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Thermal Simulation for Unconditioned Single Zone with Modified Roof

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ABSTRACT

Roof in the Iraqi houses normally flattening by a concrete panel. This concrete panel has poor thermal properties. The usage of materials with low thermal conductivity and high specific heat gives a good improvements to the thermal properties of the concrete panel, thus, the indoor room temperature improves. A Mathcad program based on a mathematical model employing complex Fourier series built for a single room building. The model input data are the ambient temperature, solar radiation, and sol-air temperature, which have been treated as a periodic function of time. While, the room construction is constant due to their materials made of it, except the roof properties are taken as a variable generated practically from the improved mixing ratios.The result showed that using concrete panel with components (cement, sand, coarse aggregate, wood ash and Alabaster aggregates) with a ratio (1:1:2:1:1) and 3-plastic layer denoted by roof No.4, gives the best improvement of the thermal performance for the building. Where, the thermal conductivity is reduced by 42% and the specific heat increased by 41.2% compared to the traditional concrete panel mixing ratio denoted by roof No.1. Also, the mechanical properties are agreed with the Iraqi standards No. 1107 on 2002.

Key words: wood ash, Alabaster aggregates, plastic layers

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الخلاصة

تسطح سقوف المباني العراقية ببلاطه خرسانية التي تمتلك خصائص حرارية رديئه. ان استخدام مواد لها موصلية حرارية منخفضة وخزن حراري عالي يحسن من الخصائص الحاراية للبلاطة الخرسانية وبالتالي يحسن من درجة الحرارة داخل المبنز برنامج الماثكاد تم اعداده اعتمادا على نموذج رياضي يستخدم سلسلة فورير المعقدة لبناء مبنى مكون من فضاء (غرفة) واحدة وقد تم اعتماد البيانات الداخلة للبرنامج من البيانات المدخله في النموذج الرياضي تتضمن درجة الحرارة الخارجية وشدة الاشعاع الشمسي ودرجة الحرارة sol-air temperature والتي تكون دالة للزمن .ان اجزاء الفرقة تعتبر ثابتة كلا حسب الخواص الحراريه للمواد المكون منها عدا السقف فقد اعتبر متغيرا نسبة الى افضل الخلطات التي تم اختيارها. النتائج اوضحت لوح الخرسانة مكون من (الاسمنت: الرمال: الركام الخشنة: رماد الخشب: مجاميع المرمر) وبنسبة (1: 1: 2: 1: 1) مع ثلاث طبقات من المشبك البلاستيكي التي رمزت بالسقف رقم 4 اعطت افضل تحسين اداء حراري للمبنى حيث ان الموصلية الحرارية قلت بنسبة 42٪ والخزن الحراري ازداد بنسبة 41.2٪ مقارنة بالسقف رقم 1 الذي يعتبر البلاط التقليدي. وأن الخواص الميكانيكية له مقبولة اعتمادا على المواصفة العراقية رقم 1107 لعام 2002

الكلمات الرئيسية: رماد الخشب, مجاميع المرمر,طبقات بلاستك

1. INTRODUCTION

The roof of the building is the most important component that contributes significantly to the quality of the indoor thermal comfort. Roof covered by the concrete panel exposed directly to the sun, and become unbearably hot during the summer and cold during winter. The thermal property improvement of the concrete panels should be done to the concrete mixture used to make it. This improvement is obtained through adding many types of additive to the concrete panel mixture

The effect of roof construction on the room thermal performance was studied by many researchers, such as: **Sodha, et al., 1979,** who investigated the periodic heat transfer through a single zone building that built from a double hollow concrete block. One face of the building envelope is exposed to solar radiation and ambient conditions, while the other is in contact with constant room air temperature. The effect of using metal sheet in the center of a hollow concrete slab on the thermal performance of the concrete slab was investigated by **Sodha, et al., 1981**. **Sodha, Kaushik, and Nayak 1981,** they investigated and analyzed the periodic heat transfer through storage walls and roof pond. The heat flux into the room has been made to assess the thermal performance of the storage of roof pond systems and walls in both summer and winter. **Eben Saleh, 1990,** investigated the effects of using different insulation materials, thicknesses and arrangements for the building thermal performance.**Mathews, 1994**, compared the thermal load predictions of six programs for 46 case studies involving 36 buildings in various climatic conditions, He found a significant differences in the results of the various methods. **Al-Sanea, 2002,** studied the performance of building roof subject to ambient temperature, which was assumed as a periodic function, as well as periodic solar radiation, and nonlinear radiation exchange. The finite-volume method with the using of the implicit formulation was developed and applied to six groups of a typical roof structure that used in the construction of the buildings. The influence of external surface color on the heat flow through a flat roof was investigated by **Granja, et al., 2003**. The heat conduction equations were solved by Fourier analysis to obtain periodic solutions. The estimation of the space heat gain through flat roofs and multilayer walls was introduced by **Yumrutaş et al., 2005,** the complex finite Fourier transform technique was used in the study. **Jain, 2006,** who analyzed the hourly temperature variation of many roofs type, namely; bare roof, insulated roof, evaporation of water above the roof and roof pond with a portable system insulation. **Khalifa, 2017,** studied the effect of the roof pond on the indoor temperature, heat flow to the building; temperature distribution through walls and roof, as well as an exergy analysis to the roof pond was achieved.

In this work the effect of concrete panel concrete that used to flat the roof of Iraqis houses is investigated mathematically. A single room building of dimensions of 6m long, 5m width and 4m height were purposed. The Complex Fourier series are used to obtain an exact solution to the variation of the indoor temperature.

2. MATHMATICAL MODEL

The time dependent heat transfer equations are solved for a proposed unconditioned single zone building. The outer four walls and the floor construction materials are kept the same for all the analysis, while the roof structure is varied depending on the experimental work. The dimensions of the zone are 6m long, 5m width and 4m height. There is a single southern window and single southern door. The walls are made of 20 mm outer cement plaster, 216 mm common bricks and 20 mm in stucco. The roof is constructed from the outside, of 30 mm stucco, 100 mm concrete and 65 mm sand, the roofs differ from each others by the 50 mm thickness external layer, which is made up of concrete for the first roof, sand panel for the second, the wood ash panel for the third, PET sand panel for the fourth and for the fifth roof is a PET wood ash panel, as shown in **Table 1**.

For the sake of simplicity, the following assumption is made;

1. The walls/roof has constant and homogeneous thermo physical properties.
2. There is no person, furniture or any supplementary heat in the room.
3. The indoor temperature is uniform throughout the room in space.
4. The solar radiation, ambient air temperature, the sol-air temperature, the temperature through the roof, walls and floor are a periodic function of time.
5. There is no heat generation within roof and walls.
6. The time required for the glass to reach it is thermal equilibrium is short compared with that of the wall, roof and floor materials, so a steady state heat transfer through the window is assumed.
7. The air leakage due to opening of window and other air-losses are assumed to be a fixed number of air changes per hour.

3. OUTDOOR CONDITION

The outdoor conditions such as ambient temperature, solar radiation and sol- air temperature can be represented by time dependent equation as follows

3.1 Ambient Temperature

The ambient air temperature can be expressed as a Fourier series as **Beerends, 2003**:

$$TA_t = TAO + \sum_{m=1}^{6} TT_m . e^{i.m.\omega.t} \quad (1)$$

The constants of the Eq.(1) are as follows:

$$TAO = \frac{1}{24} \sum_{t=0}^{23} TAM_t \quad (1.2)$$

$$TT_m = aa1_m - i.aa2_m \quad (1.3)$$

Where

$$aa1_m = \frac{1}{12} \sum_{t=0}^{23} TAM_t . cos\,(m.\omega.t) \quad (1.3a)$$

$$aa2_m = \frac{1}{12} \sum_{t=0}^{23} TAM_t . sin(m.\omega.t) \quad (1\text{-}3b)$$

