Static Analysis of Laminated Composite Plate using New Higher Order Shear Deformation Plate Theory

Dr.Ibtehal Abbas Sadiq
Instructor
College of Engineering-University of Baghdad
email-ebtialabas@yahoo.com

Haider Sami Abdul-ameer
Msc student
College of Engineering-University of Baghdad
email-haider.alam83@yahoo.com

ABSTRACT

In the present work a theoretical analysis depending on the new higher order element in shear deformation theory for simply supported cross-ply laminated plate is developed. The new displacement field of the middle surface expanded as a combination of exponential and trigonometric function of thickness coordinate with the transverse displacement taken to be constant through the thickness. The governing equations are derived using Hamilton's principle and solved using Navier solution method to obtain the deflection and stresses under uniform sinusoidal load. The effect of many design parameters such as number of laminates, aspect ratio and thickness ratio on static behavior of the laminated composite plate has been studied. The modal of the present work has been verified by comparing the results of shape functions with that were obtained by other workers. Result shows the good agreement with 3D elasticity solution and that published by other researchers.

Key words: higher order shear deformation theory, composite laminated plate, and static analysis.

التحليل السكوني للصفيحة المركبة بأستخدام نظرية قص ذات رتبة عالية جديدة

د. أبتهال عباس صادق
مدرس
جامعة بغداد-كلية الهندسة
قسم الهندسة الميكانيكية

حيدر سامي عبد الامير
طالب ماجستير
جامعة بغداد-كلية الهندسة
قسم الهندسة الميكانيكية

الخلاصة

في هذا البحث تم تطوير الحل النظري الذي يعتمد على نظرية قص جديدة ذات رتبة عالية للصفائح المركبة ذات الاسناد البسيط . مجال الازاحة الجديد للسطح الاوسط يتوسع لدمج الدالة الاسية و المثلثية و تكون الازاحة المستعرضة ثابتة خلال السمك. تم اشتقاق معادلات الحركة بأستخدام (Hamilton's principle) و تم حلها بأستخدام طريقة (Navier solution) لأيجاد الانحراف و الاجهادات تحت تأثير الحمل الجيبي المنتظم.
تأثير عدة عوامل تصميمية مثل عدد الطبقات،نسبة الطول الى العرضو نسبة السمك تم دراستها.
بينت النتائج موافقة جيدة مع الحل بأستخدام نظرية مرنة ثلاثية الابعاد و البحوث المنشورة لباحثين اخرين.
النموذج الرياضي للعمل الحالي تم اثباته بالمقارنة مع نتائج الدوال للباحثين الاخرين.

الكلمات الرئيسية: نظرية القص ذات الرتبة العالية ، الالواح الطبقية المركبة ، التحليل السكوني .

1. INTRODUCTION:

Composite materials are so necessary in many engineering applications, as vehicles parts, aero structures and medical devices industries. With the wide use of composite plate in the modern industry, static and dynamic analysis of plate structure under different types of loads and different boundary conditions become a main part in design procedure. In the few past years, many researchers resorted to the development of many theories to clearly predict the response of laminated plate composite material. Many researchers had studied static and dynamic analysis of composite plate by using higher order shear deformation theory, and other researchers have studied the static deflection and stresses of composite plates subjected to different uniform loads.

Fan and **Lin, 1998,** used an analytical solution of rectangular laminated plates by higher order theory. On the basis of the Reddy's higher-order theory of composites, this paper introduces a displacement function and transforms its three differential equations for symmetric cross-ply composites into only one order differential equation generated by the displacement-function. Which property is chosen, both solutions are obtained, namely, Navier-type solution of simply supported rectangular laminated plates and the Levy-type solution with the boundary condition, where two opposite edges are simply supported and remains are arbitrary. The numerical examples show that the results coincide well with the existing results in the references, thus validating that the method is reliable. The higher Order theory of Reddy is simpler in calculation but has higher precision than the first order shear deformation theory because the former has fewer unknowns than the latter and requires no shear coefficients. **Pervez, Al-Zebdeh** and **Farooq, 2010,** studied the effects of bboundary conditions in laminated composite plates using higher order shear deformation theory. The applicability of a modified higher order shear deformation theory to accurately determine the in-plane and transverse shear stress distributions in an orthotropic laminated composite plate subjected to different boundary conditions has been extended. A simpler, two-dimensional, shear deformable, plate theory accompanied with an appropriate set of through-thickness variations, is used to accurately predict transverse shear stresses. Finite element code was developed based on a higher order shear deformation theory to study the effects of boundary conditions on the behavior of thin-to-thick anisotropic laminated composite plates. The code was verified against three dimensional elasticity results. The study also compared the stresses and deformation results of higher order theory with those obtained using commercial software such as LUSAS, ANSYS and ALGOR. **Mantari, 2012,** used a new higher order shear deformation theory for sandwich and composite laminated plates. The proposed displacement field, which is ‘‘m’’ parameter dependent, is assessed by performing several computations of the plate governing equations. Therefore, it has been found that the results obtained are accurate and relatively close to 3D elasticity bending solutions. Plate governing equations and boundary conditions are derived by employing the principle of virtual work. The Navier-type exact solutions for static bending analysis are presented for sinusoidal and uniformly distributed loads**. Mantari and Soares, 2012,** studied bending analysis of thick exponentially graded plates using a new trigonometric higher order shear deformation theory. An analytical solution of the static governing equations of exponentially graded plates obtained by using a recently developed higher order shear deformation theory (HSDT) is presented. The mechanical properties of the plates are assumed to vary exponentially in the thickness direction. The governing equations of exponentially graded plates and boundary conditions are derived by employing the principle of virtual work. A Navier-type analytical solution is obtained for such plates subjected to transverse bi-sinusoidal loads for simply supported boundary conditions. Results are provided for thick to thin plates and for different values of the parameter n, which dictates the material variation profile through the plate thickness. The accuracy of the present code is verified by comparing it with 3D elasticity solution and with other well-known trigonometric shear deformation theory. **Lan** and **Feng, 2012,** presented an analysis of deflections and stresses for laminated composite plates based on a new higher-order shear deformation theory. Based on the new simple third-order shear deformation theory, the deflections and stresses of the simply supported symmetrical laminated composite plates are obtained by using the principle of virtual work .The solutions are compared with the solutions of three-dimensional elasticity theory, the first-order shear deformation theory and the Reddy's higher order shear deformation theory. Results show that the presented new theory is more reliable, accurate, and cost-effective in computation than the first-order shear deformation theories and other simple higher-order shear deformation theories.

Taher. etal. 2012, presented a theoretical formulation; Navier's solutions of rectangular plates based on a new higher order shear deformation model for the static response of functionally graded plates. The mechanical properties of the plate are assumed to vary continuously in the thickness direction by a simple power-law distribution in terms of the volume fractions of the constituents. Parametric studies are performed for varying ceramic volume fraction, volume fractions profiles, aspect ratios, and length to thickness ratios. It has been concluded that the proposed theory is accurate and simple in solving the static bending behavior of functionally graded plates. **Huu** and **Seung, 2013,** developed a simple higher-order shear deformation theory for bending and free vibration analysis of functionally graded plates. This theory has only four unknowns, but it accounts for a parabolic variation of transverse shear strains through the thickness of the plate. Equations of motion are derived from Hamilton's principle. Analytical solutions for the bending and free vibration analysis are obtained for simply supported plates. The obtained results are compared with 3D and quasi-3D solutions and those predicted by other plate theories. Results show that the results obtained are the same accuracy of the existing higher-order shear deformation theories which have more number of unknowns, but its accuracy is not comparable with those of 3D and quasi-3D models which include the thickness stretching effect.

Mantari, et al. 2014 developed a new tangential-exponential higher order shear deformation theory for advanced composite plates. This paper presents the static response of advanced composite plates by using a new non-polynomial higher order shear deformation theory (HSDT). The accounts for non-linear in plane displacement and constant transverse displacement through the plate thickness, complies with plate surface boundary conditions, and in this manner a shear correction factor is not required. Navier closed-form solution is obtained for functionally grade plates (FGPS) subjected to transverse loads for simply supported boundary conditions. The optimization of the shear strain function and bi-sinusoidal load is adopted in this publication. The accuracy of the HSDT is discussed by comparing the results with an existing quasi-3D exact solution and several HSDTs results. It is concluded that the present non-polynomial HSDT, is more effective than the well-known trigonometric HSDT for well-known example problems available in literature.

