_o(x)v + \delta(t,x)\} * \mathrm{sign}(s), \quad \forall s \neq 0 \end{aligned}\right\} \tag{13}$$

Where

$$\psi = \mathrm{g}_o - \ddot{x}_d + \lambda e_2 \tag{14}$$

Assumption (2) the inequality

$$\mathrm{g}_o > d^+ + |\ddot{x}_d + \lambda \dot{x}_d| \tag{15}$$

Is assumed to be satisfied in the present work.

Definition (1) the positive function denoted by $[\ \ ]_+$ is defined as follows;

$$[z]_+ = \begin{cases} 1 \ if \ \ z > 0 \\ 0 \ if \ \ z \leq 0 \end{cases}$$

Under these circumstances, the VSC law to the magnetic levitation is introduced in the following proposition.

Proposition (1) The VSC control law

$$\left.\begin{aligned} &v = \frac{1}{\varphi_o(x)(1-\alpha)} H * [H]_+ * [sign(s)]_+ \\ &H = \psi + d^+ + k_o, \ \ k_o > 0 \end{aligned}\right\} \tag{16}$$

will guarantee a global attractiveness of the sliding manifold $s = 0$ in the phase plane.

Proof: recall the time derivative of the candidate Lyapunov function (Eq. (13));

$$\begin{aligned} \dot{V} &= \{\psi - \varphi_o(x)v + \delta(t,x)\}\mathrm{sign}(s) \\ &= \big(\psi + \delta(t,x)\big)\mathrm{sign}(s) - \varphi_o(x)v * \mathrm{sign}(s) \end{aligned}$$

The attractiveness of the sliding manifold when applying the proposed control law in Eq. (16) can be proved via showing that $\dot{V}$ is negative definite for $s < 0$ and $s > 0$. From Eq. (16) it is easy to see that $v = 0$ for $s < 0$. Thus $\dot{V}$ becomes;

$$\dot{V} = -\big(\psi + \delta(t,x)\big) \le -\psi + d^+ = -(\psi - d^+)$$

To show that the sliding manifold is attractive ($\dot{V} < 0$), ψ is rewritten based on Eq. (14) and assumption (2) as follows;

$$\begin{aligned}\psi &= \mathrm{g}_o - \ddot{x}_d + \lambda e_2 = \mathrm{g}_o - \ddot{x}_d - \lambda \dot{x}_d + \lambda x_2 \\ &> d^+ + |\ddot{x}_d + \lambda \dot{x}_d| - (\ddot{x}_d + \lambda \dot{x}_d) + \lambda x_2 \\ &\ge d^+ + \lambda x_2\end{aligned}$$

or

$$\psi - d^+ > \lambda x_2$$

Therefore, when $x_2 \ge 0$, then $\dot{V} < 0$. This means that the sliding manifold is attractive when the state started in the half space $s < 0$ and $x_2 \ge 0$. When $x_2 < 0$, i.e., the ball moves upward, we take the worst case where $\psi - d^+ \le 0$. In this case the sliding manifold will be no longer attractive. Nevertheless, as the ball has negative velocity, it will decelerate until it becomes equal to zero. This behavior can be revealed by using Eq. (3) and assumption (2) as follows;

$$\begin{aligned}&\dot{x}_2 = \mathrm{g}_o + \delta(t,x) \\ \Rightarrow\ & x_2(t) - x_2(t_o) = \int_{t_o}^{t} \{\mathrm{g}_o + \delta(t,x)\}\, dt \ge \int_{t_o}^{t} \beta\, dt = \beta(t - t_o)\end{aligned}$$

where $0 < \beta = \inf\limits_{t \ge t_o}\big(\mathrm{g}_o + \delta(t,x)\big)$. Accordingly, after a certain period of time $(t - t_o \le -x_2(t_o)/\beta)$, the ball velocity will return positive, and again the sliding manifold is attractive. As a result the sliding manifold is attractive in the half space $s < 0$.

Next, we need to show that when $s > 0$, the sliding manifold is attractive also. Recall $\dot{V}$ with $s > 0$ and inequality (6)

$$\begin{aligned}\dot{V} &= \psi + \delta(t,x) - \varphi_o(x)v \le \psi + |\delta(t,x)| - \varphi_o(x)v \\ &= \psi + |\delta(t,x)| - \varphi_o(x)v \le \psi + d^+ + \alpha\varphi_o(x)v - \varphi_o(x)v \\ &= \psi + d^+ - (1-\alpha)\varphi_o(x)v\end{aligned}$$

$$\Longrightarrow \dot{V} \le \psi + d^+ - (1-\alpha)\varphi_o(x)v \tag{17}$$

Inequality (17) with the control law proposed in Eq. (16), becomes;

$$\dot{V} \le \psi + d^+ - H * [H]_+$$

Let us first examining H as follows;

$$\begin{aligned} H &= \psi + d^+ + k_o = \mathrm{g}_o - \ddot{x}_d - \lambda \dot{x}_d + \lambda x_2 + d^+ + k_o \\ &> d^+ + |\ddot{x}_d + \lambda \dot{x}_d| - (\ddot{x}_d + \lambda \dot{x}_d) + \lambda x_2 + d^+ + k_o \\ &> 2d^+ + \lambda x_2 + k_o \end{aligned}$$

If $x_2 \geq \frac{-1}{\lambda}(k_o + 2d^+)$, then $H \geq 0$, which leads to $\dot{V} \leq -k_o < 0$. On the other side, if $x_2 < \frac{-1}{\lambda}(k_o + 2d^+)$ and $H < 0$, then $\dot{V} \leq \psi + d^+$. But $H = \psi + d^+ + k_o < 0 \Rightarrow \psi + d^+ < -k_o$, hence $\dot{V} \leq -k_o < 0$. That means $\dot{V} < 0$ for $s > 0$. This ends the proof which shows that the sliding manifold is attractive in the whole space of s. Finally the stability of the proposed variable structure controller will be ensured by taking $\lambda > 0$. This guarantee the asymptotic approach of the error state to the origin. □

Remark (1) As known in sliding mode control theory the state reaches the sliding manifold at a finite time, that because the proposed controller is discontinuous at the sliding manifold. However the state will reach the desired reference only asymptotically.