3.2 Solar Intensity

The solar radiation is assumed as a periodic function time and in the same manner Eq.(1), can be expressed as Fourier series with time as:

$$SRA_{j,t} = SO_j + \sum_{m=1}^{6} (S_{j,m} . e^{i.m.\omega.t}) \quad (4)$$

The constants of the Eq.(4) are as follows:

$$SO_j = \frac{1}{24} \sum_{t=0}^{23} SOL_{j,t} \quad (4.1)$$

$$S_{j,m} = ba1_{j,m} - i.ba2_{j,m} \quad (4.2)$$

Where

$$ba1_{j,m} = \frac{1}{12} \sum_{t=0}^{23} SOL_{j,t} . cos\,(m.\omega.t) \quad (4.2a)$$

$$ba2_{j,m} = \frac{1}{12} \sum_{t=0}^{23} SOL_{j,t} . sin(m.\omega.t) \quad (4.2b)$$

3.3 Sol-Air Temperature

Sol-air temperature is actual combination effect of incident solar radiation, radiant energy exchange with sky and other outdoor surroundings and convection heat exchange with outdoor air. Sol-air temperature can be written as **ASHRAE, 2013**.

$$TSA_{j,t} = TAM_t + \alpha_j.\frac{SOL_{j,t}}{h1_j} - \varepsilon_j.\frac{\Delta R}{h1_j} \tag{5}$$

It can express this equation as Fourier series as

$$SOAT_{j,t} = TSO_j + \sum_{m=1}^{6} TSM_{j,m}.e^{i.m.\omega.t} \tag{6}$$

The constants of the Eq.(6) are as follows:

$$TSO_j = \frac{1}{24}\sum_{t=0}^{23} TSA_{j,t} \tag{6.1}$$

$$TSM_{j,m} = bb1_{j,m} - i.bb2_{j,m} \tag{6.2}$$

Where

$$bb1_{j,m} = \frac{1}{12}\sum_{t=0}^{23} TSA_{j,t}.cos\,(m.\omega.t) \tag{6.2a}$$

$$bb2_{j,m} = \frac{1}{12}\sum_{t=0}^{23} TSA_{j,t}.sin(m.\omega.t) \tag{6.2b}$$

4. ROOM AIR TEMPERATURE

The indoor temperature can be expressed as Fourier series as:

$$Troom_t = TO1 + \sum_{m=1}^{6} TO2_m.e^{i.m.\omega.t} \tag{7}$$

The variation of the indoor temperature depends on many variables, such as the outer walls, roof, windows doors and ventilation

4.1 The Walls

The wall is assumed to consist of three layered walls as shown in **Fig.1**.The periodic temperature distribution through walls are:

$$T31_{j,t,X} = A3_j.X_j + B3_j + \sum_{m=1}^{6}(AM3_{j,m}.e^{\beta 3_{j,m}.X_j} + BM3_{j,m}.e^{-\beta 3_{j,m}.X_j}).e^{i.m\omega.t} \tag{8}$$

$$\beta 3_{j,m} = \sqrt{\frac{i.m.\omega.\rho 31_j.C31_j}{K31_j}} \tag{8.1}$$

$$T32_{j,t,X} = \lambda 3_j.X_j + \theta 3_j + \sum_{m=1}^{6}(\lambda M3_{j,m}.e^{\alpha 3_{j,m}.X_j} + \theta M3_{j,m}.e^{-\alpha 3_{j,m}.X_j}).e^{i.m\omega.t} \tag{9}$$

$$\alpha 3_{j,m} = \sqrt{\frac{i.m.\omega.\rho 32_j.C32_j}{K32_j}} \tag{9.1}$$

$$T33_{j,t,x} = \eta 3_j.X_j + \delta 3_j + \sum_{m=1}^{6}(\eta M3_{j,m}.e^{\emptyset 3_{j,m}.X_j} + \delta M3_{j,m}.e^{-\emptyset 3_{j,m}.X_j}).e^{i.m\omega.t} \tag{10}$$

$$\phi 3_{j,m} = \sqrt{\frac{i.m.\omega.\rho 33_j.C33_j}{K33_j}} \tag{10.1}$$

From the inner and outer boundary condition of the walls shown in **Fig.1**

At X_j=0

$$-K31_j.\frac{dT31_{j,t,X}}{dX} = h1.\left(SOAT_{j,t} - T31_{j,t,X}\right) \quad (11)$$

At $X_j = X31_j$

$$-K31_j.\frac{dT31_{j,t,X31}}{dX} = -K32_j.\frac{dT32_{j,t,X31}}{dX} \quad (12)$$

$$T31_{j,t,X31} = T32_{j,t,X31} \quad (13)$$

At $X_j = X32_j$

$$-K32_j.\frac{dT32_{j,t,X32}}{dX} = -K33_j.\frac{dT33_{j,t,X32}}{dX} \quad (14)$$

$$T32_{j,t,X32} = T33_{j,t,X32} \quad (15)$$

At X_j=$X33_j$

$$-K33_j.\frac{dT33_{j,t,X^{33}}}{dX} = h2_j.\left(T33_{j,t,X33} - Troom_t\right) \quad (16)$$

The constants $A3_j$, $B3_j$, $AM3_{j,\,m}$, $BM3_{j,m}$, $\theta3_j$, $\lambda3_j$, $\theta M3_{j,m}$, $\lambda M3_{j,m}$, $\eta3_j$, $\delta3_j$, $\eta M3_{j,m}$ and $\delta M3_j$ are found as shown in appendix (1).

The heat flow to the room from the inner surface of the walls can be written as:

$$Qwall_{j,t} = h2_j.A_j\left(T33_j - Troom_t\right) \quad (17)$$

The final form of Eq. (17) can be expressed as:

$$Qwall_{j,t} = U3_j.A_j\left(TSO_j - TO1\right) + A_j\sum_{m=1}^{6}\left(H11_{j,m}TO2_m + H12_{j,m}TSM_{j,m}\right)e^{i.m\omega.t} \quad (18)$$

While the constants $H0_{j,m}$ to H12j,m and E0j,m to E9j,m are constants shown in appendix (1)

4.2 The Roof

The roof consists of four layers as shown in **Fig.2**. The periodic temperature distributions through roof layers are as follows:

$$T41_{j,t,X} = A4_j.X_j + B4_j + \sum_{m=1}^{6}(AM4_{j,m}.e^{\beta4_{j,m}.X_j} + BM4_{j,m}.e^{-\beta4_{j,m}.X_j}).e^{i.m\omega.t} \quad (19)$$

$$\beta4_{j,m} = \sqrt{\frac{i.m.\omega.\rho41_j.C41_j}{K41_j}} \quad (19.1)$$

$$T42_{j,t,X} = \lambda4_j.X_j + \theta4_j + \sum_{m=1}^{6}(\lambda M4_{j,m}.e^{\alpha4_{j,m}.X_j} + \theta M4_{j,m}.e^{-\alpha4_{j,m}.X_j}).e^{i.m\omega.t} \quad (20)$$

$$\alpha 4_{j,m} = \sqrt{\frac{i.m.\omega.\rho 42_j.C42_j}{K42_j}} \tag{20.1}$$

$$T43_{j,t,X} = \eta 4_j.X_j + \delta 4_j + \sum_{m=1}^{6}(\eta M4_{j,m}.e^{\phi 4_{j,m}.X_j} + \delta M_{j,m}.e^{-\phi 4_{j,m}.X_j}).e^{i.m\omega.t} \tag{21}$$

$$\phi 4_{j,m} = \sqrt{\frac{i.m.\omega.\rho 43_j.C43_j}{K43_j}} \tag{21.1}$$

$$T44_{j,t,X} = \sigma 4_j.X_j + \Omega 4_j + \sum_{m=1}^{6}(\sigma M4_{j,m}.e^{\gamma_{j,m}.X_j} + \Omega M4_{j,m}.e^{-\gamma_{j,m}.X_j}).e^{i.m\omega.t} \tag{22}$$