In the present work, a new higher order displacement field in which the displacement of the middle surface expanded as a combination of exponential and trigonometric functions of the thickness coordinate and the transverse displacement taken to be constant through the thickness, is proposed. Necessary equilibrium equations and boundary conditions are derived by employing the principle of virtual work. The theory accounts for adequate distribution of the transverse shear strains through the plate thickness and the tangential stress-free boundary conditions on the plate boundary surface, therefore a shear correction factor is not required. Exact solutions for deflections and stresses of simply supported plates are presented.

2. THEORETICAL ANALYSIS:

2.1. Displacement Field:

In the present work, a new higher order displacement field in which the displacement of the middle surface expanded as a combination of exponential trigonometric function of the thickness coordinate with the transverse displacement taken to be constant through the thickness was developed. The displacement field of the new higher order theory of laminated composite plate is: **Mantari, 2012**

$$u(x,y,z) = u(x,y) - z\,(\frac{\partial w}{\partial x}) + f(z)\theta_1(x,y)$$

$$v(x,y,z) = v(x,y) - z\,(\frac{\partial w}{\partial y}) + f(z)\theta_2(x,y)$$

$$w(x,y,z) = w(x,y) \qquad \text{(1a-c)}$$

Where:

$u(x,y), v(x,y), w(x,y), \theta_1(x,y), \theta_2(x,y)$ are the five unknown functions of middle surface of the plate as shown in the **Fig1**. While $f(z)$ represents shape functions determining the distribution of the transverse shear strains and stresses along the thickness.

The shape function derived by different researchers are given in **Table (1)**, actually the present modeling is a combination of exponential functions and polynomial as shown in **Fig2**.

With the same **Reddy** and **Liu** and generalized procedure developed by **Sadatos** and free boundary conditions at the top and bottom surfaces of the plate. The new displacement field in this paper is:

$$u(x,y,z) = u(x,y) + z\,(\frac{m\pi}{h}\theta_1 - \frac{\partial w}{\partial x}) + sin\frac{\pi z}{h}e^{\frac{m\pi z}{h}}\,\theta_1$$

$$v(x,y,z) = v(x,y) + z\,(\frac{m\pi}{h}\theta_2 - \frac{\partial w}{\partial y}) + sin\frac{\pi z}{h}e^{\frac{m\pi z}{h}}\theta_2$$

$$w(x,y,z) = w_0 \tag{2a-c}$$

where the new function used in present work is:

$$\mathbf{f(z) = sin\frac{\pi z}{h}e^{\frac{m\pi z}{h}} + yz} \tag{3}$$

$$y = \frac{\pi m}{h}\text{ , m = constant}$$

For small strains, the strain-displacement relations take the form:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y}$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} = 0$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}\gamma_{xy}$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \frac{1}{2}\gamma_{xz}$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \frac{1}{2}\gamma_{yz} \tag{4a-f}$$

The strain associated with the displacement field by substituting Eq (2a-c) into Eq. (4a-e) to give:

$$\varepsilon_{xx} = \varepsilon^0_{xx} + z\varepsilon^1_{xx} + sin\frac{\pi z}{h}e^{\frac{m\pi z}{h}}\,\varepsilon^2_{xx}$$

$$\varepsilon_{yy} = \varepsilon^0_{yy} + z\varepsilon^1_{yy} + sin\frac{\pi z}{h}e^{\frac{m\pi z}{h}}\,\varepsilon^2_{yy}$$

$$\gamma_{xy} = \varepsilon^0_{xy} + z\varepsilon^1_{xy} + sin\frac{\pi z}{h}e^{\frac{m\pi z}{h}}\,\varepsilon^2_{xy}$$

$$\gamma_{xz} = \varepsilon^0_{xz} + (m * sin\frac{\pi z}{h} + cos\frac{\pi z}{h})\frac{\pi}{h}e^{\frac{m\pi z}{h}}\varepsilon^3_{xz}$$

$$\gamma_{yz} = \varepsilon^0_{yz} + (m * sin\frac{\pi z}{h} + cos\frac{\pi z}{h})\,\frac{\pi}{h}e^{\frac{m\pi z}{h}}\,\varepsilon^3_{yz} \tag{5a-e}$$

Where:

$$\begin{Bmatrix} \varepsilon_{xx}^{0} \\ \varepsilon_{yy}^{0} \\ \gamma_{xy}^{0} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_{xx}^{1} \\ \varepsilon_{yy}^{1} \\ \gamma_{xy}^{1} \end{Bmatrix} = \begin{Bmatrix} \frac{m\pi}{h}\frac{\partial \Theta_1}{\partial x_1} - \frac{\partial^2 w}{\partial x^2} \\ \frac{m\pi}{h}\frac{\partial \Theta_2}{\partial y} - \frac{\partial^2 w}{\partial y^2} \\ \frac{m\pi}{h}\frac{\partial \Theta_2}{\partial x_1} + \frac{m\pi}{h}\frac{\partial \Theta_2}{\partial y} - 2\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_{xx}^{2} \\ \varepsilon_{yy}^{2} \\ \gamma_{xy}^{2} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \Theta_1}{\partial x} \\ \frac{\partial \Theta_2}{\partial y} \\ \frac{\partial \Theta_2}{\partial x} + \frac{\partial \Theta_1}{\partial y} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{xz}^{0} \\ \gamma_{yz}^{0} \end{Bmatrix} = \begin{Bmatrix} m\frac{\pi}{h}\Theta_1 \\ m\frac{\pi}{h}\Theta_2 \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{xz}^{3} \\ \gamma_{yz}^{3} \end{Bmatrix} = \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} \qquad (6a\text{-}e)$$

2.2. Hamilton's principles:

The equation of motion of the new higher order theory will be derived using the dynamic version of the principle of virtual displacements: **Reddy, 2003.**

$$0 = \int_0^t \delta U + \delta V - \delta K \qquad (7)$$

The virtual strain energy δU is:

$$\delta U = [\int_{\frac{-h}{2}}^{\frac{h}{2}} \{\int_{\Omega}^{k} \sigma_{xx}\delta\, \varepsilon_{xx}^{k} + \sigma_{yy}\delta\varepsilon_{yy}^{k} + \sigma_{xy}\delta\varepsilon_{xy}^{k} + \sigma_{yz}\delta\varepsilon_{yz}^{k} + \sigma_{xz}\delta\varepsilon_{xz}^{k}]\, \partial x \partial y\} \partial z] = 0 \qquad (8)$$

$$\delta U = \int (N_1\delta\, \varepsilon_{xx}^{0} + M_1\delta\varepsilon_{xx}^{1} + P_1\delta\varepsilon_{xx}^{2} + N_2\delta\varepsilon_{yy}^{0} + M_2\delta\varepsilon_{yy}^{1} + P_2\delta\varepsilon_{yy}^{2} + N_6\delta\varepsilon_{Xy}^{0} + M_6\delta\varepsilon_{Xy}^{1} + P_6\delta\varepsilon_{Xy}^{2} + Q_2\delta\varepsilon_{yz}^{0} + k_2\delta\varepsilon_{yz}^{3} + Q_1\delta\varepsilon_{\mathrm{xz}}^{0} + k_1\delta\varepsilon_{\mathrm{xz}}^{3} -)\partial x \partial y = 0 \qquad (9)$$

where:

(N_i, M_i, P_i, Q_i and K_i) are the result of the following integration:

$$(N_i, M_i, P_i) = \sum_{k=1}^{N} \int_{z^{k-1}}^{z^k} \sigma_i^k \left(1\,, z, sin\frac{\pi z}{h} e^{\frac{m\pi z}{h}}\right) dz \qquad (i = 1,2,6)$$

$$(Q_1, K1) = \sum_{k=1}^{N} \int_{z^{k-1}}^{z^k} \sigma_5^k \left(1, \frac{\pi}{h}(m * \ sin\frac{\pi z}{h} + cos\frac{\pi z}{h}) e^{\frac{m\pi z}{h}}\right) dz$$

$$(Q_2, K2) = \sum_{k=1}^{N} \int_{z^{k-1}}^{z^k} \sigma_4^k \left(1, \frac{\pi}{h}(m * \ sin\frac{\pi z}{h} + cos\frac{\pi z}{h}) e^{\frac{m\pi z}{h}}\right) dz$$