In proposition (1) the control input v was assumed unbounded. This assumption enable us to show that the attractiveness for the sliding manifold is global. In fact when considering that the control input is bounded, the attractiveness of the sliding manifold is no longer global, instead, an area of attraction is formed when the state is started in the half space $s > 0$. To derive the area of attraction the VSC law is rewritten, after considering the control saturation, as follows;

$$\left.\begin{aligned} &v = \frac{1}{\varphi_o(x)(1-\alpha)} H * [H]_+ * [sign(s)]_+ \qquad if\ v \leq v_{max} \\ &v = v_{max} \qquad\qquad\qquad\qquad\qquad\qquad if\ v > v_{max} \\ &H = \psi + d^+ + k_o, \ \ k_o > 0 \end{aligned}\right\} \tag{18}$$

Proposition (2) By using the VSC control in Eq. (16), the area of attraction is given by set Ω;

$$\Omega = \left\{(x_1, x_2): s > 0, x_2 = \frac{1}{\lambda}(-\mathrm{g}_o + \ddot{x}_d + \lambda \dot{x}_d - d^+ + (1-\alpha)\varphi_o(x) v_{max})\right\}$$

Proof: From inequality (17), and for $v > v_{max}$, we have

$$\begin{aligned} \dot{V} &\leq \psi + d^+ - (1-\alpha)\varphi_o(x) v_{max} \\ &= \mathrm{g}_o - \ddot{x}_d + \lambda e_2 + d^+ - (1-\alpha)\varphi_o(x) v_{max} \\ &= \mathrm{g}_o - \ddot{x}_d + \lambda x_2 - \lambda \dot{x}_d + d^+ - (1-\alpha)\varphi_o(x) v_{max} \end{aligned}$$

when $\dot{V} = 0$ the sliding manifold is not attractive, therefore

$$0 < \mathrm{g}_o - \ddot{x}_d + \lambda x_2 - \lambda \dot{x}_d + d^+ - (1-\alpha)\varphi_o(x) v_{max}$$

$$\Rightarrow \dot{x} < \frac{1}{\lambda}(-\mathrm{g}_o + \ddot{x}_d + \lambda \dot{x}_d - d^+ + (1-\alpha)\varphi_o(x) v_{max})$$ □

Corollary (1): The maximum desired ball position where the electromagnet force be able to stabilize the ball at it is derived from proposition (2), with $x_2 = 0$, as;

$$x_{dmax} = \varphi_o^{-1}\left(\frac{g_o - \ddot{x}_d - \lambda \dot{x}_d + d^+}{(1-\alpha)v_{max}}\right) \tag{19}$$

Corollary (2): The area of attraction for the MLS, with $x_d = const \le x_{dmax}$, is given by

$$\Omega_c = \left\{(x, \dot{x}): \dot{x} < \frac{1}{\lambda}(-g_o - d^+ + (1-\alpha)\varphi_o(x)v_{max})\right\} \tag{20}$$

If the MLS state initiated inside Ω_c, then the sliding manifold is attractive and the ball position approaches x_d asymptotically. Figure **2** depicts the invariant set according to (20). Additionally, if the desired position is varied with time, then the area of attraction is estimated as;

$$\Omega_v = \left\{(x, \dot{x}): \dot{x} < \frac{1}{\lambda}(-g_o - d^+ - \eta + (1-\alpha)\varphi_o(x)v_{max})\right\} \tag{21}$$

where $\eta = \max\limits_t |\ddot{x}_d + \lambda \dot{x}_d|$.

To ensure that the sliding manifold is attractive for a state which initiated inside Ω_c, the switching line slope λ must be is selected according to the following inequality.

$$\lambda \le \sqrt{-(1-\alpha)v_{max}\left.\frac{d\varphi_o(x)}{dx}\right|_{x_{dmax}}} \tag{22}$$

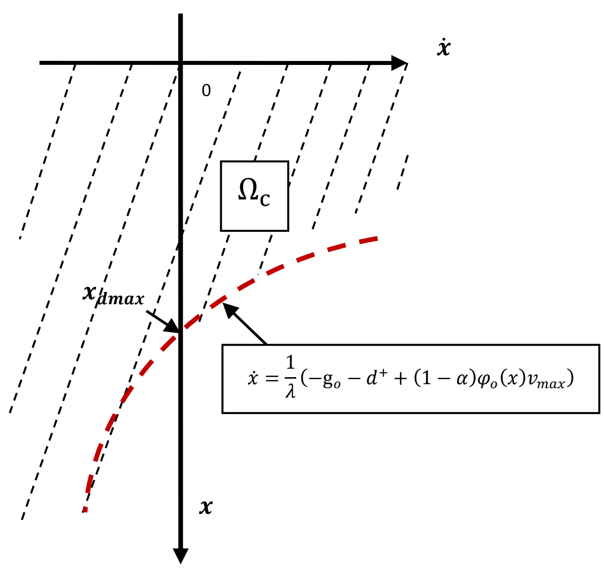

Figure 2. The set Ω- the dashed region in the phase plane.