$$\gamma 4_{j,m} = \sqrt{\frac{i.m.\omega.\rho 44_j.C44_j}{K44_j}} \tag{22.1}$$

The inner and outer boundary conditions of the roof shown in **Fig. 2**

At X_j=0

$$-K41_j.\frac{dT41_{j,t,0}}{dX} = h1.(SOAT_{j,t} - T41_{j,t,X}) \tag{23}$$

At $X_j = X41_j$

$$-K41_j.\frac{dT41_{j,t,X41}}{dX} = -K42_j.\frac{dT42_{j,t,X41}}{dX} \tag{24}$$

$$T41_{j,t,X41} = T42_{j,t,X41} \tag{25}$$

At $X_j = X42_j$

$$-K42_j.\frac{dT42_{j,t,X42}}{dX} = -K43_j.\frac{dT43_{j,t,X42}}{dX} \tag{26}$$

$$T42_{j,t,X42} = T43_{j,t,X42} \tag{27}$$

At $X_j = X43_j$

$$-K43_j.\frac{dT43_{j,t,X43}}{dX} = -K44_j.\frac{dT44_{j,t,X43}}{dX} \tag{28}$$

$$T43_{j,t,X43} = T44_{j,t,X43} \tag{29}$$

At X_j=$X44_j$

$$-K44_j.\frac{dT44_{j,t,X44}}{dX} = h2_j.(T44_{j,t,X44} - Troom_t) \tag{30}$$

The constants $A4_j$, $B4_j$, $AM4_{j,m}$, $BM4_{j,m}$, $\theta 4_j$, $\lambda 4_j$, $\theta M4_{j,m}$, $\lambda M4_{j,m}$, $\eta 4_j$, $\delta 4_j$, $\eta M4_{j,m}$, $\delta M4_j$, $\Omega 4_j$, $\sigma 4_j$, $\Omega M4_{j,m}$ and $\sigma M4_j$ can be found as shown in appendix (2):

The heat flow to the room from the inner surface of the roof can be expressed as

$$Qroof_{j,t} = h2_j.A_j\left(T44_j - TRoom_t\right) \tag{31}$$

The final form of Eq. (31) can be written as:

$$Qroof_{j,t} = U4_j.A4_j\left(TSO_j - TO1\right) + A4_j\sum_{m=1}^{6}\left(BS16_{j,m}TO2_m + BS17_{j,m}TSM_{j,m}\right)e^{i.m\omega.t} \tag{32}$$

While the constants $BS0_{j,m}$ to BS17j,m and CS0j,m to SC12j,m are constants shown in appendix (2)

4.3 The Door

The wood door is assumed that is consists of one- layers as and shown in **Fig. 3**. The periodic temperature distribution through the door is:

$$T1_{j,t,X} = A0_j.X_j + B0_j + \sum_{m=1}^{6}(AM_{j,m}.e^{\beta1_{j,m}\cdot X_j} + BM_{j,m}. \quad {}^{1_{j,m}\cdot X_j}).e^{i.m\omega.t} \tag{33}$$

$$\beta1_{j,m} = \sqrt{\frac{i.m.\omega.\rho1_j.C1_j}{K1_j}} \tag{33.1}$$

The inner and outer boundary conditions of the door shown in **Fig.3** are:
At X_j=0

$$-K1_j.\frac{dT1_{j,t,0}}{dX} = h1.\left(SOAT_{j,t} - T1_{j,t,X}\right) \tag{34}$$

At X_j=$X1_j$

$$-K1_j.\frac{dT1_{j,t,X1}}{dX} = h2_j.\left(T1_{j,t,X} - Troom_t\right) \tag{35}$$

The constants AO_j, BO_j, $AM4_{j,m}$ and $BM4_{j,m}$, can be found as shown in appendix (3):
The heat flow to the room from the inner surface of the roof can be written as

$$Qdoor_{j,t} = h2_j.A1_j\left(T1_j - Troom_t\right) \tag{36}$$

The final form of Eq.(36) can be written as:

$$Qdoor_{j,t} = U1_j.A1_j\left(TSO_j - TO1\right) + A1_j\sum_{m=1}^{6}(G3_{j,m}TO2_m + G4_{j,m}TSM_{j,m})e^{i.m\omega.t} \tag{37}$$

While the constants $G0_{j,m}$ to G4j,m and R0j,m and R1j,m are constants shown in appendix (3)

4.4 The Ground

The equation of heat conducted into the ground can be written as:

$$Qg_t = -Kg.Ag.\frac{\partial Tg_{y,t}}{\partial y}|_{y=0} \tag{38}$$

where $Tg_{y,t}$ is the ground temperature distribution, that can be obtained by solving the Fourier heat conduction equation in the ground **Davies, 2004**:

$$Kg.\frac{\partial^2 Tg_{y,t}}{\partial y^2} = \rho g.Cg.\frac{\partial Tg_{y,t}}{\partial t} \tag{39}$$

The periodic temperature distribution in the ground can be expressed as:

$$Tg_{y,t} = EO.y + FO + \sum_{m=1}^{\infty}(E_m.e^{\lambda_m\cdot y} + F_m.e^{-\lambda_m\cdot y}).e^{i.m\omega.t} \tag{40}$$

With the boundary conditions:
At y=0

$$-Kg.\frac{dTg_{y,t}}{dy} = hg.\left(TRoom_t - Tg_{y,t}\right) \quad (41)$$

At y→∞

$$Tg_{y,t} = C \quad (42)$$

$$\frac{dTg_{y,t}}{dy} = 0 \quad (43)$$

The constants EO_j, FO_j, E_m and E_m, can be found as shown in appendix (4):
The final form of Eq.(38) can be written as:

$$Qground_t = \sum_{m=1}^{6}\left(\frac{Ag}{\left(\frac{1}{hg}+\frac{1}{Kg.\lambda_m}\right)}.TO2_m\right).e^{i.m.\omega.t} \quad (44)$$

4.5 The Window

The total heat gain through a single glazing window can be expressed as **ASHRAE ,2013**:

$$Qwindow_{j,t} = \alpha a.\tau a.Aw_j.SRA_{j,t} - h3.Aw_j.(Troom_t - TA_t) \quad (45)$$

Sub Eq. (3),(4) and (6) in (45)

We obtained

$$Qwindow_{j,t} = \alpha a.\tau a.Aw_j.SO_j - h3.Aw_j.TO1 - h3.Aw_j.\sum_{m=1}^{\infty} TO2_m.e^{i.m.\omega.t} + h3.Aw_j.\sum_{m=1}^{\infty} TT_m.e^{i.m.\omega.t} + h3.Aw_j.TAO + \alpha a.\tau a.Aw_j.\sum_{m=1}^{\infty} S_{j,m}.e^{i.m.\omega.t} \quad (46)$$

4.6 Infiltration

The amount of air leakage heat can be expressed as **Jones, 1977**

$$Qinfiltration_t = Ma.Ca.wo.(TRoom_t - TA_t) + Ma.wo.\Delta H \quad (47)$$

Where (wo) is the hourly air change due to the door and window openings, the second term of Eq.(47) is the latent heat due to the differences in enthalpy between the outdoor and indoor air, which can be written as **Jones, 1977**

$$\Delta H = 1.007.(\Delta t) + 2501.\Delta g + 1.84.\Delta g.\Delta t \quad (47.1)$$

There for the heat loss by ventilation can be written as:

$$Qinfiltration_t = w3.(TO1 - TAO) + w1 + w3.\sum_{m=1}^{6}(TO2_m + TT_m)e^{i.m\omega.t} \quad (48)$$

Where

$$w1 = 2501.Ma.wo.\Delta g \quad (48.1)$$

$$w2 = Ma.wo.(1.84.\Delta g + 1.007) \quad (48.2)$$

$$w3 = Ma.Ca.wo + w2 \quad (48.3)$$

5. HEAT BALANCE OF THE ROOM

The heat balance equation for the indoor air of the room is the sum of all heat sources,and can be written as:

$$Ma.Ca\,\frac{dTRoom}{dt} = \sum_{j=1}^{4} Q_{wall} + Q_{roof} + Q_{window} + Q_{door} - Q_{ventilaltion} - Q_{ground} \qquad (49)$$