The virtual strains are known in terms of virtual displacement in Eq.(5) and then substituting the virtual strain into Eq.(9) and in integrating by parts to relative the virtual displacement (δu , δv , δw) in range of any differentiation, then we get:

$$0 = -\int [\frac{\partial N_1}{\partial x}\delta u + \frac{m\pi}{h}\frac{\partial M_1}{\partial x}\delta\Theta_1 - \frac{\partial^2 M_1}{\partial x^2}\delta w + \frac{\partial P_1}{\partial x}\delta\Theta_1 + \frac{\partial N_2}{\partial y}\delta v + \frac{m\pi}{h}\frac{\partial M_2}{\partial y}\delta\Theta_2 - \frac{\partial^2 M_2}{\partial y^2}\delta w + \frac{\partial P_2}{\partial y}\delta\Theta_2 + \frac{\partial N_6}{\partial y}\delta u + \frac{\partial N_6}{\partial x}\delta v + \frac{m\pi}{h}\frac{\partial M_6}{\partial y}\delta\Theta_1 + \frac{m\pi}{h}\frac{\partial M_6}{\partial x}\delta\Theta_2 + 2\frac{\partial^2 M_6}{\partial x\,\partial y}\delta w + \frac{\partial P_6}{\partial y}\delta\Theta_1 + \frac{\partial P_6}{\partial x}\delta\Theta_2 - \frac{m\pi}{h}Q_1\delta\Theta_1 - \frac{m\pi}{h}Q_2 - K_1\delta\Theta_1 - K_2\delta\Theta_2]\,\partial x\,\partial y = 0 \quad (10)$$

The virtual work done by applied forces δv is:

$$\delta v = -\int q\delta w\, dxdy \quad (11)$$

2.3. Equation of motion:

The Euler-Lagrange is obtained by substituting Eq.(8 – 11) into Eq.(7) and then setting the coefficient of (δu , δv, δw , $\delta\Theta_1$, $\delta\Theta_2$) over Ω_0 of Eq.(7) to zero separately, this give five equations of motion as follows:

$$\delta u: \frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = 0$$

$$\delta v: \frac{\partial N_2}{\partial y} + \frac{\partial N_6}{\partial x} = 0$$

$$\delta w: \frac{\partial^2 M_1}{\partial x^2} + \frac{\partial^2 M_2}{\partial y^2} + 2\frac{\partial^2 M_6}{\partial x\,\partial y} + p = 0$$

$$\delta\Theta_1: \frac{m\pi}{h}\frac{\partial M_1}{\partial x} + \frac{m\pi}{h}\frac{\partial M_6}{\partial y} + \frac{\partial P_1}{\partial x} + \frac{\partial P_6}{\partial y} - \frac{m\pi}{h}Q_1 - K_1 = 0$$

$$\delta\Theta_2: \frac{m\pi}{h}\frac{\partial M_2}{\partial y} + \frac{m\pi}{h}\frac{\partial M_6}{\partial x} + \frac{\partial P_2}{\partial y} + \frac{\partial P_6}{\partial x} - \frac{m\pi}{h}Q_2 - K_2 = 0 \quad (12a\text{-}e)$$

The result forces are given by:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \sum_{k=1}^{N}\int_{z^k}^{z^{k+1}} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} dz$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \sum_{k=1}^{N}\int_{z^k}^{z^{k+1}} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} z\, dz \begin{Bmatrix} P_1 \\ P_2 \\ P_6 \end{Bmatrix} = \sum_{k=1}^{N}\int_{z^k}^{z^{k+1}} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} f(z)\, dz$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \sum_{k=1}^{n} \begin{Bmatrix} \sigma_5 \\ \sigma_4 \end{Bmatrix} \partial z$$

$$\begin{Bmatrix} k_1 \\ k_2 \end{Bmatrix} = \sum_{k=1}^{n} \begin{Bmatrix} \sigma_5 \\ \sigma_4 \end{Bmatrix} f(z)\partial z \quad (13a\text{-}e)$$

The plane stress reduced stiffness Q_{ij} is:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} \;,\; Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} \;,\; Q_{11} = \frac{E_2}{1-\nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12} \;,\; Q_{44} = G_{23} \;,\; Q_{55} = G_{13} \quad (14)$$

From the constitutive relation of the lamina, the transformed stress-strain relation of an orthotropic lamina in a plane state of stress is:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} Q_{12} Q_{16} \\ Q_{12} Q_{22} Q_{26} \\ Q_{16} Q_{26} Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \qquad (15)$$

The force results are related to the strains by the relations:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{bmatrix} A_{11}A_{12}A_{16} \\ A_{12}A_{22}A_{26} \\ A_{16}A_{26}A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + \begin{bmatrix} B_{11}B_{12}B_{16} \\ B_{12}B_{22}B_{26} \\ B_{16}B_{26}B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^1 \\ \varepsilon_2^1 \\ \varepsilon_6^1 \end{Bmatrix} + \begin{bmatrix} E_{11}E_{12}E_{16} \\ E_{12}E_{22}E_{26} \\ E_{16}E_{26}E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^2 \\ \varepsilon_2^2 \\ \varepsilon_6^2 \end{Bmatrix}$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{bmatrix} B_{11}B_{12}B_{16} \\ B_{12}B_{22}B_{26} \\ B_{16}B_{26}B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + \begin{bmatrix} D_{11}D_{12}D_{16} \\ D_{12}D_{22}D_{26} \\ D_{16}D_{26}D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^1 \\ \varepsilon_2^1 \\ \varepsilon_6^1 \end{Bmatrix} + \begin{bmatrix} F_{11}F_{12}F_{16} \\ F_{12}F_{22}F_{26} \\ F_{16}F_{26}F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^2 \\ \varepsilon_2^2 \\ \varepsilon_6^2 \end{Bmatrix}$$

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_6 \end{Bmatrix} = \begin{bmatrix} E_{11}E_{12}E_{16} \\ E_{12}E_{22}E_{26} \\ E_{16}E_{26}E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + \begin{bmatrix} F_{11}F_{12}F_{16} \\ F_{12}F_{22}F_{26} \\ F_{16}F_{26}F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^1 \\ \varepsilon_2^1 \\ \varepsilon_6^1 \end{Bmatrix} + \begin{bmatrix} H_{11}H_{12}H_{16} \\ H_{12}H_{22}H_{26} \\ H_{16}H_{26}H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^2 \\ \varepsilon_2^2 \\ \varepsilon_6^2 \end{Bmatrix}$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} + \begin{bmatrix} J_{44} & J_{45} \\ J_{45} & J_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^3 \\ \gamma_{xz}^3 \end{Bmatrix}$$

$$\begin{Bmatrix} k_1 \\ k_2 \end{Bmatrix} = \begin{bmatrix} J_{44} & J_{45} \\ J_{45} & J_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} + \begin{bmatrix} L_{44} & L_{45} \\ L_{45} & L_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^3 \\ \gamma_{xz}^3 \end{Bmatrix} \qquad (16a\text{-}e)$$

Where:

$$A_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}\, dz \qquad i = (1,2,4,5,6)$$

$$(B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) =$$

$$\int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}(z, z^2, \sin(\frac{\pi z}{h}) e^{\frac{m\pi z}{h}}, \sin(\frac{\pi z}{h}) e^{\frac{m\pi z}{h}} z, \sin^2(\frac{\pi z}{h})\, e^{\frac{2m\pi z}{h}} \qquad i = (1,2,6)$$

$$J_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij} \frac{\pi}{h} e^{\frac{m\pi z}{h}} (m * \sin\frac{\pi z}{h} + \cos\frac{\pi z}{h}) dz$$

$$L_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij} \left(\frac{\pi}{h}\right)^2 e^{\frac{m\pi z}{h}} \left(m * \sin\frac{\pi z}{h} + \cos\frac{\pi z}{h}\right)^2 dz \qquad i = (4,5) \qquad (17a\text{-}d)$$

2.4. Navier's Solution

In Navier's method the generalized displacements are expanded in a double trigonometric series in terms of unknown parameters. The choice of the function in the series is restricted to those which satisfy the boundary conditions of the problem as shown in **Fig 3**. Substitution of the displacement expansion into the governing equations should give a set of algebraic equation among the parameter of the expansion.