4. CHATTERING PROBLEM

Chattering in a variable structure control system is an essential dynamical behavior during sliding motion. **Utkin, et al., 2009** clarified that chattering phenomenon is due to the discontinuous nature of the controller, and the presence of the un-modeled dynamics. Many authors like **Edwards, and Spurgeon, 1998,** solved this problem by using an approximate continuous (smooth or non-smooth) form to the signum function. The signum function is a discontinuous function of s which used in the SMC law. This solution, as showed by **Utkin, et al., 2009,** leads to a steady state error as a price to chattering removal. However, this solution is still needed in SMC design to eliminate chattering in many applications, where it is needed only to replace the signum function by an approximation.

In the present work the proposed variable structure controller has a discontinuity at $s = 0$ due to the function$[sign(s)]_+$. Replacing $[sign(s)]_+$ by an approximation will eliminate or attenuate chattering in system response. This is addressed in the following proposition.

Proposition (3) Replacing the $[sign(s)]_+$ in the control law (16) by

$$q(s) = \begin{cases} 0 \quad \text{for} \quad s \le 0 \\ (s/\varepsilon) \quad \text{for} \quad 0 < s \le \varepsilon \\ 1 \quad \text{for} \quad s > \varepsilon \end{cases} \tag{21}$$

will eliminate or attenuate the chattering in system response while the ball position is regulated to the following positively invariant set

$$\Phi = \left\{x: 0 < (x - x_d) < \frac{\varepsilon}{\lambda}\right\} \tag{22}$$

Where ε is a design parameter to be set according to the allowable steady state error. The VSC law with this approximation is named here as continuous variable structure control (CVSC) law.

Proof: By using $q(s)$ (Eq. (21)), the control law (18) becomes;

$$\left.\begin{aligned} v_{app} &= v * q(s) & if\ v \le v_{max} \\ v_{app} &= v_{max} * q(s) & if\ v > v_{max} \end{aligned}\right\} \tag{23}$$

Since $[H]_+$ and $q(s)$ are continuous functions at the sliding manifold $s = 0$, the control law in Eq. (23) is a continuous function also. As a result the chattering is eliminated. However, for a smaller value of ε the chattering is attenuated only.

To determine the invariant set, where the controller regulates the state to it, the inequality (17) is rewritten by using the control law in Eq. (23) for $v \le v_{max}$ as follows;

$$\begin{aligned}\dot{V} &\leq \psi + d^{+} - H * [H]_{+} * q(s) \\ &= \psi + d^{+} - [\psi + d^{+} + k_o](s/\varepsilon) \\ &< [\psi + d^{+} + k_o] - [\psi + d^{+} + k_o](s/\varepsilon) \\ &= [\psi + d^{+} + k_o] - [\psi + d^{+} + k_o](s/\varepsilon) \\ &= -[\psi + d^{+} + k_o]\{(s/\varepsilon) - 1\}\end{aligned}$$

That is $\dot{V} > 0$ in the region $0 < s < \varepsilon$, means that the sliding manifold is not attractive. Now by considering the dynamical behavior of $(e, \dot{e})$ in the phase plane, and the constraint $0 < s < \varepsilon$ the error state e will enter the set Φ as given in (22) and stay there for all future time. □

More details about the derivation of the invariant set for a second order system was presented by **Al-Samarraie, 2013**.

5. NUMERICAL SIMULATION RESULTS AND DISCUSSION

To validate the performance of the proposed VSC to the MLS, the numerical simulations were performed with MATLAB software. The nominal MLS parameters are as those which was used by **Matthew, et al., 2006**;

$$Q_o = 0.00145\, H.m,\ \ M_o = 0.5\, kg,\ \ X_{\infty o} = 0.0085\, m,\ \text{ and }\ \mathrm{g}_o = 9.81\, m/s^2$$

where it is assumed that the system sever from the following external disturbance;

$$d(t) = 2 * \sin(4\pi t)$$

The VSC parameters are as follows; $\alpha = 0.6$ for 20% uncertainty in system model parameters($r = 0.2$), $d^{+} = 3$, $k_o = 0.1$, $v_{max} = i_{max}^2 = 25$, $x_{dmax} = 0.0273\, m$ (Eq.(19)), and $\varphi_o(x)$ is given by Eq. (A-1). Also $\varepsilon = 0.0025$, and $\lambda = 27$, as the maximum switching line slope, which computed according to inequality (21). Therefore the steady state error is $0 < (x - x_d) < 0.1\, mm$ according to (22). Note that $\varphi_o(x)$ does not computed by using the nominal MLS parameters, but instead it was computed as the mean function for $\varphi_{max}(x)$ and $\varphi_{min}(x)$ as detailed in Appendix A.

The numerical simulations for the MLS using the proposed VSC are performed below with the system parameters; $Q = 0.00165\, H.m$, $M = 0.55\, kg$, $X_{\infty} = 0.008\, m$, and the results are presented as follows.