Substitute Eq.(18),(32),(37),(44),(46) and (48) into Eq.(49) yields:

$$i.m\omega.Ma.Ca.\sum_{m=1}^{6} TO2_m.e^{i.m\omega.t} = \sum_{j=1}^{4}\Big(U3_j.A_j\big(TSO_j - TO1\big) + A_j\sum_{m=1}^{6}\big(H11_{j,m}TO2_m + H12_{j,m}TSM_{j,m}\big)e^{i.m\omega.t}\Big) + U4_j.A4_j\big(TSO_j - TO1\big) + A4_j\sum_{m=1}^{6}\big(BS16_{j,m}TO2_m + BS17_{j,m}TSM_{j,m}\big)e^{i.m\omega.t} + \alpha a.\tau a.Aw_j.SO_j - h3.Aw_j.TO1 - h3.Aw_j.\sum_{m=1}^{6} TO2_m.e^{i.m.\omega.t} + h3.Aw_j.\sum_{m=1}^{6} TT_m.e^{i.m.\omega.t} + h3.Aw_j.TAO + \alpha a.\tau a.Aw_j.\sum_{m=1}^{6} S_{j,m}.e^{i.m.\omega.t} + U1_j.A1_j\big(TSO_j - TO1\big) + A1_j\sum_{m=1}^{6}\big(G3_{j,m}TO2_m + G4_{j,m}TSM_{j,m}\big)e^{i.m\omega.t} - w3.(TO1 - TAO) + w1 + w3.\sum_{m=1}^{6}(TO2_m + TAM_m)e^{i.m\omega.t} - \sum_{m=1}^{6}\frac{Ag}{\left(\frac{1}{hg}+\frac{1}{Kg.\lambda_m}\right)}.TO2_m.e^{i.m.\omega.t} \qquad (50)$$

Eq. (50) can split into two equations, one of them is time independent equation, which is TO1 can be found by:

$$TO1=\frac{\left(\sum_{j=1}^{4}(A_j.U3_j.TSO_j\right)+(A4_j.U4_j.TSO_j)+(A1_j.U1_j.TSO_j)+\left(\alpha a.\tau a.Aw_j.SO_j\right)+\left(h3.Aw_j.TAO\right)+w3.TAO-w1}{\left(\sum_{j=1}^{4}(A_j.U3_j\right)+(A4_j.U4_j)+(A1_j.U1_j)+\left(h3.Aw_j\right)+w3} \qquad (50.1)$$

And from the time dependent equation $TO2_m$ can be found by

$$TO2_m=\frac{\left(\sum_{j=1}^{4}(A_j.H12_{j,m}.TSM_{j,m}\right)+(A4_j.BS17_{j,m}.TSM_{j,m})+(A1_j.G4_{j,m}.TSM_{j,m})+\left(\alpha a.\tau a.Aw_j.S_{j,m}\right)+w3.TT_m+h3.Aw_j.TT_m}{\left(-\sum_{j=1}^{4}(A_j.H11_{j,m}\right)-(A4_j.BS18_{j,m})-(A1_j.G3_{j,m})+w3+h3.Aw_j+i.m.\omega.Ma.Ca+\frac{Ag}{\left(\frac{1}{hg}+\frac{1}{Kg.\lambda_m}\right)}} \qquad (50.2)$$

6. RESULTS AND DISCUSSION

Table.1 shows the mechanical and thermal properties of selected samples were selected to simulate the outer layer of the roof zone. **Fig.4** shows the variation of outdoor temperature and solar intensity on the roof, and the four wall orientations, in Baghdad for a typical hot day at 21-7-2017. **Fig.5** shows the variation of the inside surface temperature of the outer layer of the five roof types. It can be seen that, roof 5 gives the best thermal performance at the peak time. The numerical value of the temperature distributions through the roofs 1 to 5 are shown in **Fig.6**, it can be seen from the figure that the excellent thermal performance of roof 5 has faded when the ceiling performance is fully studied, i.e. the inside roof temperature for all roofs are approximately the same. However, the graph shows the effect of sand (second layer) in damping the temperature swinging through the roof. The maximum surface temperature of the roof occurs at about 20 hrs. while the minimum is at 8 hrs.

Fig.7 shows the variation of inside temperature of the zone construction with time. It can be seen that the maximum temperature inside surface is the roof temperature and the minimum temperature is the north wall temperature. The inside temperature separates the graph into many zones, each zone is marked by the difference between the inside surface temperature and room temperature. If the difference is positive, this means that the zone gains heat and the negative means that the heat flows from the room to the internal surface. **Fig.8** display this phenomenon clearly, the negative sign of heat flow means that the zone gains heat, while the positive sign means that the zone lost heat and the maximum heat flow is from the roof, while the minimum is from the north wall.

Fig.9 shows the effect of the roof type on the indoor temperature, where the roof type has a little effect on the indoor temperature, but it can be seen that roof 5 gives the minimum indoor temperature during peak load. The insignificant effect of roof on the indoor temperature is due to the rate of heat transferred from the roof represents a limited part of the total heat transferred to the building, as well as, the improvement of the roof was limited by outer layer and did not cover the whole roof layers. The heat flow from each roof type of the room is shown in **Fig.10**, it can be seen from the figure that roof 5 has the best thermal mass, due to this roof release the minimum heat during peak load, and maximum heat during dawn, this means that roof 5 store the heat during peak load. The heat flow from roof 5 represented by 0.96 of the heat flow from roof 1 (the traditional roof).

7. CONCLUSIONS

The conclusions that can be derived from the work can be summarized as follows:

1. Using roof 4 gives the best thermal performance to the zone in which:
   - There is total heat flow to the zone is reduced by 4%.
   - The thermal conductivity is reduced by 42%
   - The specific heat increased by 41.2%.
2. The peak indoor temperature occurs at about 18 hr. for all roof types this means that the roof has a little effect on the room thermal performance.

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NOMENCLATURE

A	Area, m^2
C	Specific heat, kJ/kg.K
h1, $h2_j$	Heat transfer coefficient by convection and radiation from the outer surface of the wall and roof to the ambient and from the inner surface of the wall to the indoor air, kJ/hr.m^2. K
h3	Heat transfer coefficient between the inside air and ambient air through window glass, kJ/hr.m^2. K
hg	Heat transfer coefficient between the inside air and ground, kJ/hr.m^2. K
k	Thermal conductivity coefficient, kJ/hr.m.K
Ma	Mass of inside air, kg
m	Number of harmonics
Q	Heat transfer , kJ/hr.
$S_{j,m}$	Amplitude of the m th harmonic of SOj, kJ/hr.m^2
$SOAT_{j,t}$	Sol-air temperature of the walls/roof, ^oC
SO_j	Average value of solar intensity, kJ/hr.m^2
$SOL_{j,t}$	Readable data of solar radiation on walls and roof, kJ/hr.m^2
$SRA_{j,t}$	Solar intensity incident on walls and roof, kJ/hr.m^2

T	Temperature, ^oC
t	Time coordinate, hr.
TAM_t	Readable data of ambient air temperature, ^oC
TAO	Average value of ambient air temperature, ^oC
TA_t	Ambient air temperature, ^oC
TO1	Average value of inside air temperature, ^oC
$TO2_m$	Amplitude of the m th harmonic of TO1, ^oC
$Troom_t$	Inside air temperature, ^oC
$TSA_{j,t}$	Sol-air temperature, ^oC
$TSM_{j,m}$	Amplitude of the m th harmonic of TSOj, ^oC
TSO_j	Average value of so-lair temperature, ^oC
TT_m	Amplitude of the m th harmonic of TAO, ^oC
U	Heat transfer coefficient, $kJ/hr.m^2.K$
V	Air changes per hour, hr^{-1}
X	Wall/roof thickness, m
x	Co-ordinate normal to the walls and roof, m
y	Co-ordinate normal to the ground, m

SUBSCRIPTS:

$1_{j,t,x}$	door
$3_{j,t,x}$	Wall
$4_{j,t,x}$	Roof
$31_{j,t,x}$	1st layer of j th wall
$32_{j,t,x}$	2nd layer of j th wall
$33_{j,t,x}$	third layer of j th wall
$41_{j,t,x}$	1st layer of roof
$42_{j,t,x}$	2nd layer of roof
$43_{j,t,x}$	third layer of roof
$44_{j,t,x}$	fourth layer of roof
a	inside air
j	Number of the surface, walls and roof. j=0 for roof , wall, j=1 for Eastern wall, j=2 for Southern wall, j=3 for Western wall and j=4 for Northern wall
g	ground
W_j	Wall
w_j	window glass

GREEK SYMBOLS:

ρ	Density , kg/m^3
α_a	Absorptivity of inside air
α_j	Absorptivity of j- surface
ε_j	Emissivity of j- surface
τa	Transmittance of the glass window
ω	Frequency of the time-harmonic signal, hr^{-1}

Δg Different in moisture content between room air and door air, kg_w/kg_a

ΔR_j Difference between long-wave radiation incident on j-surface from sky and surroundings and radiation emitted by the blackbody at outdoor air temperature, $kJ/hr.m^2$

Table 1. The mechanical and thermal properties of selected samples experimentally tested.