Simply supported boundary conditions are satisfied by assuming the following form of displacements: **Reddy, 2003**

$$u(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn} \cos(\alpha x) \sin(\beta y)$$

$$v(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn} \sin(\alpha x) \cos(\beta y)$$

$$w(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} \sin(\alpha x) \sin(\beta y)$$

$$\theta_1(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \theta_{1_{mn}} \cos(\alpha x) \sin(\beta y)$$

$$\theta_2(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \theta_{2_{mn}} \sin(\alpha x) \cos(\beta y) \qquad (18)$$

Where:

$\alpha = \frac{m\pi}{h}$, $\beta = \frac{n\pi}{h}$,($U_{mn}, V_{mn}, W_{mn}, \theta_{1mn}, \theta_{2mn}$) are arbitrary constants .

The Navier solution exists if the following stiffnesses are zero, $A_{16} = B_{16} = D_{16} = E_{16} = F_{16} = H_{16} = A_{26} = B_{26} = D_{26} = E_{26} = F_{26} = H_{26} = A_{45} = J_{45} = L_{45} = 0$

The equation of motion in Eq. (12) can be expressed in terms of displacements by substituting the force and moment resultants from Eqs.(16 and 17) and substituting Eq. (18a-e) into Eq. (12a-e), the following equations are obtained:

$$k_{ij}d_{ij} = F_{ij} \quad (i = j = 1 \dots\dots 5) and\ k_{ij} = k_{ji} \tag{19}$$

$$\{d_{ij}\} = \{U_{mn}, V_{mn}, W_{mn}, \theta_{1mn}, \theta_{2mn}\} \tag{20}$$

$$\{F_{ij}\} = \{0,0, Q_{mn}, 0,0\} \tag{21}$$

Where Q_{mn} are the coefficients in the double Fourier expansion of the transverse load.

$$q(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn} \sin(\alpha x) \sin(\beta y) \tag{22}$$

Where the stiffness element of k_{ij} are:

$$C_{11} = A_{11}\alpha^2 - A_{66}\beta^2$$
$$C_{12} = A_{12}\alpha\,\beta - A_{66}\alpha\,\beta$$
$$C_{13} = B_{11}\alpha^3 + B_{12}\alpha\beta^2 + 2B_{66}\alpha\beta^2$$
$$C_{14} = -B_{11}\frac{m\pi}{h}\alpha^2 - E_{11}\alpha^2 - B_{66}\frac{m\pi}{h}\beta^2 - E_{66}\beta^2$$
$$C_{15} = -B_{12}\frac{m\pi}{h}\alpha\,\beta - E_{12}\alpha\,\beta - B_{66}\frac{m\pi}{h}\alpha\,\beta - E_{66}\alpha\,\beta$$
$$C_{21} = A_{12}\alpha\,\beta - A_{66}\alpha\,\beta$$
$$C_{22} = -A_{22}\beta^2 - A_{66}\alpha^2$$
$$C_{23} = B_{12}\alpha^2\beta + B_{22}\beta^3 + 2B_{66}\alpha^2\beta$$
$$C_{24} = -B_{12}\frac{m\pi}{h}\alpha\,\beta - E_{12}\alpha\,\beta - B_{66}\frac{m\pi}{h}\alpha\,\beta - E_{66}\alpha\,\beta$$
$$C_{25} = -E_{22}\beta^2 - B_{66}\frac{m\pi}{h}\alpha^2 - E_{66}\alpha^2 - B_{22}\frac{m\pi}{h}\beta^2$$
$$C_{31} = B_{11}\alpha^3 + B_{12}\alpha\beta^2 + 2B_{66}\alpha\beta^2$$
$$C_{32} = B_{12}\alpha^2\beta + B_{22}\beta^3 + 2B_{66}\alpha^2\beta$$
$$C_{33} = -D_{11}\alpha^4 - 2D_{12}\alpha^2\beta^2 - D_{22}\beta^4 - 4D_{66}\alpha^2\beta^2$$
$$C_{34} = D_{11}\frac{m\pi}{h}\alpha^3 + D_{12}\frac{m\pi}{h}\alpha\beta^2 + F_{12}\alpha\beta^2 + F_{11}\alpha^3 + 2D_{66}\frac{m\pi}{h}\alpha\,\beta^2 + 2F_{66}\alpha\,\beta^2$$
$$C_{35} = D_{12}\frac{m\pi}{h}\alpha^2\beta + F_{12}\alpha^2\beta + D_{22}\frac{m\pi}{h}\beta^3 + F_{22}\beta^3 + 2D_{66}\frac{m\pi}{h}\alpha^2\beta^2 + F_{66}\alpha^2\beta$$
$$C_{41} = -B_{11}\frac{m\pi}{h}\alpha^2 - E_{11}\alpha^2 - B_{66}\frac{m\pi}{h}\beta^2 - E_{66}\beta^2$$
$$C_{42} = -B_{12}\frac{m\pi}{h}\alpha\,\beta - E_{12}\alpha\,\beta - B_{66}\frac{m\pi}{h}\alpha\,\beta - E_{66}\alpha\,\beta$$
$$C_{43} = D_{11}\frac{m\pi}{h}\alpha^3 + D_{12}\frac{m\pi}{h}\alpha\beta^2 + F_{12}\alpha\beta^2 + F_{11}\alpha^3 + 2D_{66}\frac{m\pi}{h}\alpha\,\beta^2 + 2F_{66}\alpha\,\beta^2$$

$$C_{44} = -D_{11}\frac{m^2\pi^2}{h^2}\alpha^2 - 2F_{11}\frac{m\pi}{h}\alpha^2 - D_{66}\frac{m^2\pi^2}{h^2}\beta^2 - 2F_{66}\frac{m\pi}{h}\beta^2 - H_{11}\alpha^2 - B66\beta^2 - F_{66}\beta^2 - A_{55}\frac{m^2\pi^2}{h^2} - 2J_{55}\frac{m\pi}{h} - L_{55}$$

$$C_{45} = -D_{12}\frac{m^2\pi^2}{h^2}\alpha\beta - 2F_{12}\frac{m\pi}{h}\alpha\beta - D_{66}\frac{m^2\pi^2}{h^2}\alpha\beta - 2F_{66}\frac{m\pi}{h}\alpha\beta - H_{12}\alpha\beta - H_{66}\alpha\beta$$

$$C_{51} = -B_{12}\frac{m\pi}{h}\alpha\beta - E_{12}\alpha\beta - B_{66}\frac{m\pi}{h}\alpha\beta - E_{66}\alpha\beta$$

$$C_{52} = -E_{22}\beta^2 - B_{66}\frac{m\pi}{h}\alpha^2 - E_{66}\alpha^2 - B_{22}\frac{m\pi}{h}\beta^2$$

$$C_{53} = D_{12}\frac{m\pi}{h}\alpha^2\beta + F_{12}\alpha^2\beta + D_{22}\frac{m\pi}{h}\beta^3 + F_{22}\beta^3 + 2D_{66}\frac{m\pi}{h}\alpha^2\beta^2 + F_{66}\alpha^2\beta$$

$$C_{54} = -D_{12}\frac{m^2\pi^2}{h^2}\alpha\beta - 2F_{12}\frac{m\pi}{h}\alpha\beta - D_{66}\frac{m^2\pi^2}{h^2}\alpha\beta - 2F_{66}\frac{m\pi}{h}\alpha\beta - H_{12}\alpha\beta - H_{66}\alpha\beta$$

$$C_{55} = -D_{22}\frac{m^2\pi^2}{h^2}\beta^2 - 2F_{22}\frac{m\pi}{h}\beta^2 - D_{66}\frac{m^2\pi^2}{h^2}\alpha^2 - 2F_{66}\frac{m\pi}{h}\alpha^2 - H_{22}\beta^2 - H_{66}\alpha^2 - A_{44}\frac{m^2\pi^2}{h^2} - J_{44}\frac{m\pi}{h} - J_{44} - L_{44}$$

The main computer program has been built to carry out the analysis required for solving the equations of motion and determine the deflection and stresses of composite laminated simply supported plate using new higher order shear deformation plate theory. A computer code written in (Matlab 13). The flow chart of computer programming shown in **Fig 4.**

3-RESULT AND CONCULATION

3-1-Result

The stresses and deflection of composite laminated plate under uniform sinsundiol load with different design parameters for simply supported boundary condition, are analyzed and solved using Matlab 13 programming. To examine the validly of the derived equation and performance of computer programming for bending and stress analysis of composite laminated simply supported plate, a comparison[3D elasticity &J.Raddy & J.L.Mantari] for square plate [h=1 and a=b] for two, three and four layers cross ply laminated simply supported on all edge, while the mechanical properties of each layers are (E_1=175 Gpa, E_2=E_3=7Gpa, v_{12}=v_{13}=0.25, v_{23}=0, G_{12}=G_{13}=3.5Gpa, G_{23}=1.5Gpa).