5.1 The First set of numerical simulations: In this set of numerical simulations, three different results for three cases are obtained. In the first two cases, the CVSC is used with initial conditions given by; **a)** $x(0) = 0.008\, m, \dot{x}(0) = 0.01\, m/s$ and **b)** $x(0) = 0.02\, m,\ \dot{x}(0) = 0.01\, m/s,$ while in the third case **c)** the initial condition and the desired position are as in **a)** but with discontinuous VSC law.

For case **a)** the phase plot is shown in **Fig. 3**, where the VSC forces the state towards the sliding surface ($s = 0$), and then slide to the desired position ($x = 0.01\ m$ and $\dot{x} = 0$). This figure reveal the ability of the proposed controller. As a result, the ball position reaches asymptotically to the desired position as depicted in **Fig. 4**. The powerful features of the proposed CVSC to the MLS are deducted also from the simulation results shown in **Figs. 5, 6** and **7**. Due to use CVSC law, (instead of discontinuous) the ultimate steady state error is determined from inequality (22), which it equal to $\frac{\varepsilon}{\lambda} = \frac{0.0025}{27} = 0.1mm$. The maximum error after a small period of time is clearly bounded by the ultimate bound as can be shown clearly in **Fig. 5**. Also the state is directed to the sliding manifold (**Fig. 6**) in a small period of time ($< 0.005\ sec.$), and without chattering as shown when plotting the control input (the current= $\sqrt{v}$) in **Fig. 7**, and from the phase plot in **Fig. 3**.

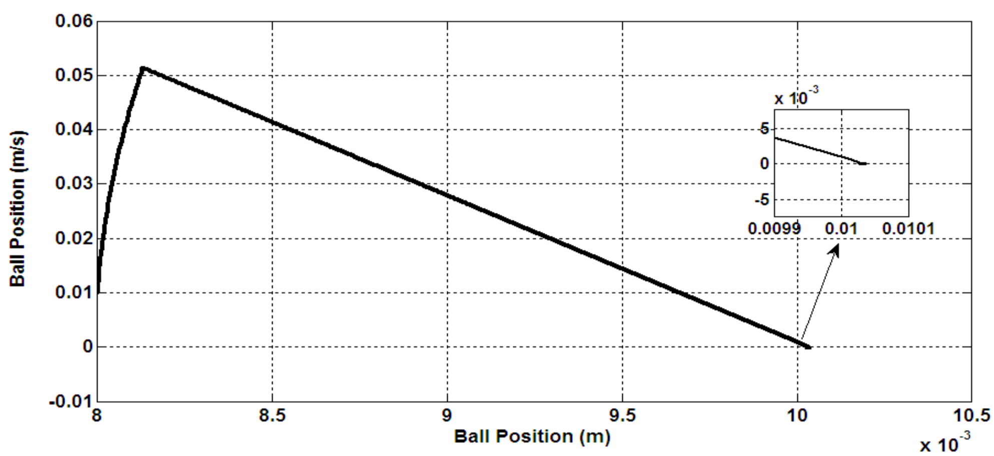

Figure 3. The phase plot (case **a)**).

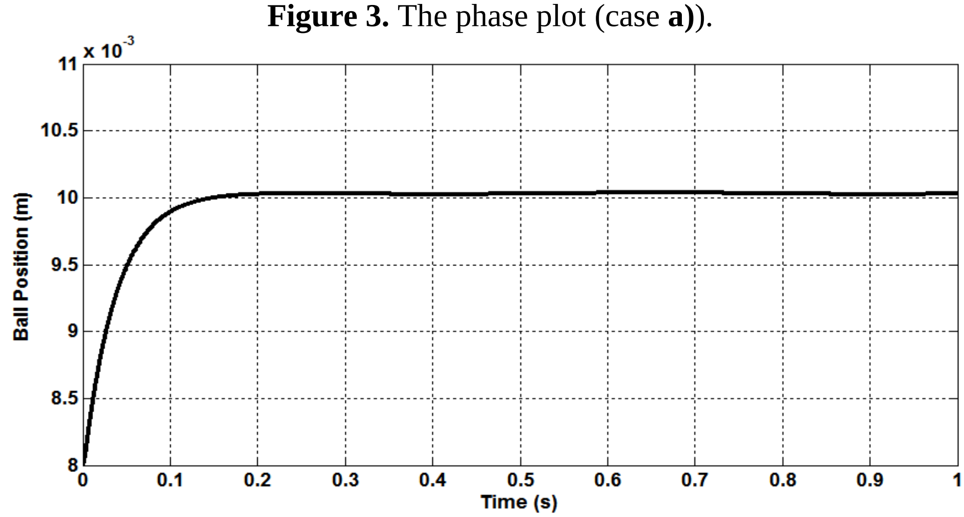

Figure 4. The ball position (case **a)**).

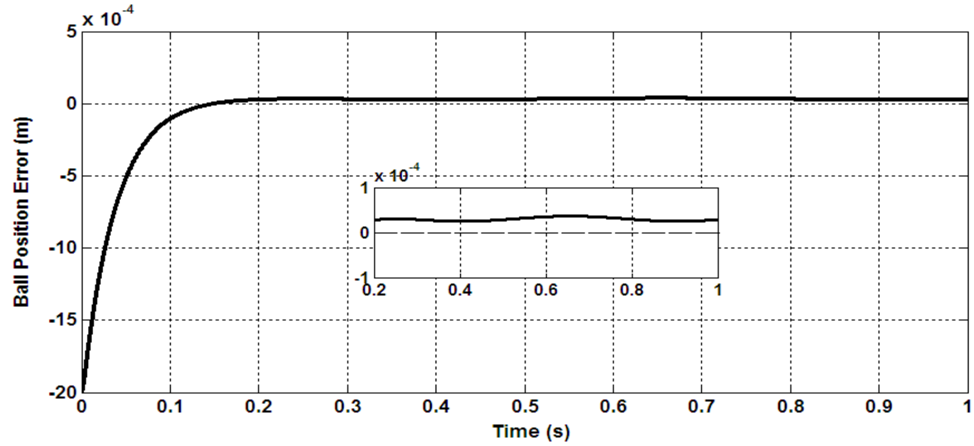

Figure 5. Ball position error (case **a)**).