Roof No.	Fracture forces* KN	Absorption water percentage**	cp kJ/ kg .K	ρ kg/m^3	k W/ m.K	k kJ/hr. m.K
1 (traditional)	2.73	5.7	0.72	2124	1.58	5.68
2	1.7	6.2	0.87	1822	1.36	4.89
3	0.9	7.6	1.14	1776	1.01	3.63
4	2.9	5.9	1.01	1335	0.91	3.27
5	2.3	7.0	1.28	1517	0.74	2.66

* Fracture forces, according to the Iraqi standards No. 1107 on 2002 is 2.7 KN or above this limit.
** Absorption water percentage according to the Iraqi standards No. 1107 on 2002 is 10% or less this limit.

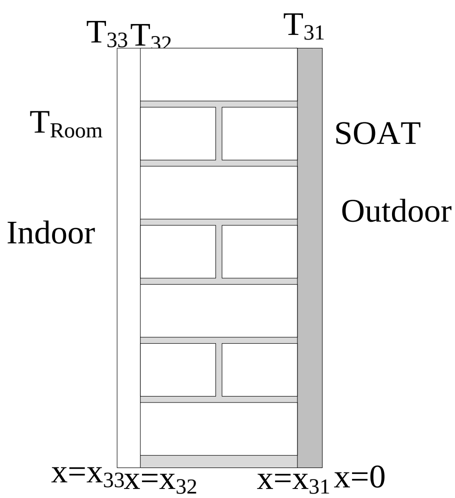


Figure 1. The cross section in typical three layered wall.

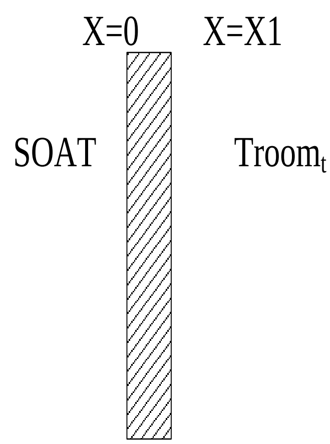


Figure 3. The door cross section.

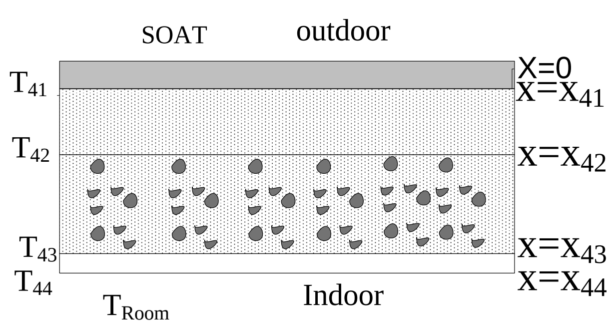


Figure 2. The cross section in 4 layered roof.

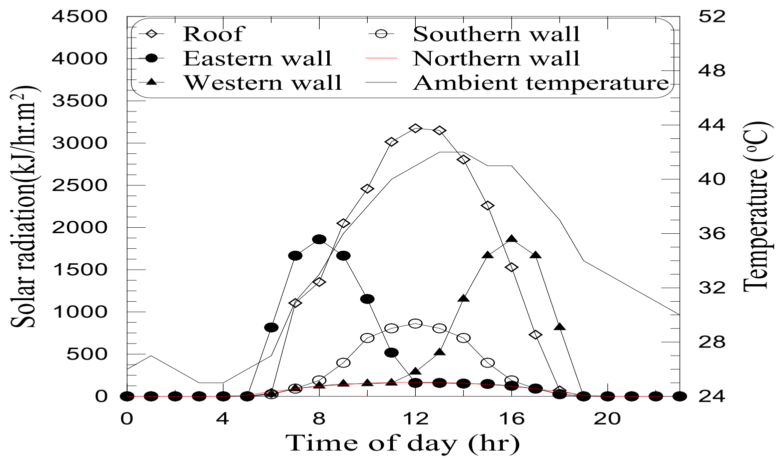


Figure 4. The Solar Radiation Intensity on Different Surfaces and ambient Temperature in Baghdad.

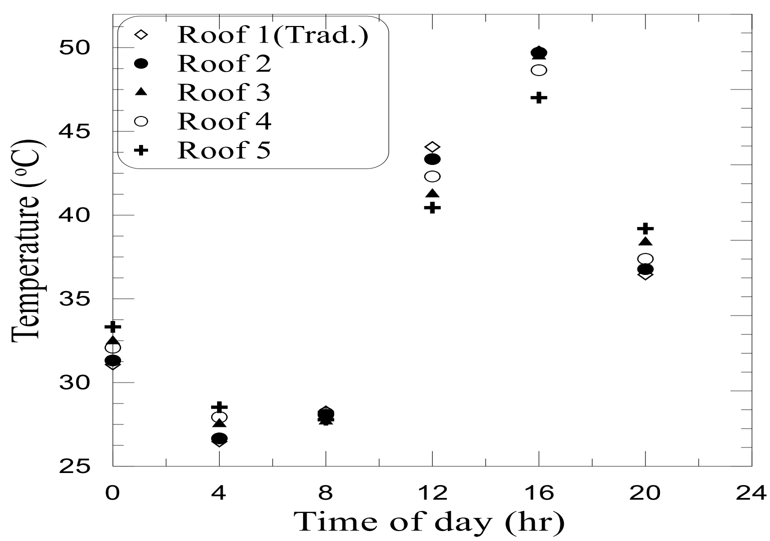
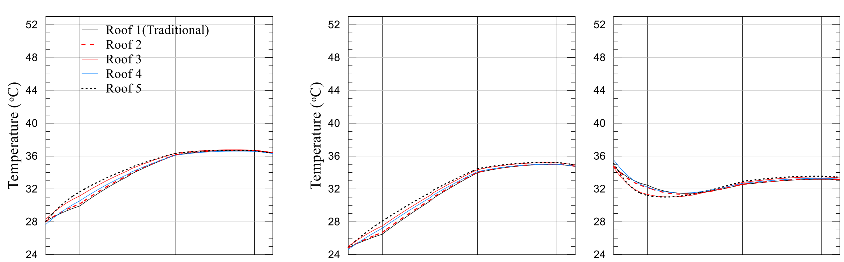
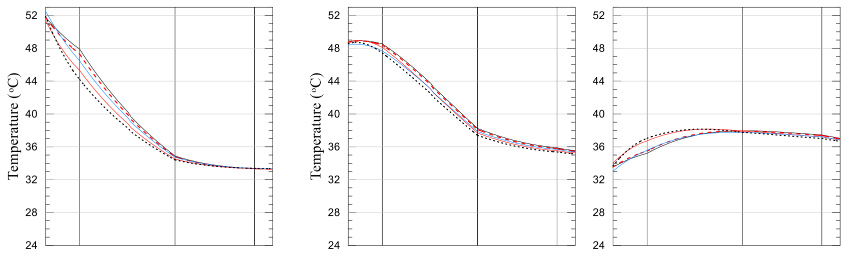


Figure 5. The inside surface measured temperature along the day for the roofs under study.



Time 0 hr. 4 hr. 8 hr.



12 hr. 16 hr. 20 hr.

Figure 6. The temperature distribution through the roof.