Table 2 shows the non-dimensional maximum deflections and stresses for symmetric and unsymmetric laminated plate in four layers (0/90/90/0) (a=b). The results of the present theory and other theaories such as (Reddy and Mantari) are compared with the three dimentional elasticity results (3D) for simply supported symmetric cross ply laminated plate which shows that the present results are in good agreement with 3D elasticity solution in deflection and normal stresses, However there is a considerable difference with 3D elasticity solution for (γ_{yz}) srtress for both thick and thin plate.

To examine the comparion between symmetric and unsymmetric four layer laminated square plate, **Fig 5** showes the non dimensional central deflection versus side-to-thickness (a/h) for (0/90/90/0) and (0/90/0/90) for the same mechanical properties under sinsoidal load.

Table 3 shows the non-dimensional maximum deflections and stresses in three layers (0/90/0) for the same square plate and mechanical properties .The performance of the present theory is evaluated by calculating the error compared 3D exact solution. The results of the present method give better results for shear stresses than in normal stresses for thick plate (a/h=4). Additionally, for a/h>=10, the proposed theory performs best in terms normal and shear stresses and the error decreases with increase of (a/h) ratio.

Table 4 shows the maximum central deflection and stresses in three layers (0/90/0) for the same mechanical properties of rectangular simply supported composite laminated plates (b=3a) under sinusoidal load similar conclusion compared with the square plate can be inferred.

Fig 6. Shows the non dimentional deflection versus side-to-thickness ratio (a/h) for cross –ply (0/90) laminated plate compared with **Mantari 2012** and **Reddy 2003**. And **Fig 7** shows the nondimentional deflection versus modules ratio (E_1/E_2) for cross –ply (0/90) compared with, the present work shows closed results with that published with the above theories.

3.2. Conclusions

A new higher order shear deformation theory of simply supported composite laminated plate is developed. The displacement of the middle surface is expanded as combination of exponential and trigonometric functions of the thickness coordinate and the transverse displacement taken to be constant through the thickness, the theory accounts for adequate distribution of the transverse shear strains though the plate thickness and tangential stress-free boundary conditions on the plate boundary surface, therefore a shear correction factor is not required.

The results obtained from present theory give an accurate results for thick, and moderately thick and thin plate when comparing it with that published from other research.

Nomenclature

Symbol	Discretion	Units
A	Plate dimension in x-direction	m
A_{ij} , B_{ij} , D_{ij} , E_{ij} F_{ij} , H_{ij}	Extension, bending extension coupling, bending and additional stiffness	-
B	Plate dimension in y-direction	m
E_1 , E_2 , E_3	Elastic modulus components	GPa
G_{12} , G_{23} , G_{13}	Shear modulus components	GPa
H	Plate thickness	m

K_1 , K_2	Transverse shear force result(HSDT)	N
M_1 , M_2 , M_6	Moment result per unit length	N.m/m
N_1 , N_2 , N_6	In-plane force result	N/m
N	Total number of plate layers	-
P_1 , P_2 , P_6	Result force per unit length	N/m
Q_1 , Q_2	Transverse shear force result	N
x,y,z	Cartesian coordinate system	M
z_k , z_{k+1}	Upper and lower lamia surface coordinates along z-direction	M
ε_{xx} , ε_{yy} , ε_{xy}	Strain components	m/m
γ_{xz} , γ_{yz}	Transverse shear strain	m/m
ν_{12}	Poisson's ratio components	-
σ_{xx} , σ_{yy} , σ_{xy} , σ_{yz} σ_{xz}	Stress components	GPa
θ	Fiber orientation angle	degree
U_{mn} , V_{mn} , W_{mn} , ${\phi^1}_{mn}$ ${\phi^2}_{mn}$	Arbitrary constant	-
u(x, y)	Flexural displacement	-
v(x, y)	Flexural displacement	-
w(x, y)	Flexural displacement	-
A	$\frac{m\pi}{h}$	-
β	$\frac{n\pi}{h}$	-
C_{ij}	Stiffness matrix	-
1,2,3	Principal material coordinate system	-
W	Deflection	M
Z	Distance from neutral axis	M

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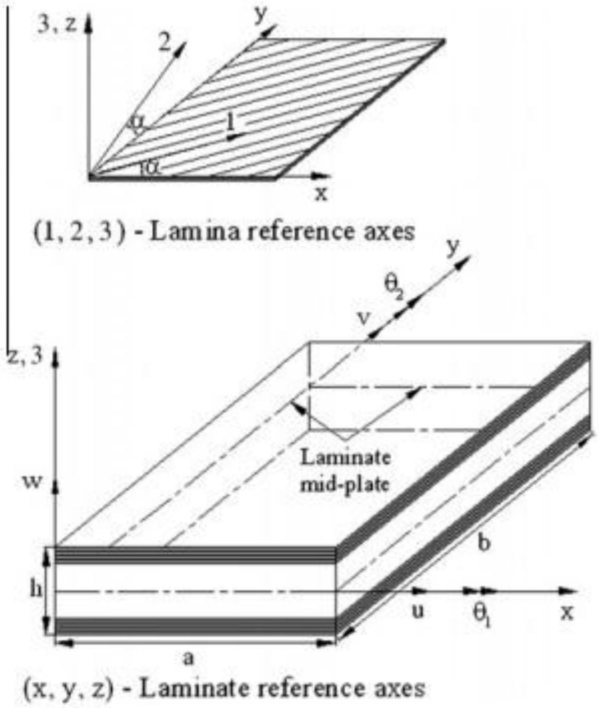


Figure 1. Laminate geometry with positive set of lamina/laminate reference axes, displacement components and fiber orientation.

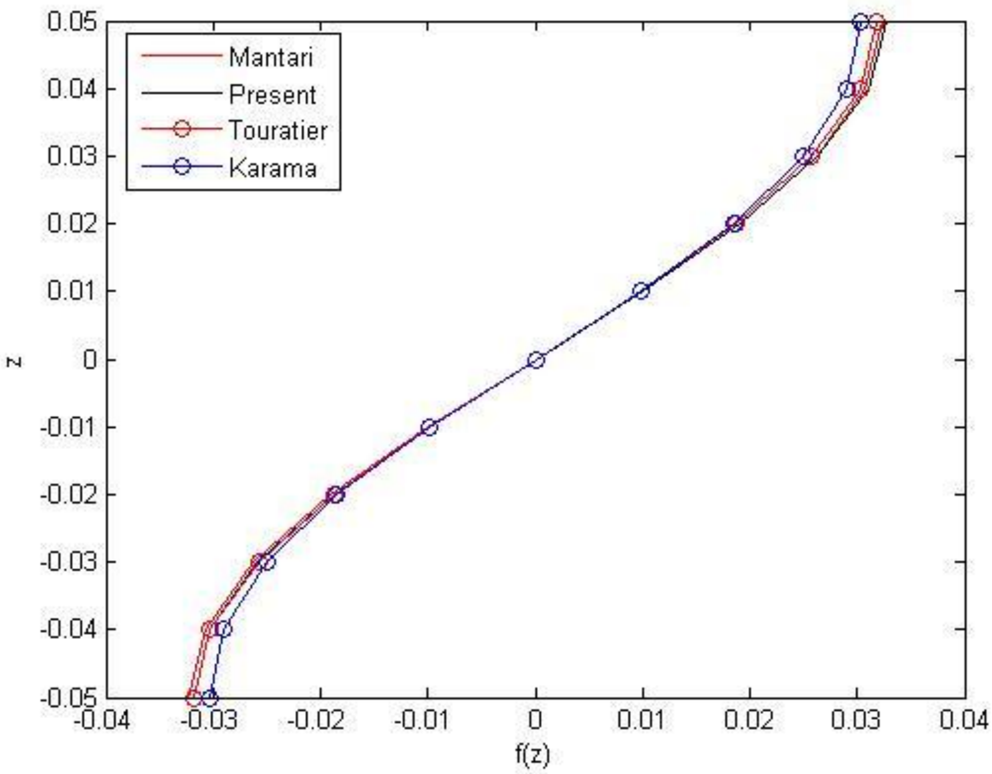
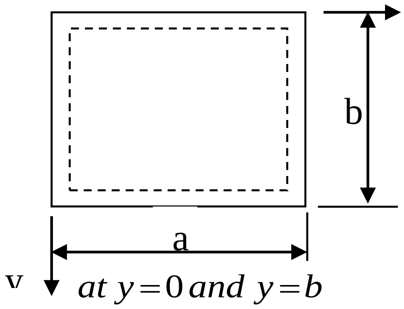