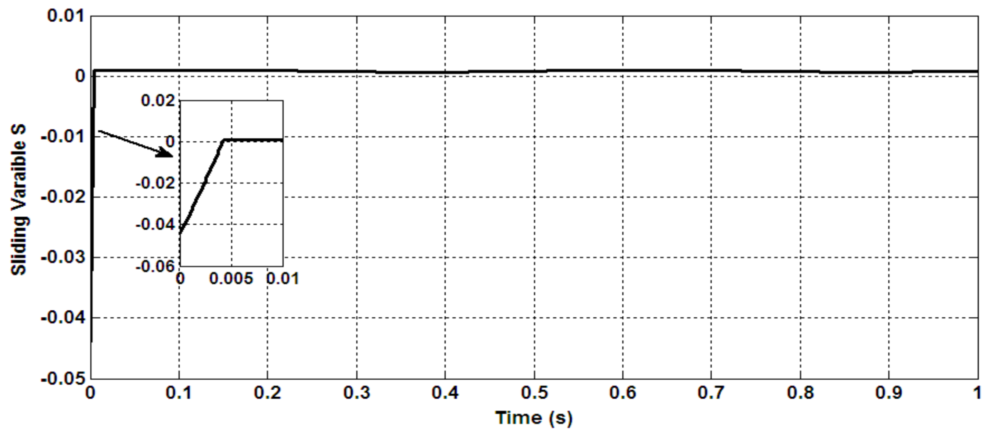

Figure 6. The sliding variable (case **a)**).

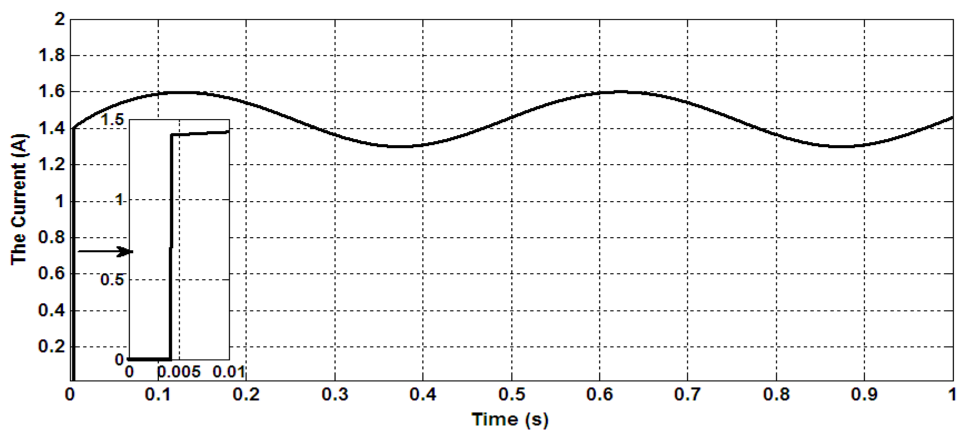

Figure 7. The control input (case **a)**).

The ability of the proposed CVSC for controlling the MLS is tested again with different initial condition given in **b)**. The obtained results in **Figs. 8, 9,** and **10**, demonstrate the effectiveness of the CVSC to deal with different initial conditions.

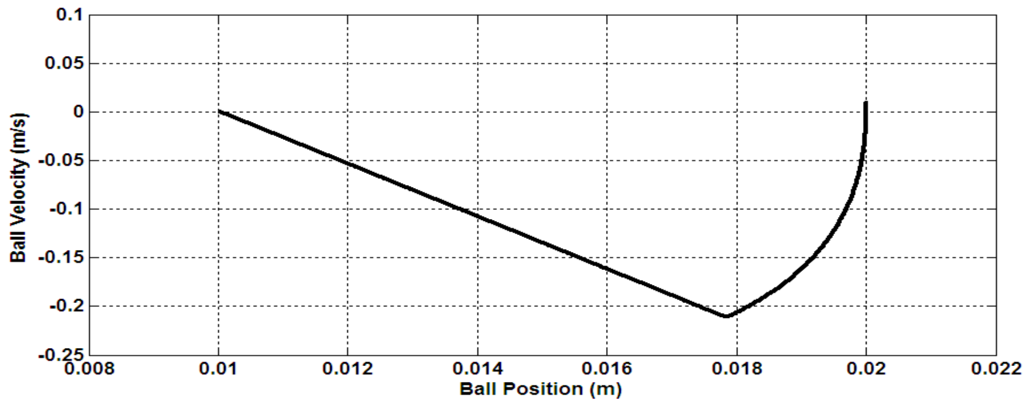

Figure 8. The phase plane (case **b)**).