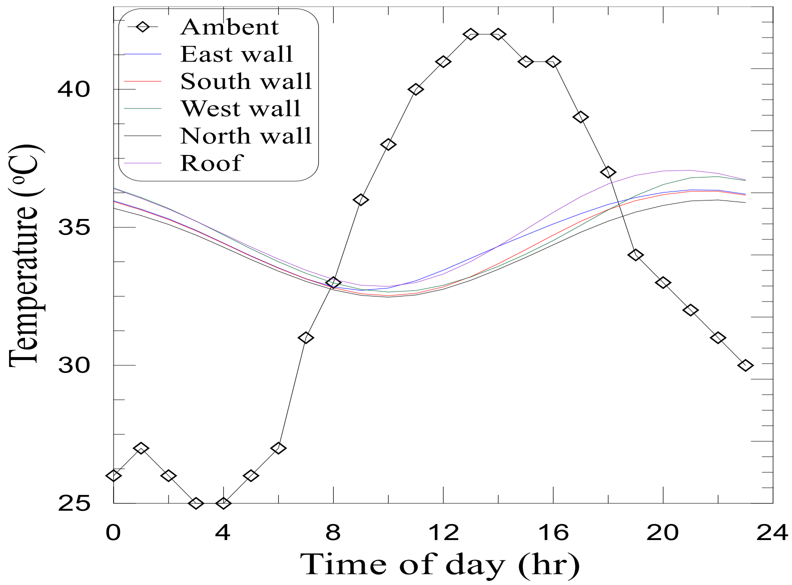


Figure 7. The Variation of inside surface temperature of the zone construction with time.

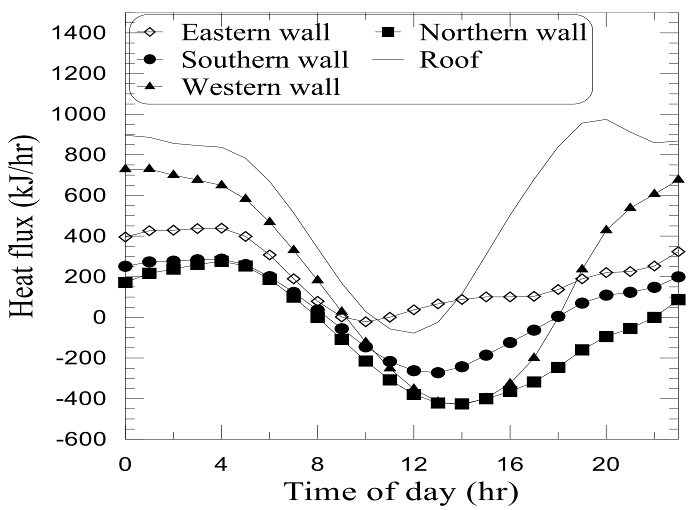


Figure 8. The Variation of the heat flow rate from zone construction to the room with time.

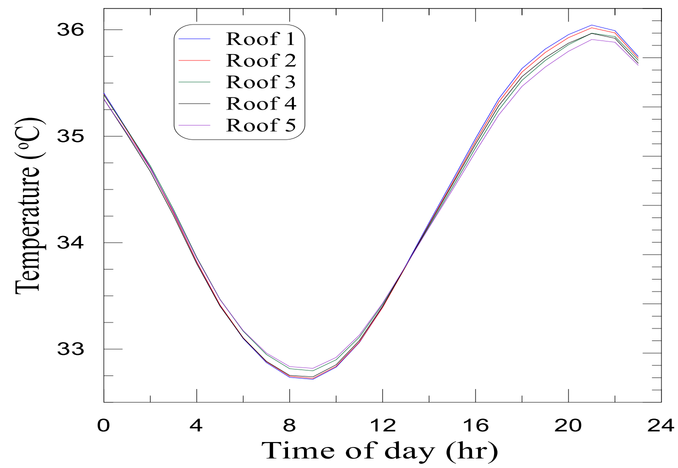


Figure 9. The effect of roof type on the inside room temperature.

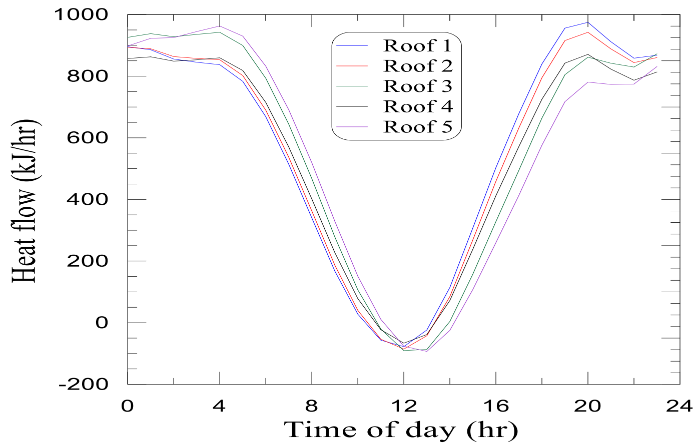


Figure 10. The effect of roof type on the heat flow to the room from the roof.

APPENDIX 1

Constants for temperature distribution through three layered walls

$$\mathrm{HO}_{j,m}=\frac{1-\frac{h2_j}{k33_j.\emptyset3_{j,m}}}{1+\frac{h2_j}{k33_j.\emptyset3_{j,m}}}.\,e^{-2.\emptyset3_{j,m}.X33_j}$$

$$H1_{j,m}=HO_{j,m}.\,e^{\emptyset3_{j,m}.X32_j}+e^{-\emptyset3_{j,m}.X32_j}$$

$$H2_{j,m}=\frac{k33_j.\,\emptyset3_{j,m}}{k32_j.\,\alpha3_{j,m}}.\left(HO_{j,m}.\,e^{\emptyset3_{j,m}.X32_j}-e^{-\emptyset3_{j,m}.X32_j}\right)$$

$$H3_{j,m}=\frac{1}{2}.\left(H1_{j,m}+H2_{j,m}\right).\,e^{-\alpha3_{j,m}.X32_j}$$

$$H4_{j,m}=\frac{1}{2}.\left(H1_{j,m}-H2_{j,m}\right).\,e^{\alpha3_{j,m}.X32_j}$$

$$H5_{j,m}=H3_{j,m}.\,e^{\alpha3_{j,m}.X31_j}+H4_{j,m}.\,e^{-\alpha3_{j,m}.X31_j}$$

$$H6_{j,m}=\frac{k32_j.\,\alpha3_{j,m}}{k31_j.\,\beta3_{j,m}}.\left(H3_{j,m}.\,e^{\alpha3_{j,m}.X31_j}-H4_{j,m}.\,e^{-\alpha3_{j,m}.X31_j}\right)$$

$$H7_{j,m}=\frac{1}{2}.\left(H5_{j,m}+H6_{j,m}\right).\,e^{-\beta3_{j,m}.X31_j}$$

$$H8_{j,m}=\frac{1}{2}.\left(H5_{j,m}-H6_{j,m}\right).\,e^{\beta3_{j,m}.X31_j}$$

$$H9_{j,m}=\left(1+\frac{h1}{k31_j.\,\beta3_{j,m}}\right).H8_{j,m}-\left(1-\frac{h1}{k31_j.\,\beta3_{j,m}}\right).H7_{j,m}$$

$$H10_{j,m} = HO_{j,m}.E9_{j,m} + H9_{j,m}.EO_{j,m}$$

$$H11_{j,m} = h2_j.\left(\frac{H10_{j,m}.e^{\emptyset 3_{j,m}.X33_j} + E9_{j,m}.e^{-\emptyset 3_{j,m}.X33_j}}{H9_{j,m}} - 1\right)$$

$$H12_{j,m} = \frac{h2_j.h1}{k31_j.\beta 3_{j,m}}.\frac{(HO_{j,m}.e^{\emptyset 3_{j,m}.X33_j} + e^{-\emptyset 3_{j,m}.X33_j})}{H9_{j,m}}$$

$$EO_{j,m} = \frac{\frac{h2_j}{k33_j.\emptyset 3_{j,m}}}{1 + \frac{h2_j}{k33_j.\emptyset 3_{j,m}}}.e^{-\emptyset 3_{j,m}.X33_j}$$