Figure 2. Shape strain functions of different shear deformation theories.

$$at\ x=0\ and\ x=a$$

$$v_0 = w_0 = \frac{\partial w_0}{\partial y} = 0$$

$$N_{xx} = M_{xx} = 0$$



$$at\ y=0\ and\ y=b$$

$$u_0 = w_0 = \frac{\partial w_0}{\partial x} = 0$$

$$N_{yy} = M_{yy} = 0$$

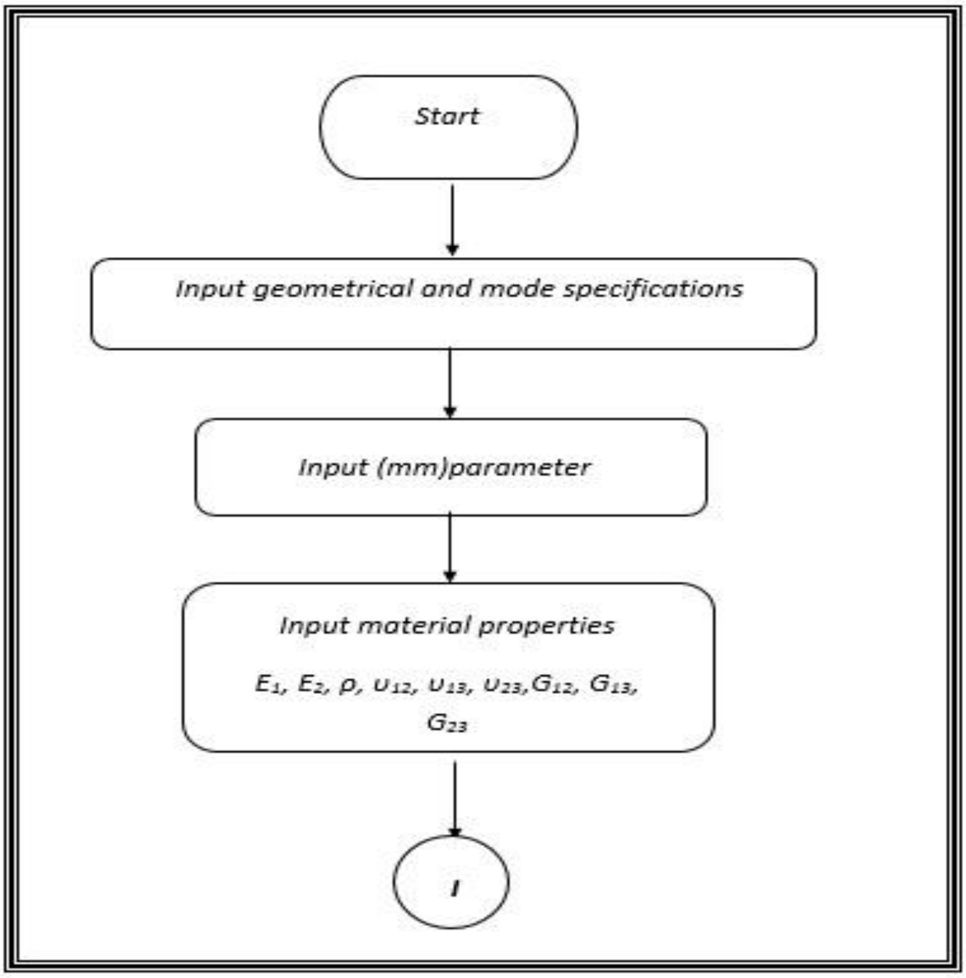


Figure 3. Boundary condition for simply supported plate.

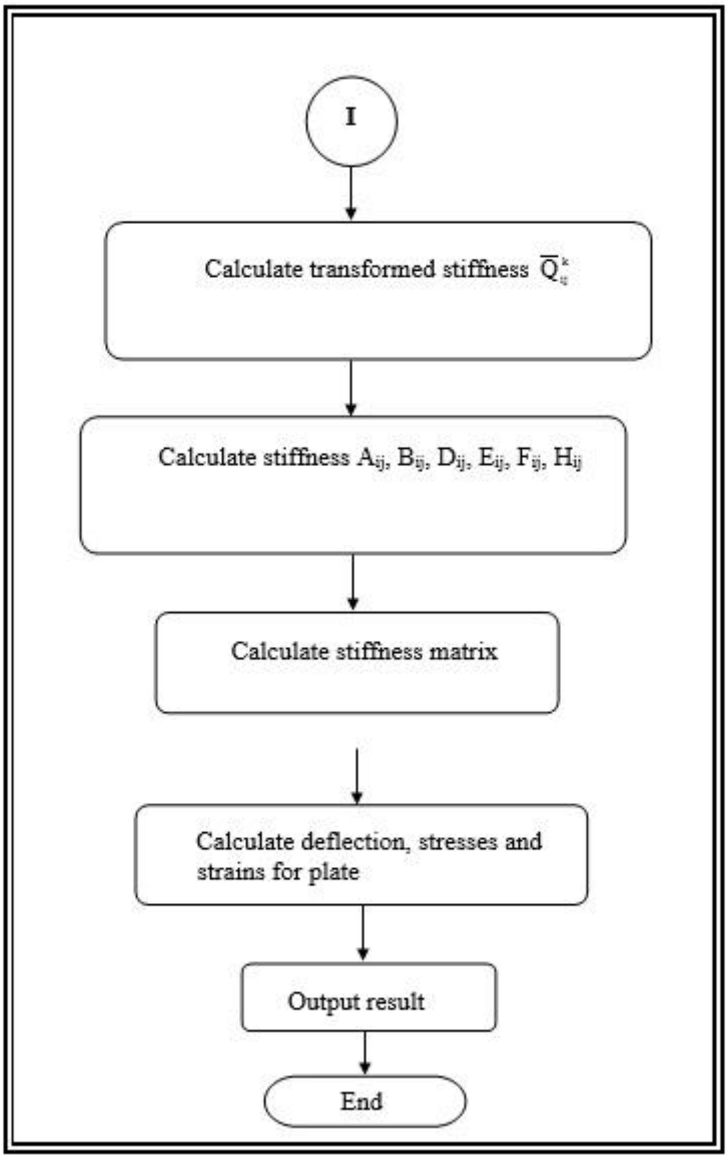


Figure 4. The flow chart of computer programming.