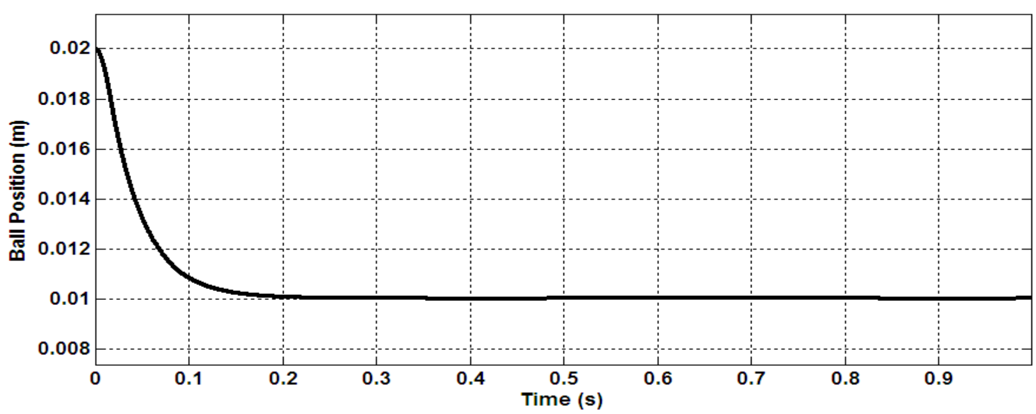

Figure 9. The ball position (case **b)**).

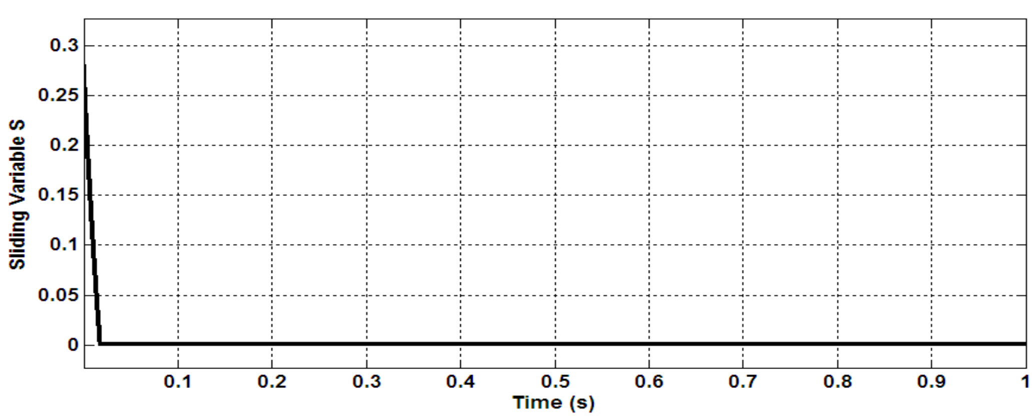

Figure 10. The sliding variable (case **b)**).

As an important note from the results of the above two cases, one can see that the state in both cases, initiated in the area of attraction Ω_c, which derived in corollary (2). The area of attraction represent the region in which the CVSC is able to regulate the state to the desired position. Another situation which can be noted from the plot the current in **Fig. 7**. In this figure the control input is zero for approximately $0.005\ sec.$, where the sliding variable is negative and accordingly $v = 0$. During this period the ball velocity is positive, and accordingly it becomes responsible of directing the state towards the sliding manifold. This situation does not found in case b), because the sliding variable initiated in the positive side. Additionally, it shows the importance of using the variable structure control scheme for this sort of system that use one directional control input.

The chattering behavior which induced due to the discontinuity in the proposed VSC is explored in the third case **c)**. **Figs. 11** and **12** show the chattering behavior in the phase plane and the sliding variable plot. However the effect of chattering is more destructive for the control input provider as shown in **Fig. 13**, where it is required to switches between zero and a positive value with high frequency.

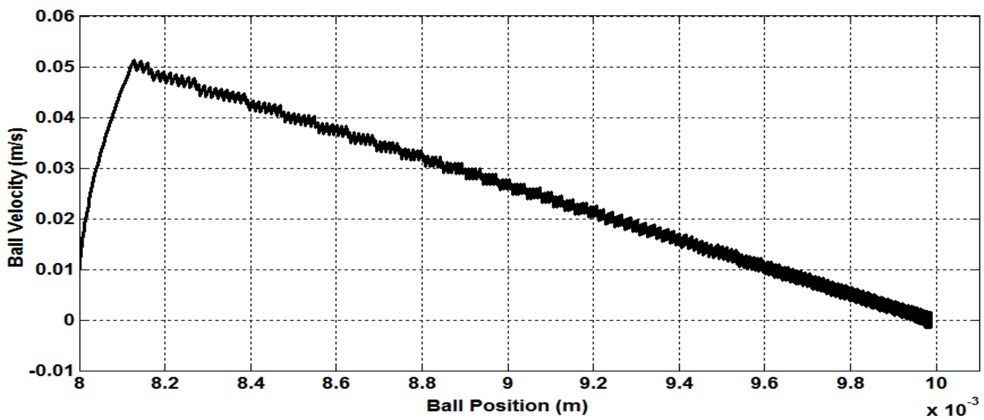

Figure 11. The phase plane (case **c)**).