$$E1_{j,m} = EO_{j,m}.e^{\emptyset 3_{j,m}.X32_j}$$

$$E2_{j,m} = \frac{k33_j.\emptyset 3_{j,m}}{k32_j.\alpha 3_{j,m}}.EO_{j,m}.e^{\emptyset 3_{j,m}.X32_j}$$

$$E3_{j,m} = \frac{1}{2}.\left(E1_{j,m} + E2_{j,m}\right).e^{-\alpha 3_{j,m}.X32_j}$$

$$E4_{j,m} = \frac{1}{2}.\left(E1_{j,m} - E2_{j,m}\right).e^{\alpha 3_{j,m}.X32_j}$$

$$E5_{j,m} = E3_{j,m}.e^{\alpha 3_{j,m}.X31_j} + E4_{j,m}.e^{-\alpha 3_{j,m}.X31_j}$$

$$E6_{j,m} = \frac{k32_j.\alpha 3_{j,m}}{k31_j.\beta 3_{j,m}}.(E3_{j,m}.e^{\alpha 3_{j,m}.X31_j} - E4_{j,m}.e^{-\alpha 3_{j,m}.X31_j})$$

$$E7_{j,m} = \frac{1}{2}.\left(E5_{j,m} + E6_{j,m}\right).e^{-\beta 3_{j,m}.X31_j}$$

$$E8_{j,m} = \frac{1}{2}.\left(E5_{j,m} - E6_{j,m}\right).e^{\beta 3_{j,m}.X31_j}$$

$$E9_{j,m} = \left(1 - \frac{h1}{k31_j.\beta 3_{j,m}}\right).E7_{j,m} - \left(1 + \frac{h1}{k31_j.\beta 3_{j,m}}\right).E8_{j,m}$$

$$\eta M3_{j,m} = \frac{H10_{j,m}.TO2_m + \frac{h1.H0_{j,m}.TSM_{j,m}}{k31_j.\beta 3_{j,m}}}{H9_{j,m}}$$

$$\delta M3_{j,m} = \frac{E9_{j,m}.TO2_m + \frac{h1.TSM_{j,m}}{k31_j.\beta 3_{j,m}}}{H9_{j,m}}$$

$$AM3_{j,m} = H7_{j,m}.\delta M3_{j,m} + E7_{j,m}.TO2_m$$

$$BM3_{j,m} = H8_{j,m}.\delta M3_{j,m} + E8_{j,m}.TO2_m$$

$$\lambda M3_{j,m} = H3_{j,m}.\delta M3_{j,m} + E3_{j,m}.TO2_m$$

$$\theta M3_{j,m} = H4_{j,m}.\delta M3_{j,m} + E4_{j,m}.TO2_m$$

$$A30_j = \frac{(TO1 - TSO_j)}{k31_j}.U3_j$$

$$\lambda 30_j = \frac{(TO1 - TSO_j)}{k32_j}.U3_j$$

$$\eta 30_j = \frac{(TO1 - TSO_j)}{k33_j}.U3_j$$

$$B30_j = \left(\left(\frac{X31_j}{k31_j} + \frac{X33_j - X32_j}{k33_j} + \frac{X32_j - X31_j}{k32_j} + \frac{1}{h2_j}\right).TSO_j + \frac{1}{h1}.TO1\right).U3_j$$

$$\theta 30_j = \left(\left(\frac{X33_j - X32_j}{k33_j} + \frac{X32_j}{k32_j} + \frac{1}{h2_j}\right).TSO_j + \left(\frac{1}{h1} + \frac{X31_j}{k31_j} - \frac{X31_j}{k32_j}\right).TO1\right).U3_j$$

$$\delta 30_j = \left(\left(\frac{X33_j}{k33_j} + \frac{1}{h2_j}\right).TSO_j + \left(\frac{1}{h1} + \frac{X31_j}{k31_j} + \frac{X32_j - X31_j}{k32_j} - \frac{X32_j}{k33_j}\right).TO1\right).U3_j$$

APPENDIX 2

Constants for temperature distribution through four layered roof

$$BS0_{j,m} = \frac{1 - \frac{h2_j}{k44_j.\gamma 44_{j,m}}}{1 + \frac{h2_j}{k44_j.\gamma 44_{j,m}}}.e^{-2.\gamma 44_{j,m}.X44_j}$$

$$BS1_{j,m} = BS0_{j,m}.e^{\gamma 44_{j,m}.X43_j} + e^{-\gamma 44_{j,m}.X43_j}$$

$$BS2_{j,m} = \frac{k44_j.\gamma 44_{j,m}}{k43_j.\emptyset 43_{j,m}}.(BS0_{j,m}.e^{\gamma 44_{j,m}.X43_j} - e^{-\gamma 44_{j,m}.X43_j})$$

$$BS3_{j,m} = \frac{1}{2}.\left(BS1_{j,m} + BS2_{j,m}\right).e^{-\emptyset 43_{j,m}.X43_j}$$

$$BS4_{j,m} = \frac{1}{2}.\left(BS1_{j,m} - BS2_{j,m}\right).e^{\emptyset 43_{j,m}.X43_j}$$

$$BS5_{j,m} = BS3_{j,m}.e^{\emptyset 43_{j,m}.X42_j} + BS4_{j,m}.e^{-\emptyset 43_{j,m}.X42_j}$$

$$BS6_{j,m} = \frac{k43_j.\emptyset 43_{j,m}}{k42_j.\alpha 42_{j,m}}.(BS3_{j,m}.e^{\emptyset 43_{j,m}.X42_j} - BS4_{j,m}.e^{-\emptyset 43_{j,m}.X42_j})$$

$$BS7_{j,m} = \frac{1}{2}.\left(BS5_{j,m} + BS6_{j,m}\right).e^{-\alpha 42_{j,m}.X42_j}$$

$$BS8_{j,m} = \frac{1}{2}.\left(BS5_{j,m} - BS6_{j,m}\right).e^{\alpha 42_{j,m}.X42_j}$$

$$BS9_{j,m} = BS7_{j,m}.e^{\alpha 42_{j,m}.X41_j} + BS8_{j,m}.e^{-\alpha 42_{j,m}.X41_j}$$

$$BS10_{j,m} = \frac{k42_j.\alpha 42_{j,m}}{k41_j.\beta 41_{j,m}}.(BS7_{j,m}.e^{\alpha 42_{j,m}.X41_j} - BS8_{j,m}.e^{-\alpha 42_{j,m}.X41_j})$$

$$BS11_{j,m} = \frac{1}{2}.\left(BS9_{j,m} + BS10_{j,m}\right).e^{-\beta 41_{j,m}.X41_j}$$

$$BS12_{j,m}=\frac{1}{2}.\left(BS9_{j,m}-BS10_{j,m}\right).\,e^{\beta41_{j,m}.X41_j}$$

$$BS13_{j,m}=\left(1-\frac{h1}{k41_j.\,\beta41_{j,m}}\right).\,CS11_{j,m}-\left(1+\frac{h1}{k41_j.\,\beta41_{j,m}}\right).\,CS12_{j,m}$$

$$BS14_{j,m}=\left(1+\frac{h1}{k41_j.\,\beta41_{j,m}}\right).\,BS12_{j,m}-\left(1-\frac{h1}{k41_j.\,\beta41_{j,m}}\right).\,BS11_{j,m}$$

$$BS15_{j,m}=BS0_{j,m}.\,BS13_{j,m}+BS14_{j,m}.\,CS0_{j,m}$$

$$BS16_{j,m}=h2_j.\left(\frac{BS15_{j,m}.\,e^{\gamma44_{j,m}.X44_j}+BS13_{j,m}.\,e^{-\gamma44_{j,m}.X44_j}}{BS14_{j,m}}-1\right)$$

$$CS0_{j,m}=\frac{\dfrac{h2_j}{k44_j.\,\gamma44_{j,m}}}{1+\dfrac{h2_j}{k44_j.\,\gamma44_{j,m}}}.\,e^{-\gamma44_{j,m}.X44_j}$$

$$CS1_{j,m}=CS0_{j,m}.\,e^{\gamma44_{j,m}.X43_j}$$

$$CS2_{j,m}=\frac{k44_j.\,\gamma44_{j,m}}{k43_j.\,\emptyset43_{j,m}}.\,CS0_{j,m}.\,e^{\gamma44_{j,m}.X43_j}$$