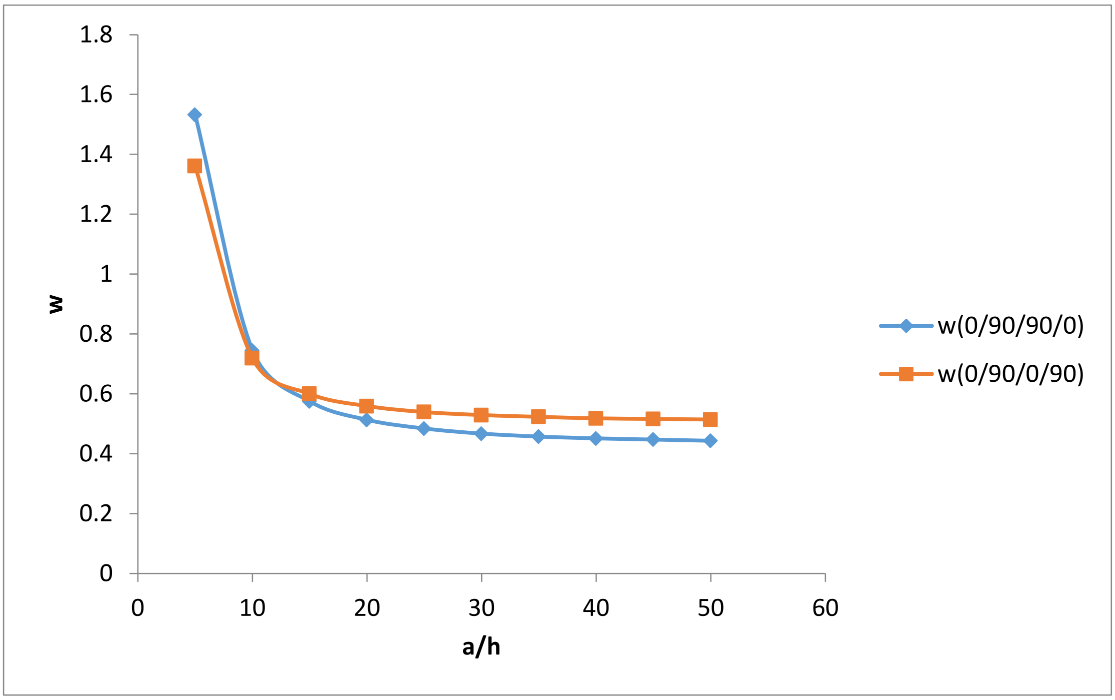


Figure 5. Nondimentionalized deflection versus side-to-thickness ratio (a/h) for symmetrical cross-ply (0/90/90/0) and unsymmetrical cross-ply (0/90/0/90) laminate under sinusoidal load

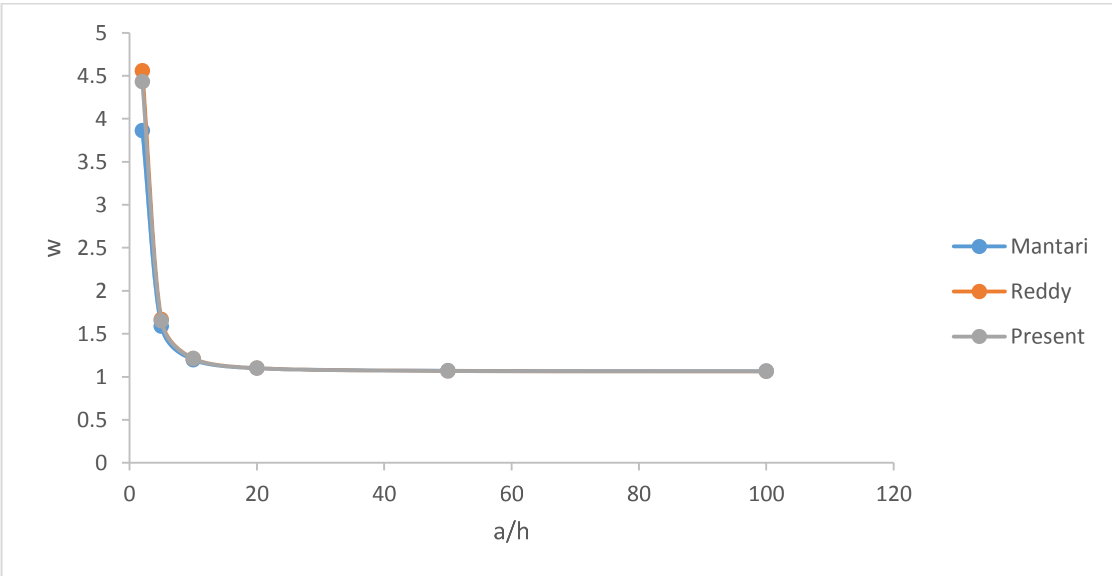


Figure 6. Nondimentionalized deflection versus side-to-thickness ratio (a/h) for cross –ply (0/90) for different modals

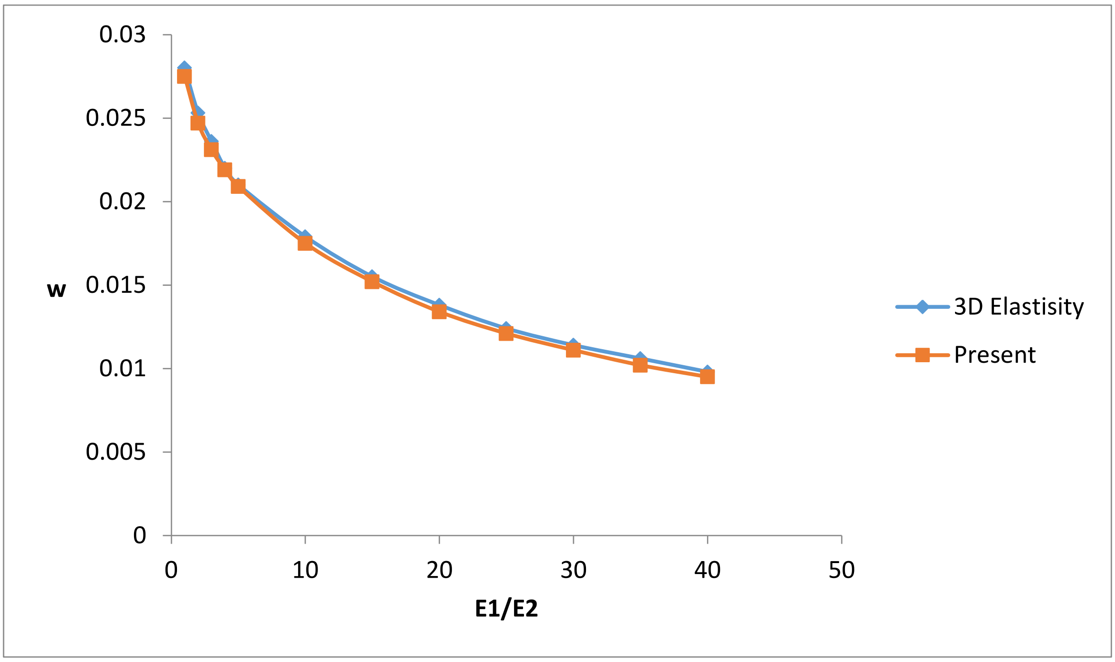


Figure 7. Nondimentionalized deflection versus modules ratio (E_1/E_2) for cross –ply (0/90) for 3Delastisity comparison with present work

Table 1. Different shear shape strain functions.

Modal	f(z)function
Touratier 1991	$f(z) = \frac{h}{\pi}\sin\frac{\pi z}{h}$
Karma 2003	$f(z) = ze^{-2(z/h)^2}$
Mantari 2012	$f(z) = \sin\frac{m\pi z}{h} e^{\cos\frac{m\pi z}{h}} + yz$
Present	$f(z) = \sin\frac{\pi z}{h} e^{\frac{m\pi z}{h}} + yz$

Table 2. Non-dimensional deflections and stresses in four layers (0/90/90/0) square plate (a=b) under sinusoidal load, $= \frac{10^2 w_0*\left(\frac{a}{2},\frac{b}{2}\right)*E_2*h^3}{a^4*q_0}$, $\sigma_{xx} = \frac{\sigma_{xx}*\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right)*h^2}{a^2*q_0}$, $\sigma_{yy} = \frac{\sigma_{yy}*\left(\frac{a}{2},\frac{b}{2},-\frac{h}{4}\right)*h^2}{a^2*q_0}$ $\sigma_{xy} = \frac{\sigma_{xy}*\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right)*h^2}{a^2*q_0}$, $\sigma_{yz} = \frac{\sigma_{yz}*\left(\frac{a}{2},0,0\right)*h}{a*q_0}$, $\sigma_{xz} = \frac{\sigma_{xz}*\left(0,\frac{b}{2},0\right)*h}{a*q_0}$