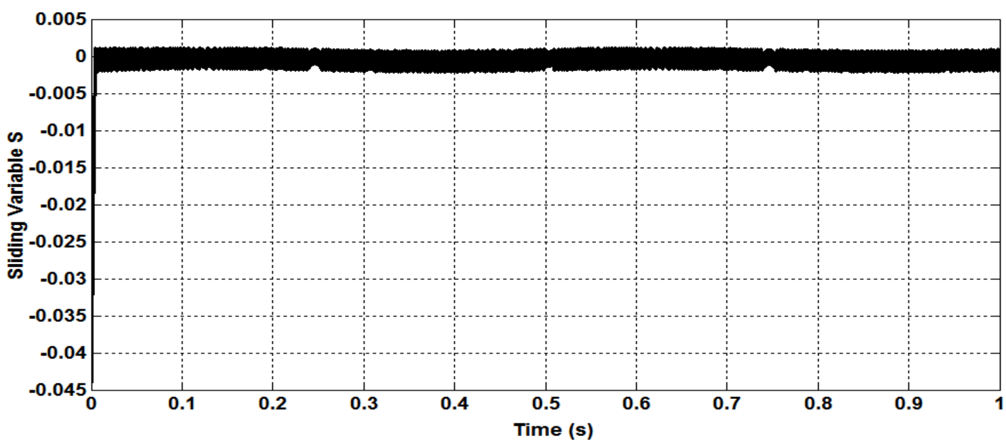

Figure 12. The sliding variable (case **c)**).

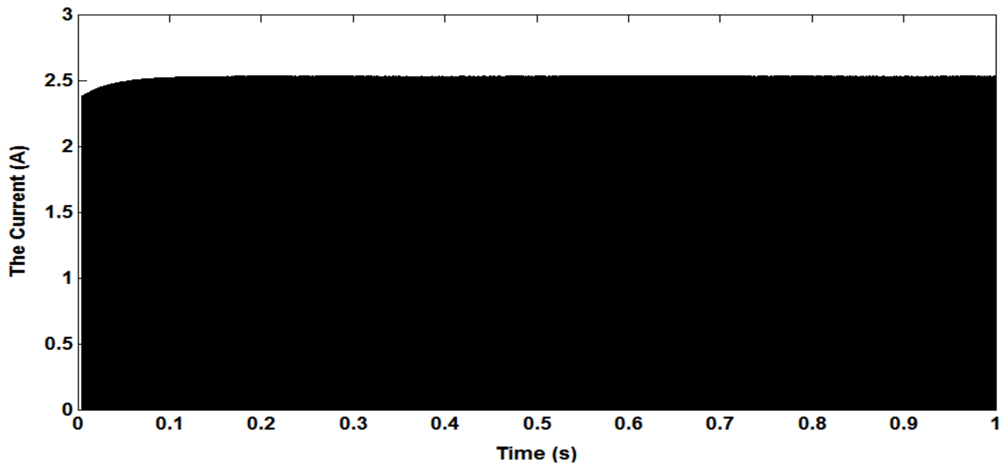

Figure 13. The control input (case **c)**).

5.2 The second numerical simulation: The simulations results in this set are devoted to prove the validation of the derived area of attraction Ω. To do that, two cases are considered. The initial condition for the first case lies outside Ω (**d)** $x(0) = 0.035\,m > x_{dmax}$, $\dot{x}(0) = 0.4\,m/s$), while in the second case the initial condition is taken inside Ω in spite of that the initial ball position is greater than the maximum desired value according to Eq. (19) (**e)** $x(0) = 0.035\,m > x_{dmax}$, $\dot{x}(0) = -0.4\,m/s$).

Figs. 14 and **15**, show clearly that when the state initiated outside the area of attraction Ω, the controller is no longer able to direct the state toward the sliding manifold, hence the MLS is unstable, **Fig. 14**. Alternatively, the ball is directed to the desired value when its initial condition is inside Ω as in **Fig. 15**.

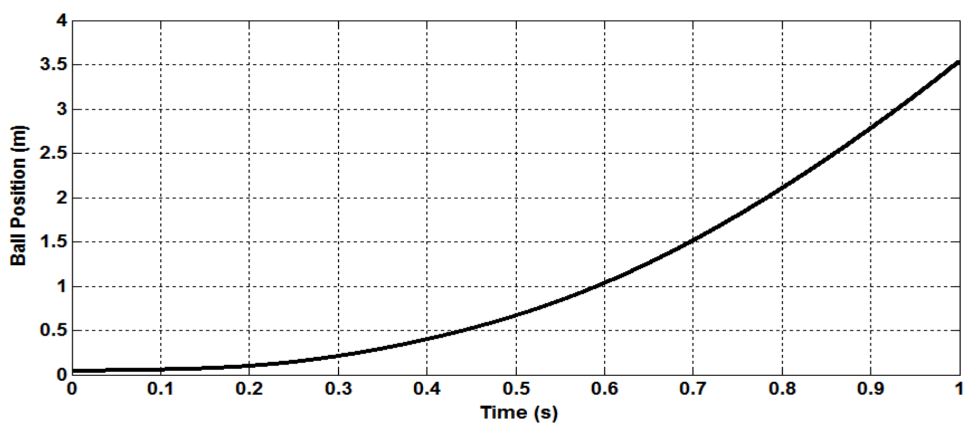

Figure 14. The ball position (case **d)**).