$$CS3_{j,m}=\frac{1}{2}.\left(CS1_{j,m}+CS2_{j,m}\right).\,e^{-\emptyset43_{j,m}.X43_j}$$

$$CS4_{j,m}=\frac{1}{2}.\left(CS1_{j,m}-CS2_{j,m}\right).\,e^{\emptyset43_{j,m}.X43_j}$$

$$CS5_{j,m}=CS3_{j,m}.\,e^{\emptyset43_{j,m}.X42_j}+CS4_{j,m}.\,e^{-\emptyset43_{j,m}.X42_j}$$

$$CS6_{j,m}=\frac{k43_j.\,\emptyset43_{j,m}}{k42_j.\,\alpha42_{j,m}}.\left(CS3_{j,m}.\,e^{\emptyset43_{j,m}.X42_j}-CS4_{j,m}.\,e^{-\emptyset43_{j,m}.X42_j}\right)$$

$$CS7_{j,m}=\frac{1}{2}.\left(CS5_{j,m}+CS6_{j,m}\right).\,e^{-\alpha42_{j,m}.X42_j}$$

$$CS8_{j,m}=\frac{1}{2}.\left(CS5_{j,m}-CS6_{j,m}\right).\,e^{\alpha42_{j,m}.X42_j}$$

$$CS9_{j,m}=CS7_{j,m}.\,e^{\alpha42_{j,m}.X41_j}+CS8_{j,m}.\,e^{-\alpha42_{j,m}.X41_j}$$

$$CS10_{j,m}=\frac{k42_j.\,\alpha42_{j,m}}{k41_j.\,\beta41_{j,m}}.\left(CS7_{j,m}.\,e^{\alpha42_{j,m}.X41_j}-CS8_{j,m}.\,e^{-\alpha42_{j,m}.X41_j}\right)$$

$$CS11_{j,m}=\frac{1}{2}.\left(CS9_{j,m}+CS10_{j,m}\right).\,e^{-\beta41_{j,m}.X41_j}$$

$$CS12_{j,m}=\frac{1}{2}.\left(CS9_{j,m}-CS10_{j,m}\right).\,e^{\beta41_{j,m}.X41_j}$$

$$\sigma M4_{j,m}=\frac{BS16_{j,m}.\,TO2_m+\dfrac{h1.\,BS0_{j,m}.\,TSM_{j,m}}{k41_j.\,\beta41_{j,m}}}{BS14_{j,m}}$$

$$\Omega M3_{j,m}=\frac{BS13_{j,m}.\,TO2_m+\dfrac{h1.\,TSM_{j,m}}{k41_j.\,\beta41_{j,m}}}{BS41_{j,m}}$$

$$AM4_{j,m}=BC11_{j,m}.\,\Omega M4_{j,m}+CS11_{j,m}.\,TO2_m$$

$$BM4_{j,m}=BC12_{j,m}.\,\Omega M4_{j,m}+CS12_{j,m}.\,TO2_m$$

$$\lambda M4_{j,m}=BC7_{j,m}.\,\Omega M4_{j,m}+CS7_{j,m}.\,TO2_m$$

$$\theta M4_{j,m}=BC8_{j,m}.\,\Omega M4_{j,m}+CS8_{j,m}.\,TO2_m$$

$$\eta M4_{j,m}=BC3_{j,m}.\,\Omega M4_{j,m}+CS3_{j,m}.\,TO2_m$$

$$\delta M4_{j,m}=BC4_{j,m}.\,\Omega M4_{j,m}+CS4_{j,m}.\,TO2_m$$

$$A40_j=\frac{(TO1-TSO_j)}{k41_j}.\,U4_j$$

$$\lambda40_j=\frac{(TO1-TSO_j)}{k42_j}.\,U4_j$$

$$\eta40_j=\frac{(TO1-TSO_j)}{k43_j}.\,U4_j$$

$$\sigma40_j=\frac{(TO1-TSO_j)}{k44_j}.\,U4_j$$

$$B40_j=\left(\left(\frac{X41_j}{k41_j}+\frac{X42_j-X41_j}{k42_j}+\frac{X43_j-X42_j}{k43_j}+\frac{X44_j-X43_j}{k44_j}+\frac{1}{h2_j}\right).\,TSO_j+\frac{1}{h1}.\,TO1\right).\,U4_j$$

$$\theta40_j=\left(\left(\frac{X43_j-X42_j}{k43_j}+\frac{X44_j-X43_j}{k44_j}+\frac{X42_j}{k42_j}+\frac{1}{h2_j}\right).\,TSO_j+\left(\frac{1}{h1}+\frac{X41_j}{k41_j}-\frac{X41_j}{k42_j}\right).\,TO1\right).\,U4_j$$

$$\delta40_j=\left(\left(\frac{X43_j}{k43_j}+\frac{X44_j-X43_j}{k44_j}+\frac{1}{h2_j}\right).\,TSO_j+\left(\frac{1}{h1}+\frac{X41_j}{k41_j}+\frac{X42_j-X41_j}{k42_j}-\frac{X42_j}{k43_j}\right).\,TO1\right).\,U4_j$$

$$\Omega40_j=\left(\left(\frac{X44_j}{k44_j}+\frac{1}{h2_j}\right).\,TSO_j+\left(\frac{1}{h1}+\frac{X41_j}{k41_j}+\frac{X42_j-X41_j}{k42_j}+\frac{X43_j-X42_j}{k43_j}-\frac{X43_j}{k44_j}\right).\,TO1\right)$$

APPENDIX 3

Constants for temperature distribution through one layer door

$$R0_{j,m}=\frac{h2_j}{\left(1+\dfrac{h2_j}{k1_j.\,\beta1_{j,m}}\right).\,k1_j.\,\beta1_{j,m}}.\,e^{-\beta1_{j,m}.X1_j}$$

$$R1_{j,m}=R0_{j,m}.\left(1-\frac{h1}{k1_j.\,\beta1_{j,m}}\right)$$

$$G0_{j,m}=\frac{1-\dfrac{h2_j}{k1_j.\beta1_{j,m}}}{1+\dfrac{h2_j}{k1_j.\beta1_{j,m}}}.e^{-2\beta1_{j,m}.X1_j}$$

$$G1_{j,m}=\left(1+\frac{h1}{k1_j.\beta1_{j,m}}\right)-\left(1-\frac{h1}{k1_j.\beta1_{j,m}}\right).G0_{j,m}$$

$$G2_{j,m}=G0_{j,m}.R1_{j,m}+G1_{j,m}.R0_{j,m}$$

$$G3_{j,m}=h2_j.\left(\frac{G2_{j,m}.e^{\beta1_{j,m}.X1_j}+R1_{j,m}.e^{-\beta1_{j,m}.X1_j}}{G1_{j,m}}-1\right)$$

$$G4_{j,m}=\frac{h1.h2_j}{k1_j.\beta1_{j,m}}.\left(\frac{G0_{j,m}.e^{\beta1_{j,m}.X1_j}+e^{-\beta1_{j,m}.X1_j}}{G1_{j,m}}\right)$$

$$Ao_j=\frac{(TO1-TSO_j)}{k1_j}.U1_j$$

$$Bo_j=\left(\left(\frac{X1_j}{k1_j}+\frac{1}{h2_j}\right).TSO_j+\left(\frac{1}{h1}\right).TO1\right).U1_j$$

$$Am_{j,m}=\frac{G2_{j,m}.TO2_m+\dfrac{h1.G0_{j,m}.TSM_{j,m}}{k1_j.\beta1_{j,m}}}{G1_{j,m}}$$

$$Bm_{j,m}=\frac{R1_{j,m}.TO2_m+\dfrac{h1.TSM_{j,m}}{k1_j.\beta1_{j,m}}}{G1_{j,m}}$$

APPENDIX 4

Constants for temperature distribution through the ground

$EO=0$, $E_m=0$

$FO=TO1$

$$F_m=\frac{Ag}{\left(\dfrac{1}{hg}+\dfrac{1}{Kg.\lambda_m}\right)}.TO2_m$$