Method	a/h	W	Diff %	σ_{xx}	Diff %	σ_{yy}	Diff %	σ_{xy}	Diff %	σ_{xz}	Diff %	σ_{yz}	Diff %
3D elasticity		1.954	-	0.720	-	0.663	-	0.047	-	0.219	-	0.291	-
Reddy 2003	4	1.893	3.12	0.665	7.63	0.632	4.67	0.044	6.38	0.206	5.93	0.239	17.8
Mantari2012		1.921	1.68	0.740	2.77	0.635	4.22	0.048	2.12	0.254	15.9	0.269	7.56
Present		1.909	2.30	0.682	5.27	0.635	4.22	0.045	4.25	0.216	1.36	0.246	15.4
3D elasticity		0.743	-	0.559	-	0.401	-	0.027	-	0.301	-	0.196	-
Reddy 2003	10	0.715	3.76	0.546	2.32	0.389	2.99	0.026	3.70	0.264	12.2	0.153	21.9
Mantari2012		0.730	1.74	0.561	0.35	0.395	1.49	0.028	3.70	0.335	11.2	0.177	9.69
Present		0.720	3.09	0.549	1.78	0.391	2.49	0.027	0	0.279	7.30	0.158	19.3
3D elasticity		0.517	-	0.543	-	0.308	-	0.023	-	0.328	-	0.156	-
Reddy2003	20	0.506	2.12	0.539	0.73	0.304	1.29	0.023	0	0.283	13.7	0.123	21.1
Mantari2012		0.511	1.16	0.543	0	0.306	0.64	0.023	0	0.362	10.3	0.142	8.97
Present		0.507	1.93	0.540	0.55	0.305	0.97	0.023	0	0.299	8.84	0.127	18.5
3D elasticity		0.438	-	0.539	-	0.276	-	0.021	-	0.337	-	0.141	-
Reddy2003	100	0.434	0.91	0.539	0	0.273	0.79	0.021	0	0.290	13.9	0.112	20.5
Mantari2012		0.435	0.68	0.539	0	0.271	1.81	0.021	0	0.372	10.3	0.128	9.21
Present		0.434	0.91	0.539	0	0.271	1.81	0.021	0	0.307	8.90	0.115	18.4

Table 3. Non-dimensional maximum deflections and stresses in three layers (0/90/0) square plate (a=b) under sinusoidal load, $\sigma_{yy} = \frac{\sigma_{yy}*\left(\frac{a}{2},\frac{a}{2},-\frac{h}{6}\right)*h^2}{a^2*q_0}$.

Method	a/h	W	σ_{xx}	Diff %	σ_{yy}	Diff %	σ_{xy}	Diff %	σ_{xz}	Diff %	σ_{yz}	Diff %
3D elasticity	4	-	0.755	-	0.556	-	0.0505	-	0.282	-	0.217	-
Mantar2012		1.943	0.823	9.00	0.497	13.8	0.0536	6.13	0.245	13.1	0.201	7.37
Karama2003		1.944	0.775	2.64	0.502	9.71	0.0516	2.17	0.220	21.9	0.191	11.9
Present		1.930	0.754	0.13	0.503	9.53	0.0507	0.39	0.211	25.1	0.188	13.3
3D elasticity	10	-	0.590	-	0.288	-	0.028	-	0.357	-	0.123	-
Mantar2012		0.734	0.588	0.33	0.276	4.16	0.028	0	0.314	12.0	0.115	6.50
Karama2003		0.723	0.576	2.37	0.272	5.55	0.028	0	0.272	23.0	0.108	12.1
Present		0.718	0.572	3.05	0.271	5.90	0.028	0	0.258	27.7	0.106	13.8
3D elasticity	20	-	0.552	-	0.210	-	0.023	-	0.385	-	0.094	-
Mantari2012		0.511	0.551	0.18	0.206	1.90	0.023	0	0.331	14.0	0.090	4.25
Karama2003		0.508	0.540	2.17	0.205	2.38	0.023	0	0.285	25.9	0.086	8.51
Present		0.506	0.547	0.90	0.205	2.38	0.023	0	0.270	29.8	0.084	10.6
3D elasticity	50	-	0.541	-	0.185	-	0.0216	-	0.393	-	0.084	-
Mantari2012		0.445	0.541	0	0.184	0.54	0.0217	0.46	0.336	14.5	0.082	2.38
Karama2003		0.444	0.540	0.18	0.183	1.08	0.0216	0	0.289	26.4	0.079	5.95
Present		0.443	0.540	0.18	0.184	0.54	0.0216	0	0.273	30.5	0.077	8.33
3D elasticity	100	-	0.539	-	0.181	-	0.0213	-	0.395	-	0.083	-
Mantari2012		0.435	0.539	0	0.181	0	0.0214	0.46	0.337	14.6	0.081	2.40
Karama2003		0.434	0.538	0.27	0.180	0.55	0.0213	0	0.289	26.8	0.078	6.02
Present		0.434	0.539	0	0.181	0	0.0213	0	0.274	30.7	0.077	7.22

Table 4. Non-dimensional maximum deflections and stresses in three layers (0/90/0) square plate (a=b) under sinusoidal load (b=3a),$w = \frac{10^2 w_0 * \left(\frac{a}{2}, \frac{b}{2}\right) * E_2 * h^3}{a^4 * q_0}$,$\sigma_{xx} = \frac{\sigma_{xx} * \left(\frac{a}{2}, \frac{b}{2}, -\frac{h}{2}\right) * h^2}{a^2 * q_0}$, $\sigma_{yy} = \frac{\sigma_{yy} * \left(\frac{a}{2}, \frac{b}{2}, -\frac{h}{4}\right) * h^2}{a^2 * q_0}$ $\sigma_{xy} = \frac{\sigma_{xy} * \left(\frac{a}{2}, \frac{b}{2}, -\frac{h}{2}\right) * h^2}{a^2 * q_0}$, $\sigma_{yz} = \frac{\sigma_{yz} * \left(\frac{a}{2}, 0, 0\right) * h}{a * q_0}$, $\sigma_{xz} = \frac{\sigma_{xz} * \left(0, \frac{b}{2}, 0\right) * h}{a * q_0}$

Method	a/h	W	Diff %	σ_{xx}	Diff %	σ_{yy}	Diff %	σ_{xy}	Diff %	σ_{xz}	Diff %	σ_{yz}	Diff %
3D elasticity		2.820	-	1.100	-	0.119	-	0.028	-	0.387	-	0.033	-
Mantari2012	4	2.963	5.07	1.165	5.9	0.103	13.4	0.028	0	0.333	13.95	0.037	12..0
Karama2003		2.683	4.85	1.097	0.27	0.104	12.6	0.027	3.57	0.298	22.9	0.036	6.06
Present		2,664	5.53	1.070	2.72	0.104	12.6	0.027	3.57	0.285	26.35	0.035	
3D elasticity		0.919	-	0.725	-	0.044	-	0.012	-	0.420	-	0.015	-
Mantari2012	10	0.892	2.93	0.719	0.82	0.041	6.81	0.012	0	0.369	12.14	0.018	20
Karama2003		0.876	4.67	0.704	2.89	0.040	9.09	0.011	8.33	0.319	24.04	0.018	20
Present		0.868	5.54	0.699	3.58	0.040	9.09	0.012	0	0.302	28	0.017	13.3
3D elasticity		0.610	-	0.650	-	0.030	-	0.0093	-	0.434	-	0.012	-
Mantari2012	20	0.603	1.14	0.648	0.30	0.029	3.33	0.0092	1.07	0.375	13.59	0.015	25
Karama2003		0.597	2.13	0.644	0.92	0.029	3.33	0.0092	1.07	0.323	25.57	0.014	16.6
Present		0.595	2.45	0.642	1.23	0.029	3.33	0.0091	2.15	0.304	29.9	0.014	16.6
3D elasticity		0.520	-	0.628	-	0.026	-	0.0084	-	0.439	-	0.011	-
Mantari2012	50	0.520	0	0.627	0.15	0.030	15.3	0.0085	1.19	0.376	14.35	0.014	27.2
Karama2003		0.519	0.19	0.626	0.31	0.026	0	0.0084	0	0.323	26.42	0.013	18.1
Present		0.517	0.57	0.626	0.31	0.026	0	0.0084	0	0.305	30.52	0.013	18.1
3D elasticity		0.508	-	0.624	-	0.025	-	0.0083	-	0.439	-	0.011	-
Mantari2012	100	0.508	0	0.624	0	0.025	0	0.0083	0	0.376	14.35	0.014	27.2
Karama2003		0.508	0	0.620	0.64	0.025	0	0.0083	0	0.323	26.42	0.013	18.1
Present		0.506	0.39	0.624	0	0.025	0	0.0083	0	0.306	30.29	0.013	18.1