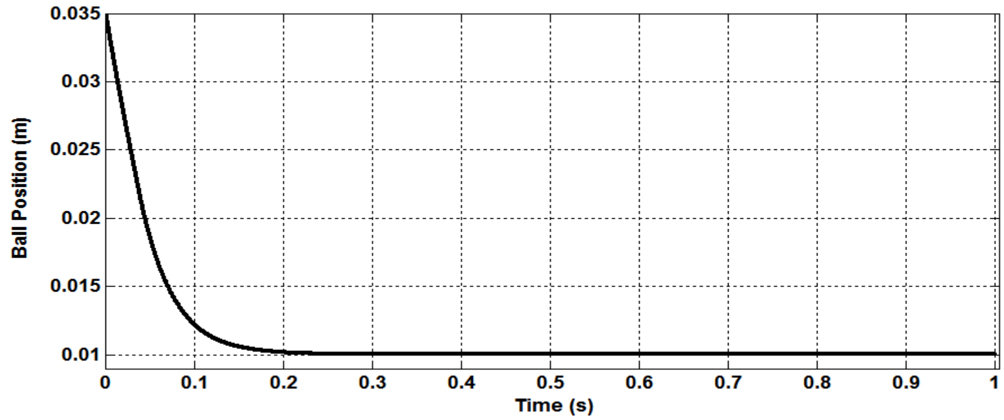

Figure 15. The ball position (case **d)**).

5.3 The third numerical simulation: In this simulation the initial condition is as in case **a),** but with desired position given by a sinusoidal function

$$x_d = 0.01 + 0.007 * \sin(2\pi t)$$

The desired ball position satisfies assumption (2), and additionally, the ball state is initiated inside Ω_v (21) with $\eta = 1.22$.

The obtained result for this simulation is shown in **Fig. 16**. This figure, prove without doubt the ability of the CVSC to make the ball position tracks the desired sinusoidal one effectively.

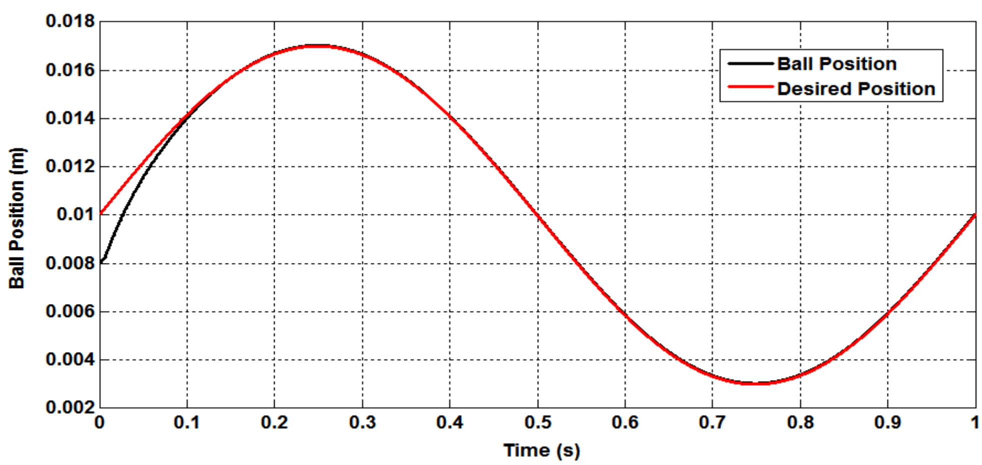

Figure 16. The ball position for sinusoidal reference.

6. CONCLUSIONS

The variable structure controller was effectively designed to the magnetic levitation system, based on Lyapunov function approach, in propositions (1) and (3). The proposed CVSC was effectively attenuate the chattering effect while forcing the ball position to various desired positions as shown in **Figs. 4**, **9**, and **16**. Additionally, the ability to control the ball position was guaranteed by the obtained area of attraction Ω, which derived in proposition (2). Also an ultimate bound was derived in terms of the control design parameters. This enables the designer to effectively squeeze the steady state error for the ball position to a reasonable value as shown in **Fig. 5**, where the steady state error does not exceed 0.1 mm.

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Appendix A

The uncertainty parameter α is estimated in this appendix. First the nominal function $\varphi_o(x)$ is defined as;

$$\varphi_o(x) = \frac{\varphi_{max}(x)+\varphi_{min}(x)}{2} \tag{A-1}$$

where $\varphi_{max}(x)$ and $\varphi_{min}(x)$ are the maximum and minimum functions of $\varphi(x)$ which are defined by

$$\left.\begin{aligned} \varphi_{max}(x) &= \frac{Q_{max}}{2m_{min}(X_{\infty min}+x)^2} \\ \varphi_{min}(x) &= \frac{Q_{min}}{2m_{max}(X_{\infty max}+x)^2} \end{aligned}\right\} \tag{A-2}$$

The maximum and minimum system parameters values are calculated by using their nominal values and the uncertainty percent r as follows;

$$\left.\begin{aligned} Q_{max,min} &= (1 \pm r)Q_o \\ m_{max,min} &= (1 \pm r)m_o \\ X_{\infty max,min} &= (1 \pm r)X_{\infty o} \end{aligned}\right\} \tag{A-3}$$

Accordingly from Eq. (5) the bound on $\Delta\varphi(x)$ can be calculated as follows

$$\begin{aligned} \Delta\varphi(x) &= \varphi(x) - \varphi_o(x) \le \varphi_{max}(x) - \varphi_o(x) \\ &= \left(\frac{\varphi_{max}(x)-\varphi_o(x)}{\varphi_o(x)}\right)\varphi_o(x) \le \alpha\varphi_o(x) \end{aligned} \tag{A-4}$$

where

$$\alpha \ge \frac{\varphi_{max}(x)-\varphi_o(x)}{\varphi_o(x)} = \frac{\varphi_{max}(x)}{\varphi_o(x)} - 1 \tag{A-5}$$